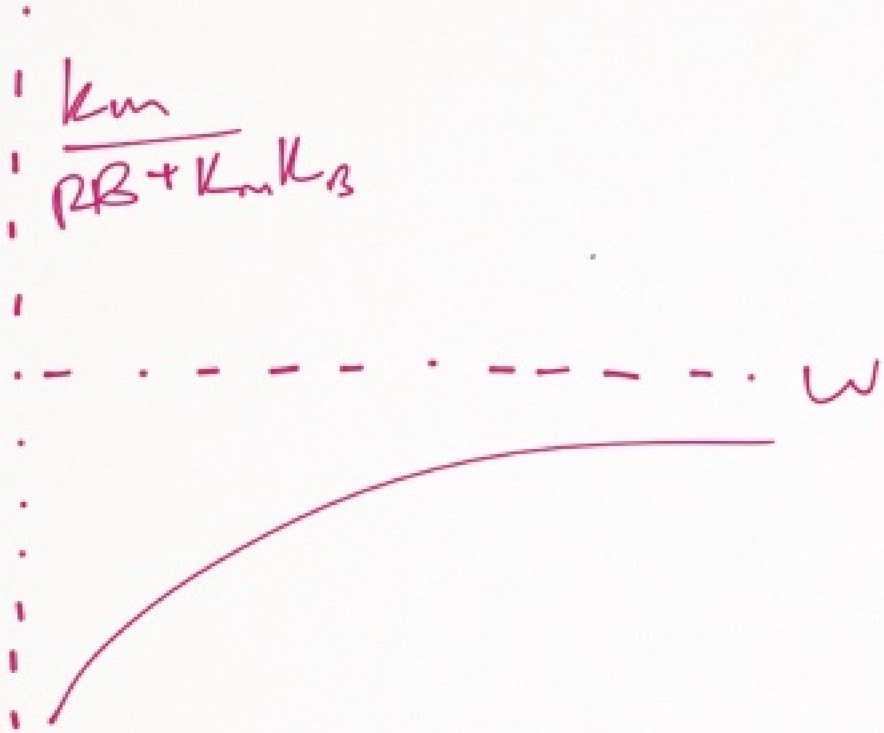
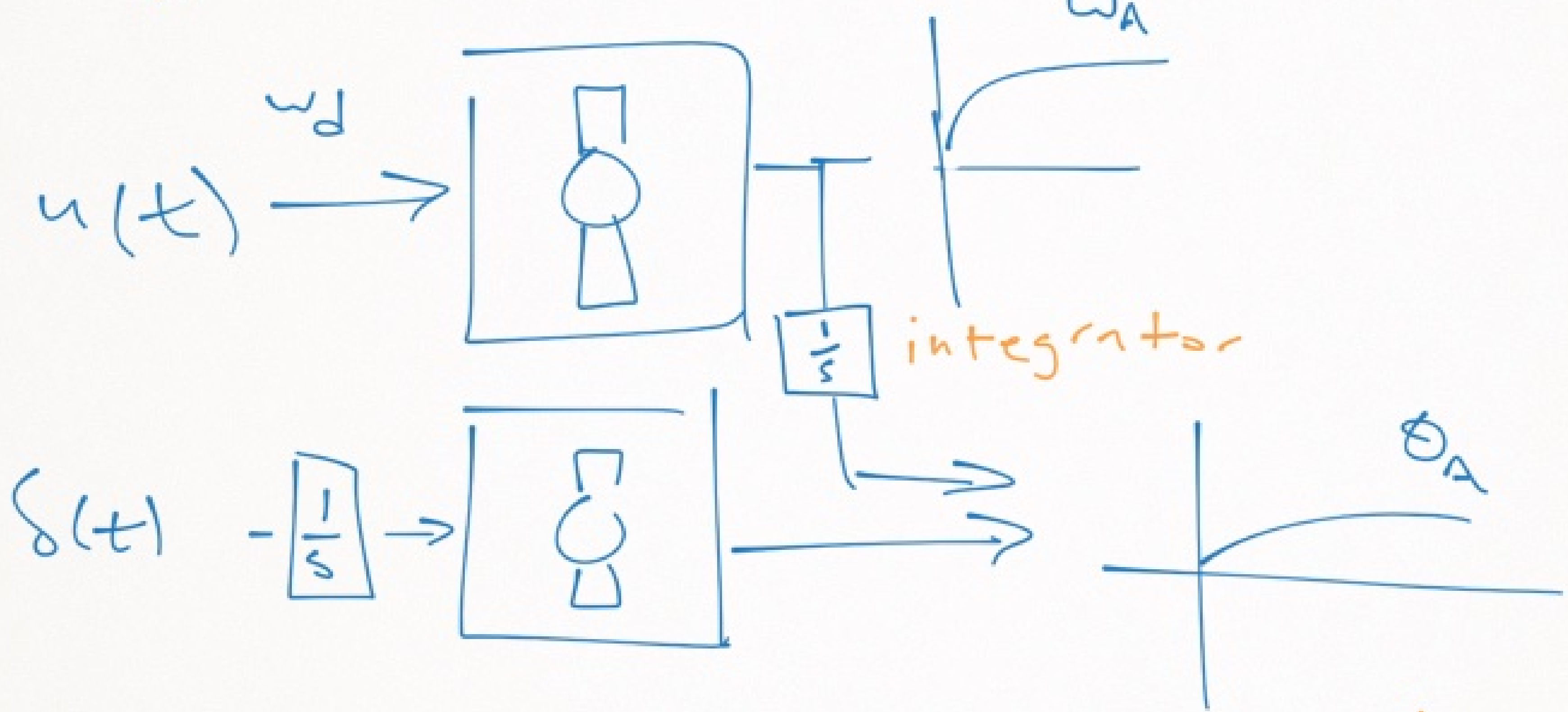


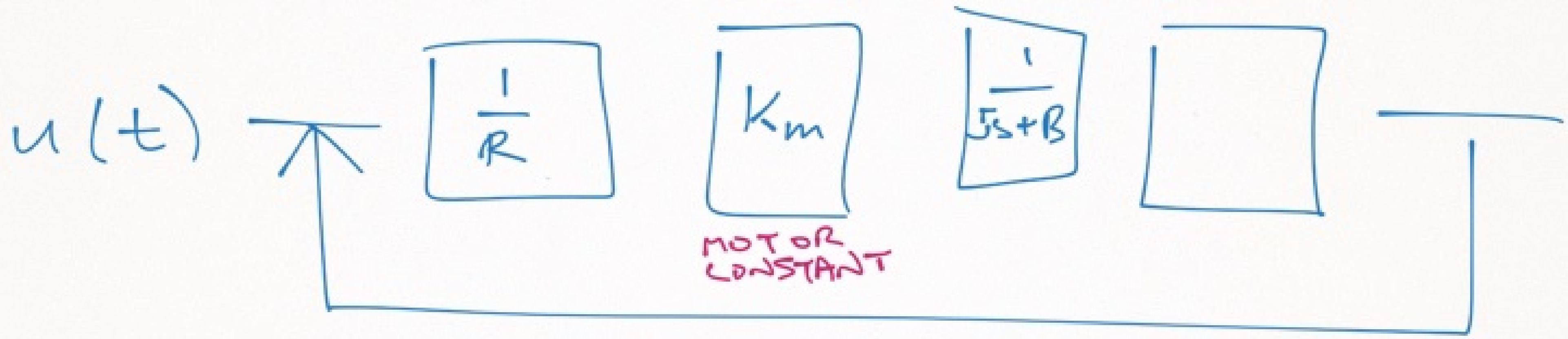
IMPULSE

$$w(t) = \frac{K_m}{RB+K_mK_B}\left(u(t) - e^{-\frac{(RB+K_mK_B)t}{RJ}}\right)$$

//calculus in laplace is commutative



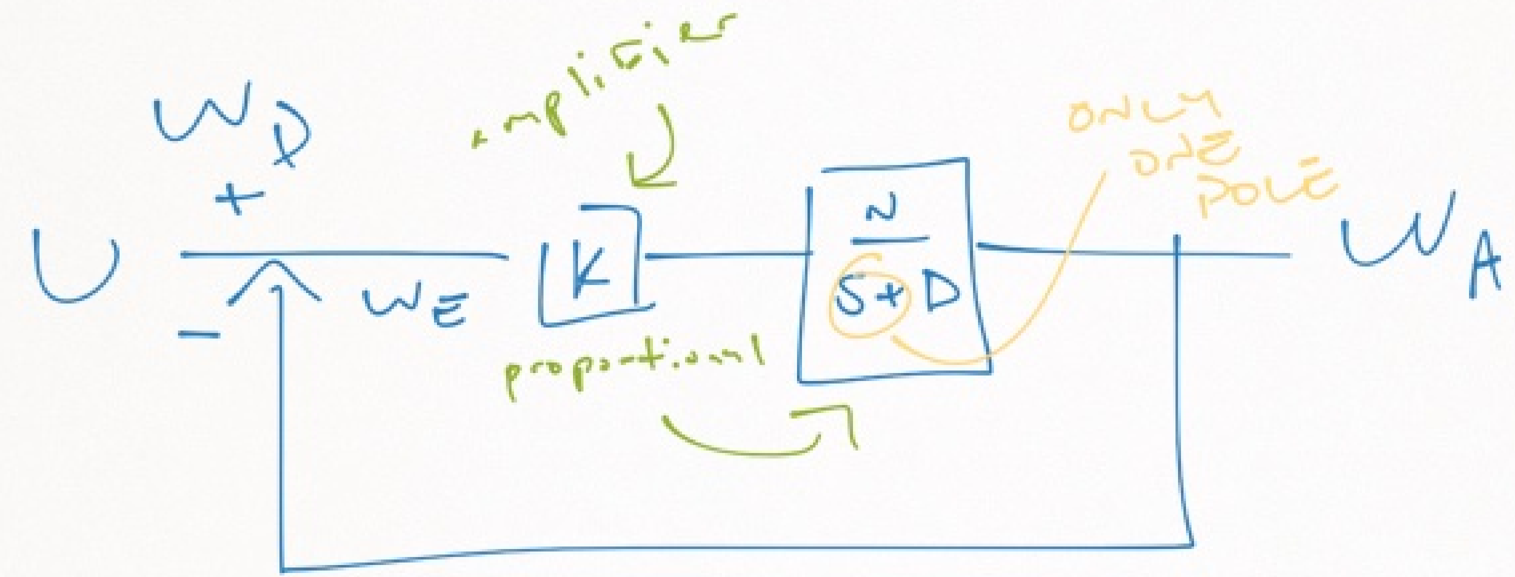
MOTOR MODEL // online



TAKE MOTOR & CONTROL

(BRUNT OF COURSEWORK)

MOST FUNDAMENTAL CONTROL SYS.



$$TF = \frac{KN/S+D}{1 + KN/S+D} = \frac{KN}{S+KN+D}$$

separate DC GAIN

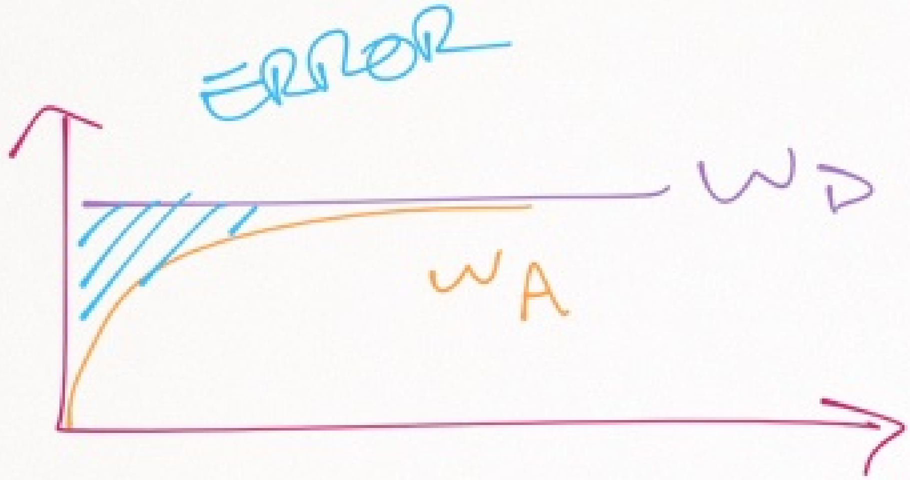
$$= \frac{KN}{KN+D}\,\frac{KN+D}{(S+KN+D)}$$

$$K_{DC} = \frac{KN}{KN+D}$$

YOU PICK K

K_{DC}

$$\frac{a}{s+a}$$



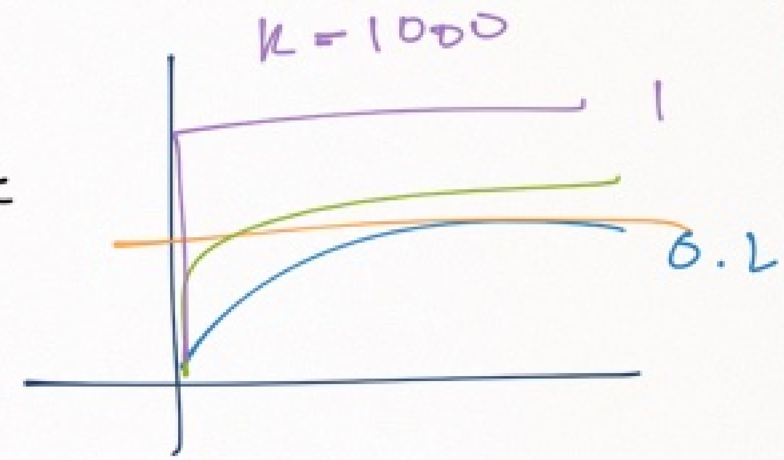
N=1 D=3 K=1

$$\frac{1}{4}\left(u(t) - e^{-4t}\right)$$

N=1 D=3 K=3

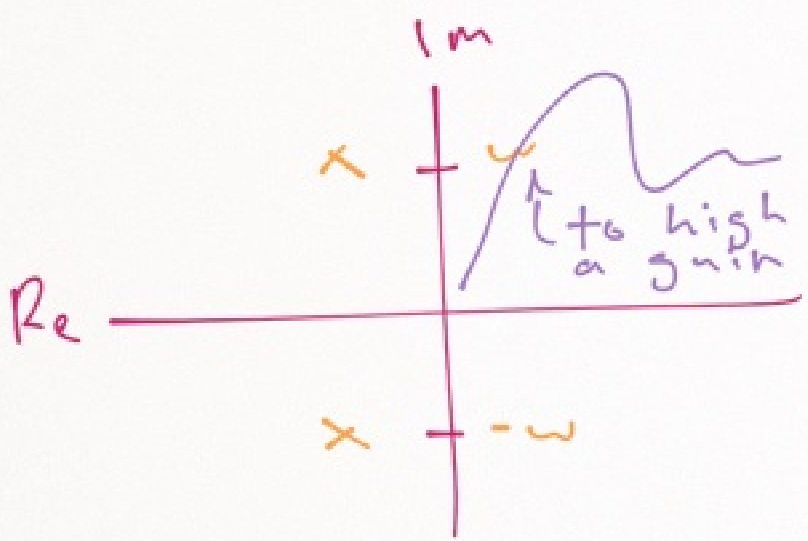
BIGGER GETS TO w_D FASTER

VALUE OF K_{DC} SHOWS YOU VALUE, $e^{-(KN+D)t}$ SHOWS ROUTE TO GET THERE.

