Assignment 1 (Ch. 1-2)

Monday, May 23, 2016 6:57 PM

1.6. The set $E = \{2x : x \in \mathbf{Z}\}$ can be described by listing its elements, namely $E = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$. List the elements of the following sets in a similar manner.

(a) $A = \{2x + 1 : x \in \mathbf{Z}\}$
(b) $B = \{4n : n \in \mathbf{Z}\}$
(c) $C = \{3q + 1 : q \in \mathbf{Z}\}$

a) $A = \{\ldots -3, -1, 1, 3 \ldots\}$

b) $B = \{\ldots -8, -4, 0, 4, 8 \ldots\}$

c) $C = \{\ldots -5, -2, 1, 4, 7 \ldots\}$

1.16. Find $\mathcal{P}(\mathcal{P}(\{1\}))$ and its cardinality.

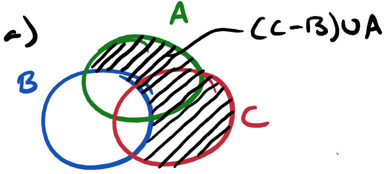
$P(\{1\}) = \{\{1\}, \emptyset\}$

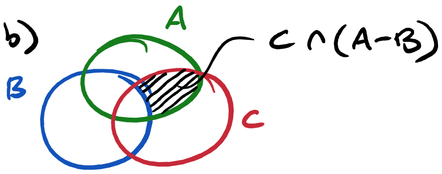
$P(\{\{1\}, \emptyset\}) = \{\{\{1\}\}, \{\emptyset\}, \emptyset\}, \{\emptyset, \{1\}\}\}$

$|P(P(\{1\}))| = \boxed{4}$

1.28. Let A, B and C be nonempty subsets of a universal set U. Draw a Venn diagram for each of the following set operations.

(a) $(C - B) \cup A$
(b) $C \cap (A - B)$.





1.36. For a real number r, define S_r to be the interval $[r - 1, r + 2]$. Let $A = \{1, 3, 4\}$. Determine $\bigcup_{\alpha \in A} S_\alpha$ and $\bigcap_{\alpha \in A} S_\alpha$.

$\bigcup_{\alpha \in A} S_\alpha = \{x : x \in S_\alpha \text{ for all } \alpha \in A\}$

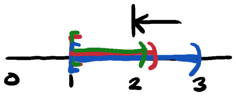
$S_1 = [0, 3]$ $S_3 = [2, 5]$ $S_4 = [3, 6]$

$S_1 \cup S_3 \cup S_4 = \boxed{[0, 6]}$

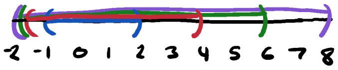
$S_1 \cap S_3 \cap S_4 = \boxed{[3]}$

1.42. For each of the following collections of sets, define a set A_n for each $n \in \mathbf{N}$ such that the indexed collection $\{A_n\}_{n \in \mathbf{N}}$ is precisely the given collection of sets. Then find both the union and intersection of the indexed collection of sets.

(a) $\{[1, \overset{A}{2} + 1), [1, \overset{B}{2} + 1/2), [1, \overset{C}{2} + 1/3), \ldots\}$
(b) $\{(-1, 2), (-3/2, 4), (-5/3, 6), (-7/4, 8), \ldots\}$.



$A \cup B \cup C = \boxed{[1, 3)}$ $A \cap B \cap C = \boxed{[1, 2)}$



$A \cup B \cup C = \boxed{(-2, \infty)}$ $A \cap B \cap C = \boxed{(-1, 2)}$

1.46. Which of the following are partitions of $A = \{a, b, c, d, e, f, g\}$? For each collection of subsets that is not a partition of A, explain your answer.

