Zach Ambur
CS 540
HW 4

Problem 2

a) $P(R|B) = \frac{P(R,B)}{P(B)}$

$P(B) = P(B|S) * P(S) + P(B|\neg S) * P(\neg S)$
$= 0.01 \times 0.4 + 0.5 \times 0.6$
$= 0.304$

$P(R|B) = \frac{0.1 \times 0.304}{0.304} = \boxed{0.1}$

b) $P(S|B) = \frac{P(S,B)}{P(B)}$

$P(S,B) = P(B|S) * P(S)$
$= 0.01 \times 0.4$
$= 0.004$

$P(S|B) = \frac{0.004}{0.304} = \boxed{0.01315}$

c) $P(W,B) = P(W,R,S,B) + P(W,R,\neg S,B) + P(W,\neg R,S,B) + P(W,\neg R,\neg S,B)$

$P(W,R,S,B) = P(B|W,R,S) * P(W,R,S)$
$= P(B|W,R,S) * P(W|R,S) * P(R,S)$
$= 0.01 \times 1 \times 0.1 \times 0.4$
$= 0.0004$

$P(W,R,\neg S,B) = P(B|W,R,\neg S) * P(W|R,\neg S) * P(R,\neg S)$
$= 0.03$

$P(W,\neg R,S,B) = P(B|W,\neg R,S) * P(W|\neg R,S) * P(\neg R,S)$
$= 0.036$

$P(W,\neg R,\neg S,B) = P(B|W,\neg R,\neg S) * P(W|\neg R,\neg S) * P(\neg R,\neg S)$
$= 0.027$

$P(W,B) = 0.0004 + 0.03 + 0.036 + 0.027 = 0.0934$

$P(W|B) = \frac{P(W,B)}{P(B)} = \frac{0.0934}{0.304} = \boxed{0.3072}$

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d) $P(\neg W, \neg R, \neg S \mid B) = \dfrac{P(\neg W, \neg R, \neg S, B)}{P(B)}$

$P(\neg W, \neg R, \neg S, B) = P(\neg W, \neg R, \neg S) * P(B \mid \neg S)$

$P(\neg W, \neg R, \neg S) = 1 - P(W) - P(R) - P(S) + P(W, R) + P(W, S)$

$= 1 - 0.514 - 0.1 - 0.4 + 0.1 + 0.4$

$= 0.486$

$P(\neg W, \neg R, \neg S, B) = 0.486 \times 0.5 = 0.243$

$P(\neg W, \neg R, \neg S \mid B) = \dfrac{0.243}{0.304} = \boxed{0.799}$

e) $P(W) = P(W, R, S) + P(W, \neg R, S) + P(W, R, \neg S) + P(W, \neg R, \neg S)$

$P(W, R, S) = P(W \mid R, S) * P(R, S) = 0.04$

$P(W, \neg R, S) = P(W \mid \neg R, S) * P(\neg R, S) = 0.36$

$P(W, R, \neg S) = P(W \mid R, \neg S) * P(R, \neg S) = 0.06$

$P(W, \neg R, \neg S) = P(W \mid \neg R, \neg S) * P(\neg R, \neg S) = 0.054$

$P(W) = 0.04 + 0.36 + 0.06 + 0.054 = \boxed{0.514}$

f) $P(B) = P(B \mid S) * P(S) + P(B \mid \neg S) * P(\neg S)$

$= 0.01 \times 0.4 + 0.5 \times 0.6$

$= \boxed{0.304}$

g) $P(W, B) = P(W \mid B) * P(B)$

$= 0.3072 \times 0.304$

$= \boxed{0.09338}$

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h) $P(B|W) = \frac{P(W,B)}{P(W)} = \frac{0.09338}{0.514} = \boxed{0.1816}$

i) $P(R|W) = \frac{P(W,R)}{P(W)} = \frac{P(W,R,S) + P(W,R,\neg S)}{P(W)} = \frac{0.1}{0.514} = \boxed{0.1945}$

j) $P(R|W,S) = \frac{P(R,W,S)}{P(W,S)} = \frac{.04}{.4} = \boxed{0.1}$