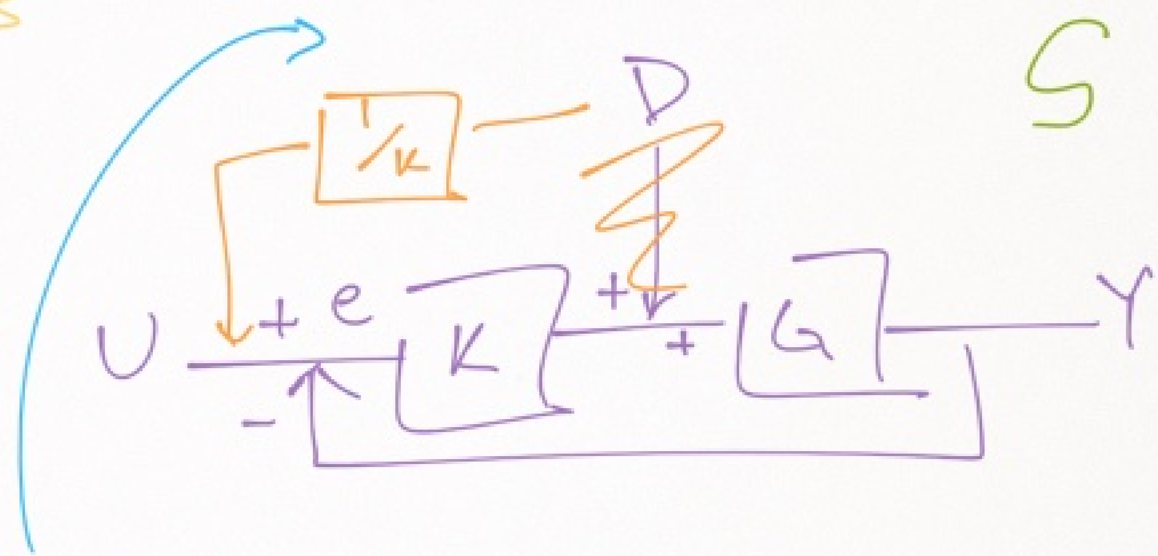


If you have one pole, it has to be on left of y-axis

THE BIGGER THE K THE BETTER



S



reduces the disturbances based on how big you pick K

$R = 2$

OTHERS = 1

$$\text{MOTOR TF} = \frac{K_m}{RJs + RB + K_m K_B}$$

$$= \frac{1}{2s+3} = \frac{1/2}{s + 3/2} \rightarrow \text{POLE @ } -\frac{3}{2}$$

ADD F.B.

$$TF = \frac{\overbrace{KK_M}^{G_N H_D}}{\underbrace{RJs + RB + K_m K_B}_{G_D H_D} + \underbrace{KK_M}_{G_N H_N}}$$

$$= \frac{K}{2s + 2 + 1 + K}$$

$$= \frac{K/2}{s + \frac{3}{2} + \frac{K}{2}}$$

$\rightarrow$ POLES @ $-\left(\frac{3}{2} + \frac{K}{2}\right)$

NO FB:

$\rightarrow$ pole @ $-\frac{3}{2}$

FB

$\rightarrow$ pole @ $-\frac{3}{2} - \frac{K}{2}$

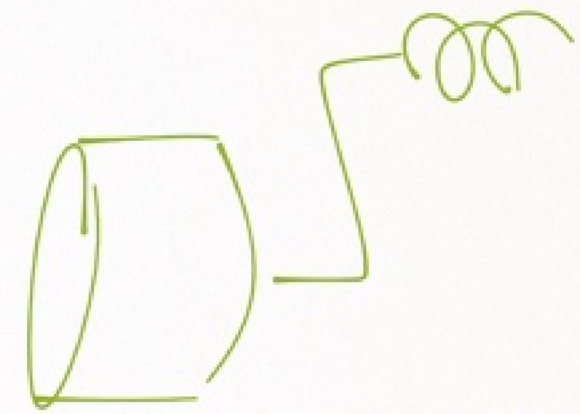
To make it sharper feedback can move pole to left!

SECOND ORDER SYS.

first order systems never oscillate // you can model gravity as spring

→ mass introduces derivative term

→ spring introduces integral term



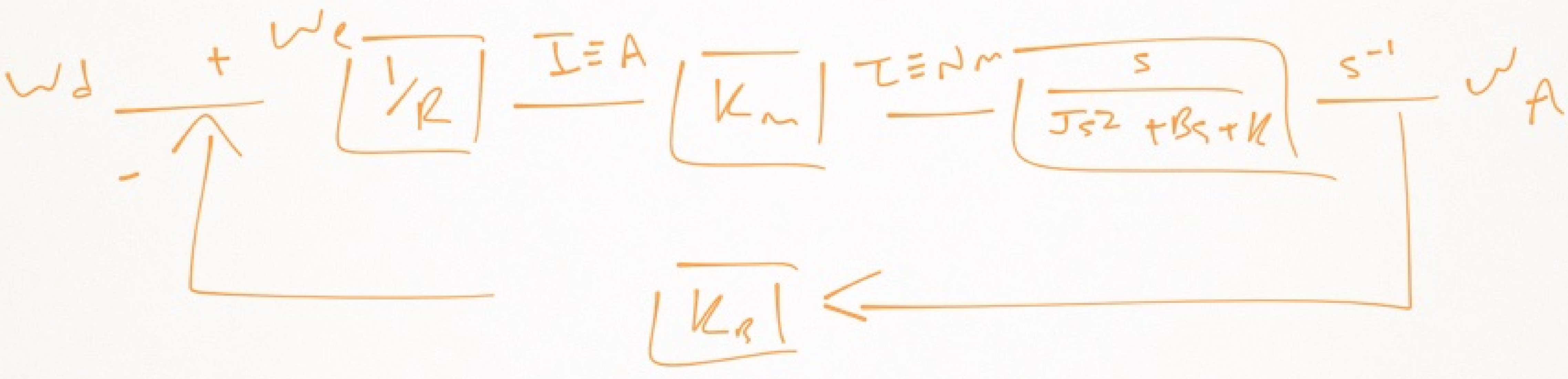
$\tau = Js\omega = B\omega$

$= \underbrace{\text{SPRING}}_{K} \cdot \frac{1}{s}\omega$

$\tau = \left(Js + B + \frac{K}{s}\right)\omega$

$\frac{\ }{\tau} = \frac{s}{Js^2 + Bs + K}$

MOTOR MODEL



TRANSFER FUNCTION

MOTOR MODEL

$$G(s) = \frac{K_m s}{RJs^2 + RBs + RK}$$

$$= \frac{K_m s}{RJs^2 + (RB + K_A K_B) + RK}$$

$$H(s) = \frac{K_B}{1}$$

$R = 2\,\Omega$

$K_m = 6\,\frac{Nm}{A}$

$J = 3\,Nm^2$

$B = 9\,Nms$

$K = 3\,Nm$

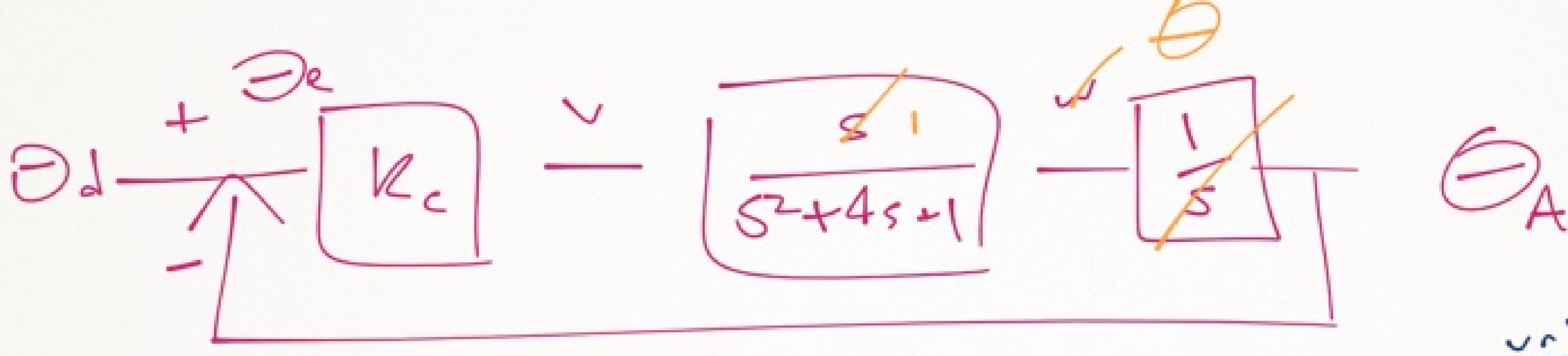
$K_B = 1\,Vs$

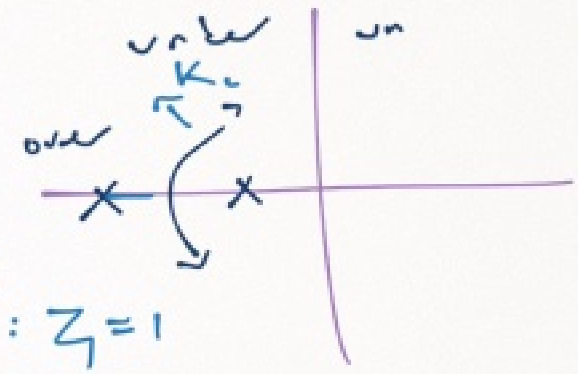
$$TF = \frac{6s}{6s^2 + 24s + 6}$$

$$= \frac{s}{s^2 + 4s + 1}$$

poles at

$$\frac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$$





critical damping : $\zeta = 1$

$$TF = \frac{K_c}{s^2+4s+1+K_c}$$

$\omega_n = ?$ (NAT. f) $= \sqrt{1+K_c}$

natural f increases w/ K_c

$2\zeta\sqrt{1+K_c} = 4$ $\quad \zeta = \frac{2}{\sqrt{1+K_c}}$

K_c compensates for friction
∴ reduces damping

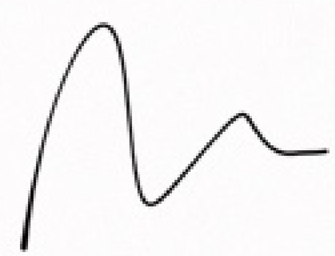
$$TF = \frac{K_c}{1+K_c}\,\frac{1+K_c}{s^2+4s+1+K_c}$$

controller

$\frac{2}{\sqrt{1+K_c}} = 1$ ∴ $K_c = 3$ ∴ critically damped (ideal)

ie. as fast as possible w/o overshooting

if $K_c = 1$, underdamped



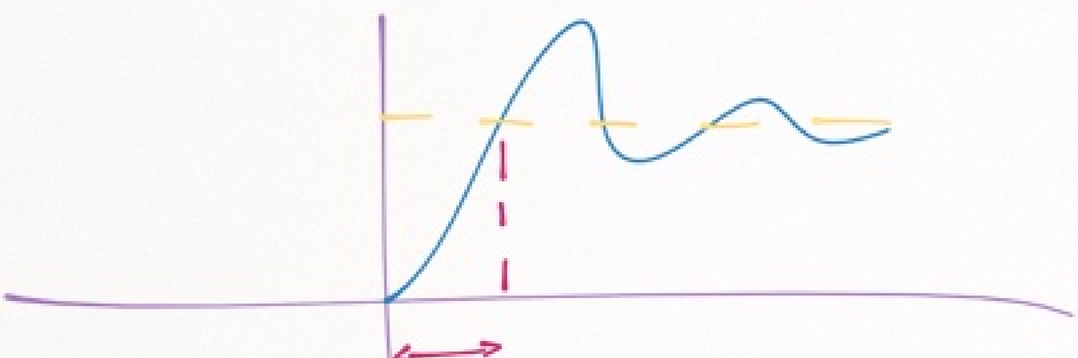
if $\zeta > 1$, overdamped

note: get simulink
get MATLAB

$$\frac{\omega_n^2}{s^2+2\zeta\omega_n^2 s+\omega_n^2}$$

$$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

he sturred these laplace functions



rise time, T_R

sometimes you just care about how long it takes to reach target

ex. racing

for rise time, we're interested in step response

$$\frac{1}{s}\cdot\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}=T(s)$$

@ t=? such that $f(t)=1$

T_R is the time $f(t)=1$

$$f(t)=1-\frac{1}{\beta}e^{-\zeta\omega_n t}\underbrace{\sin(\beta\omega_n t+\cos^{-1}\zeta)}_{=\emptyset}$$

$e^{-\zeta\omega_n t} = \emptyset$ @ $t=\infty$

$=\emptyset$ @ $\beta\omega_n t+\cos^{-1}\zeta = \emptyset, \pi, 2\pi$

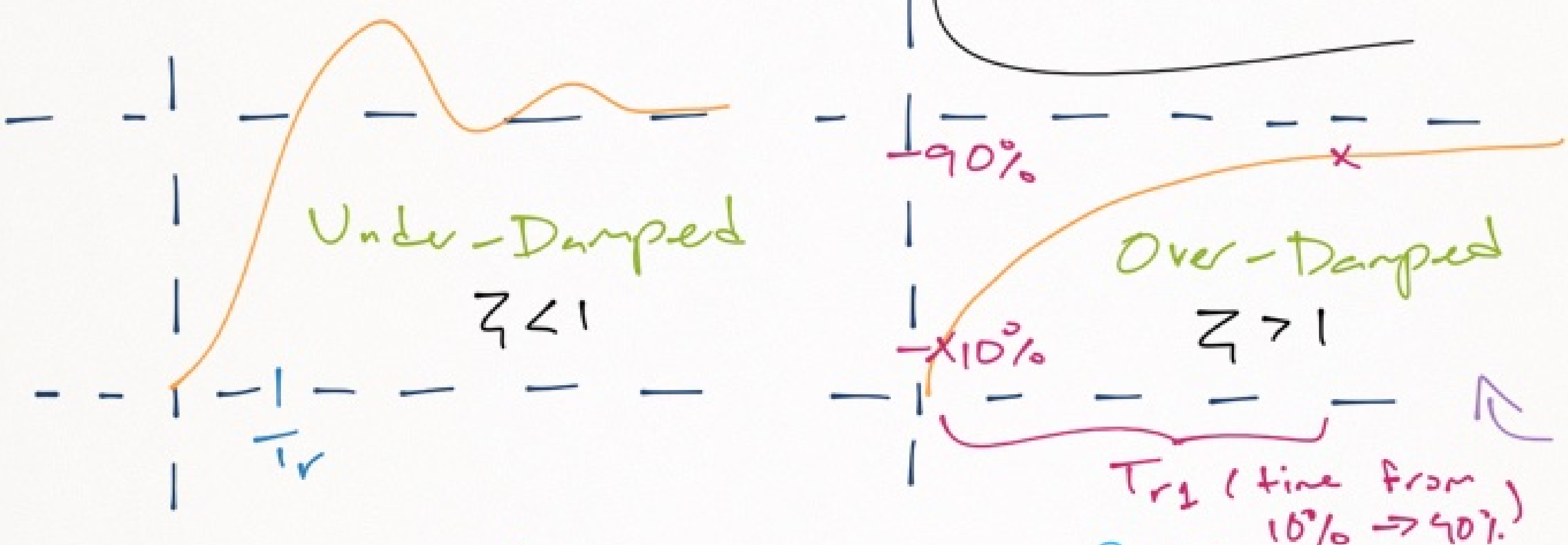
$\zeta^2+\beta^2=1$

$\sin\theta=\beta$

$\cos\theta=\zeta$

ϕ : $t = \dfrac{-\cos^{-1}\zeta}{\beta\omega_n}$ $\cancel{t < \phi}$ doesn't satisfy our conditions

π : $t = \dfrac{\pi - \cos^{-1}\zeta}{\beta\omega_n} = \boxed{\dfrac{\pi - \tan^{-1}\frac{\beta}{\zeta}}{\beta\omega_n}}$ by trig identity



$\beta = j\#$ ie. never reach T_r

$$T_r = \frac{1}{\beta\omega_n}\left[\pi - \tan^{-1}\frac{\beta}{\zeta}\right] \qquad \beta = \sqrt{1-\zeta^2}$$

only parameters are ω_n & ζ, can determine rise-time

Oct 6/2014

Computing T_{r1} (Tr for overdamped sys)

Take step response

$$y(t) \quad \underbrace{1 - \frac{1}{\beta} e^{-\zeta \omega_n t} \sin(\beta \omega_n t - \cos^{-1} \zeta)}_{0.9/0.1}$$

β is still imaginary, doesn't make sense

this is only the function for underdamped.