(a) $S_1 = \{\{a, c, e, g\}, \{b, f\}, \{d\}\}$ (b) $S_2 = \{\{a, b, c, d\}, \{e, f\}\}$
(c) $S_3 = \{A\}$ (d) $S_4 = \{\{a\}, \emptyset, \{b, c, d\}, \{e, f, g\}\}$
(e) $S_5 = \{\{a, c, d\}, \{b, g\}, \{e\}, \{b, f\}\}$.

conditions: a) all sets are subsets of A
b) no subsets are empty
c) union of all subsets is original set
d) intersects of any two subsets is nothing

a) yes, meets all conditions above
b) no, violates condition c
c) yes, meets all conditions
d) no, violates condition b
e) no, violates condition d

1.60. For $A = \{\emptyset, \{\emptyset\}\}$, determine $A \times \mathcal{P}(A)$.

$P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

$A \times P(A) = \{(\emptyset, \emptyset), (\emptyset, \{\emptyset\}), (\emptyset, \{\{\emptyset\}\}), (\emptyset, \{\emptyset, \{\emptyset\}\}),$
$\cancel{(\{\emptyset\}, \emptyset)}, (\{\emptyset\}, \{\emptyset\}), (\{\emptyset\}, \{\{\emptyset\}\}), (\{\emptyset\}, \{\emptyset, \{\emptyset\}\})\}$

2.2. Consider the sets A, B, C and D below. Which of the following statements are true? Give an explanation for each false statement. prove it?

$$A = \{1, 4, 7, 10, 13, 16, \ldots\} \quad C = \{x \in \mathbf{Z} : x \text{ is prime and } x \neq 2\}$$
$$B = \{x \in \mathbf{Z} : x \text{ is odd}\} \quad D = \{1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$$

(a) $25 \in A$ (b) $33 \in D$ (c) $22 \notin A \cup D$ (d) $C \subseteq B$ (e) $\emptyset \in B \cap D$ (f) $53 \notin C$.

a) $\{\ldots 19, 22, 25 \ldots\}$ ∴ $\boxed{T}$

b) 33 is an odd integer ∴ $\boxed{T}$

c) 22 is in A, so it is in $A \cup D$ ∴ $\boxed{F}$

d) all primes are odd except 2 ∴ $\boxed{T}$

e) all sets have $\emptyset$ as a subset ∴ $\boxed{T}$

f) 53 is a prime ∴ $\boxed{F}$

2.6. For the open sentence $P(A) : A \subseteq \{1, 2, 3\}$ over the domain $S = \mathcal{P}(\{1, 2, 4\})$, determine:

(a) all $A \in S$ for which $P(A)$ is true.
(b) all $A \in S$ for which $P(A)$ is false.
(c) all $A \in S$ for which $A \cap \{1, 2, 3\} = \emptyset$.

$S = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}, \{1,2,4\}\}$

a) $\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

b) $\{\{4\}, \{2,4\}, \{1,4\}, \{1,2,4\}\}$

c) $\{\emptyset, \{4\}\}$

2.18. Let $S = \{1, 2, \ldots, 6\}$ and let

$$P(A) : A \cap \{2, 4, 6\} = \emptyset. \text{ and } Q(A) : A \neq \emptyset.$$

be open sentences over the domain $\mathcal{P}(S)$.

(a) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.
(b) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \vee (\sim Q(A))$ is true.
(c) Determine all $A \in \mathcal{P}(S)$ for which $(\sim P(A)) \wedge (\sim Q(A))$ is true.

$S = \{1, 2, 3, 4, 5, 6\}$

a) $\boxed{\{\{1\}, \{3\}, \{5\}, \{1,3\}, \{1,5\}, \{3,5\}, \{1,3,5\}\}}$

b) $A = \emptyset$ ∴ $\boxed{\{\emptyset\}}$

c) $A = \emptyset$ & $A \cap \{2, 4, 6\} \neq \emptyset$ ∴ impossible ∴ $\boxed{\emptyset}$

2.20. For statements P and Q, construct a truth table for $(P \Rightarrow Q) \Rightarrow (\sim P)$.

P	Q	$P \Rightarrow Q$	$\sim P$	$(P \Rightarrow Q) \Rightarrow (\sim P)$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	T
T	T	T	F	F

2.22. Consider the statements:

P: $\sqrt{2}$ is rational. Q: $\frac{2}{3}$ is rational. R: $\sqrt{3}$ is rational.

Write each of the following statements in words and indicate whether the statement is true or false.
(a) $(P \wedge Q) \Rightarrow R$ (b) $(P \wedge Q) \Rightarrow (\sim R)$
(c) $((\sim P) \wedge Q) \Rightarrow R$ (d) $(P \vee Q) \Rightarrow (\sim R)$.

