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ASSOCIATION OF SATELLITE OBSERVATIONS USING BAYESIAN INFERENCE

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When observing satellites in an increasingly cluttered space environment, ambiguity in measurement association often arises. By using principles of probabilistic, or Bayesian, inference, we can assign numerical values of probability to the different possible associations. Several methods for handling ambiguous satellite observations are discussed, including Probabilistic Data Association (PDA) and Multiple Hypothesis Tracking (MHT). We present a comparison of these methods for a small number of observations and satellites, with a simplified motion and observation models. Filtering performance is characterized for four representative scenarios, including satellite breakups and conjunctions.

INTRODUCTION

Data quality is a fundamental influence on the orbit determination process. Biased, exceptionally noisy, or otherwise poor-quality data can have a cascading effect which may may result in a product which is less precise or even misleading. Techniques that ensure the maximum amount of information is extracted from the observations are well known and widely used. However, when the *origin* of the observation (i.e. the would-be satellite) is uncertain, the problem becomes more difficult. Associating measurements of observed quantities to a specific object in order to update the object's hidden state—while a crucial function of any surveillance system—is a difficult problem.[1]

The US Space Surveillance Network currently catalogs and tracks over 16,000 objects,[2] and upgrades to current sensors, along with deployment of new sensors with greater capability, will only increase this number. This higher spatial density will make the issue of data association more difficult. In addition, a larger number of objects will also result in more on-orbit collisions, exacerbating the issue. A larger catalog inherently stresses the surveillance network further, driving the need to extract as much information as possible out of a sparse set of data. While standard processes for observation association exist, there is no strong, comprehensive theory in place which can handle the range of different scenarios.[3] Satellite breakups may take hours or days to be fully characterized, as enough measurements must be obtained to produce reliable orbit estimates. Additionally, missions that involve formation/cluster flight[4] and rendezvous and proximity operations[5] all require the ability to differentiate between closely-spaced objects. This is especially true in cases which involve non-cooperative "target" objects, such as debris or non-working satellites. All of these issues motivate the work presented here.

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Generally, the sparse Earth-orbit environment has resulted in a low spatial density of observations, facilitating the data association problem.[6,*] However, there are many fields in which the observation density is exceptionally high, such as air traffic radar systems. In these systems, the data association problem is often the most difficult part to solve. As a result, there exists a large body of literature which addresses this issue, which is generally referred to as multiple target tracking (MTT).[7,8,9,10] The goal of this paper is to apply several of these techniques to the astrodynamics realm, and to compare their relative performance for a representative set of different scenarios.

Others have applied these concepts in the past. DeMars and coworkers used the Probabilistic Data Association (PDA) method for orbit determination at geosynchronous orbits with angle and angle rate measurements with good results.[11,12] They also used multiple hypothesis methods for range-binning, which differs from the multiple hypothesis tracking approach to data association, but uses the same basic tenets. Miller looked at applying Multiple Hypothesis Tracking methods to the uncorrelated track (UCT) problem,[13] which is outside the scope of this paper. Most of the work in observation association with respect to orbit determination is, in fact, focused on the UCT problem—few explicit references to the problem of multiple target tracking in clutter over shorter time spans are to be found in the literature.

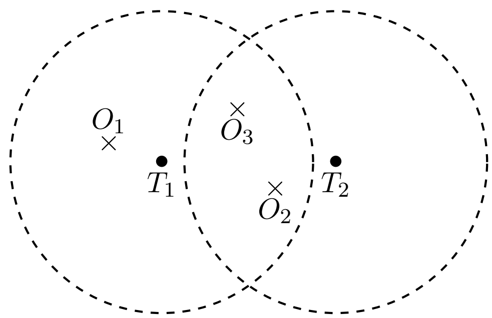

Figure 1. Basic example of an ambiguous observation situation.[14] O's are observations, T's are targets, and dashed lines are target gates.

Figure 1 illustrates the type of problem addressed by traditional MTT techniques. In a given "frame" of data (such as an image from a telescope), multiple observations are recorded. There may be multiple expected returns (several closely-spaced satellites being tracked), and there may be several more extraneous returns. A key insight to this problem is the fact that each observation has *some* origin (even if that origin is sensor noise), and that each target produces only one observation. Thus, the aim is to correctly associate each observation with its source, be it an expected satellite, an unexpected satellite, or a false target (sensor noise or some other obstruction, also referred to in the literature as "clutter"). If the observations are sufficiently separated, it is trivial to make the correct (or at least most likely to be correct) associations as with O_1. However, when the observations are closely-spaced as with O_2 and O_3, these decisions become significantly more difficult.