↳ "gets ugly when overdamped"

don't do mathy stuff, just plot & read

as a last resort:

when $\|a\| < \|10.b\|$

just ignore b-term

Actual & Approx. Rise Time (underdamped)

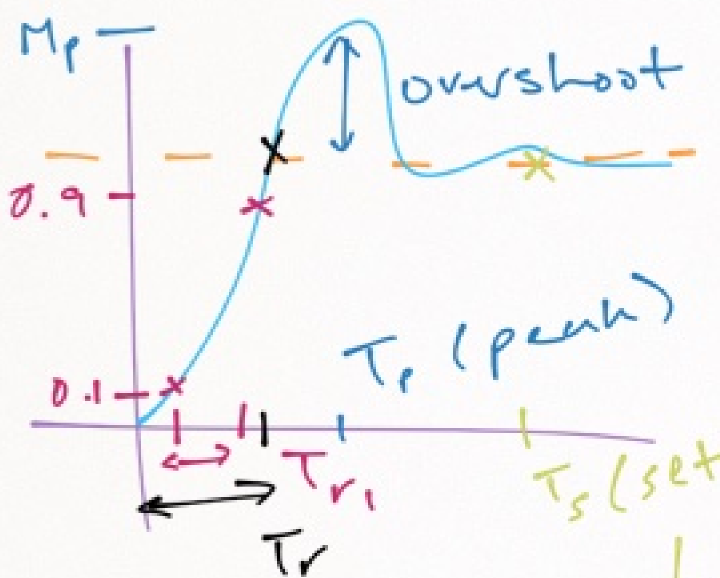
When $0.3 < \zeta < 0.8$

$$T_{r_1} \simeq \frac{2.16\zeta + 0.60}{\omega_n}$$

When $\zeta \geq 0.8$

$$T_{r_1} \simeq \frac{4.65\zeta - 1.3}{\omega_n}$$

just reasonable estimates



T_s (settling)

↳ within $\pm 2\%$

2nd order underdamped

$$Y(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Oct. 15, 2014

Peak time, T_p

Same as 1st year calc., take derivative and find zeros

$\cancel{s} \cdot \frac{1}{\cancel{s}} T(s) \longrightarrow t(t) = 0$

derivative

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \longrightarrow \frac{\omega_n}{\beta} e^{-\zeta\omega_n t} \underbrace{\sin(\beta\omega_n t)}_{= 0, \pi, 2\pi} = 0$$

flat slope at $t = 0$ (trivial)

$\beta\omega_n t = \pi$

$$\boxed{T_p = \frac{\pi}{\beta\omega_n}}$$

find val. @ T_p

$\frac{1}{s} T(s) \rightarrow t(t) = 0$

sub T_p

$$\underbrace{t(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin\left[\beta\omega_n t + \cos^{-1}\zeta\right]}_{\%\ \text{overshoot}}$$

$$\hookrightarrow = \frac{1}{\beta} e^{-\zeta\pi/\beta} \sin(\pi + \cos^{-1}\zeta)$$

$$\sin(\pi + \#) = -\sin(\#)$$

$$\therefore \; -\sin(\cos^{-1}\zeta) = \beta$$

$$\frac{1}{\cancel{\beta}} e^{-\zeta\pi/\beta} \cdot (-\cancel{\beta}) = -e^{-\zeta\pi/\beta}$$

$$t(t) = 1 + \underbrace{e^{-\zeta\pi/\beta}}_{\%\text{ overshoot}}$$

Settle time, T_s

time to get 98% of destination (±2%)

$$y(t) = 1 - \underbrace{\frac{1}{\beta} e^{-\zeta \omega_n t}}_{\text{envelope of oscillation}} \sin(\beta \omega_n t + \cos^{-1}\zeta)$$

T_s, when magnitude of envelope < 0.2

$$\frac{1}{\beta} e^{-\zeta \omega_n t} = 0.2$$

$$e^{-\zeta \omega_n t} = \beta \cdot 0.2$$

$$-\zeta \omega_n t = \ln(0.2 \cdot \beta)$$

$\frac{1}{50}$

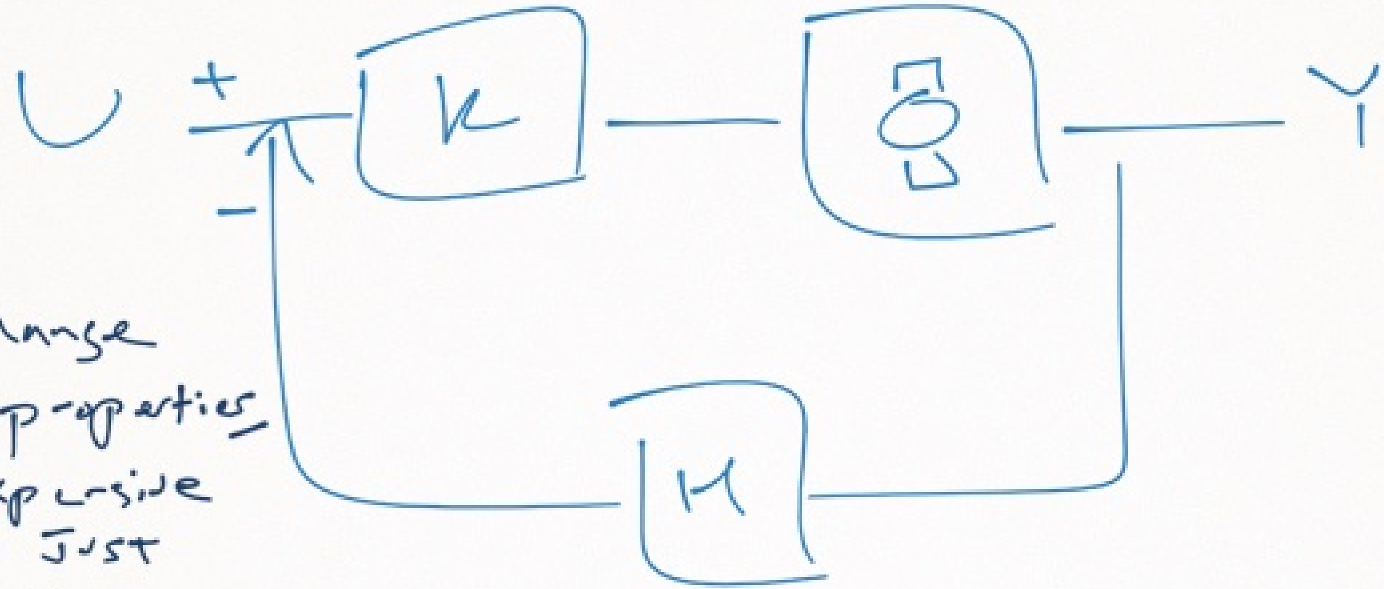
$$t = \frac{\ln(50/\beta)}{\zeta \omega_n}$$

swap to cancel (-)ive

$\simeq 4$ $\qquad$ $\simeq \emptyset$ when ζ is small

$$t = \frac{\ln 50 - \ln \beta}{\zeta \omega_n}$$

$$T_s = t \simeq \frac{4}{\zeta \omega_n}$$



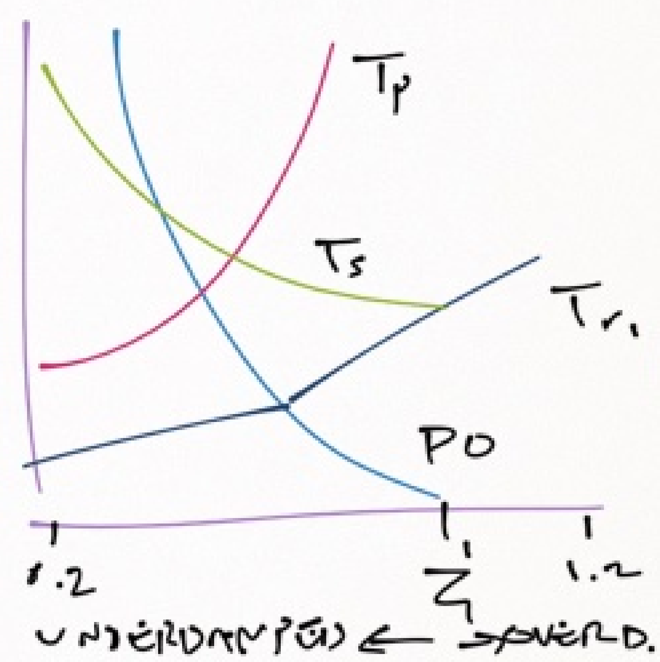
Don't change physical properties that's expensive and hard. Just change using control.

$$T(s) = \frac{N(k,s)}{D(k,s)}$$

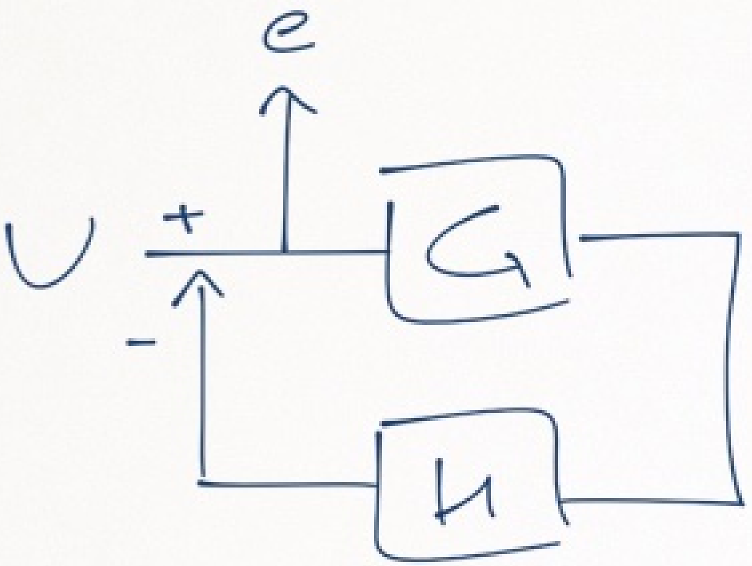
ex. can change a number (K) in C code and increase damping or whatever without changing bearings or physical motor.

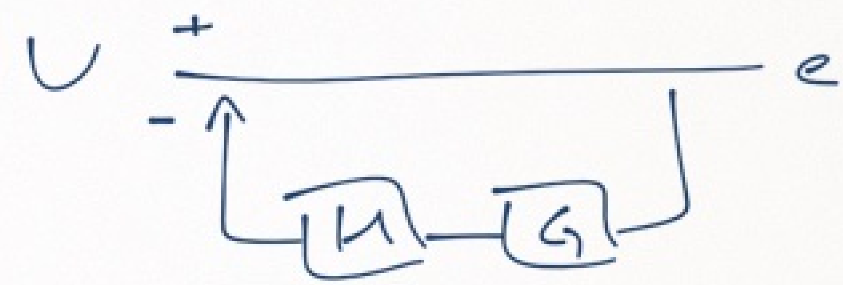
$$D_{(k,s)} = s^2 + 2\zeta(k)\omega_n(k)s + \omega_n(k)^2$$

2nd ORDER STEP RESP.



OCT 8, 2014



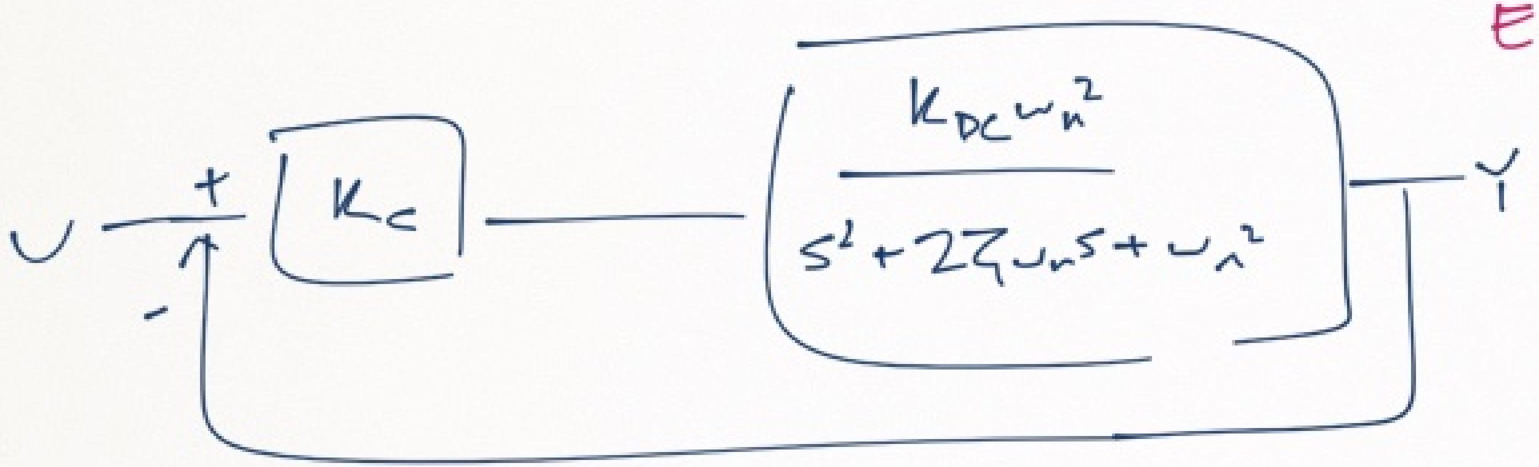


STEADY-STATE ERROR

= DC Gain of error

$$\frac{e}{u} = \frac{1}{1+GH}$$

$$E_{ss} = \frac{e}{u}\Big|_{s=\emptyset} = \frac{1}{1+\underbrace{G(s=0)H(s=0)}_{\text{O-L DC GAIN}}}$$



$$E_{ss} = \frac{1}{1+\frac{K_c K_{DC}\cancel{\omega_n^2}}{\cancel{\omega_n^2}}} = \frac{1}{1+K_c K_{DC}}$$

If you increase K_c, E_{ss} decreases

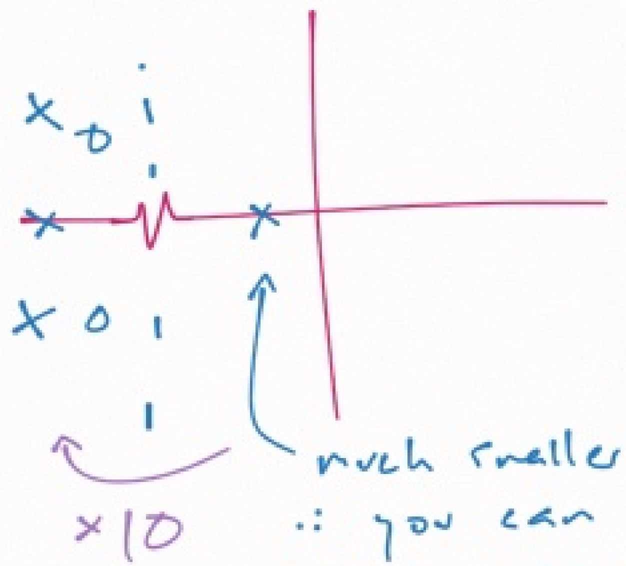
↳ pretending error is bigger ∴ compensates faster

3rd order sys $\quad \dfrac{N}{s^3 + As^2 + Bs + C}$

$\zeta = ? \quad \omega_n = ?$

$$\frac{D}{s+a} \quad \frac{E}{s+b} \quad \frac{F}{s+c}$$

DOMINANT
ROOTS
SLIDES



much smaller
∴ you can
ignore others
and give yourself
single order
system

Design controller taking into account changes in system we have no control over. (Differences in motors for instance)

ex. different friction, heat, worn bearings, etc.

