Así, la solución es

$$y = e^z(C_1\cos\sqrt{3}\,z + C_2\,\text{sen}\,\sqrt{3}\,z) + \frac{1}{13}(3\cos z - 2\,\text{sen}\,z) + \frac{1}{2}e^z\,\text{sen}\,z$$

$$= x(C_1\cos\sqrt{3}\cdot\ln x + C_2\,\text{sen}\,\sqrt{3}\cdot\ln x) + \frac{1}{13}(3\cos\ln x - 2\,\text{sen}\,\ln x) + \frac{1}{2}x\,\text{sen}\,\ln x.$$

**4.** Resolver $(x+2)^2\dfrac{d^2y}{dx^2} - (x+2)\dfrac{dy}{dx} + y = 3x + 4.$

Póngase $x + 2 = e^z$: entonces, la ecuación dada se convierte en

$$\{\mathcal{D}(\mathcal{D}-1) - \mathcal{D} + 1\}y = (\mathcal{D}-1)^2 y = 3e^z - 2.$$

La función complementaria es $y = C_1e^z + C_2ze^z$, y una integral particular es

$$y = \frac{1}{(\mathcal{D}-1)^2}(3e^z - 2) = 3e^z\iint(dz)^2 - 2\,\frac{1}{(\mathcal{D}-1)^2}\,e^{0z} = \frac{3}{2}z^2e^z - 2.$$

La solución es $y = C_1e^z + C_2ze^z + \frac{3}{2}z^2e^z - 2$ o, ya que $z = \ln(x+2)$,

$$y = (x+2)[C_1 + C_2\ln(x+2) + \frac{3}{2}\ln^2(x+2)] - 2.$$

**5.** Resolver $\{(3x+2)^2D^2 + 3(3x+2)D - 36\}y = 3x^2 + 4x + 1.$

La transformación $3x + 2 = e^z$ reduce la ecuación a

$$\{9\mathcal{D}(\mathcal{D}-1) + 9\mathcal{D} - 36\}y = 9(\mathcal{D}^2-4)y = \frac{1}{3}(9x^2 + 12x + 3) = \frac{1}{3}(e^{2z} - 1) \quad\text{o}\quad (\mathcal{D}^2-4)y = \frac{1}{27}(e^{2z} - 1).$$

La solución completa es $y = C_1e^{2z} + C_2e^{-2z} + \dfrac{1}{27}\left(\dfrac{1}{\mathcal{D}^2-4}e^{2z} - \dfrac{1}{\mathcal{D}^2-4}e^{0z}\right)$

$$= C_1e^{2z} + C_2e^{-2z} + \frac{1}{108}(ze^{2z} + 1)$$

o bien $y = C_1(3x+2)^2 + C_2(3x+2)^{-2} + \dfrac{1}{108}[(3x+2)^2\ln(3x+2) + 1].$

# PROBLEMAS PROPUESTOS

Resolver

6. $(x^2D^2 - 3xD + 4)y = x + x^2\ln x$ *Sol.* $y = C_1x^2 + C_2x^2\ln x + x + \frac{1}{6}x^2\ln^3 x$

7. $(x^2D^2 - 2xD + 2)y = \ln^2 x - \ln x^2$ *Sol.* $y = C_1x + C_2x^2 + \frac{1}{2}(\ln^2 x + \ln x) + \frac{1}{4}$

8. $(x^3D^3 + 2x^2D^2)y = x + \text{sen}(\ln x)$ *Sol.* $y = C_1 + C_2x + C_3\ln x + x\ln x + \frac{1}{2}(\cos\ln x + \text{sen}\ln x)$

9. $x^3y''' + xy' - y = 3x^4$ *Sol.* $y = C_1x + C_2x\ln x + C_3x\ln^2 x + x^4/9$

10. $[(x+1)^2D^2 + (x+1)D - 1]y = \ln(x+1)^2 + x - 1$ *Sol.* $y = C_1(x+1) + C_2(x+1)^{-1} - \ln(x+1)^2 + \frac{1}{2}(x+1)\cdot\ln(x+1) + 2$

11. $(2x+1)^2y'' - 2(2x+1)y' - 12y = 6x$ *Sol.* $y = C_1(2x+1)^{-1} + C_2(2x+1)^3 - 3x/8 + 1/16$