It is helpful here to define several terms. First, the uncertainty associated with a predicted state defines a *gate*. It may be useful to think of this in terms of the usual covariance ellipsoid centered on the predicted position; the gating is usually performed based on some statistical distance from the prediction. There is considered to be ambiguity in the system if either an observation is within the

[*] Note that while the uncorrelated track (UCT) problem is essentially an issue of data association, it is not the type of problem addressed in this paper.

gates of multiple targets, or if there are multiple observations within a single gate. The techniques described in this paper apply only to these ambiguous situations. If there exists a pair consisting of a single observation within a single target's gate, that is considered a positive association and no further action is taken.

Because there is rarely any identifying information within the observations which distinguishes one target from another, associations are made probabilistically. There are two main ways in which the associations are made. The first is based on a pure minimum mean square error (MMSE) approach.[15] In this approach, each observation is essentially weighted based on the magnitude of deviation from the predicted state, and the filter update step is performed using all of these weighted measurements. No absolute associations are made, and while the process of weighting, in theory, prevents less probable associations from influencing the filter, this method will never result in a perfect update (i.e. the update that would be made in the absence of clutter). The resulting covariance may be unrealistic, as the measurement origin adds uncertainty. Methods that use the pure MMSE approach are said to make "soft decisions".[15] The probabilistic data association (PDA) method is an example of a MMSE method.

The other class of data association methods make what are called (perhaps predictably) "hard decisions".[15] Here, a single observation is associated to each target. Obviously, this may result in the maximum amount of information being extracted from the set of observations, but only if the correct associations are made. This association can be done in several different ways. One of the most basic (and commonly used) approaches is called Global Nearest Neighbor (GNN). In GNN, the most likely *combination* of observation associations is made. Essentially, some metric is used to "score" each observation/target pair, resulting in a matrix of values. The combination of associations which results in the highest score is chosen, taking into account the fact that a single target may only result in one observation. In other words, it is a combinatorial optimization problem which must be solved with any one of several methods, such as Munkres' algorithm,[16] which uses a series of matrix row operations to solve for the optimal assignments in polynomial time. Nearest Neighbor (NN) is the less-sophisticated version of GNN. It is very similar to GNN, except that it does not take into account the fact that an observation may be associated with another target. The Air Force Space Surveillance System ("the Fence") uses NN at the sensor level to associate cataloged objects with observations based on fence crossing time.[6]

Multiple hypothesis tracking (MHT) is a method which also belongs in this latter class of hard decision-makers, but with several important distinctions. Rather than treating each data frame as an independent event, MHT choses the best[*] *set* of associations over multiple frames of data.[14] This is done by scoring all possible observation/source combinations for each frame, and then using each of these combinations to seed the next data frame. This is done for as many frames as are available or required to resolve the ambiguity. Each set of associations (across all frames) is termed a "hypothesis", and a running score is kept for each. Obviously this results in an exponential growth in the number of hypotheses, but techniques exist to keep this number manageable.[14,17,18,19] MHT essentially defers decisions until more information is available; hard decisions are not made until they need to be, and then these decisions are made based on more information than just the current data frame.

Before going into the details of each of these data association methods, it is useful to first qualitatively compare them. As mentioned previously, the soft decision-making algorithms have the

[*]In either the maximum *a posteriori* (MAP) sense or the maximum likelihood (ML) sense .

benefit of never being completely wrong, but they are also never completely correct. The overall result is a slightly inflated uncertainty after the update due to the unknown source origin.[15] The hard decision methods run the risk of misassociation, which could have a much more profound impact on the overall estimate and uncertainty. However, if the correct associations are made, there is no added uncertainty injected into the process, and a more accurate result may be obtained. This is especially important in situations in which data is sparse. Within the hard decision class of algorithms, NN and GNN have the advantage of being much less complex and therefore easier to implement. However, a single incorrect association could skew the system such that future observations are misassociated. MHT reconciles this by deferring the decision: if an observation is close enough to be ambiguous, all possible outcomes are carried forward, and the following data is used to adjust the likelihoods of the ambiguous association. Of course, the large number of hypotheses involved in MHT necessitates more a significantly more complex algorithmic structure.

METHODOLOGY

The data association methods discussed thus far do not rely on any particular dynamics, measurement, or estimation model*; these are treated as independent components for our purposes. A simple unperturbed, two-body dynamics model is implemented here using Lagrange's coefficients.[20] It is coupled with a measurement model that assumes full position state knowledge at each measurement time.

The dynamics and measurement models are coupled with a basic Extended Kalman Filter (EKF).[21] While often troublesome in estimation of higher-fidelity orbit dynamical systems,[22] it is sufficient for our purposes in that it handles the nonlinearity inherent in the orbit propagation process and provides a reasonable state and covariance estimate without a significant impact to overall processing time.