Sensitivity to Sys Δs

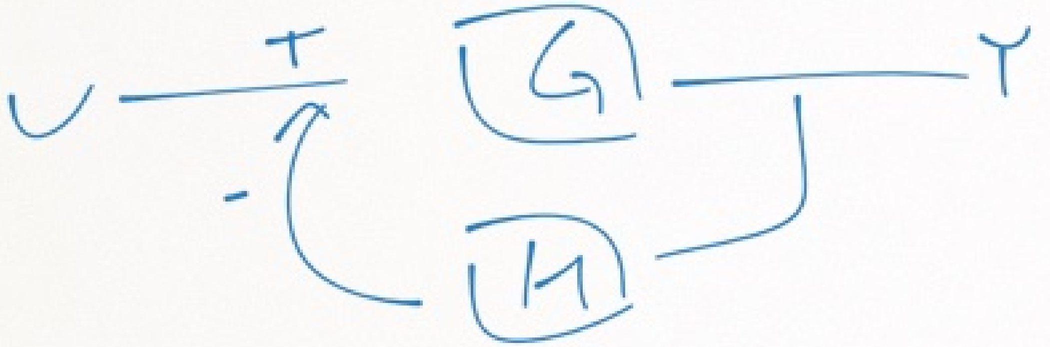
$$S = \frac{\%\Delta \text{ in } T(s)}{\%\Delta \text{ in } G(s)}$$

Smaller S, the better

ie. if big change in G(s)

only causes small change in T(s)

$$\lim_{\Delta \to \emptyset} \frac{\Delta T}{\Delta G} = \frac{\delta T}{\delta G} \qquad \boxed{S = \frac{G}{T} \cdot \frac{\delta T}{\delta G}}$$



$$T = \frac{G}{1+GH} = G(1+GH)^{-1}$$

$$S_G^T = \frac{G}{T} \frac{\delta T}{\delta G}$$

$$\frac{\partial T}{\partial G} = (1+GH)^{-1} - G(1+GH)^{-2} H$$

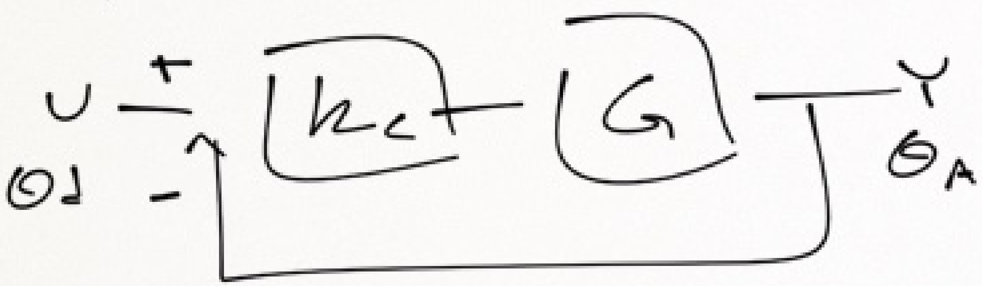
$$= \frac{1+GH}{(1+GH)^2} - \frac{GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

cont'd

$$S_G^T = \frac{G(1+GH)}{G}\frac{\partial T}{\partial G} = \frac{1}{1+GH}$$

sensitivity of T w.r.t. G

Ex)



$$G = \frac{1}{Ms^2+Bs}$$ MASS & DAMP FREQ.

fric B ↑ over time, $S_B^T = ?$

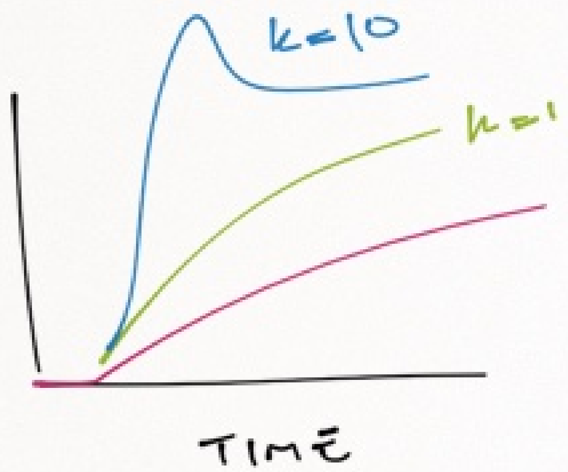
$$T(s) = \frac{K_c/(Ms^2+Bs)}{1+\frac{K_c}{Ms^2+Bs}} = \frac{K_c}{Ms^2+Bs+K_c} = K_c(Ms^2+Bs+K_c)^{-1}$$

$$\frac{\partial T}{\partial B} = -K_c(Ms^2+Bs+K_c)^{-2}s = \frac{-K_c s}{(Ms^2+Bs+K_c)^2}$$

$$S_B^T = \underbrace{\frac{B\cancel{(Ms^2\cdots)}}{\cancel{K_c}}}_{\frac{G(1+GH)}{G}} \cdot \underbrace{\frac{-\cancel{K_c}s}{(Ms^2\cdots\cancel{)^2}}}_{\frac{\partial T}{\partial G}} = \underbrace{\frac{-Bs}{Ms^2+Bs+K_c}}_{\text{freq dependency, inv. proportional to } K_c}$$

OUTPUT

Trajectory



% Change

RHP ZEROS OK

RHP POLES = INSTABILITY

CE: $(s+a)(s+b)(s+c) = \emptyset$

$$s^3 + \underbrace{(a+b+c)}_{A}s^2 + \underbrace{(ab+ac+bc)}_{B}s + \underbrace{abc}_{C}$$

if any co-eff $< \emptyset$ then a, b, c must $< \emptyset$

A, B, C

ex. CE: $s^5 + 4s^4 + s^3 - 9s^2 + 2s + 8$

if negative anywhere, we know a pole in RHP $\therefore$ UNSTABLE

UNSTABLE

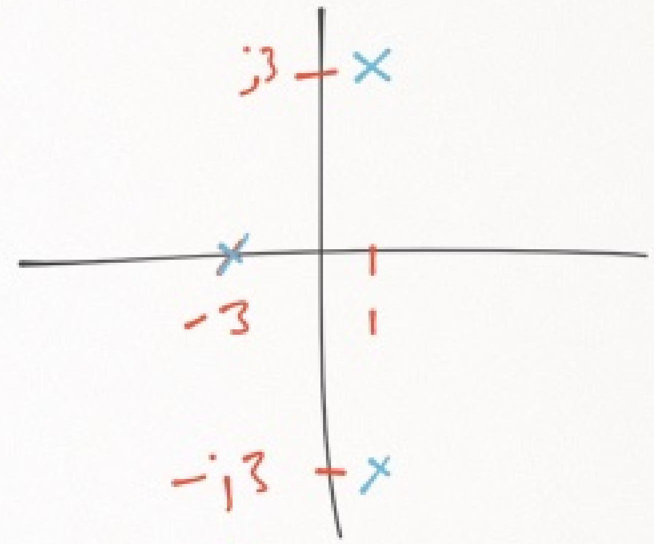
All coeffs need to be $> \emptyset$

(but that's not enough to know if it's stable) ex. $(s+3)(s-1+j3)(s-1-j3)$

$(s+3)(s^2+2s+10)$

$s^3-2s^2+10s+3s^2-6s+30$

$s^3+s^2+4s+30=\emptyset$



unstable

but doesn't look it.

you can't determine stability this way.

ROUTH HURWITZ CRITERIA (1895)

CE: $as^7+bs^6+cs^5+ds^4+es^3+fs^2+gs+h$

orders					
7	a	c	e	g	$\emptyset$
6	b	d	f	h	$\emptyset$
5	m_1	m_2	m_3	$\emptyset$	
4	n_1	n_2	n_3		
3	o_1	o_2			
2	p_1	p_2			
1	q_1				
0	r_1				

if the sign changes between any of these ∴ UNSTABLE

$m_1 = \frac{-1}{b}\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ ← determinant

$m_2 = \frac{-1}{b}\begin{vmatrix} a & e \\ b & f \end{vmatrix}$

$m_3 = \frac{-1}{b}\begin{vmatrix} a & g \\ b & h \end{vmatrix}$

$n_1 = \frac{-1}{m_1}\begin{vmatrix} b & d \\ m_1 & m_2 \end{vmatrix}$

$n_2 = \frac{-1}{m_1}\begin{vmatrix} b & f \\ m_1 & m_3 \end{vmatrix}$

$n_3 = \frac{-1}{m_1}\begin{vmatrix} b & h \\ m_1 & \emptyset \end{vmatrix}$

$7s^7 + 6s^6 + 5s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1$

7	5	3	1
6	4	2	1
$\frac{1}{3}$	$\frac{2}{3}$		
-14			

$-\frac{1}{6}(28-30) = \frac{1}{3}$

$-\frac{1}{6}(14-18) = \frac{2}{3}$

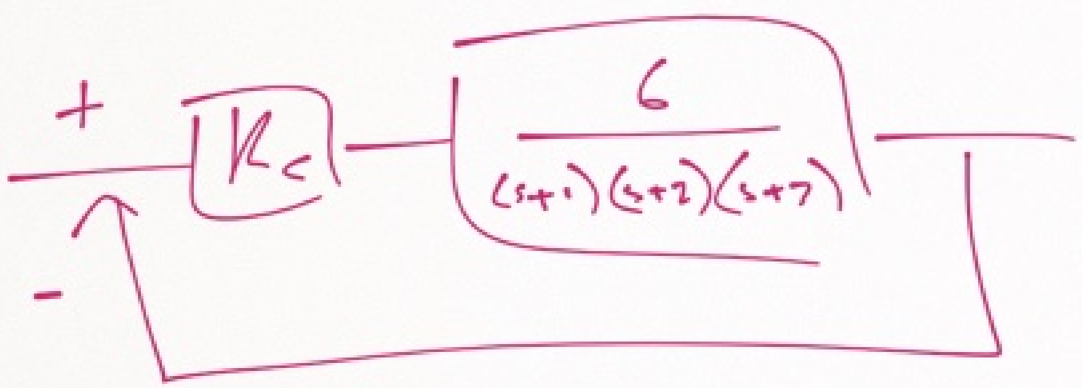
$-3\left(\frac{18}{7} - \frac{4}{3}\right) = -\frac{3 \cdot 14}{3} = -14$

UNSTABLE B/C SIGN

R-H CRITERIA EXCEPTION

→ WHEN C.E. IS UNFACTORABLE & NO COEFFICIENT SIGNS CHANGE

1. Check if numerator order < denominator order (if not what?)
2. check if sign Δ in denominator



$$\frac{Y}{U} = \frac{1}{6K_c + s^3 + 6s^2 + 11s + 6}$$

$$= \frac{1}{s^3 + 6s^2 + 11s + 6(K_c + 1)}$$

what's value of K_c such that system is stable?

↳ can change ζ by changing K_c

K_c at least needs to be > -1

s^3	1	11
s^2	6	$6(k_c+1)$
s^1	$10-k_c$	
s^0	$6(k_c+1)$	

$$-\frac{1}{6}\left(6(k_c+1)-66\right) = 11-(k_c+1) > \emptyset$$

$$k_c < 10$$

STABILITY:

$k_c > -1$

$k_c < 10$

if either of these $= \emptyset$ then MARGINAL STABILITY

$$k_c = -1 \quad \frac{1}{s}\,\frac{6k_c}{(s^2+6s+11)}$$

$=$

CE:

$$D(s) = s^3 + s^2 + 7s + 7$$

0 in engineering is $\lim_{x\to 0}$ in mathematics

zero is not nothing, its insignificance, but you can amplify it to being finite.

s^3	1	7
s^2	1	7
s^1	$\cancel{\emptyset}\ \varepsilon$	$\emptyset$
s^0	$\cancel{\frac{\emptyset}{\emptyset}}$	7

$$-\frac{1}{1}(7-7)$$

$$-\frac{1}{\emptyset}(\emptyset-\emptyset)$$

$$-\frac{1}{\varepsilon}(\emptyset-7\varepsilon) = 7$$

can't push a rope

two ways to scratch nose

$\frac{\emptyset}{\emptyset} = 7$ (or something finite)

$0 = \varepsilon$ (small insignificant value that could multiplied to something)

$s^4 + 3s^3 + 2s^2 \longrightarrow s^2(s^2 + 3s^2 + 2)$

$$\begin{array}{ll} 1 & 2 \\ 3 & \\ 2 & \end{array}$$

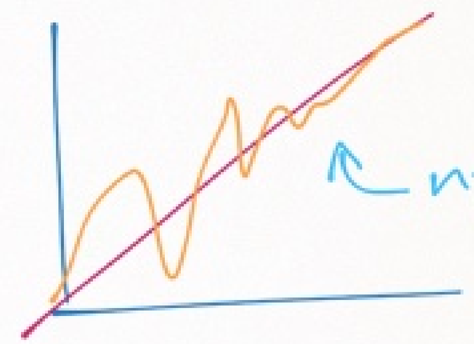
✓ looks stable.

$$\frac{1}{s} \cdot \frac{Num}{s^2 + 3s + 2}$$

$\hookrightarrow (s+2)(s+1)$

(?)

HEAVYSIDE



not stable!

have to do

$s^4 + 3s^2 + 2s^2 + \varepsilon s + \varepsilon$

$$\begin{array}{lll} 1 & 2 & \emptyset \\ 3 & \emptyset & \\ 2 & & \\ -\frac{\varepsilon}{2} & & \end{array}$$

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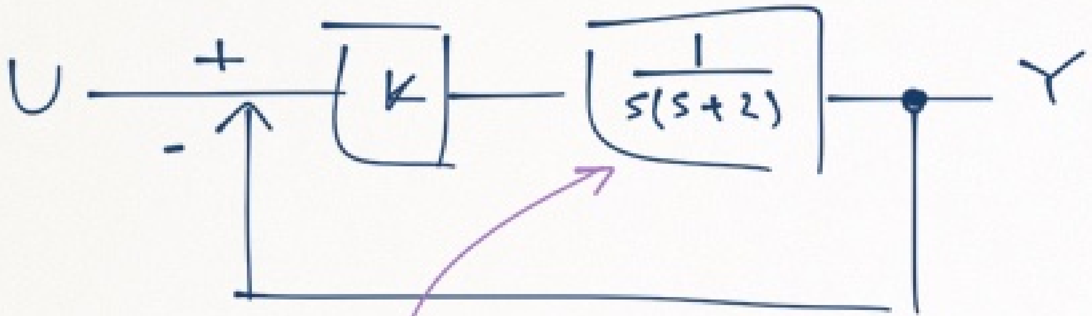


$$\frac{Y}{U} = \frac{K}{s^2+2s+K}$$

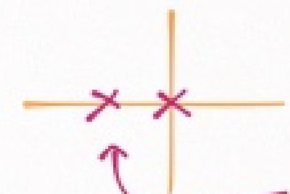
CE: $s^2+2s+K=\emptyset$

values of s that makes this true defines our poles, depends on K

poles of open loop transfer function = poles of closed loop transfer function

test $K=\emptyset$ (not mathematical $\emptyset$, a negligible #)

$\hookrightarrow s=\emptyset, -2 \longrightarrow$ poles @ $K=\emptyset$

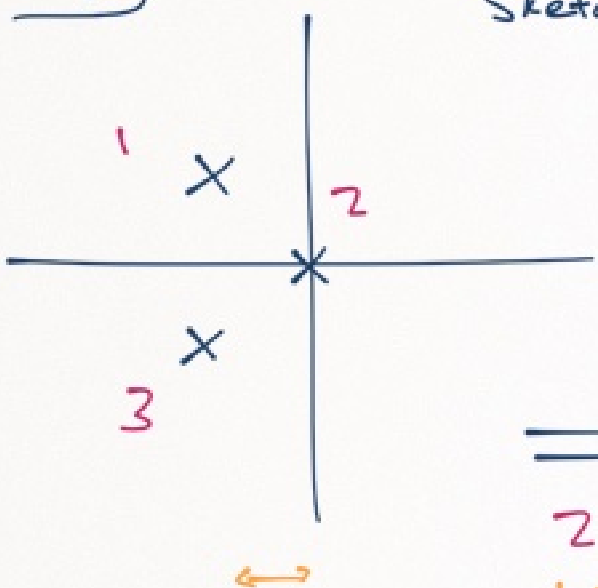


if you have a pole here w/ a mag 10x bigger than the inside pole, you can ignore it.

