$$\frac{d^2u}{dx^2} + (\frac{2}{x} - \frac{3x^2/2}{x^3/2})\frac{du}{dx} - \frac{1}{2x} \cdot \frac{x^3}{2}u = 0 \quad \text{o bien} \quad \frac{d^2u}{dx^2} - \frac{1}{x}\frac{du}{dx} - \frac{1}{4}x^2u = 0.$$

A su vez, la sustitución $\frac{dz}{dx} = \sqrt{\frac{-S}{\frac{1}{4}}} = \sqrt{\frac{\frac{1}{4}x^2}{\frac{1}{4}}} = x$ reduce esta ecuación a

$$\frac{d^2u}{dz^2} - \frac{1}{4}u = 0 \quad \text{cuya solución es} \qquad u = C_1 e^{\frac{1}{2}Z} + C_2 e^{-\frac{1}{2}Z}.$$

Entonces,
$$y = \frac{1}{Qu} \frac{du}{dx} = \frac{2}{x^3} \cdot \frac{\frac{1}{2} (C_1 e^{\frac{1}{2}z} - C_2 e^{-\frac{1}{2}z})}{C_1 e^{\frac{1}{2}z} + C_2 e^{-\frac{1}{2}z}} x = \frac{1}{x^2} \cdot \frac{e^{\frac{1}{4}x^2} - ke^{-\frac{1}{4}x^2}}{e^{\frac{1}{4}x^2} + ke^{-\frac{1}{4}x^2}}, \text{ donde } k = \frac{C_2}{C_1}.$$

20. Resolver
$$\frac{dy}{dx} = (\lg x + 3\cos x)y + y^2\cos^2 x = -2$$
.

La sustitución
$$y = \frac{1}{Qu} \frac{du}{dx} = \frac{\sec^2 x}{u} \frac{du}{dx} \text{ reduce la ecuación a}$$
$$\frac{d^2 u}{dx^2} + (\operatorname{tg} x - 3\cos x) \frac{du}{dx} + 2u\cos^2 x = 0.$$

A su vez, la sustitución $\frac{dz}{dx} = \sqrt{\frac{2 \cos^2 x}{2}} = \cos x$, o sea, $z = \sin x$, reduce esta ecuación a $\frac{d^2u}{dz^2} - 3\frac{du}{dz} + 2u = 0$ cuya solución es $u = C_1e^z + C_2e^{2z}$.

Entonces.
$$y = \frac{1}{Qu} \frac{du}{dx} = \frac{\sec^2 x (C_1 e^2 + 2C_2 e^{2z})}{C_1 e^2 + C_2 e^{2z}} \cos x = \sec x \frac{e^{\sec x} + 2ke^2 \sec x}{e^{\sec x} + ke^2 \sec x}.$$

PROBLEMAS PROPUESTOS

Resolver

21.
$$xy'' - (x+2)y' + 2y = 0$$

22.
$$(1+x^2)y'' - 2xy' + 2y = 2$$

23.
$$(x^2 + 4)y'' - 2xy' + 2y = 8$$

24.
$$(x+1)y'' - (2x+3)y' + (x+2)y = (x^2 + 2x + 1)e^{2x}$$

25.
$$y'' - 2 tg x y' - 10y = 0$$

26.
$$x^2y'' - x(2x+3)y' + (x^2+3x+3)y = (6-x^2)e^{x}$$

27.
$$4x^2y'' + 4x^3y' + (x^2 + 1)^2y = 0$$

$$x^2y'' + (x-4x^2)y' + (1-2x+4x^2)y = (x^2-x+1)e^x$$

29.
$$xy'' - y' + 4x^3y = 0$$

30.
$$x^4y'' + 2x^5y' + y = (1+x)/x$$

Sol.
$$y = C_1 e^x + C_2 (x^2 + 2x + 2)$$

$$y = C_1 x + C_2 (x^2 - 1) + x^2$$

$$y = C_1(x^2 - 4) + C_2x + x^2$$

$$y = C_1 e^x + C_2 e^x (x+1)^2 + x e^{2x}$$

$$y = (C_1e^{3x} + C_2e^{-3x})\sec x$$

$$y = C_1 x^3 e^x + C_2 x e^x + e^x (x^2 + 2)$$

$$y = \sqrt{x} e^{-x^2/4} (C_1 + C_2 \ln x)$$

$$y = e^{2x}(C_1 \cos \ln x + C_2 \sin \ln x) + e^x$$

$$y = C_4 \operatorname{sen} x^2 + C_0 \operatorname{cos} x^2$$

$$y = C_1 \cos(1/x) + C_2 \sin(1/x) + (1+x)/x$$

Estimple La expueste $z^{\frac{1}{2}} \frac{d^2y}{dz^2} + 2 \frac{d^2y}{dz^2} \frac{dy}{dz} + 3z(\frac{dy}{dz})^2 + z^2 = 0$ de orden

Is somittuded at y = 0, $y = \frac{d^2y}{dx^2} = \frac{dq}{dx}$, ... reductra of order on x are odes.

ECUACIONES EN LAS QUE FALTA LA VARIAME. (NDEPENDIENTE, SI la comoci del término en x, esto as si os de la formada.

 $\frac{d^2y}{dx} = \frac{d^2y}{dx}(a\frac{dx}{dy}) + \frac{d^2y}{dx}(a\frac{dy}{dy}) + \frac{$

 $\frac{d^2}{dt} + 2p \frac{d}{dt} - 2sp^2 - s^2 = 0$, de orden des mudames la residencia $\frac{ds}{dt} + p_1 \frac{d^2s}{dt} = 1$.

31.
$$x^8y'' + 4x^7y' + y = 1/x^5$$

31.
$$x^8y'' + 4x^7y' + y = 1/x^5$$
 * Sol. $y = C_1\cos(1/3x^5) + C_2\sin(1/3x^5) + 1/x^5$

32.
$$(x \sin x + \cos x)y'' - x \cos x y' + y \cos x = x$$
 $y = C_1x + C_2\cos x - \sin x$

$$y = C_1 x + C_0 \cos x - \sin x$$

33.
$$xy'' - 3y' + 3y/x = x + 2$$

$$y = C_1 x + C_2 x^3 - x^2 - x \ln x$$

34. Resolver el Problema 21 mediante descomposición en factores.

35.
$$[(x+1)D^2 - (3x+4)D + 3]y = (3x+2)e^{5x}$$

$$y = C_1(3x+4) + C_2e^{3x} + xe^{3x}$$

36.
$$x^2y'' - 4xy' + (6 + 9x^2)y = 0$$

$$y = x^2 (C_1 \cos 3x + C_2 \sin 3x)$$

37.
$$xy'' + 2y' + 4xy = 4$$

$$y = (C_1 \cos 2x + C_2 \sin 2x + 1)/x$$

38.
$$(1+x^2)y'' - 2xy' + 2y = (1-x^2)/x$$

$$y = C_1(x^2 - 1) + C_2x + x \ln x$$