Nearest Neighbor

As one of the simplest data association methods, nearest neighbor is also one of the easiest to implement. First, we define a normalized statistical distance[9]†

$$d^2 = \tilde{\mathbf{y}}^T S^{-1} \tilde{\mathbf{y}}, \tag{1}$$

where $\tilde{\mathbf{y}}$ is the residual vector that results from using an observation to update the predicted state, and S is the measurement residual covariance matrix ($S = HPH^T + R$, where H is the measurement matrix, P is the covariance matrix, and R is the measurement noise). The value of d^2 is used to score each association. So for each known target, the EKF produces a predicted mean and covariance at the observation time. Then, the NN routine loops over all observations in the frame for each target, using each target/ob pair to update the EKF, and d^2 is calculated for all possible associations‡. Finally, the association with each target is made with the observation that has the lowest d^2 value. Note that there is no check for shared observations among targets. The state and covariance are again propagated to the next observation time, and the process repeats.

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*Although most require some sequential estimation method.

†d is also known as Mahalanobis distance.

‡It is assumed that all observations passed to the NN routine are within some gate, and thus there is some ambiguity in the association.

Global Nearest Neighbor

As with NN, GNN uses d^2 to score each possible association. However, instead of just finding the minimum d^2 value for each target, this routine stores each value in a two-dimensional matrix, with targets in one dimension and observations in the other. Once this matrix is produced, the optimal set of target/ob associations is made, *with* the constraint that targets may not share observations. For small assignment matrices, it is not computationally prohibitive to simply step over each possible combination; for larger matrices, more efficient methods such as Munkres' algorithm[16] may be used*. The optimal set of associations is used to update each target's state and covariance, and the process continues.

Probabilistic Data Association

PDA is a soft decision association method, meaning that instead of choosing a specific observation to associate with each target, it uses a weighted sum of all possible observations to update the state and covariance. For each target, the following process is performed. First, the gating step is performed. This consists simply of checking whether d^2 for the target/ob pair is less than some specified value. Generally, this value is chosen to be large enough to accept all feasible observations. If the observation j is within the statistical gate of the current target of interest i, the associated likelihood (assuming Gaussian statistics) is[9]

$$g_{ij} = \frac{\exp\left(-d^2/2\right)}{\left(2\pi^{3/2}\sqrt{\det S}\right)}. \tag{2}$$

Assuming uniform priors (more on this later), we may use Bayes' theorem to go from likelihood to probability ratio

$$p'_{ij} \propto g_{ij}. \tag{3}$$

Then, normalizing the probability ratios p'_{ij}, we may compute the probabilities

$$p_{ij} = \frac{p'_{ij}}{\sum\limits_{k=0}^{N} p'_{ik}}, \tag{4}$$

where N is the total number of observations within the target i's gate. Now, we must modify the filter update steps in the following ways. First, instead of using a single residual, we use the weighted sum of all residuals:

$$\tilde{\mathbf{y}}_i = \sum_{j=1}^{N} p_{ij}\tilde{\mathbf{y}}_{ij}. \tag{5}$$

The state is updated in the usual way[21] using this new weighted sum of residuals. The covariance P is updated by

$$P = P^* + K\left[\sum_{j=1}^{N} p_{ij}\tilde{\mathbf{y}}_{ij}\tilde{\mathbf{y}}_{ij}^T - \tilde{\mathbf{y}}_i\tilde{\mathbf{y}}_i^T\right]K^T, \tag{6}$$

where P^* is the standard Kalman filter covariance and K is the Kalman filter gain, neither of which depends on the measurement itself. Thus, these are computed in the usual way.[21]

*Matlab code written and published by Buehren[23] was used to solve the matrix assignment problem in this paper.

Multiple Hypothesis Tracking

The version of MHT implemented in this work is known as "hypothesis-oriented" MHT, which is a maximum *a posteriori* (MAP) approach.[15] That is, it makes hard association decisions by selecting the one with the highest posterior probability. Other variations on the MHT framework may be found in the literature.[8,9,15]

One of the first things needed in any MHT framework is a method to create the hypotheses*. As stated earlier, the number of hypotheses grows at a very high rate. For example, for a single target and single feasible (gated) observation, there exist three possibilities: (1) the observation belongs to the target, (2) the observation belongs to a new object and the target was not observed, and (3) the observation is spurious sensor noise and the target was not observed. Reid[24] developed an algorithm that creates all possible hypotheses for any number of existing targets or observations. This algorithm is widely used in MHT, and it is what we use here. Details of the algorithm may be found in the original paper.

The second major element involves evaluating each hypothesis. Blackman's treatment of this[9] is used for our work, and presented below. First, for convenience, we wish to work with log-likelihood ratios instead of likelihoods or probabilities:

$$LLR = \ln\left[P_T/P_F\right], \tag{7}$$

where $P_T \propto p(D|H_T)P_0(H_T)$ and $P_F \propto p(D|H_F)P_0(H_F)$ are the conditional probabilities of a true and false association, respectively, given data D. True hypotheses are denoted by a T subscript, and false alarm hypotheses with F, and P_0 is a prior probability of a given hypothesis. Taking the ratio of P_T to P_F allows us to cancel the normalizing term ($P(D)$) that arises from Bayes' Law. Calculating this quantity will allow us to keep a running score for each hypothesis simply by adding a score update to the existing score. Let the priors for false targets, new targets, and detection probability be represented by P_{FT}, P_{NT}, and P_D respectively.