(not w/ $\emptyset$ though, so not now)

Ex)

Sketch the impulse response.

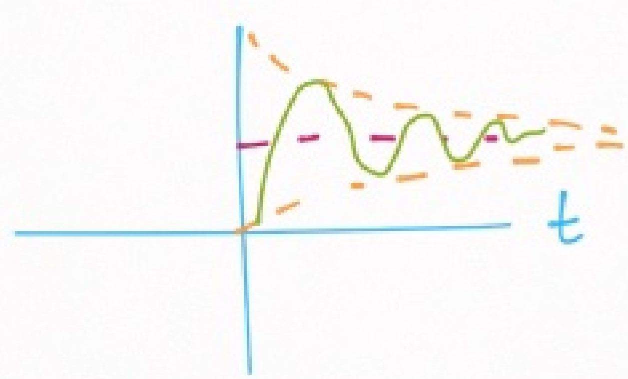


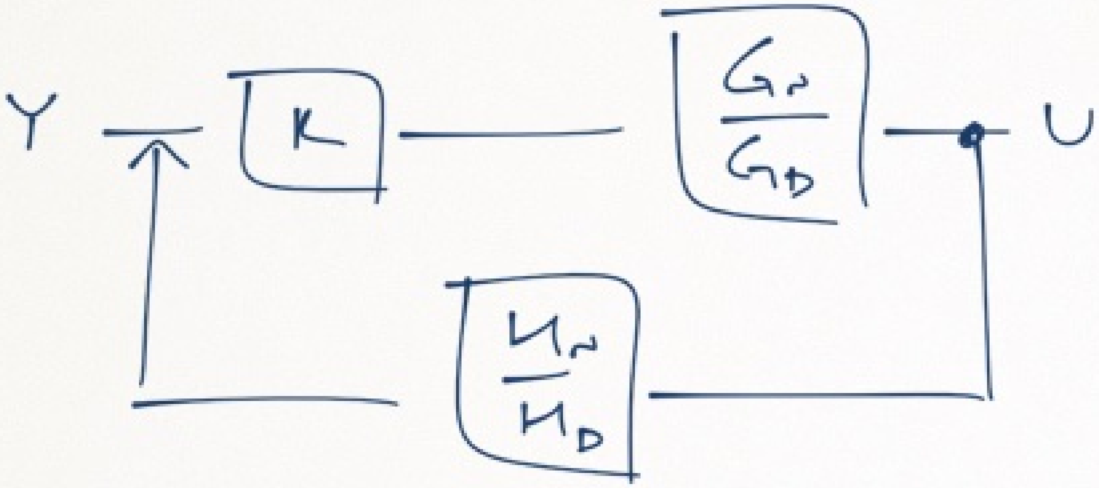
=

2 means step response

1 & 3 being on left means decaying envelope

being on top & bottom gives us w so there's a sign





KGH = open loop transfer function

$$TF = \frac{KG_NH_D}{KG_NH_N + G_DH_D}$$

$\boxed{\text{when } K = \emptyset}$ (small)

$$TF = \frac{1}{G_DH_D}$$

G_DH_D = open loop characteristic equation

closed loop poles = open loop poles

$\boxed{\text{when } K \to \infty}$

TF

closed loop poles = open loop zeros

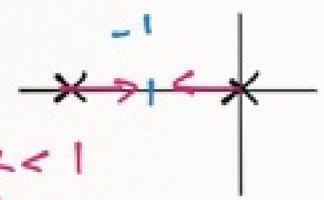
CE: $s^2 + 2s + k = \varnothing$

$$s = \frac{-2 \pm \sqrt{4 - 4k}}{2} = \underbrace{-1}_{\sigma} \pm \sqrt{1-k}$$

from $k = \varnothing \to 1$, poles are real

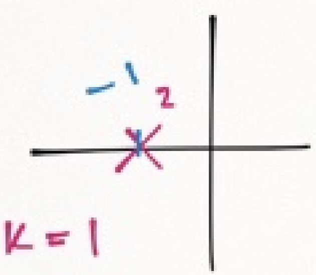
$k > 1$, imaginery component

↳ corresponds to frequency

① 

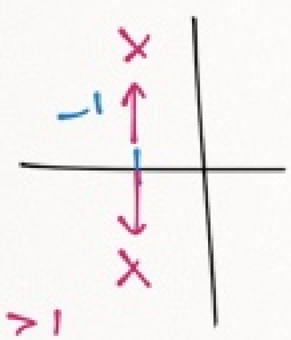
$k < 1$

$k \uparrow$

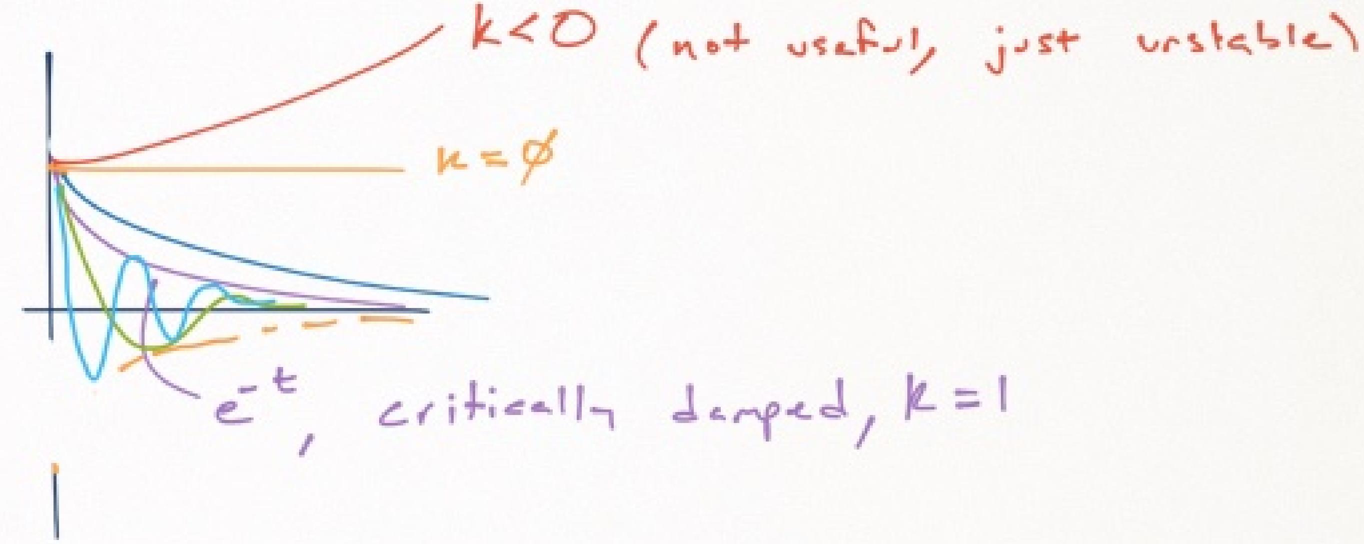
② 

$k = 1$

CRITICAL DAMPING

③ 

$k > 1$



WHEN YOU HAVE MANY POLES

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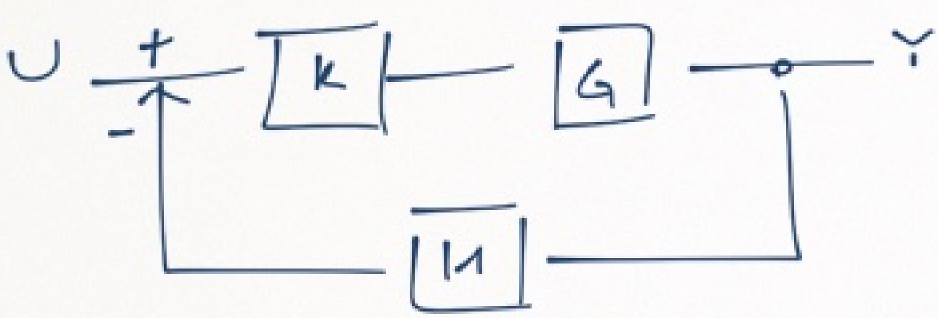


$$TF = \frac{KG}{1+KGH}$$

← function of s which is a complex number, $s = \sigma + j\omega$

$$CE: \quad 1 + \underbrace{KG(s)H(s)} = \emptyset$$

this is just a complex number we need to be equal to -1

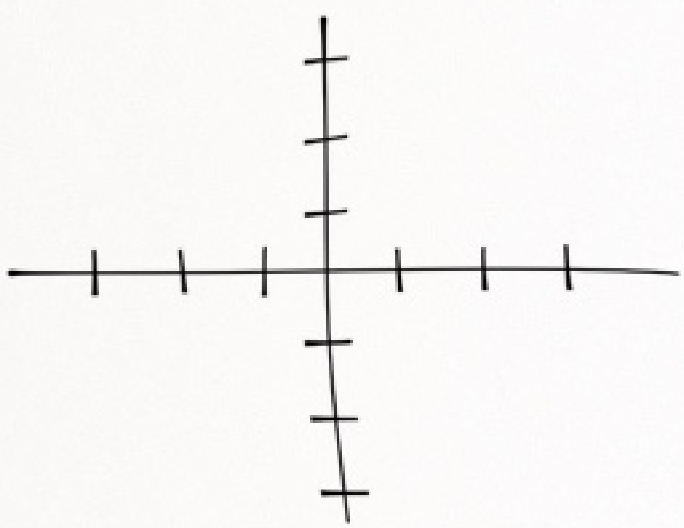
ie. $= 1 \angle 180^\circ$

$$\frac{A\angle\alpha}{B\angle\beta \; C\angle\gamma} = \frac{A}{BC}\underbrace{\angle \alpha - \beta - \gamma}_{\text{ZEROS - POLES}}$$

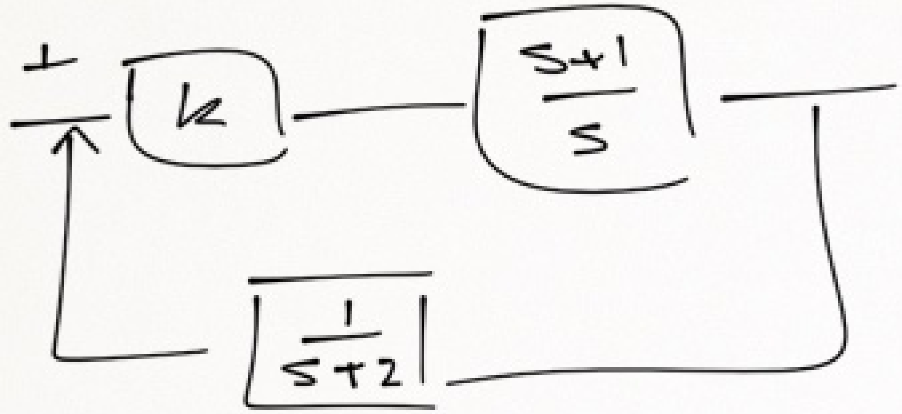
$|KGH| = 1$

$\angle KGH = \underbrace{180^\circ \pm n360^\circ}$

we can extract many rules from this criteria



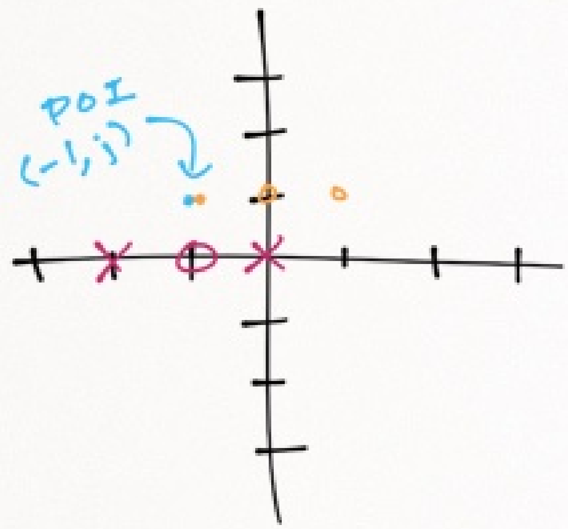
$$\angle \frac{C_1}{C_2 C_3} = \angle C_1 - \angle C_2 - \angle C_3$$



$$KGH = \frac{k(s+1)}{s(s+2)} \qquad CE: \; 1 + KGH = \emptyset$$

$$KGH\Big|_{s=POI} = -1 = 1\angle 180^\circ$$

ie. $|KGH| = 1, \; \angle KGH = 180^\circ$

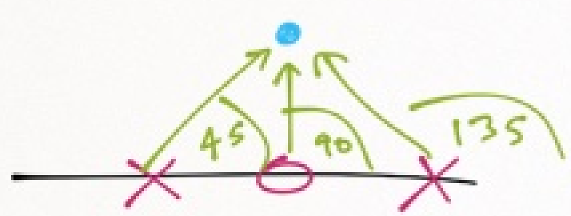


POI

ZERO: $s+1\Big|_{POI} = (-1+j+1) = j$

POLES: $s\Big|_{POI} = -1+j$

$(s+2)\Big|_{POI} = -1+j+2 = 1+j$

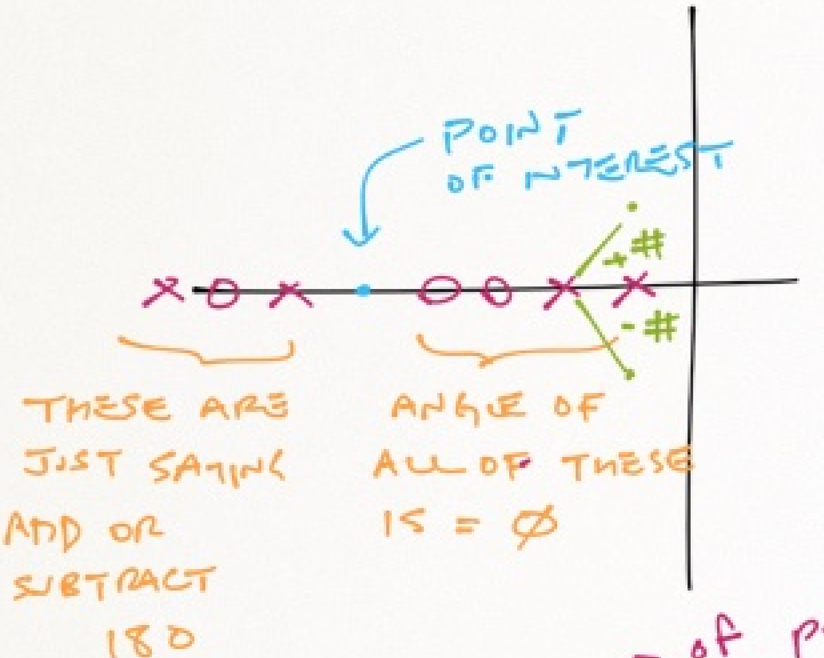


$\sum \angle ZEROS = \sum \angle POLES = 90^\circ - 45^\circ - 135^\circ = -90^\circ \neq 180$

∴ NO, POINT IS NOT ON ROOT LOCUS

how the roots of system change w/ variation in k

HOW ABOUT REAL AXIS?



