Q2

Given $k_1 = [I \mid 0]$

$k_2 = [R \mid b]$

The fundamental matrix $F = A'^{-T} S_{b_{12}} A''^{-1}$

here $A' = I$

$A'' = R$

$b'_{12} = A''^{-1} a'' - A'^{-1} a' = R^{-1} b$

$= R^{-1} b - I \cdot 0 = R^{-1} b$

The projection matrix P general notation is $[kR \mid -kRX_0]$

Since $k_1 = [I \mid 0]$ $X_0 = 0$

Hence, the centre of the camera 1 $= 0$

Now,

epipole is the image of the projection centre hence the projection centre of camera 1 $= 0$

$$e_2 = K_2 (0)^T = [R \mid b] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$= b$

hence, b is one of the epipole (on Image 2)

Hence,

$F = S_{b_{12}} R^{-1}$

(or)

$F = (R^{-1} b) \times R^{-1}$

Now, for epipole e_2

$F e_2 = 0$

hence

$(R^{-1} b) \times (R^{-1} e_2) = 0$

for this to happen

$e_2 = b$

Assignment-6

Q3) Let u_{fp}, v_{fp} be the points on the image of frame f pointnum p

i_f, j_f → unit vectors as given in slides.

(P – no. of features)

Now, ~~suppose~~.

$\tilde{u}_{fp} = u_{fp} - a_f$ $\quad a_f = \frac{1}{P}\sum_{p=1}^{P} u_{fp}$ (Similarly $\tilde{v}_{fp}$) —— ①

$b_f = \frac{1}{P}\sum_{p=1}^{P} v_{fp}$.

Now,

$u_{fp} = i_f^T(s_p - t_f)$ $\quad \left(\begin{array}{l} s_p = \text{position of feature in 3D} \\ t_f = (a_f, b_f, c_f) \end{array} \right)$

—— ②

Now,

$\tilde{u}_{fp} = u_{fp} - a_f$

$= i_f^T(s_p - t_f) - \frac{1}{P}\sum_{p=1}^{P} i_f^T(s_q - t_f)$

$= i_f^T s_p$ —— ③

Combining ①, ②, ③

$\tilde{u}_{fp} = u_{fp} - a_f$

$\Rightarrow i_f^T s_p = i_f^T(s_p - t_f) - a_f$

$\Rightarrow a_f = -i_f^T t_f$

~~We already~~

Similarly, $b_f = -j_f^T t_f$

~~from~~ we already know, a_f, b_f hence we can find out c_f from these two equations.

① → defined $\tilde{u}_{fp}$ as that

② → from geometry, the image feature position onto frame 'f' → projection of point s_p

③ → ~~Similarly~~ using ② in ①

$t_f = (a_f, b_f, c_f)$

a_f → avg of u_{fp}; b_f → avg of v_{fp}

↓

The translation vector from world origin to image frame f.

After obtaining t_i and t_{i-1}

$$\boxed{T_i = t_i - t_{i-1}}$$

Translation of camera b/w frame i & $i-1$

Assignment – 6

Q4) Given images I_1, I_2, I_3
and for a point X the correspondent points are x_1, x_2, x_3 on I_1, I_2, I_3 respectively.

also fundamental matrices F_{13}, F_{23} are given.

Now,

from coplanarity constraint.

$$x_1^T F_{13} x_3 = 0$$

$$x_2^T F_{23} x_3 = 0$$

Now, $x_1^T = [x_{11} \ x_{12} \ 1]_{1\times3}$

$F_{13} = [\quad]_{3\times3}$

Hence $x_1^T \cdot F_{13} = [1\times3]$

$= $ a line.

$(x_1^T F_{13})^T$ is the epipolarline on I_3 ~~of~~ which is the image of the ray ~~OP~~ O_1X. Similarly $(x_2^T F_{23})^T$ is the epipolar line of the ray O_2X.

Since image point lies on epipolar line.

Intersection of both epipolar lines

$x_3 =$ Intersection of both epipolar lines

$$= [(x_1^T F_{13})^T] \underset{\text{crossproduct}}{\times} [(x_2^T F_{23})^T]$$

$$\boxed{x_3 = (F_{13}^T x_1) \times (F_{23}^T x_2)}$$

Q1) Given;

$(0,0,0,1)$ is the centre of camera 1

and transformation b/w two cameras is described by rotation matrix R and t st. $X' = RX + t$

camera matrix $-K$, camera calibration matrix K'

(a) epipoles are the images of projection centres of

$e = P$ (proj. centre of cam 2)

$= P \cdot R^{-1} t$

$e' = P' (0\ 0\ 0\ 1)^T$

(b) $F = [e']_x K' R K^{-1}$

To prove: $e' \times x' = Fx$.

Given the point x' the epipolar line l' pass through x' and e' can be written as $l' = e' \times x' = [e']_x x'$

Now, $l' = Fx$

~~Hence proved~~ $[e']_x \cdot K' \cdot R \cdot \underbrace{K^{-1} \cdot x}_{\text{gives } \boxed{3D \text{ point } X}}$ gives x'

$= e' \times x'$