For new targets, the score is initialized by

$$LLR(1) = \ln\left[\frac{P_D\beta_{NT}}{\beta_{FT}}\right], \tag{8}$$

where β is the ratio of an initial probability (that a measurement is caused by a new target or false alarm) to the detection volume. Score updates are computed by

$$\Delta LLR = \ln\left[\frac{P_D}{(2\pi)^{(3/2)}\beta_{FT}\sqrt{\det S}}\right] - \frac{d^2}{2} \tag{9}$$

when assuming a Gaussian likelihood (see Equation 2) for true target associations and a uniform distribution for false targets. It should be noted here that we use only kinematic contributions to the hypothesis score—signal-related data may be incorporated, as shown in Blackman.[9] Now, the score for each hypothesis may be updated by

$$LLR(k+1) = LLR(k) + \Delta LLR. \tag{10}$$

In order to keep the number of hypotheses manageable, a step called "pruning" is performed immediately after scoring. In our implementation, two types of pruning are performed: hypotheses

*As with the other methods, gating is really the first action required. It does not differ from the method presented earlier.

with a score lower than some threshold are deleted, and only the N best hypotheses are kept. N must be chosen carefully, as it directly influences the total number of hypotheses that must be stored, and thus has a large impact on code performance. For our purposes in this paper, N is generally on the order of hundreds. The surviving hypotheses are propagated to the next measurement time, and the process is repeated*. Figure 2 outlines this entire cycle.

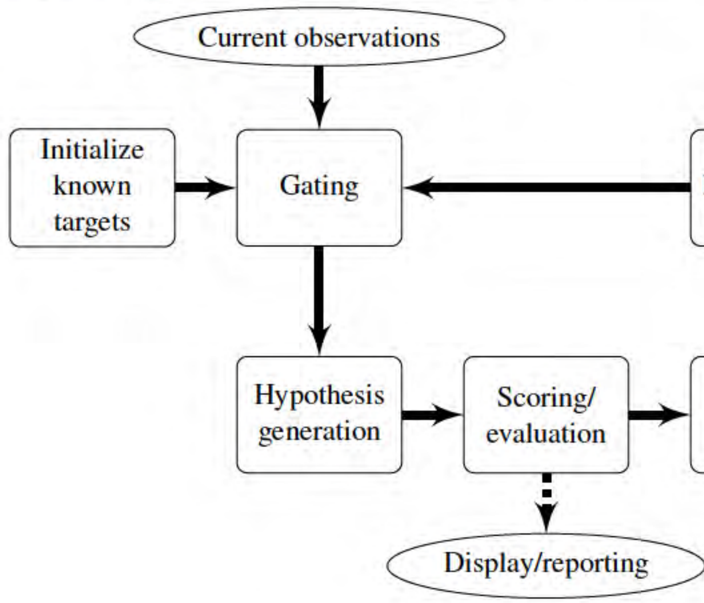

Figure 2. Major MHT functions.

Prior Distributions

A very common difficulty in Bayesian analysis is the selection of prior probability distributions, otherwise known simply as "priors". In Bayes' law,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \tag{11}$$

the prior is the term $P(A)$. It has direct influence on the posterior distribution, and so a great deal of attention must be paid to its selection. In this work, we specify a ratio of the probability of an event (e.g. a given measurement is in fact a false target) to the total observing area or volume; this ratio is denoted β_{event}, or the "(event) density". In the MTT problem, there are several prior probabilities we are concerned with. Usually, they are uniform over the observing area or volume, but spatial correlation may be more appropriate for certain situations†. The probability of detection may generally be thought to be close to unity (especially for high-quality sensors), but could be a more complicated distribution depending on conditions (lighting, weather), sensor quality, or other factors. Likewise, the false target/false alarm density should be very low, but may be higher based on sensor quality, environmental effects, or poor knowledge of nearby objects. The new target density

*Note that each hypothesis is a collection of target states and covariances, and the filter updates each target independently.

†See our other paper[25] in this conference for more discussion on this.

is perhaps the most interesting in the scope of this paper. For widely-spaced targets, of course, this could be assumed to be very low. However, this assumption does not hold in breakup or deployment scenarios, and must be chosen accordingly. Again, the importance of choosing priors must not be underestimated, and there remains work to be done in constructing these. In this paper we assume uniform spatial distributions for all priors.

As an aside, while most modern dedicated sensors are of high quality (low noise, etc.), the selection of priors becomes more interesting in the case of very low-quality sensors. If the undesirable characteristics of these sensors (high false alarm rate, missed detections, etc.) can be handled algorithmically, the desirable aspects (low cost, possibly smaller size, lower power) could make these sensors valuable in providing useful data for space situational awareness.