RL (root locus) exists to left of all odd #s of poles and/or zeros

ANYTIME COMPLEX CONJUGATES, THEIR ANGLES WILL CANCEL.

IF EVEN # RIGHT OF POI (of poles & zeros), $\angle = \phi^\circ$ $\longrightarrow$ RL doesn't exist

IF ODD # RIGHT OF POI, $\angle = 180^\circ$ $\longrightarrow$ RL does exist

More on root locus

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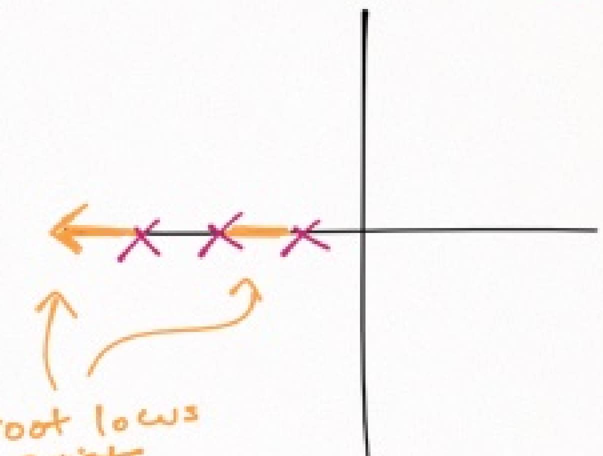
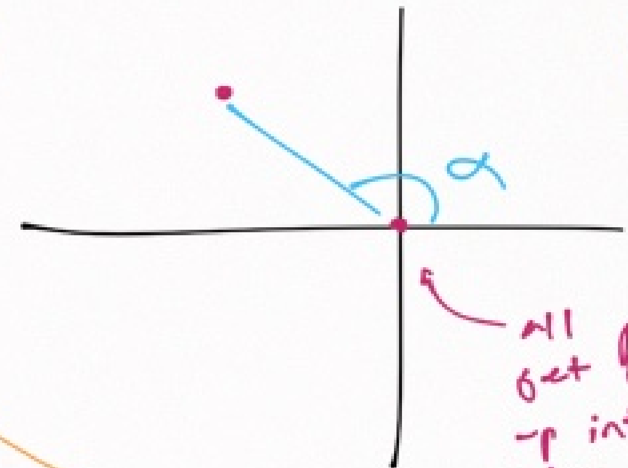
$$KGH = \frac{1}{(s+1)(s+2)(s+3)}$$



As $k \to \infty$, $\alpha_1 = \alpha_2 = \alpha_n$

$(\#Z - \#P)\alpha = -180$

$\underbrace{(\#P - \#Z)}_{XS}\alpha = 180$



$k = 10000$

All poles get bunched up into one point when we zoom out of a big value of k

$$\alpha = \frac{180}{XS}$$

$$\frac{180}{XS} = \frac{180}{3} = 60^\circ$$

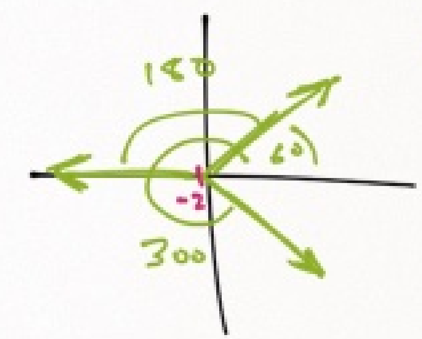
$$\frac{180 + n360}{3} = 60 + n120$$

WE CAN ALREADY SEE THIS HERE

$\underbrace{60, 180, 300}$

VALUES SATISFYING ANGLE CRITERIA

↳ "ASSYMPTOTES"



← 120° between each, evenly spaced.

→ SYMMETRIC → 60 & 300 have to be complex conjugates

→ EVENLY SPACED

$$\sigma_A = \frac{\Sigma P - \Sigma Z}{XS} = \frac{-1 + (-2) + (-3)}{3} = -2$$

↖ $\#P - \#Z$

CE: $1 + kGH = 0$

SLOPE OF ROOT LOCUS $= \infty$ @ CORNER

$$k = \frac{-1}{GH}$$

$$\therefore \quad \frac{\delta s}{\delta k} = \infty \ , \quad \frac{\delta k}{\delta s} = 0$$

$$\underbrace{\frac{\delta}{\delta s}\left(\frac{1}{GH}\right) = 0}_{(?)}$$

Ex)

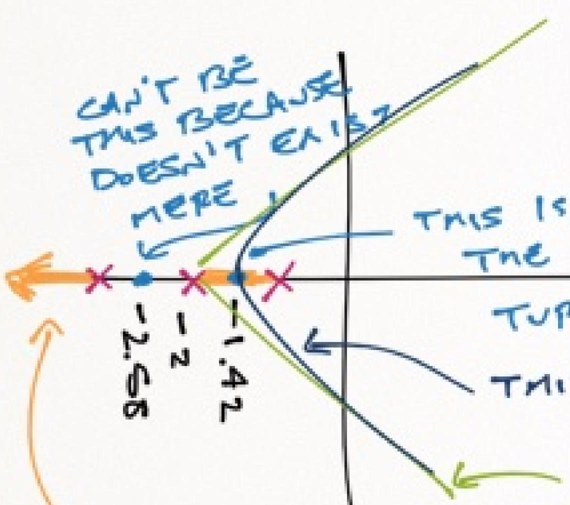
$$GH = \frac{1}{(s+1)(s+2)(s+3)} = \frac{1}{s^3 + 6s^2 + 11s + 6}$$

$$\frac{\delta}{\delta s}\frac{1}{GH} = 3s^2 + 12s + 11 = 0 \longrightarrow s = \frac{-12 \pm \sqrt{144 - 132}}{6} = -2 \pm 0.58$$

$$= \begin{cases} -2.58 \\ -1.42 \end{cases}$$

WE KNOW FROM PREVIOUS ANGLE CRITERIA THIS IS WRONG

(NOT FALSE, JUST CORRESPONDS TO -IVE k, WHICH WE'RE NOT CONCERNED WITH)



CAN'T BE THIS BECAUSE DOESN'T EXIST HERE

THIS IS THE POINT WHERE THE POLES COLLIDE & TURN AROUND

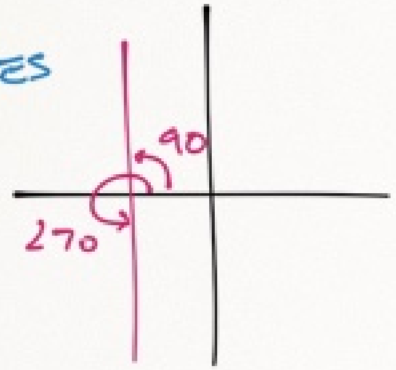
THIS IS THE REAL ROOT LOCUS

FROM ANGLE CRITERIA (PREVIOUS PAGE)

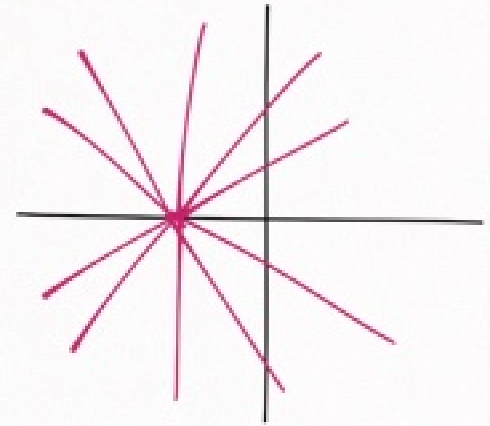
$$\phi_A = \frac{2q+1}{XS} \times 180^\circ \quad , \quad q = \text{index } 0, 1, 2 \ldots$$

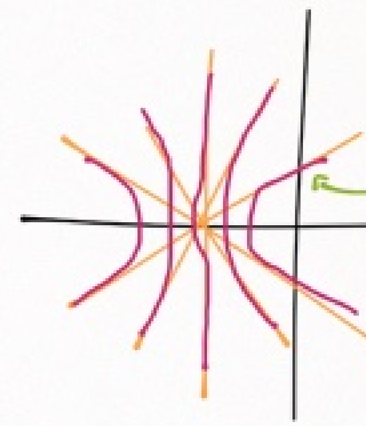
$P \rightarrow k$
$+$
I
$+$
D

2 POLES



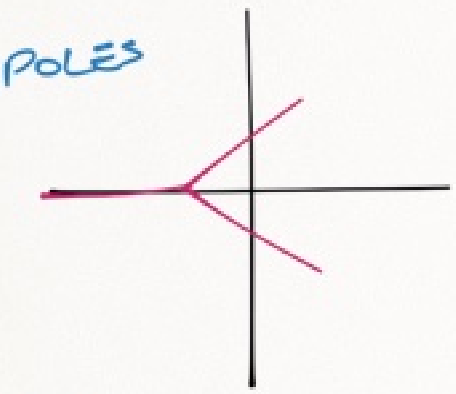
10 POLES





The system goes unstable when poles pass over to right. So it's important to understand @ what k this happens.

3 POLES



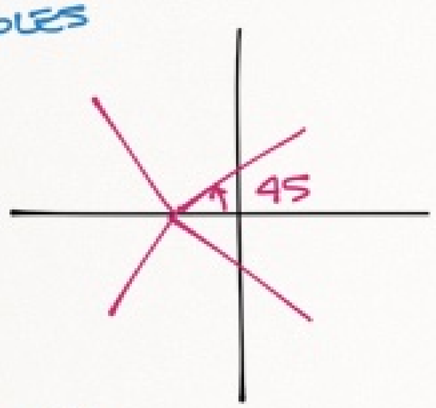
BREAK POINT @ −1.42, FIND K

$\cancel{s=\emptyset}$? no, that's only for finding DC gain @ $\omega = \emptyset$

PLUG INTO C.E. (closed loop)

USE ROUTH-HURWITZ TO FIND K THAT MAKES SYSTEM UNSTABLE.

4 POLES



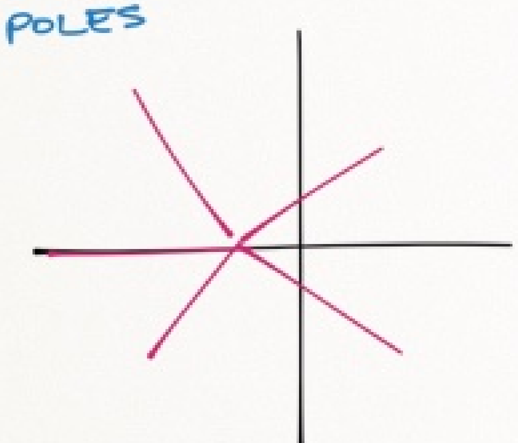
$$(-1.42)^3 + 6(-1.42)^2 + 11(-1.42) + 6 + k = \emptyset$$

$\hookrightarrow k = 0.4$

WHAT K DOES THIS GO UNSTABLE AT?

R.H. $s^3 + 6s^2 + 11s + 6 + k$

5 POLES



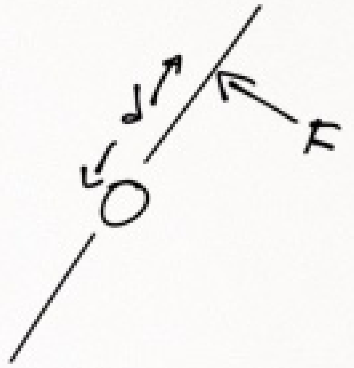
1	11
6	6+k
$\frac{60-k}{6}$	
6+k	

SYSTEM UNSTABLE @ $k = 60$ or $k = -6$

ONLY CARE ABOUT POSITIVE K VALUES

PROJECT HINTS (NOV. 3rd)

- making sense of control
- remove diode
- figure out units
- assume wind resistance is F_{point} @ $\frac{\text{propeller length}}{2}$
- start soon



DETERMINE TIME CONSTANT EXPERIMENTALLY

DESIRED ALTITUDE m

$$GH = \frac{s+5}{s^2+6s+8} = \frac{s+5}{(s+2)(s+4)}$$

DRAW ROOT LOCUS

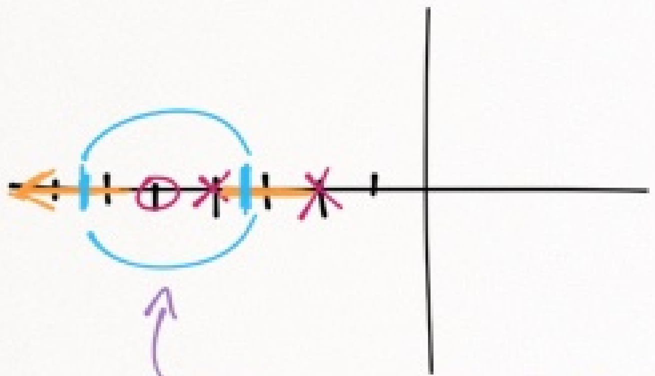
POLES: $-2, -4$

ZERO: -5

$XS = \#P - \#Z = 1$

\# OF ASYMPTOTES

BY ODD # TEST



BP

$$\frac{d}{ds}\frac{1}{GH} = \emptyset \qquad \frac{d}{ds}(s+5)^{-1}(s^2+6s+8) = \emptyset$$

....

$$s^2 + 10s + 22 = \emptyset$$

$$\underbrace{s = -6.7, -3.3}_{\text{BOTH VALID B/C AGREE W/ ODD \# TEST}}$$

AND WE JUST SORT OF CONNECT THE POINTS. WE COULD PROVE IT DOING THE ANGLE CRITERIA, BUT IT'S A BUNCH OF WORK.

Anyway, that's the root locus.

