1-61 An oil company is considering adding an additional grade of fuel at its service stations. To do this, an additional 12,000-litre tank must be buried at each station. Discussions with tank fabricators indicate that the least expensive tank would be cylindrical with minimum surface area. What size of tank should be ordered?

$$A = 2(\pi r^2) + 2\pi r h$$

$$V = 12000 = \pi r^2 h \implies h = \frac{12}{\pi r^2}$$

$$12000 L = 12\, m^3$$

$$A = 2\pi r^2 + 2\pi r \frac{12}{\pi r^2}$$

to find minimum, take derivative

$$\emptyset = \frac{d}{dr}\left(2\pi r^2 + 2\frac{12}{r}\right)$$

$$= 4\pi r - \frac{24}{r^2} \implies \boxed{r = 1.241\, m}$$

$$\hookrightarrow h = \frac{12}{\pi(1.241)^2} = \boxed{2.481\, m}$$

1-64 Jim Jones, a motel owner, noticed that just down the street the Motel 36 advertises a $36-per-night room rental rate on its sign. As a result, this competitor rents all 80 rooms every day by late afternoon. Jim, on the other hand, does not advertise his rate, which is $54 per night, and he averages only a 68% occupancy of his 50 rooms.

There are a lot of other motels nearby, but only Motel 36 advertises its rate on its sign. (Rates at the other motels vary from $48 to $80 per night.) Jim estimates that his actual incremental cost per night for each room rented, rather than remaining vacant, is $12. This $12 pays for all the cleaning, laundering, maintenance, utilities, and so on. Jim believes his eight alternatives are:

Alternative		Resulting Occupancy Rate
	Advertise and Charge	
1	$35 per night	100%
2	42 per night	94
3	48 per night	80
4	54 per night	66
	Do Not Advertise and Charge	
5	$48 per night	70%
6	54 per night	68
7	62 per night	66
8	68 per night	56

What should Jim do? Show how you reached your conclusion.

Price, P	Occp. Rate, R	Occp. Rooms, Q	Revenue, P*Q	Cost, 12*Q	Income, Revenue-Cost
35	100	50	1750	600	1150
42	94	47	1974	564	1410
48	80	40	1920	480	1440
54	66	33	1782	396	1386
48	70	35	1680	420	1260
54	68	34	1836	408	1428
62	66	33	2046	396	1650
68	56	28	1904	336	1568

Capacity	50
Cost	12

Jim has the most net income when he charges $62/night and doesn't advertise.

2-15 A small company manufactures a certain product. Variable costs are $20 per unit and fixed costs are $10,875. The price-demand relationship for this product is $P = -0.25D + 250$, where P is the unit sales price of the product and D is the annual demand. Use the data (and helpful hints) that follow to work out answers to parts (a)–(e).

- Total cost = Fixed cost + Variable cost
- Revenue = Demand × Price
- Profit = Revenue − Total cost

Set up your graph with dollars on the y axis (between 0 and $70,000) and, on the x axis, demand (units produced or sold, between 0 and 1,000 units).

(a) Develop the equations for total cost and total revenue.
(b) Find the break-even quantity (in terms of profit and loss) for the product.
(c) What profit would the company obtain by maximizing its total revenue?
(d) What is the company's maximum possible profit?
(e) Graph neatly the solutions to parts (a), (b), (c), and (d).

$C_V = 20/\text{unit}$ $\qquad\qquad C_T = C_F + DC_V$

$C_F = 10875$ $\qquad\qquad R = DP$

$P = -0.25D + 250$ $\qquad$ Profit $= R - C_T$
$\qquad\qquad \underset{\text{annual demand}}{\underbrace{}}$

a) $\boxed{\begin{array}{l} C_T = 10875 + 20D \\ R = -0.25D^2 + 250D \end{array}}$

break-even @ Profit $= \emptyset \to R = C_T$

$10875 + 20D = -0.25D^2 + 250D$

b) $\hookrightarrow \boxed{D = 50}$ or $\underline{D = 870}$

max R @ $\frac{d}{dD}R = \emptyset$

$-0.50D + 250 = \emptyset \to \underline{D = 500}$

Profit $= -0.25(500)^2 + 250(500) - (20(500) + 10875)$

c) $= \boxed{41.63k}$

max Profit @ $\frac{d}{dD}(R - c) = \emptyset$

$\frac{d}{dD}(-0.25D^2 + 250D - 20D - 10875) = \emptyset$

$-0.5D - 230 = \emptyset \to D = 460$

Profit $= -0.25(460)^2 + 250(460) - (20(460) + 10875)$

d) $= \boxed{42.03k}$

e)