Test Setup

The entire test setup for this paper exists in the Matlab computing environment, including observation simulations and the observation association methods. First, each "secondary" target is defined relative to the reference target. These relative states (explained below) are displayed in Table 2, and the reference state is shown in Table 1. Then, all of these orbits are propagated over the observing period, and observations are recorded at specified intervals. These observations are then corrupted with Gaussian noise. A driver program is then used to sequentially process each set of observations for each known target, given an initial (perturbed) state estimate and covariance. For consistency, each of the four methods is tested against a single set of generated observations. This is repeated 25 times, and results are averaged across all runs. Each scenario is run both with and without process noise.

Table 1. Reference orbit used for observation generation.

[Earth-Fixed, km and s]	
x	7278.00
y	0.000
z	0.000
$\dot{x}$	0.000
$\dot{y}$	7.4005
$\dot{z}$	0.000

A set of four representative scenarios is presented here. In the first, a fragmentation even occurs within the observation span, resulting in several new targets to be tracked. The second includes several closely-spaced targets in very similar orbits, with all targets known *a priori*. The third represents a close-approach between several objects, with three near-intersecting tracks. The fourth is a simple conjunction, with two targets intersecting and two trackable debris pieces appearing afterwards. The sets of initial states for all scenarios are shown in Table 2, and plotted in Figure 3.

RESULTS AND DISCUSSION

Error and Uncertainty

Due to the significant differences in the way each of these four methods actually handles observation association (for instance, there is no explicit association in PDA), we will look at the end effect, i.e. state error and RMS uncertainty. For cases with multiple tracked targets, results for the

Table 2. Initial conditions for all scenarios.

Relative States (km and s in Hill's Frame)

Scenario 1						
Object	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
1	0.000	0.000	0.000	1.000	0.000	0.000
2	0.000	0.000	0.000	0.100	0.000	0.000
Scenario 2						
Object	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
1	10.000	0.000	0.000	0.000	0.000	0.000
2	-15.000	0.000	0.000	0.000	0.000	0.000
Scenario 3						
Object	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
1	5.000	0.000	0.000	-0.100	0.000	0.000
2	-5.000	0.000	-5.000	0.250	0.000	0.000
Scenario 4						
Object	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
1	60.00	0.000	0.000	-1.200	0.000	0.000
2	0.000	0.000	0.000	-0.500	0.000	0.000
3	0.000	0.000	0.000	0.900	0.000	0.000