ROOT LOCUS SUMMARY SO FAR

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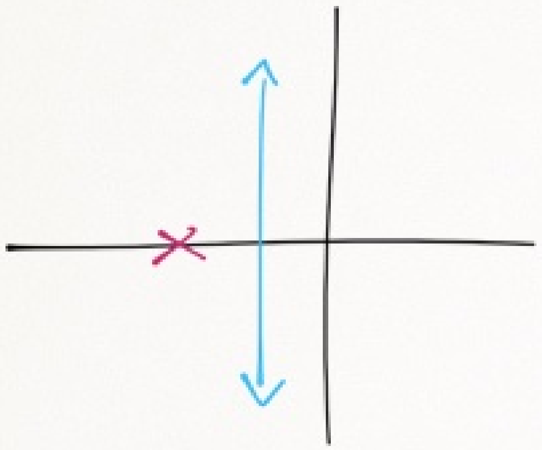
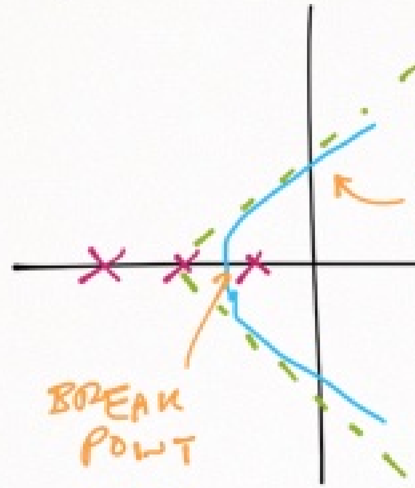
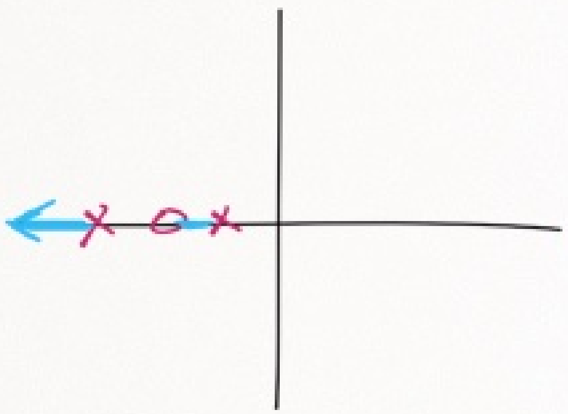
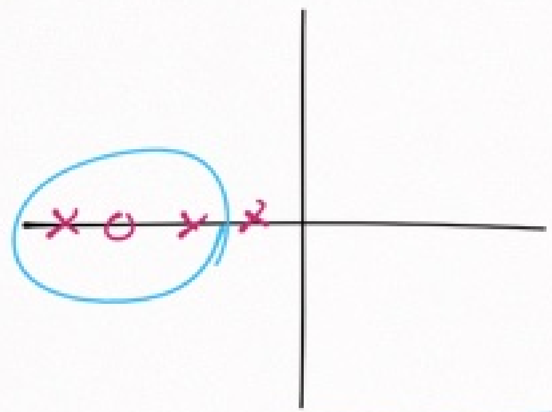


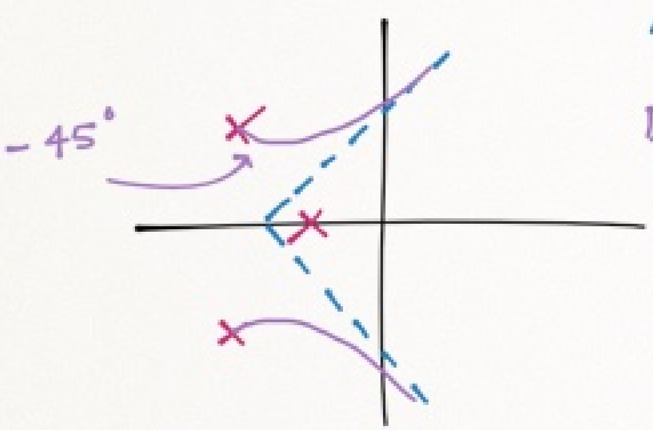






OKAY, WELL WHAT ABOUT $\boxed{\text{COMPLEX POLES}}$?

Ex) $$\frac{1}{(s+1)(s+2+j)(s+2-j)} = \frac{1}{(s+1)(s^2+4s+5)}$$ ROOT LOCUS?

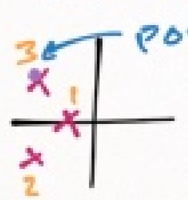


ASYMPTOTE CENTER $= \frac{-1-2-2}{3} = \frac{-5}{3} = -1\frac{2}{3}$

DEPARTURE ANGLE =

ANGLE CRITERIUM (PICK POINT, ADD ANGLE FROM ALL POINTS, SEE IF = 180)

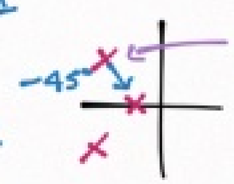
LETS PICK A POINT WHERE K IS TINY, SO VERY CLOSE TO POLE



∠ FOR POI

$\underset{1}{135} + \underset{2}{90} + \underset{3}{\theta} = 180$

↳ $\theta = -45$ ∴

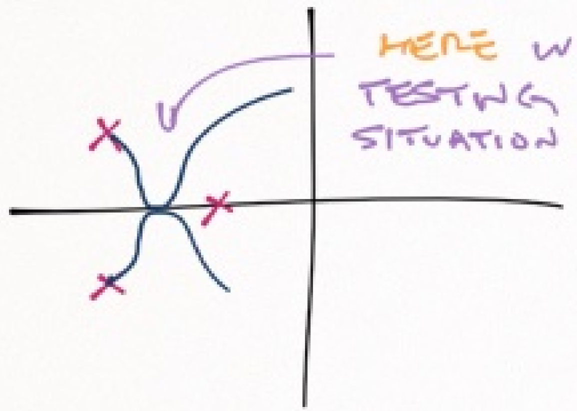


SO THIS IS DIRECTION OF ROOT LOCUS

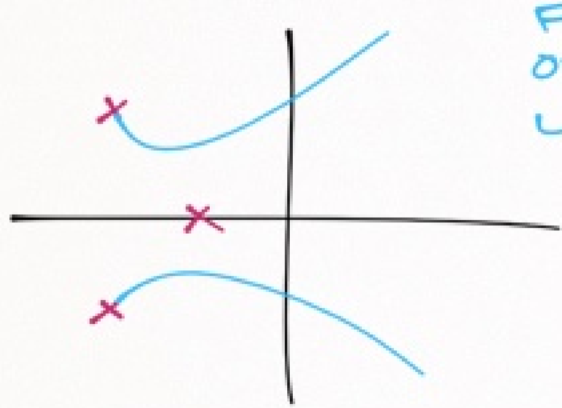
EX CONT'D

FIND BREAK POINTS $\frac{d}{ds}\frac{1}{GH} = 3s^2 + 10s + 9 = \emptyset$

$\hookrightarrow \frac{-10 \pm \sqrt{100-108}}{6}$ ← NO BREAKPOINTS ON REAL AXIS



HERE WE WERE TESTING FOR A SITUATION LIKE THIS, BUT WE'RE OKAY. IT DIDN'T HAPPEN.

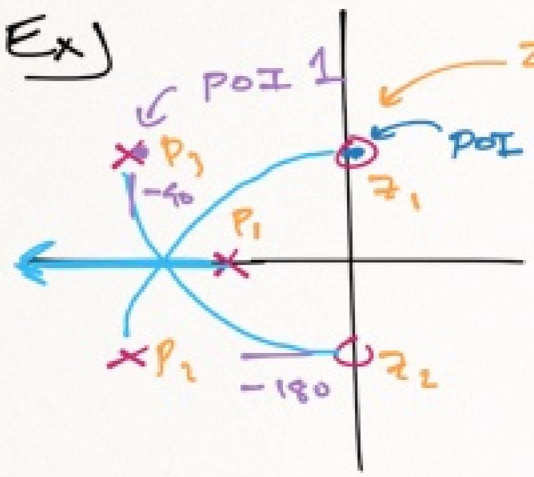


FINAL ANSWER OF WHAT ROOT LOCUS LOOKS LIKE.

WHAT ABOUT $\boxed{\text{COMPLEX ZEROS}}$?

$$\frac{k(s^2+1)}{(s+1)(s+2+j)(s+2-j)} = KGH \quad \text{ROOT LOCUS?}$$

Ex)



ZEROS PULL POLES TOWARDS THEM

→ SO WE'RE INTERESTED IN ARRIVAL ANGLE (OF ZERO)

→ THE ZEROS ALSO AFFECT DEPARTURE ANGLE (OF POLE)

POI ANGLE CRITIRIUM DEPARTURE

$$\underset{Z_1}{180} + \underset{Z_2}{135} - \underset{P_1}{135} - \underset{P_2}{90} - \underset{P_3}{\Theta} = 180$$

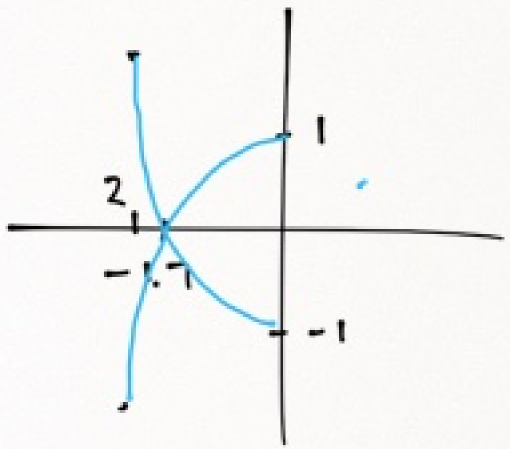
(SUBTRACTING POLES, ADDING ZEROS) ↳ ARBITRARY

$\Theta = -90$

POI ANGLE ARRIVAL

$$\underset{Z_1}{\Theta} + \underset{Z_2}{90} - \underset{P_1}{45} - \underset{P_2}{45} - \underset{P_3}{0} = 180$$

↳ $\Theta = 180$



ROOT LOCUS FINAL ANSWER

BREAK POINTS

$$\frac{d}{ds}\frac{1}{GH} = \frac{d}{ds}(s^2+1)^{-1}(s^3+5s^2+9s+5) = \emptyset$$

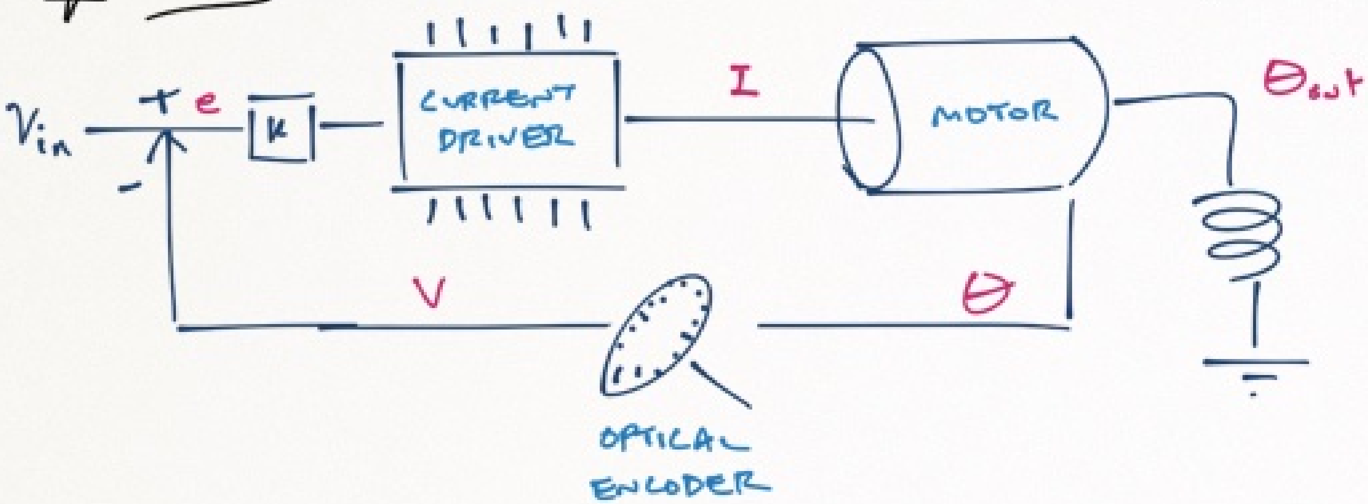
↳ $(s^2-3)^2 = \emptyset$ $\quad s = \pm\sqrt{3} \simeq \pm 1.7$

CAN'T BE +1.7 BECAUSE DOESN'T EXIST

$\therefore$ BREAK P $= -1.7$

ONLY ONE BREAK POINT

★ Ex) FOR THE FOLLOWING SYSTEM, DRAW ROOT LOCUS | OCT 27 2014



CURRENT DRIVER: $\frac{1}{s+1}$

MOTOR: $J=1$
$B=5$

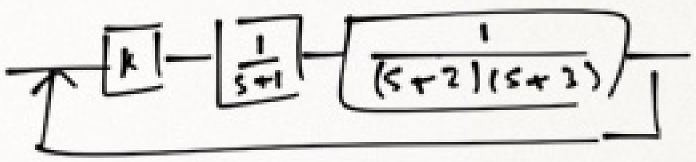
$k=6$

=

$T=(Js^2+Bs+k)\theta$

$\frac{\theta}{T}=\frac{1}{(s+2)(s+3)}$

MOTOR $\rightarrow$ $\boxed{\frac{1}{(s+2)(s+3)}}$



$KGH=\frac{k}{(s+1)(s+2)(s+3)}=\frac{1}{s^3+6s^2+11s+6}$, $\frac{Y}{U}=\frac{1}{s^3+6s^2+11s+6+k}$

CE: $s^3+6s^2+11s+6+k$ DC GAIN $=K_{DS}=\left.\frac{V}{U}\right|_{s=\emptyset}=\frac{k}{k+6}$

STEADY STATE ERROR $=e_{ss}=1-K_{DS}=\frac{k+6}{k+6}-\frac{k}{k+6}=\frac{6}{k+6}$

MAKE K BIG TO REDUCE ERROR $\rightarrow$ TRADEOFF B/W OVERSHOOT & STEADY STATE ERROR

HOW TO GET RID OF STEADY STATE ERROR → FUNCTION LANDING IN WRONG PLACE

PI CONTROL

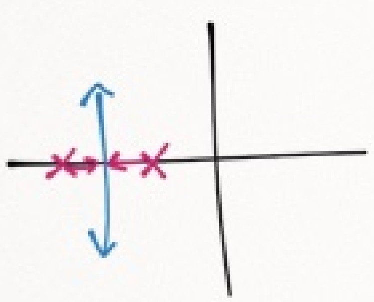
$$KGH = \frac{N}{D} \qquad e_{ss} = \frac{1}{1+\frac{N}{D}}\Bigg|_{s=\emptyset} = \frac{D}{D+N}\Bigg|_{s=\emptyset}$$

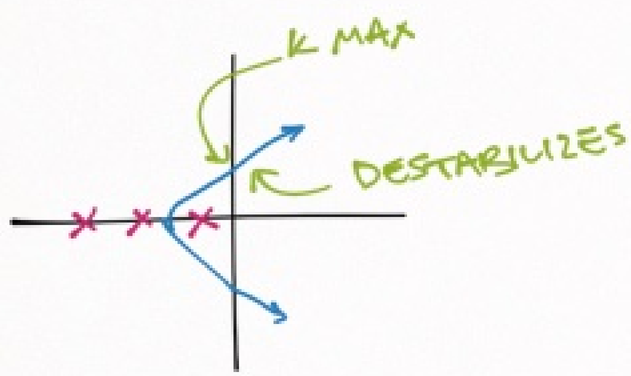
← CAN MAKE D SMALL OR MAKE N LARGE TO REDUCE.