100000 ... 42.03k (d) ... 41.63k (c) ... 0 (b) ... D (100 units)

(x from −543.8 to 1100)
— $(250 − 0.25 x) x$
— $20 x + 10875$
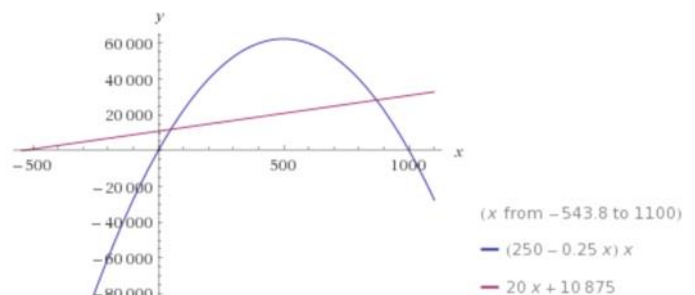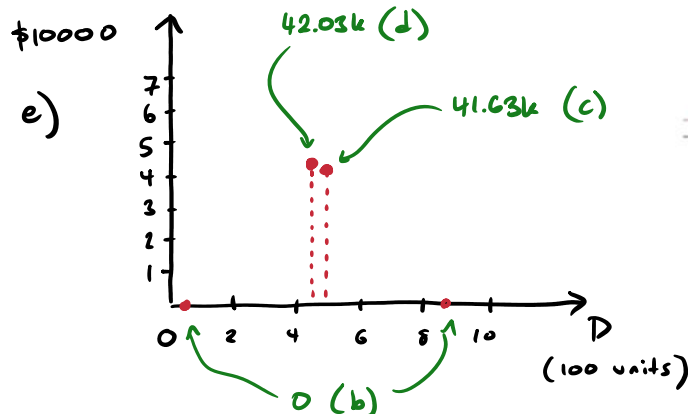
2-33 Northern Tundra Telephone (NTT) has received a contract to install emergency phones along a new 100-km section of the Snow-Moose Highway. Fifty emergency phone systems will be installed about 2 km apart. The material cost of a unit is $125. NTT will need to run underground communication lines that cost NTT $7,500 per kilometre (including labour) to install. There will also be a one-time cost of $10,000 to network these phones into NTT's current communication system.

(a) Develop a cost estimate of the project from NTT's perspective.

(b) If NTT adds a profit margin of 35% to its costs, how much will it cost the province to fund the project?

$$C_T = \$125 \frac{100\text{km}}{2\text{km}} + \frac{\$7500}{\text{km}}(100\text{km}) + \underbrace{\$10000}_{C_F}$$

a) $\boxed{= \$766250}$

a 35% profit margin means $0.35 of income for each dollar spent

$$\text{profit margin} = \frac{\text{revenue} - \text{cost goods sold}}{\text{revenue}}$$

$\hookrightarrow 0.35 = \frac{R - 766250}{R} \rightarrow \boxed{R = \$1.179M}$

2-41 A refinisher of antiques named Constance has been so successful with her small business that she is planning to expand her shop and buy all new equipment. She is going to start enlarging her shop by purchasing the following equipment.

Equipment	Original Capacity	Cost of Original Equipment	Power Sizing Exponent	Capacity of New Equipment
Varnish bath	50 gal	$3,500	0.80	75 gal
Power scraper	3/4 hp	250	0.22	1.5 hp
Paint booth	3 ft³	3,000	0.6	12 ft³

What would be the *net* cost to Constance to obtain this equipment (assume that she can trade the old equipment in for 15% of its original cost)? Assume also that there has been no inflation in equipment prices.