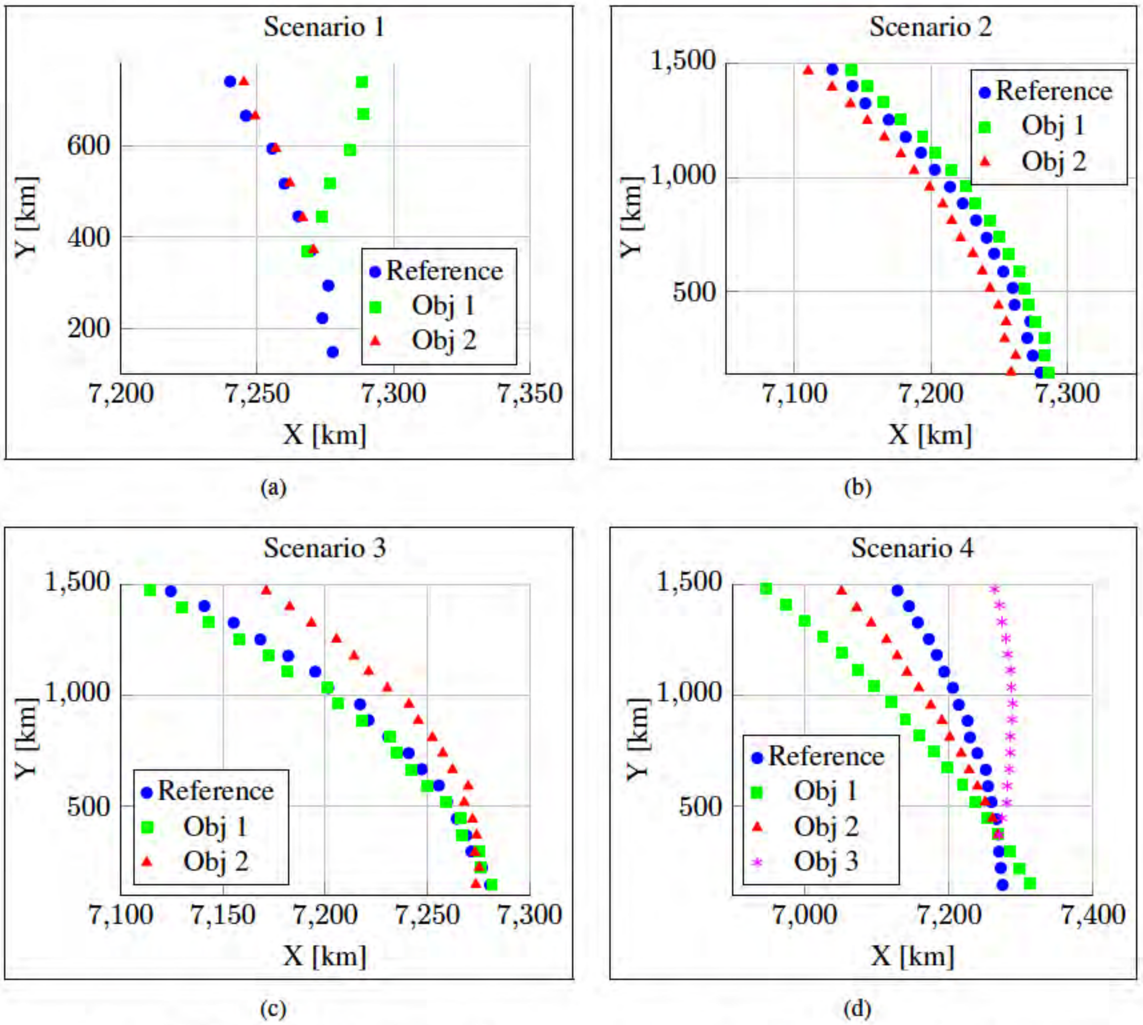

Figure 3. Inertial X-Y projection of observations in the four test cases: (a) breakup, (b) cluster flight, (c) conjunction, and (d) collision. Note that axes do not have equal scale in order to more clearly show separation.

estimate of the reference orbit are shown (in fact, performance is similar across all targets for a given method). In Table 3, we see that although all methods produce a final estimate with < 10 km position error, the relatively simple GNN performs the best, followed very closely by PDA. Uncertainty for all four methods is very close (< 1 km)—no single method shows a strong advantage. The MHT routine often "picks up" the wrong target—confusing the known one with one of the new debris objects.

Table 3. Results for test scenario 1 [km and s].

	NN mean (std. dev.)	**GNN** mean (std. dev.)	**PDA** mean (std. dev.)	**MHT** mean (std. dev.)
Position Error	5.4174 (6.4803)	3.1920 (4.7690)	3.1990 (5.0308)	9.0235 (6.6392)
Position Uncertainty	0.9578 (0.0000)	0.8922 (0.0000)	0.9193 (0.0068)	0.9132 (0.0029)
Velocity Error	0.0397 (0.0417)	0.0204 (0.0270)	0.0223 (0.0297)	0.0569 (0.0395)
Velocity Uncertainty	0.0096 (0.0000)	0.0076 (0.0000)	0.0083 (0.0001)	0.0082 (0.0001)

The geometry of Scenario 2 differs significantly from Scenario 1, as do the results, shown in Table 4. MHT exhibits a huge advantage for the problem of tracking multiple parallel objects. The performance of the other methods is roughly in line with their relative complexity. The reasoning behind this is that in the beginning of the observation period, when uncertainty and ambiguity is highest, neighboring observations are able to influence the NN, GNN, and PDA estimators. MHT, meanwhile, recognizes this period of ambiguity and is able to go back and make the correct associations once the uncertainty is reduced.

Table 4. Results for test scenario 2 [km and s].

	NN mean (std. dev.)	**GNN** mean (std. dev.)	**PDA** mean (std. dev.)	**MHT** mean (std. dev.)
Position Error	7.4871 (6.5769)	3.9381 (4.7415)	2.4913 (2.3037)	0.9504 (0.3267)
Position Uncertainty	0.9346 (0.0000)	0.9346 (0.0000)	0.9346 (0.0000)	0.9355 (0.0038)
Velocity Error	0.0205 (0.0195)	0.0123 (0.0048)	0.0125 (0.0116)	0.0095 (0.0039)
Velocity Uncertainty	0.0088 (0.0000)	0.0088 (0.0000)	0.0088 (0.0000)	0.0089 (0.0001)

Scenario 3 produces similar results (Table 5), except the more basic NN and GNN perform even worse. Here, there is added complexity in that the targets have near-intersecting orbits, allowing for more confusion within the estimator. Again, MHT exhibits the best performance, with PDA giving similar results. NN and GNN both have error near 20 km (however, there is a large spread in these results, and in some individual runs they exhibit similar performance to PDA).

Scenario 4 presents the most challenging case to all of the association methods (Table 6). As a combination of closely-spaced similar targets and newly-introduced clutter, it is a mix of two very different, difficult problems in multiple-target tracking. The simpler NN and GNN methods actually perform better than MHT here, while PDA again results in the lowest error. The rapidly-diverging targets after the conjunction do not give the MHT method lots of useful information, and association decisions need to made based on only the first couple frames of data. As can be seen from the results, MHT often infers the wrong decision based on this limited data. It is also worth noting the relatively high standard deviations in the position error for NN, GNN, and MHT—in

Table 5. Results for test scenario 3 [km and s].

