Assignment 2 (Ch. 2-3)

Monday, May 30, 2016 2:44 PM

CHARLES CLAYTON
#21518139
JUNE 2, 2015

2.48. For statements P and Q, show that $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a tautology. Then state $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ in words. (This is an important logical argument form, called **modus ponens**.)

$[P \wedge (P \to Q)] \to Q$ is a tautology if it true for all inputs

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$[P \wedge (P \to Q)] \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

As you can see $[P \wedge (P \to Q)] \to Q$ is true for all possible combinations of P and Q.

2.52. Let P and Q be statements.

(a) Is $\sim (P \vee Q)$ logically equivalent to $(\sim P) \vee (\sim Q)$? Explain.

(b) What can you say about the biconditional $\sim (P \vee Q) \Leftrightarrow ((\sim P) \vee (\sim Q))$?

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	T
F	T	T	F	T	F	T
F	F	F	T	T	T	T

a) as you can see $\sim(P \vee Q)$ is equivalent to $\sim P \vee \sim Q$

$\sim(P \vee Q)$	$\sim P \vee \sim Q$	$\sim(P \vee Q) \to (\sim P \vee \sim Q)$	$(\sim P \vee \sim Q) \to \sim(P \vee Q)$
F	F	T	T
F	T	T	F
F	T	T	F
T	T	T	T

the two implications do not match so the biconditional is false.

2.54. For statements P and Q, show that $(\sim Q) \Rightarrow (P \wedge (\sim P))$ and Q are logically equivalent.

$$
\begin{aligned}
\sim Q \to (P \wedge \sim P) &= \sim(\sim Q) \vee (P \wedge \sim P) && \text{by theorem 2.17} \\
&= Q \vee (P \wedge \sim P) && \text{by double negation} \\
&= Q \vee F && \text{by tautology} \\
&= Q && \text{by identity laws}
\end{aligned}
$$

$= Q \vee (P \wedge \sim P)$ by double negation
$= Q \vee F$ by tautology
$= Q$ by identity laws

2.60. Consider the implication: If x and y are even, then xy is even.

(a) State the implication using "only if."
(b) State the converse of the implication.
(c) State the implication as a disjunction (see Theorem 2.17).
(d) State the negation of the implication as a conjunction (see Theorem 2.21(a)).

a) "if P then Q" is equal to "Q only if P"

$\boxed{\text{xy is even only if x and y are even}}$

b) the converse is the impliation backwards

$\boxed{\text{if xy is even then x and y are even}}$

c) theorem 2.17 states $P \to Q \equiv \sim P \vee Q$

$\boxed{\text{x or y is odd, or xy is even}}$

d) theorem 2.21 states $\sim(P \to Q) \equiv P \wedge \sim Q$

$\boxed{\text{x and y are even, and xy is odd}}$

2.64. For which biconditional is its negation the following?
n^3 and $7n+2$ are odd or n^3 and $7n+2$ are even.

$\underbrace{\text{x and y are odd}}_{P}$ or $\underbrace{\text{x and y are even.}}_{Q}$

$\sim P \vee Q \equiv P \to Q$ $\quad$ $P \vee Q \equiv \sim P \to Q$

$\boxed{\text{if x or y is even then x and y are even}}$

2.68. State the negations of the following quantified statements:

(a) For every rational number r, the number $1/r$ is rational.
(b) There exists a rational number r such that $r^2 = 2$.

negation of quantified statements

$\exists x \in S$ s.t. $P(x) \equiv \sim(\forall x \in S, \sim P(x))$
$\forall x \in S, P(x) \equiv \sim(\exists x \in S,$ s.t. $\sim P(x))$

a) there doesn't exists some rational number r such that $1/r$ is irrational

b) for not all rational numbers r, $r^2 \neq 2$

2.74. Consider the open sentence

$$P(x, y, z):\ (x-1)^2 + (y-2)^2 + (z-2)^2 > 0.$$

where the domain of each of the variables x, y and z is $\mathbf{R}$.

(a) Express the quantified statement $\forall x \in \mathbf{R}, \forall y \in \mathbf{R}, \forall z \in \mathbf{R}, P(x, y, z)$ in words.
(b) Is the quantified statement in (a) true or false? Explain.
(c) Express the negation of the quantified statement in (a) in symbols.
(d) Express the negation of the quantified statement in (a) in words.
(e) Is the negation of the quantified statement in (a) true or false? Explain.

a) for all real numbers x, y, z, $(x-1)^2+(y-2)^2+(z-2)^2 > 0$

b) it is $\boxed{\text{false}}$ because if $x=1$, $y=2$, $z=2$
then it is not >0

c) $\sim(\exists x \in S$ s.t. $(x-1)^2+(y-2)^2+(z-2)^2 \leq 0)$

d) there does not exist real numbers x, y, z
such that $(x-1)^2+(y-2)^2+(z-2)^2 \leq 0$

e) It is $\boxed{\text{false}}$ because if $x=1$, $y=2$, and $z=2$
then $(x-1)^2+(y-2)^2+(z-2)^2 = 0$

3.14. Let $S = \{1, 5, 9\}$. Prove that if $n \in S$ and $\frac{n^2+n-6}{2}$ is odd, then $\frac{2n^3+3n^2+n}{6}$ is even.

We will solve this using proof by cases for the cases where $n=1$, $n=5$, and $n=9$.

CASE 1: if $n=1$, then $\frac{n^2+n-6}{2} = \frac{1+1-6}{2} = -2$, which is even.

Since the antecedant of the implication is false, by definition, the implication is true.

CASE 2: if $n=5$, then $\frac{n^2+n-6}{2} = \frac{5^2+5-6}{2} = 12$, which is even.

Since the antecedant of the implication is false, by definition, the implication is true.

CASE 3: if $n=9$, then $\frac{n^2+n-6}{2} = \frac{9^2+9-6}{2} = 42$, which is even.

Since the antecedant of the implication is false, by definition, the implication is true.

As demonstrated, all possible cases for n result in the implication being true, so the statement is true.