$$\frac{d^2u}{dx^2} + \left(\frac{2}{x} - \frac{3x^2/2}{x^3/2}\right)\frac{du}{dx} - \frac{1}{2x}\cdot\frac{x^3}{2}u = 0 \quad \text{o bien} \quad \frac{d^2u}{dx^2} - \frac{1}{x}\frac{du}{dx} - \frac{1}{4}x^2u = 0.$$

A su vez, la sustitución $\dfrac{dz}{dx} = \sqrt{\dfrac{-S}{\frac{1}{4}}} = \sqrt{\dfrac{\frac{1}{4}x^2}{\frac{1}{4}}} = x$ reduce esta ecuación a

$$\frac{d^2u}{dz^2} - \frac{1}{4}u = 0 \quad \text{cuya solución es} \quad u = C_1e^{\frac{1}{2}z} + C_2e^{-\frac{1}{2}z}.$$

Entonces, $y = \dfrac{1}{Qu}\dfrac{du}{dx} = \dfrac{2}{x^3}\cdot\dfrac{\frac{1}{2}(C_1e^{\frac{1}{2}z} - C_2e^{-\frac{1}{2}z})}{C_1e^{\frac{1}{2}z} + C_2e^{-\frac{1}{2}z}}\,x = \dfrac{1}{x^2}\cdot\dfrac{e^{\frac{1}{4}x^2} - ke^{-\frac{1}{4}x^2}}{e^{\frac{1}{4}x^2} + ke^{-\frac{1}{4}x^2}}$, donde $k = \dfrac{C_2}{C_1}$.

20. Resolver $\dfrac{dy}{dx} - (\text{tg } x + 3\cos x)y + y^2\cos^2 x = -2.$

La sustitución $y = \dfrac{1}{Qu}\dfrac{du}{dx} = \dfrac{\sec^2 x}{u}\dfrac{du}{dx}$ reduce la ecuación a

$$\frac{d^2u}{dx^2} + (\text{tg } x - 3\cos x)\frac{du}{dx} + 2u\cos^2 x = 0.$$

A su vez, la sustitución $\dfrac{dz}{dx} = \sqrt{\dfrac{2\cos^2 x}{2}} = \cos x$, o sea, $z = \text{sen } x$, reduce esta ecuación a

$$\frac{d^2u}{dz^2} - 3\frac{du}{dz} + 2u = 0 \quad \text{cuya solución es} \quad u = C_1e^z + C_2e^{2z}.$$

Entonces, $y = \dfrac{1}{Qu}\dfrac{du}{dx} = \dfrac{\sec^2 x(C_1e^z + 2C_2e^{2z})}{C_1e^z + C_2e^{2z}}\cos x = \sec x\,\dfrac{e^{\text{sen } x} + 2ke^{2\text{ sen } x}}{e^{\text{sen } x} + ke^{2\text{ sen } x}}.$

PROBLEMAS PROPUESTOS

Resolver

21. $xy'' - (x+2)y' + 2y = 0$ *Sol.* $y = C_1e^x + C_2(x^2 + 2x + 2)$

22. $(1+x^2)y'' - 2xy' + 2y = 2$ $y = C_1x + C_2(x^2 - 1) + x^2$

23. $(x^2+4)y'' - 2xy' + 2y = 8$ $y = C_1(x^2 - 4) + C_2x + x^2$

24. $(x+1)y'' - (2x+3)y' + (x+2)y = (x^2 + 2x + 1)e^{2x}$ $y = C_1e^x + C_2e^x(x+1)^2 + xe^{2x}$

25. $y'' - 2\text{ tg } x\, y' - 10y = 0$ $y = (C_1e^{3x} + C_2e^{-3x})\sec x$

26. $x^2y'' - x(2x+3)y' + (x^2 + 3x + 3)y = (6 - x^2)e^x$ $y = C_1x^3e^x + C_2xe^x + e^x(x^2+2)$

27. $4x^2y'' + 4x^3y' + (x^2+1)^2y = 0$ $y = \sqrt{x}\,e^{-x^2/4}(C_1 + C_2\ln x)$

28. $x^2y'' + (x - 4x^2)y' + (1 - 2x + 4x^2)y = (x^2 - x + 1)e^x$ $y = e^{2x}(C_1\cos\ln x + C_2\text{sen }\ln x) + e^x$

29. $xy'' - y' + 4x^3y = 0$ $y = C_1\text{ sen } x^2 + C_2\cos x^2$

30. $x^4y'' + 2x^3y' + y = (1+x)/x$ $y = C_1\cos(1/x) + C_2\text{ sen}(1/x) + (1+x)/x$

31. $x^8y'' + 4x^7y' + y = 1/x^3$ *Sol.* $y = C_1\cos(1/3x^3) + C_2\,\mathrm{sen}\,(1/3x^3) + 1/x^3$

32. $(x\,\mathrm{sen}\,x + \cos x)y'' - x\cos x\,y' + y\cos x = x$ $y = C_1x + C_2\cos x - \mathrm{sen}\,x$

33. $xy'' - 3y' + 3y/x = x + 2$ $y = C_1x + C_2x^3 - x^2 - x\ln x$

34. Resolver el Problema 21 mediante descomposición en factores.

35. $[(x+1)D^2 - (3x+4)D + 3]y = (3x+2)e^{3x}$ $y = C_1(3x+4) + C_2e^{3x} + xe^{3x}$

36. $x^2y'' - 4xy' + (6+9x^2)y = 0$ $y = x^2(C_1\cos 3x + C_2\,\mathrm{sen}\,3x)$

37. $xy'' + 2y' + 4xy = 4$ $y = (C_1\cos 2x + C_2\,\mathrm{sen}\,2x + 1)/x$

38. $(1+x^2)y'' - 2xy' + 2y = (1-x^2)/x$ $y = C_1(x^2-1) + C_2x + x\ln x$