	NN mean (std. dev.)	**GNN** mean (std. dev.)	**PDA** mean (std. dev.)	**MHT** mean (std. dev.)
Position Error	16.889 (18.816)	15.694 (16.686)	2.7256 (2.2233)	1.6892 (2.8141)
Position Uncertainty	0.9346 (0.0000)	0.9346 (0.0000)	0.9346 (0.0000)	0.9349 (0.0021)
Velocity Error	0.1062 (0.1085)	0.1006 (0.0959)	0.0137 (0.0112)	0.0147 (0.0164)
Velocity Uncertainty	0.0088 (0.0000)	0.0088 (0.0000)	0.0088 (0.0000)	0.0089 (0.0001)

fact, all three perform the correct associations almost as often as the incorrect ones. Therefore these higher numbers are less a question of ability and more a question of robustness and consistency. Additionally, further investigation into the MHT tracker reveals that the cases which show high position error have merely swapped targets incorrectly—swapping them back brings the error back down to about 1 km. However, in a real scenario, the "truth" states and identities are not known, thus this type of error would not be caught.

Table 6. Results for test scenario 4 [km and s].

	NN mean (std. dev.)	**GNN** mean (std. dev.)	**PDA** mean (std. dev.)	**MHT** mean (std. dev.)
Position Error	15.304 (16.587)	25.179 (17.816)	2.8730 (2.3815)	30.356 (17.247)
Position Uncertainty	0.9346 (0.0000)	0.9346 (0.0000)	0.9346 (0.0000)	0.9553 (0.0837)
Velocity Error	0.0975 (0.0935)	0.1536 (0.1002)	0.0144 (0.0120)	0.1883 (0.0955)
Velocity Uncertainty	0.0088 (0.0000)	0.0088 (0.0000)	0.0088 (0.0000)	0.0099 (0.0042)

Another interesting aspect of the results is that in nearly all of the scenarios, the position and velocity uncertainty matches very closely among all four methods. This is an especially important result for PDA—one of the main arguments against its use is the fact that, in a very uncertain system, the soft decisions could artificially inflate the covariance. However, we show this not to be the case. In fact, the length of the observation span used here (20 observations at 10 second intervals) is sufficient for PDA to reduce the state uncertainty to the steady-state limit for the Kalman filter. The problem of inflated covariance still exists for shorter observation spans.

Effects of Process Noise

Results for the runs with a modest process noise (on the order of tens of meters in position and tens of centimeters per second in velocity) are shown in the Appendix. Overall, there are no significant differences between the runs with and without process noise, aside from the final state uncertainty. It is expected that with more complex gating functions and more targets, the process noise will play an increasingly important factor in observation association by allowing the estimator to accept more observations as valid. The relatively low number of targets, and the fact that these scenarios were chosen to be very ambiguous from the start, means that process noise doesn't alter the results much—the gating routines are already allowing nearly all nearby observations.

Timing

When evaluating these different methods, it is of course important to look at the amount of processing time required for each. All runs were performed using Matlab on a single core of an Intel i7 quad-core with 2.67 GHz clock speed and 8 GB of RAM. NN, GNN, and PDA all took less than 500 ms to complete, while the much more complex MHT code took 1–3 minutes. Of course, for some systems, this kind of processing time is unacceptable. However, in the context of space surveillance, this may not be prohibitive, and may even be an improvement over current protocol. Additionally, there are myriad opportunities to speed up the MHT code, including parallelization, which could result in orders of magnitude speed increases.

CONCLUSIONS AND FUTURE WORK

Several well-known probabilistic observation association methods were applied to the space surveillance realm, with mostly good results. While more basic nearest-neighbor approaches offered acceptable state estimate errors for cases with simple geometries, they are shown to break down when the problem gets more complex. The more sophisticated Probabilistic Data Association and Multiple Hypothesis Tracking methods performed well over the entire range of scenarios shown in this paper. However, only the PDA method consistently made the correct associations, and with no noticeable adverse effects on the state uncertainty estimates. These results are consistent with others who have used the method in a similar setting.[11] PDA has the added benefit of running orders of magnitude faster than MHT.

Although many techniques exist for improving MHT performance, the complexity is another disadvantage when compared to PDA. However, one aspect of MHT which was not investigated in-depth—the ability to initialize and track new targets—may be of enough use in some systems to warrant consideration. See below for discussion on the difficulties of implementing this in an orbit determination setting.

While the EKF was sufficient for demonstrating the capability of these observation association methods, the next logical step is to couple them with a more capable filter, such as an Unscented Kalman Filter (UKF).[26] While the methods discussed require only some sequential estimator, the way these more complex filters handle nonlinearities (particularly with uncertainty propagation) may influence their performance. In addition to more complex filtering methods, the promulgation of more sophisticated uncertainty representations will require new methods of statistically gating observations.[26,27]

More capable filtering methods also enable several other important aspects, including high-fidelity modeling of perturbations and more realistic measurement modeling. It will be particularly interesting to investigate how non-conservative perturbations such as drag or solar radiation pressure could affect the observation association problem. The ability to do higher-fidelity modeling also allows us to attempt to expand to longer time spans for association, perhaps over one or more orbits.