trade in price $= \$0.15(3500 + 250 + 3000)$
$= \$1013$

power sizing model: power sizing exponent

$$\text{cost B} = \text{cost A}\left(\frac{\text{size B}}{\text{size A}}\right)^x$$

$$C = 3500\left(\frac{75}{50}\right)^{0.8} + 250\left(\frac{1.5}{0.75}\right)^{0.22} + 3000\left(\frac{12}{3}\right)^{0.6}$$

$= \$12.02k$

net cost $= 12024 - 1013 = \boxed{\$11.01k}$

2-42 Refer to Problem 2-41 and now assume the prices for the equipment that Constance wants to replace have not been constant. Use the cost index data for each piece of equipment to update the costs to the price that would be paid today. Develop the overall cost for Constance, again assuming the 15% trade-in allowance for the old equipment. Trade-in is based on original cost.

Original Equipment	Cost Index When Originally Purchased	Cost Index Today
Varnish bath	154	171
Power scraper	780	900
Paint booth	49	76

line model: $\text{cost B} = \text{cost A} \dfrac{\text{index B}}{\text{index A}}$

$$C = 3500\left(\frac{75}{50}\right)^{0.8}\frac{171}{154}$$
$$+ 250\left(\frac{1.5}{0.75}\right)^{0.22}\frac{900}{780}$$
$$+ 3000\left(\frac{12}{3}\right)^{0.6}\frac{76}{49} = \$16.4k$$

net cost $= 16.4k - 1013 = \boxed{\$15.39k}$

unclear what this means, assuming no cost index on 15%

3-10 How long will it take for an investment to double at 4% a year compounding annually?

$F = P(1+i)^n$ where $F = 2P$, $i = 0.04$

$2P = P(1 + 0.04)^n \rightarrow 2 = (1.04)^n \rightarrow \boxed{n = 17.67 \text{ years}}$

$$2P = P(1 + 0.04)^n \quad \rightarrow \quad 2 = (1.04)^n \rightarrow \boxed{n = 17.67 \text{ years}}$$

3-13 A manufacturing company made an investment 10 years ago that is now worth $1,500,000. How much was the initial investment:

(a) At a simple interest rate of 10% a year?

(b) At an interest rate of 10% a year compounding annually?

$$F = P(1 + in) \rightarrow P = \frac{1.5M}{1 + 0.1(10)} = \boxed{\$750k}$$

simple

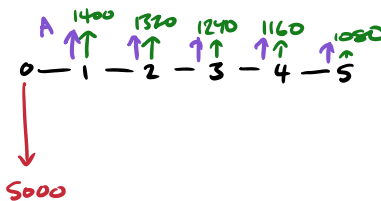$$F = P(1+i)^n \rightarrow P = \frac{1.5M}{(1+0.1)^{10}} = \boxed{\$578.3k}$$

compound

3-28 The following series of payments will repay a present sum of $5,000 at an 8% interest rate. Using single payment factors, what present sum is equivalent to this series of payments at a 10% interest rate?

Year	End-of-Year Payment
1	$1,400
2	1,320
3	1,240
4	1,160
5	1,080

$G = 80$

$i = 0.08$

using only P/F

$$P = 1400(P/F, 10\%, 1) + 1320(P/F, 10\%, 2) + 1240(P/F, 10\%, 3)$$
$$+ 1160(P/F, 10\%, 4) + 1080(P/F, 10\%, 5)$$
$$= 1273 + 1091 + 931.6 + 792.3 + 670.6 = 4758$$

using A - G

$$P = 1400(P/A, 10\%, 5) - 80(P/G, 10\%, 5) = 5307 - 548.9 = \boxed{4758}$$

3-47 Mr Sansome withdrew $1,000 from a savings account and invested it in common shares. At the end of five years, he sold the shares and received a cheque for $1,307. If Mr Sansome had left his $1,000 in the savings account, he would have received an interest rate of 5%, compounded quarterly. Mr Sansome would like to compute a comparable interest rate on his common shares investment. If compounding is quarterly, what nominal annual interest rate did Mr Sansome receive on his investment in shares? What effective annual interest rate did he receive?