P: F, Q: T, R: F

a) if $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational then $\sqrt{3}$ is rational, $F \Rightarrow F$, $\boxed{T}$

b) if $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational then $\sqrt{3}$ is irrational, $F \Rightarrow T$, $\boxed{T}$

c) if $\sqrt{2}$ is irrational and $\frac{2}{3}$ is rational then $\sqrt{3}$ is rational, $T \Rightarrow F$, $\boxed{F}$

d) if $\sqrt{2}$ is rational or $\frac{2}{3}$ is rational then $\sqrt{3}$ is irrational, $T \Rightarrow T$, $\boxed{T}$

2.32. In each of the following, two open sentences $P(x)$ and $Q(x)$ over a domain S are given. Determine all $x \in S$ for which $P(x) \Rightarrow Q(x)$ is a true statement.

(a) $P(x): x - 3 = 4;\ Q(x): x \geq 8;\ S = \mathbf{R}$.
(b) $P(x): x^2 \geq 1;\ Q(x): x \geq 1;\ S = \mathbf{R}$.
(c) $P(x): x^2 \geq 1;\ Q(x): x \geq 1;\ S = \mathbf{N}$.
(d) $P(x): x \in [-1, 2];\ Q(x): x^2 \leq 2;\ S = [-1, 1]$.

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

a) P is T if $x = 7$
Q is T if $x \geq 8$

$x \neq 7$ or $(x = 7$ and $x \geq 8)$
$\hookrightarrow \boxed{x \neq 7}$

b) P is T if $x \geq 1$ or $x \leq 1$
Q is T if $x \geq 1$

$-1 \leq x \leq 1$ or $[(x \geq 1$ or $x \leq 1)$ and $x \geq 1]$
$\hookrightarrow -1 \leq x \leq 1$ or $x \geq 1 \rightarrow \boxed{x \geq -1}$

c) P is T if $x \neq 0$
Q is T if $x \geq 1$

$x = 0$ or $(x \neq 0$ and $x \geq 1)$
$\hookrightarrow \boxed{x = 0 \text{ or } x \geq 1}$

d) P is T if $x \in [-1, 1]$
Q is T if $x \geq -\sqrt{2}$ or $x \leq \sqrt{2}$
$\hookrightarrow x \geq -1$ or $x \leq 1$

$(x > -1$ or $x < 1)$ or $(-1 \leq x \leq 1$ and $-1 \leq x \leq 1)$
$\hookrightarrow -1 < x < 1$ or $-1 \leq x \leq 1 \rightarrow \boxed{-1 \leq x \leq 1}$

2.34. Each of the following describes an implication. Write the implication in the form "if, then."

(a) Any point on the straight line with equation $2y + x - 3 = 0$ whose x-coordinate is an integer also has an integer for its y-coordinate.
(b) The square of every odd integer is odd.
(c) Let $n \in \mathbf{Z}$. Whenever $3n + 7$ is even, n is odd.
(d) The derivative of the function $f(x) = \cos x$ is $f'(x) = -\sin x$.
(e) Let C be a circle of circumference 4π. Then the area of C is also 4π.
(f) The integer n^3 is even only if n is even.

if P then Q

a) if an x-coordinate on the line $2y + x - 3 = 0$ is an integer, then the y-coordinate is an integer

b) if an intege is odd, then its squre is odd

c) if n is an odd integer, $3n + 7$ is even

d) if $f(x)$ is $\cos(x)$, then $\frac{d}{dx} f(x)$ is $-\sin x$

e) if the circumference of a circle is 4π, then its area is 4

f) if n is even, n^3 is even

2.42. Determine all values of n in the domain $S = \{2, 3, 4\}$ for which the following is a true statement: The integer $\underbrace{\frac{n(n-1)}{2}}_{P}$ is odd if and only if $\underbrace{\frac{n(n+1)}{2}}_{Q}$ is even.

$S = \{2, 3, 4\}$

P	Q	P⇒Q	Q⇒P	P⇔Q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

@n=2   odd(T) $\frac{2(2-1)}{2} = 1$ odd(F) $\frac{2(2+1)}{2} = 3$ ✗

$\boxed{@n=3}$ odd(T) $\frac{3(3-1)}{2} = 3$ even(T) $\frac{3(3+1)}{2} = 6$ ✓

@n=4   even(F) $\frac{4(4-1)}{2} = 6$ even(T) $\frac{4(4+1)}{2} = 10$ ✗