In addition to the four methods discussed here, many other variations and new techniques exist for the observation association problem.[15,28] In particular, the Identity Management Kalman Filter[28] looks to offer a compact solution integrated directly within a filtering framework. Additionally, there are many different “flavors” of MHT and PDA, including Joint Probabilistic Data Association (JPDA) and track-oriented MHT. Any of these may offer improvement over current methods in the astrodynamics community.

One of the most useful aspects of MHT—the ability to recognize and begin tracking new targets—was not discussed at length in this paper. This is because initializing one of these new targets requires an initial orbit determination (IOD) to be performed. Historic methods typically require three or more observations to even begin to compute (and even then sometimes produce dubious results), although recent work has shown to produce good results, often with fewer required measurements.[11,12] Implementing this in an MHT framework would add considerable complexity, and to the best of the authors' knowledge has not been attempted previously.

As discussed in the Prior Distributions section, characterizing the observation environment and sensors is an important problem. Under normal circumstances, new/false target densities may be very small, but it could be more accurate to parameterize them based on the region of the sky for a given observing site. For instance, the probability of additional, unknown measurements may be higher when observing the GEO belt. Several off-nominal cases must be characterized as well: breakups, conjunctions, and deployments will all have a unique priors. While the noise characteristics of most operational sensors are understood, it is not always clear how to incorporate these statistics into the Bayesian association framework. Additionally, it is interesting to think about inexpensive, low-quality sensors and what could be done with them. If one of these association methods is able to successfully filter out most false measurements, it could be a good way of expanding space situational awareness capabilities.

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APPENDIX

Table 7. Results for test scenario 1 with process noise [km and s].

	NN mean (std. dev.)	**GNN** mean (std. dev.)	**PDA** mean (std. dev.)	**MHT** mean (std. dev.)
Position Error	8.5807 (29.556)	3.0701 (4.9287)	4.7755 (6.1268)	9.8346 (6.7596)
Position Uncertainty	1.0360 (0.0000)	1.0062 (0.0000)	1.0205 (0.0086)	1.0146 (0.0015)
Velocity Error	0.0620 (0.1940)	0.0234 (0.0305)	0.0336 (0.0368)	0.0651 (0.0417)
Velocity Uncertainty	0.0174 (0.0000)	0.0169 (0.0000)	0.0171 (0.0000)	0.0170 (0.0000)

Table 8. Results for test scenario 2 with process noise [km and s].

	NN mean (std. dev.)	GNN mean (std. dev.)	PDA mean (std. dev.)	MHT mean (std. dev.)
Position Error	8.6744 (5.7130)	5.5313 (5.6597)	2.5493 (1.8263)	0.9261 (0.3065)
Position Uncertainty	1.0240 (0.0000)	1.0240 (0.0000)	1.0240 (0.0000)	1.0244 (0.0024)
Velocity Error	0.0189 (0.0124)	0.0149 (0.0079)	0.0128 (0.0092)	0.0092 (0.0172)
Velocity Uncertainty	0.0172 (0.0000)	0.0172 (0.0000)	0.0172 (0.0000)	0.0172 (0.0001)

Table 9. Results for test scenario 3 with process noise [km and s].

	NN mean (std. dev.)	GNN mean (std. dev.)	PDA mean (std. dev.)	MHT mean (std. dev.)
Position Error	15.725 (16.449)	14.817 (16.984)	3.2011 (2.5729)	2.2577 (3.8420)
Position Uncertainty	1.0240 (0.0000)	1.0240 (0.0000)	1.0240 (0.0000)	1.0267 (0.0050)
Velocity Error	0.1018 (0.0943)	0.0952 (0.0987)	0.0161 (0.0129)	0.0196 (0.0270)
Velocity Uncertainty	0.0172 (0.0000)	0.0172 (0.0000)	0.0172 (0.0000)	0.0172 (0.0001)

Table 10. Results for test scenario 4 with process noise [km and s].

	NN mean (std. dev.)	GNN mean (std. dev.)	PDA mean (std. dev.)	MHT mean (std. dev.)
Position Error	19.779 (18.394)	20.867 (18.940)	3.1208 (2.3788)	35.389 (15.189)
Position Uncertainty	1.0240 (0.0000)	1.0240 (0.0000)	1.0240 (0.0000)	1.0845 (0.1837)
Velocity Error	0.1251 (0.1043)	0.1306 (0.1072)	0.0157 (0.0119)	0.2151 (0.0822)
Velocity Uncertainty	0.0172 (0.0000)	0.0172 (0.0000)	0.0172 (0.0000)	0.0181 (0.0038)