

$$D^2 y = -4Ax^3 \cos 2x - 4Bx^3 \sin 2x + (12B-4C)x^2 \cos 2x + (-12A-4E)x^2 \sin 2x \\ + (6A+8E-4F)x \cos 2x + (6B-8C-4G)x \sin 2x + (2C+4G) \cos 2x + (2E-4F) \sin 2x,$$

$$(D^2 + 4)y = 12Bx^2 \cos 2x - 12Ax^2 \sin 2x + (6A+8E)x \cos 2x + (6B-8C)x \sin 2x \\ + (2C+4G) \cos 2x + (2E-4F) \sin 2x = x^2 \sin 2x.$$

Igualando los coeficientes de los términos semejantes $-12A = 1$, $12B = 0$, $6A + 8E = 0$, $6B - 8C = 0$, $2C + 4G = 0$, $2E - 4F = 0$; de donde $A = -1/12$, $B = 0$, $C = 0$, $E = 1/16$, $F = 1/32$, $G = 0$.

Una integral particular es $y = -\frac{1}{12}x^3 \cos 2x + \frac{1}{16}x^2 \sin 2x + \frac{1}{32}x \cos 2x,$

y la primitiva es $y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{12}x^3 \cos 2x + \frac{1}{16}x^2 \sin 2x + \frac{1}{32}x \cos 2x.$

PROBLEMAS PROPUESTOS

Resolver, utilizando el método de variación de parámetros.

10. $(D^2 + 1)y = \operatorname{cosec} x$ Sol. $y = C_1 \cos x + C_2 \sin x + \sin x \ln |\sin x| - x \cos x$
11. $(D^2 + 4)y = 4 \sec^2 2x$ Sol. $y = C_1 \cos 2x + C_2 \sin 2x - 1 + \sin 2x \ln |\sec 2x + \tan 2x|$
12. $(D^2 - 4D + 3)y = (1 + e^{-x})^{-1}$ Sol. $y = C_1 e^x + C_2 e^{3x} + \frac{1}{2}e^{2x} + \frac{1}{2}(e^x - e^{3x}) \ln(1 + e^{-x})$
13. $(D^2 - 1)y = e^{-x} \sin e^{-x} + \cos e^{-x}$ Sol. $y = C_1 e^x + C_2 e^{-x} - e^x \sin e^{-x}$
14. $(D^2 - 1)y = (1 + e^{-x})^{-2}$ Sol. $y = C_1 e^x + C_2 e^{-x} - 1 + e^{-x} \ln(1 + e^x)$

Resolver, utilizando el método de coeficientes indeterminados.

15. $(D^2 + 2)y = e^x + 2$ Sol. $y = C_1 \cos \sqrt{2}x + C_2 \sin \sqrt{2}x + e^x/3 + 1$
16. $(D^2 - 1)y = e^x \sin 2x$ Sol. $y = C_1 e^x + C_2 e^{-x} - e^x (\sin 2x + \cos 2x)/8$
17. $(D^2 + 2D + 2)y = x^2 + \sin x$ Sol. $y = e^{-x}(C_1 \cos x + C_2 \sin x) + \frac{1}{2}(x-1)^2 + \frac{1}{5}(\sin x - 2 \cos x)$
18. $(D^2 - 9)y = x + e^{2x} - \sin 2x$ Sol. $y = C_1 e^{3x} + C_2 e^{-3x} - x/9 - e^{2x}/5 + \frac{1}{13} \sin 2x$
19. $(D^3 + 3D^2 + 2D)y = x^2 + 4x + 8$ (Emplear $Ax^3 + Bx^2 + Cx$.) Sol. $y = C_1 + C_2 e^{-x} + C_3 e^{-2x} + \frac{1}{6}x^3 + \frac{1}{4}x^2 + \frac{11}{4}x$
20. $(D^2 + 1)y = -2 \sin x + 4x \cos x$ Sol. $y = C_1 \cos x + C_2 \sin x + 2x \cos x + x^2 \sin x$
1. $(D^3 - D^2 - 4D + 4)y = 2x^2 - 4x - 1 + 2x^2 e^{2x} + 5x e^{2x} + e^{2x}$ Sol. $y = C_1 e^x + C_2 e^{2x} + C_3 e^{-2x} + \frac{1}{2}x^2 + \frac{1}{6}x^3 e^{2x}$