Ch. 2 Logic

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2.1 STATEMENTS

STATEMENT → declarative sentence that is T or F
→ don't need to know if T or F
→ usually represented with P, Q, R, S, T

OPEN SENTENCE → statement with variables, called domain

NEGATION, ~ $\sim T = F$, $\sim F = T$

AND, $\wedge$
OR, $\vee$

P	Q	$P \wedge Q$	$P \vee Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

2.4 THE IMPLICATION

IMPLICATION, →

↳ if P, then Q $\equiv P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

only false when P (hypothesis) is T and Q (conclusion) is false

if hypothesis is false, we don't care about conclusion, so assume T

note: $\boxed{P \to Q \equiv \sim P \vee Q}$ $\boxed{\sim(P \to Q) \equiv P \wedge \sim Q}$

theorem 2.17 theorem 2.21

ENGLISH EXAMPLES OF $P \to Q$
- if P then Q
- P implies Q
- P is sufficient for Q
- whenever P, Q
- P only if Q
- Q if P
- Q is a necessary for P

2.6 BICONDITIONAL

CONVERSE
↳ converse of $P \to Q$ is $Q \to P$

BICONDITIONAL, ↔
↳ true when both implication & converse are true

$$\boxed{P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)}$$

P	Q	$P \to Q$	$Q \to P$	$P \leftrightarrow Q$
T	T	T	T	T

P	Q	$P \to Q$	$Q \to P$	$P \leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

↳ True if both F or both T

ENGLISH
- P is equivalent to Q
- P if and only if Q
- P is necessary and sufficient condition for Q

2.7 TAUTOLOGIES AND CONTRADICTIONS

↳ TAUTOLOGY → statement that is true for all possible inputs

$\boxed{P \vee \sim P \equiv T}$ $\boxed{\sim Q \vee (P \to Q) \equiv T}$

↳ CONTRADICTION → statement false for all possible inputs

$\boxed{P \wedge \sim P \equiv F}$ $\boxed{(P \wedge Q) \wedge (Q \to \sim P)}$

2.9 SOME FUNDAMENTAL PROPERTIES OF LOGICAL EQUIVALENCE

COMMUTATIVE LAWS $P \vee Q \equiv Q \vee P$, $P \wedge Q \equiv Q \wedge P$

ASSOCIATIVE LAWS $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

DISTRIBUTIVE LAWS $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

DEMORGAN'S LAWS $\boxed{\sim (P \vee Q) \equiv \sim P \wedge \sim Q}$

same if you swap $\vee$'s and $\wedge$'s

2.10 QUANTIFIED STATEMENTS

QUANTIFICATION → method of converting open sentence into a statement

universal quantifier, $\forall$ → "for all"

$\forall x \in S, P(x)$ → true if every x in S makes $P(x)$ true

↑ comma

existential quantifier, $\exists$ → "there exists"

$\exists x \in S$ s.t. $P(x)$ → true if any x in S makes $P(x)$ true

↑ such that

note: it is bad form to use symbolic quantifiers in an English sentence

Negating Quantifiers

$$\boxed{\begin{array}{l} \sim (\forall x \in S, P(x)) \equiv \exists x \in S \text{ s.t. } \sim P(x) \\ \sim (\exists x \in S \text{ s.t. } P(x)) \equiv \forall x \in S, \sim P(x) \end{array}}$$