because compounding quarterly

$$F = P\left(1 + \frac{i}{n}\right)^{nt} \leftarrow \text{\# years}$$

$\leftarrow$ # times compounded = 4 per year

$n = 4, \quad P = 1000, \quad F = 1307, \quad t = 5, \quad i = ?$

$$1307 = 1000\left(1 + \frac{i}{4}\right)^{4 \cdot 5} \rightarrow i = 0.053907$$

nominal, $\boxed{i = 5.39\%}$

$$1307 = 1000\left(1 + \frac{i}{4}\right)^{4.5} \rightarrow i = 0.053907$$

nominal, quarterly $\boxed{i = 5.39\%}$

$$1307 = 1000(1+i)^5 \rightarrow i = 0.055$$

effective $\boxed{i = 5.5\%}$

4-22 Liam dreams of starting his own business for importing consumer electronic products to his home country. He estimates that he can earn 5% on his investments and will need to have $300,000 at the end of the 10th year if he wants to give his business a good, solid foundation. He now has $28,850 in his account, and he believes he can save $12,000 from his income each year, beginning now. He plans to marry at about the end of the sixth year and will skip the investment contribution that year. How far below or above his $300,000 goal will he be?

$i = 0.05$, $A = 12k$, $n = 10$, $P = 28850$, $F_n = 300k$

0 1 2 3 4 5 6 7 8 9 10 — F (up), P (down), A (down arrows), note missing year

$$P\left(\tfrac{F}{P}, 5\%, 10\right) + A\left(\tfrac{F}{A}, 5\%, 10\right) - P\left(\tfrac{F}{P}, 5\%, 4\right) = F_R$$

$$28850(1.6289) + 12k(12.5779) - 12k(1.216) = F_R$$

$$46993.6 + 150934.7 - 14586.1 = F_R = 183342$$

Liam is $\boxed{116659}$ short

4-26 Determine the break-even resale price 10 years from now of an apartment house that can be bought today for $449,000. Its annual net income is $54,000. The owner wants a 10% annual return on her investment.
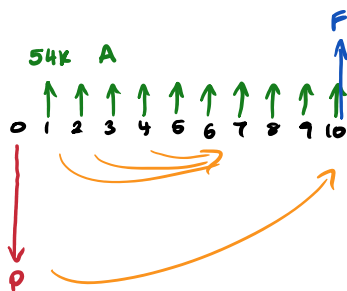
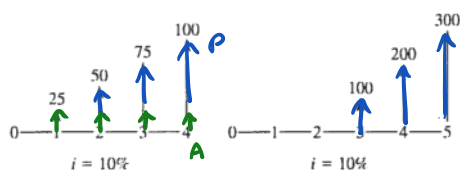$i = 0.1$
$n = 10$
$A = 54k$
$P = 449k$

assuming investing 54k

54k A — 0 1 2 3 4 5 6 7 8 9 10 — F (up), P (down)

investment, P, must be cancelled by net income & resale price, F

$$P\left(\tfrac{F}{P}, 10\%, 10\right) = A\left(\tfrac{F}{A}, 10\%, 10\right) + F$$

$$F = 449k(1.1)^{10} - 54k\frac{(1.1)^{10} - 1}{0.1} = 303969$$

break even resale price $\boxed{F = \$304k}$

4-55 For diagrams (a) to (d), compute the present values of the cash flows.
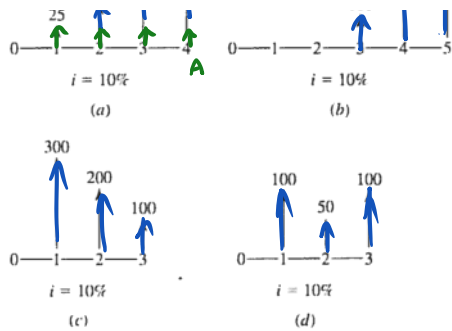

(a) diagram: 25, 50, 75, 100 at periods 1-4, P, A, i = 10%

(b) diagram: 100, 200, 300 at periods 3, 4, 5, i = 10%

a) $P = 25\left(\tfrac{P}{A}, 10\%, 4\right) + 25\left(\tfrac{P}{G}, 10\%, 4\right)$
   $= 79.25 + 109.45 = \boxed{188.7}$

b) $P = 100\left(\tfrac{P}{F}, 10\%, 3\right) + 200\left(\tfrac{P}{F}, 10\%, 4\right) + 300\left(\tfrac{P}{F}, 10\%, 5\right)$
   $= 75.13 + 136.6 + 186.3 = \boxed{398}$

