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Aim: RX for $C + PCD$ random variables; Change of Density for C+PCD random variables (case I)

Thm. $\mathbb{P}\{X \in RX\} = 1$ for any random variable X.

Open subsets of $\mathbb{R}$: $\mathcal{U}$, where $\forall u \in \mathcal{U}$, $\exists\, \varepsilon > 0$ such that $u - \varepsilon, u + \varepsilon \subseteq \mathcal{U}$. $\varnothing$ is open vacuously.

Thm. If $\mathcal{U}$ is open and $\mathcal{U} \neq \varnothing$, then $\exists \Lambda \subseteq \mathbb{Z}$ and $\exists$ sequences $(a_n | n \in \Lambda)$ and $(b_n | n \in \Lambda)$ such that $\bigcup_{n \in \mathcal{A}} (a_n, b_n)$.

Closed subsets of $\mathbb{R}$: $\mathcal{C} = U^c$ where $\mathcal{U}$ is open. [Note: $\varnothing$ and $\mathbb{R}$ are closed]

Thm. If $\mathcal{C}$ is closed, and $c_n \in \mathcal{C}$ for all $n \geq 1$, and if $c_n \to \ell \in \mathbb{R}$, then $\ell \in \mathcal{C}$.

Thm. $(\mathcal{R}X)^c$ is an open subset of $\mathbb{R}$.

Cor. $\mathcal{R}X$ is a closed subset of $\mathbb{R}$.

$\mathbb{P}\{X \in (a,b)\} = \mathbb{P}\{X < b\} - \mathbb{P}\{X \leq a\} = F_X(b-) - F_X(a)$

Thm.

$$\begin{aligned}
\mathbb{P}\{X \in (\mathcal{R}X)^c\} &= \mathbb{P}\left\{X \in \bigcup_{n \in \Lambda} (a_n, b_n)\right\} \\
&= \mathbb{P}\left\{\bigcup_{n \in \Lambda} \{X \in (a_n, b_n)\}\right\} \\
&= \sum_{n \in \Lambda} \mathbb{P}\{\mathrm{X} \in (\mathrm{a_n}, \mathrm{b_n})\} \\
&= \sum_{n \in \Lambda} [F_X(b_n-) - F_X(a_n)] \\
&= \sum_{n \in \Lambda} 0 = 0
\end{aligned}$$

Cor. If $\mathcal{A} \subseteq \mathbb{R}$ is "manageable" $\mathcal{A} = \bigcup\limits_{n \in \Lambda} ([a_n, b_n]), \Lambda \subseteq \mathbb{Z}$, then $\mathbb{P}\{X \in \mathcal{A}\} = \mathbb{P}\{X \in \mathcal{A} \cap \mathcal{R}X\}$.

$\mathbb{P}\{X \in \mathcal{A}\} = \mathbb{P}\{X \in \mathcal{A} \cap \mathcal{R}X\} + \mathbb{P}\{X \in \mathcal{A} \cap (\mathcal{R}X)^c\}$

C+PCD random variables: Let X be a C+PCD random variable [F_X is globally continuous and is piecewise continuously differentiable].

Thm. $\mathcal{R}X = \overline{\{x \in \mathbb{R} | f_X(x) > 0\}}$ (for any subset $S \subseteq \mathbb{R}$, $\overline{S}$ is the closure of S which means $S \cup \operatorname{bd} S$).