Diagrams (a), (b), (c), (d) with $i = 10\%$.

b) $P = 100(\frac{F}{F}, 10\%, 3) + 200(\frac{1}{F}, 10\%, 4) + 300(\frac{1}{F}, 10\%, 5)$
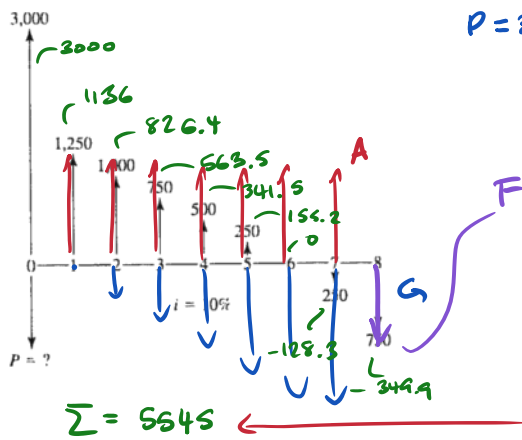
$= 75.13 + 136.6 + 186.3 = \boxed{398}$

c) $P = 300(\frac{P}{F}, 10\%, 1) + 200(\frac{P}{F}, 10\%, 2) + 100(\frac{P}{F}, 10\%, 3)$

$= 272.7 + 165.3 + 75.13 = \boxed{513}$

d) $P = 100(\frac{P}{F}, 10\%, 1) + 50(\frac{P}{F}, 10\%, 2) + 100(\frac{P}{F}, 10\%, 3)$

$= 90.91 + 41.32 + 75.13 = \boxed{207}$

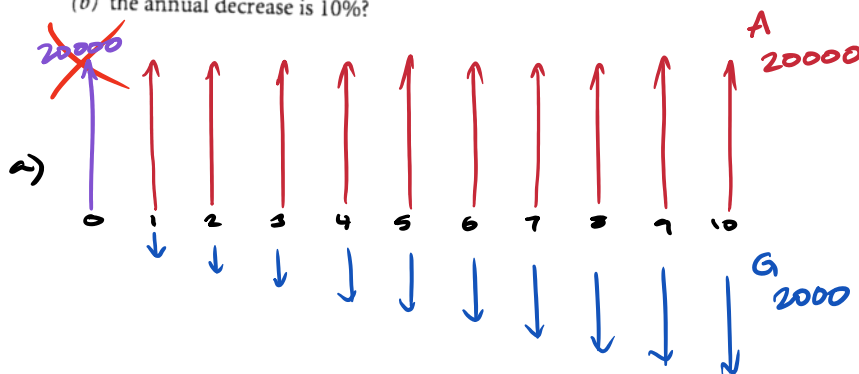**4-63** Find the value of P for the following cash flow diagram.

$P = 3000 + 1250\left(\frac{P}{A}, 10\%, 7\right)$ — 4.868

$- 250\left(\frac{P}{G}, 10\%, 7\right)$ → 12.763

$- 750\left(\frac{P}{F}, 10\%, 8\right)$ → 0.4665

$\boxed{\$5644}$

✓ checks out

$\Sigma = 5545$

4-85 A set of cash flows begins at \$20,000 the first year, with a decrease each year until $n = 10$ years. If the interest rate is 8%, what is the present value when
(a) the annual decrease is \$2,000?
(b) the annual decrease is 10%?

$i = 0.08$

a)

$P = 20000 + 20000\left(\frac{P}{A}, 8\%, 10\right) - 2000\left(\frac{P}{G}, 8\%, 10\right)$

$= 20000 + 20000(6.710) - 2000(25.977)$

$= \boxed{102246} = \boxed{82246}$

4.6683

b) $P = 20000 + 20000(6.710) - A_1\left[\dfrac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right]$

b) $P = \cancel{20000} + 20000(6.710) - A_1 \left[\dfrac{1-(1+g)^n(1+i)^{-n}}{i-g} \right]$

$P = \cancel{61033} = \boxed{41034}$

$$i = 0.08 \qquad g = -0.1$$

$$93166$$