OR SET LOW POLE AT ∅

$$KGH = \frac{K}{S} \cdot \frac{G_N}{G_D} \cdot \frac{H_N}{H_D}$$

→ sends $e_{ss} \to \emptyset$



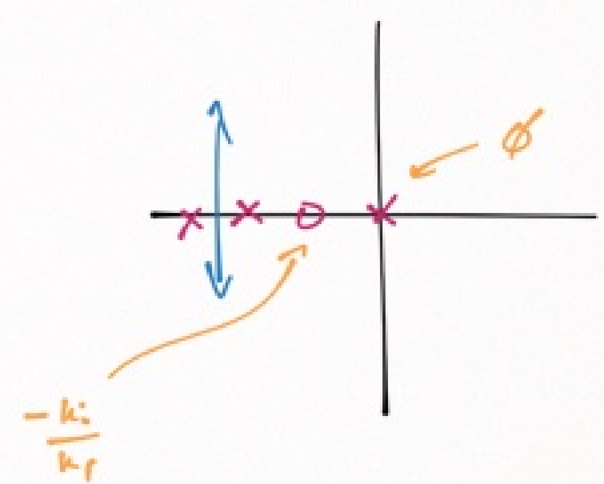


$$K = K_p + \frac{1}{S}K_i = \frac{sK_p + K_i}{s} = K_p \frac{S + K_i/K_p}{S}$$

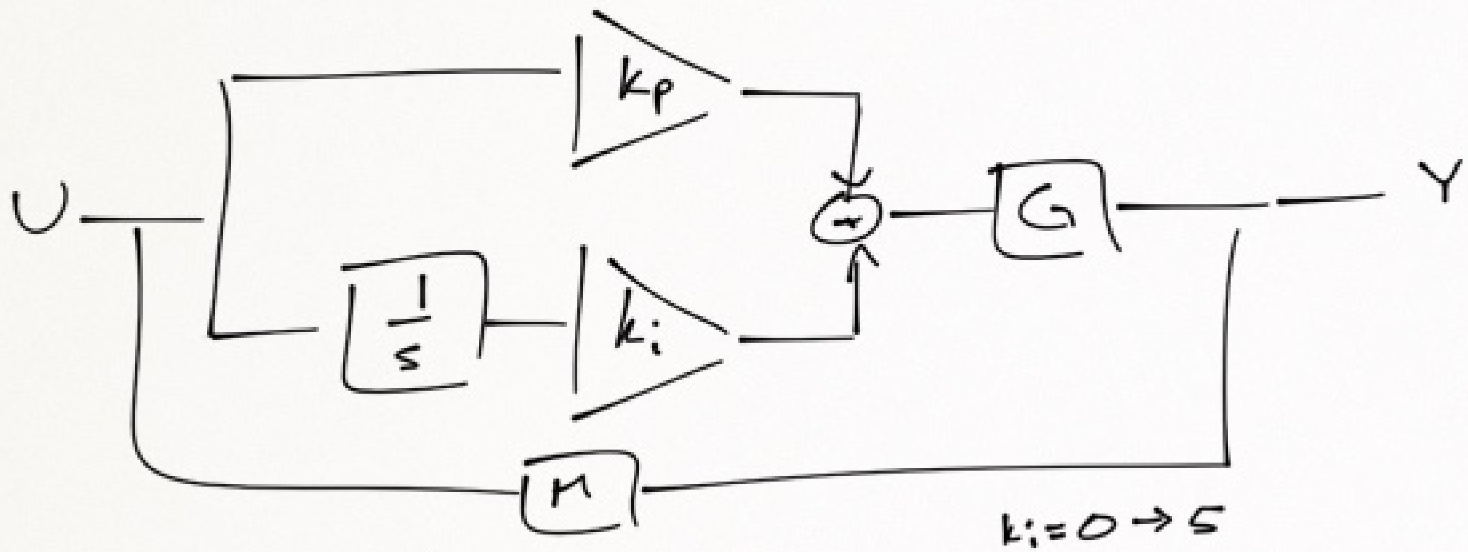
ZERO AT $-\frac{K_i}{K_p}$

PROPORTIONAL K

POLE AT ∅



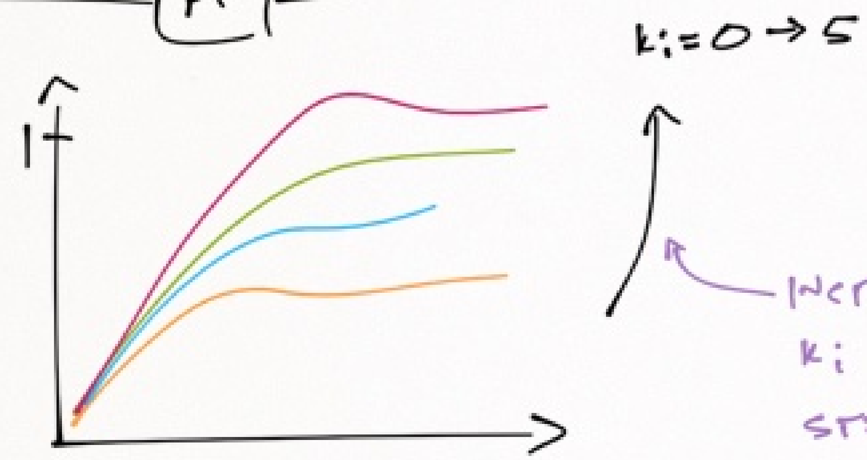
PI CONTROL



$$k = k_p \frac{s + \frac{k_i}{k_p}}{s}$$

$$k = k_p \frac{s+z}{s} GH$$

$$k = k_p + \frac{k}{s}$$



INCREASING k_i REDUCES STEADY STATE ERROR

Z-N METHOD (ZIEGLER - NICHOLS)

$k_u \rightarrow$ max K for stability
∴ proportional control ($k_i = \emptyset$)

ultimate gain, gain you can have b/f you need an integrator

by R-H, $k_u = 6\emptyset$

$$\boxed{T_u = \frac{2\pi}{w} \quad @\ k = k_u}$$

CE: $s^3 + 6s^2 + 11s + 6 + k_p = \emptyset$

@ k_u CE: $s^3 + 6s^2 + 11s + 66 = \emptyset$

has 1 real solution and 2 purely imaginary ones

imag real imag. real

$s = \sigma$
$s = \pm jw$

BIG Ki REDUCES STEADY STATE ERROR FASTER

↳ OFFSET IN OSCILLATING

BUT DESTABILIZES

↳ MAKES IT OVERSHOOT AND FLUCTUATE

$$6(jw)^2 + 66 = \emptyset$$
$$-6w^2 + 66 = \emptyset$$
$$w = \sqrt{11}$$

$$(jw)^{\cancel{3}\,2} + 11\cancel{j}\cancel{w} = \emptyset$$
$$-w^2 + 11 = \emptyset$$
$$w = \sqrt{11}$$

Z-N FORMULAS

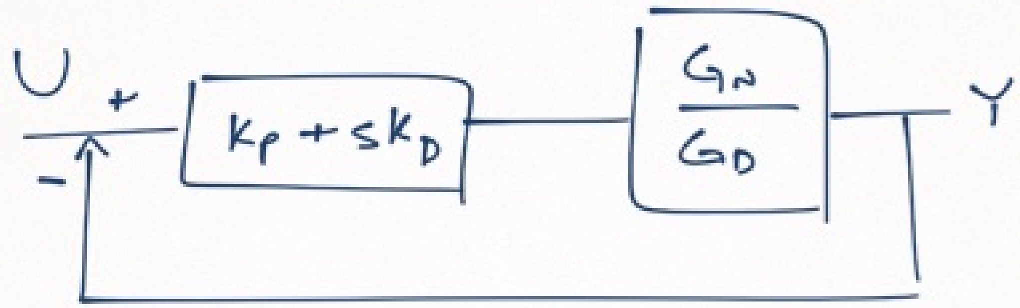
$$\boxed{\begin{aligned} k_p &= 0.45\ k_u \\ k_I &= 0.54\ \frac{k_u}{T_u} \end{aligned}}$$

$$T_u = \frac{2\pi}{\sqrt{11}} = 1.9\ s$$

period at frequency where marginal stability (ie. "sustained oscillation")

PD CONTROL

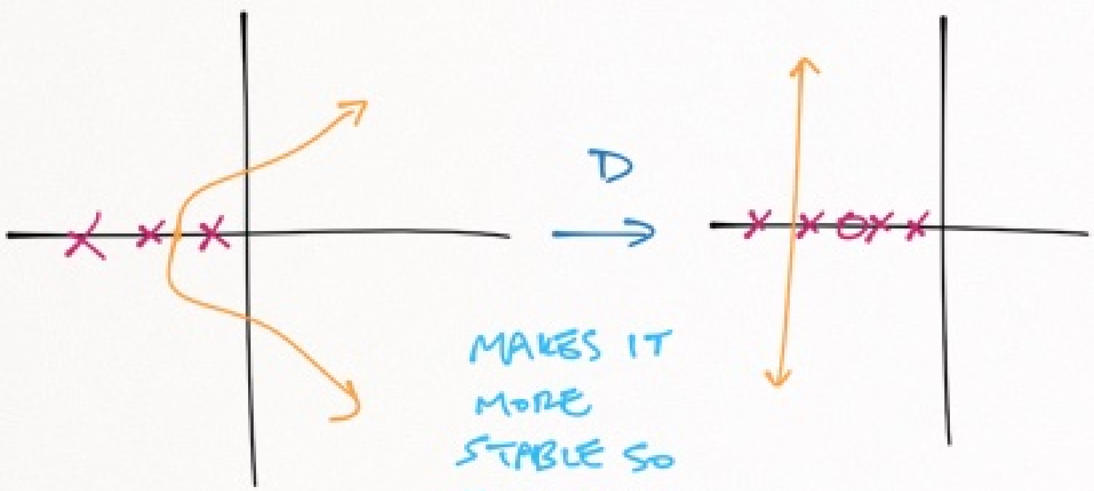
$K = K_p + sK_D$



$$\frac{Y}{U} = \frac{(K_p + sK_D)G_N}{(K_p + sK_D)G_N + G_D} = \frac{K_D\left(s + \frac{K_p}{K_D}\right)G_N}{\sim}$$

ADDED A NEW ZERO

POLES MOVED



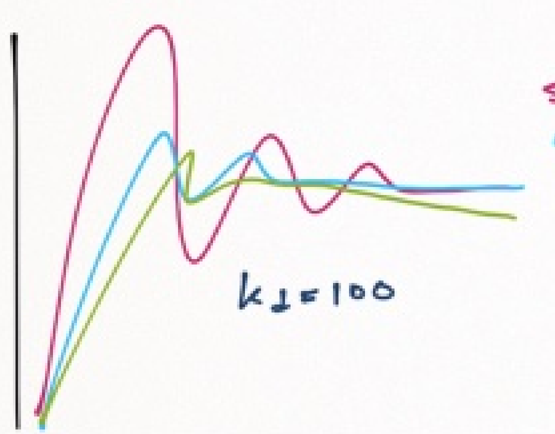
MAKES IT MORE STABLE SO YOU CAN INCREASE GAINS

K_D IS A STABILIZING GAIN

$$e_{ss} = \frac{1}{1 + KG_H}\bigg|_{s=0} = \frac{1}{1 + (K_p + \cancel{sK_D})G_H}\bigg|_{s=0}$$

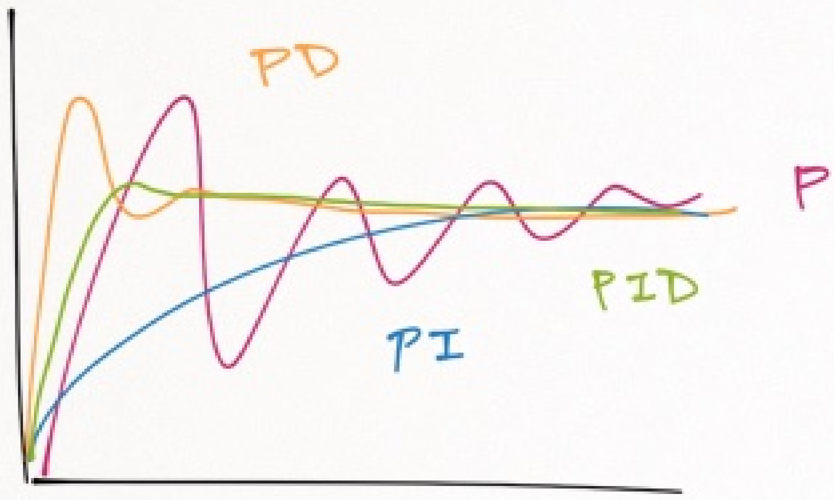
K_D DOES NOT DIRECTLY EFFECT SS ERROR

QUICKER RISE TIME, REDUCED S.S. ERROR etc.



SMALL K_D
MIDDLE K_D
LARGE K_D

PI VS. PD



NONE ARE PERFECT,
BUT LET'S COMBINE THEM ALL

GAIN	T_R	over shoot	settle time	SS error
P	↓	↑	↑	↓
I	↓	↑	↑	Ø
D	↑	↓	↓	N/A

want everything to be ↓

PID CONTROL

3 POLE SYSTEM EXAMPLE

MOTOR
CURRENT DRIVER
SPRING ← constant force reading to be readjusted

↳ creates steady state error

integrator tells system to push back until at right level

↳ for example, if gravity is pulling down on robot arm

PID CONTROL

$$G = \frac{1}{(s+1)(s+2)(s+3)} = \frac{1}{s^3+6s^2+11s+6}$$

$$\left.\frac{Y}{U} = \frac{k}{s^3+6s^2+11s+k}\right\}$$ P CONTROL

$k = k_p + s k_D$

$$\frac{Y}{U} = \frac{k_p + s k_D}{s^3+6s^2+(11+k_D)s+6+k_p}$$ ← closed loop poles

$$k = k_p + \frac{1}{s}k_i + s k_D = \frac{s^2 k_D + s k_p + k_i}{s} = k_D \frac{s^2 + s\frac{k_p}{k_D} + \frac{k_i}{k_D}}{s} = k_D \frac{(s+z_1)(s+z_2)}{s}$$

k_D IS ACTUALLY THE PROPORTIONAL GAIN

POLE AT ZERO (integrator)

HAS 2 ZEROS THAT YOU CAN CHOOSE

(REAL, COMPLEX OR IMAGINARY BASED ON WHAT YOU CHOOSE)

↳ CAN PUT ZEROS ON POLES

STEADY STATE ERROR IS Ø

STABILITY IMPROVED

↳ DECREASED # OF ASYMPTOTES ($xs' = xs - 1$)

Z - N

GET K_U } set by when all non-proportional gain set to zero
GET T_U

$K_U = 60$
$T_U = 1.9$

$K_P = 0.6 K_U$

$K_I = 1.2 \frac{K_U}{T_U}$ ← differentiator allows you to become more aggressive w/ integrator

$K_D = 0.75 K_U T_U$

kind of like adding fake friction

QUIZ #3 REVIEW

E7.1

$$1 + \frac{ks(s+4)}{s^2+2s+2} = \emptyset$$

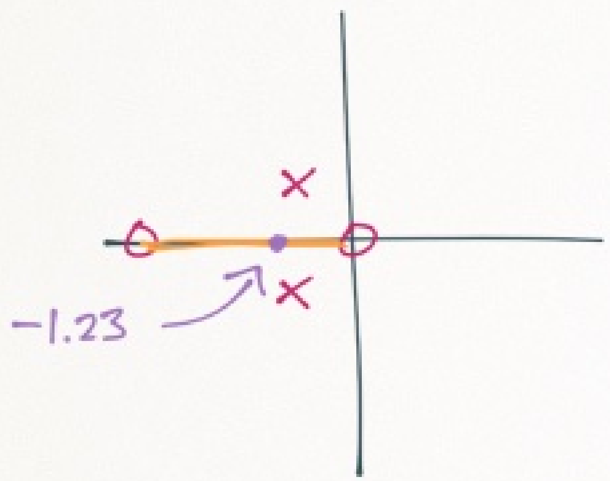
CHARACTERISTIC EQN IF NEGATIVE FEEDBACK

a) SKETCH ROOT LOCUS
b) FIND THE GAIN WHEN ROOTS ARE EQUAL
c) FIND THE TWO EQUAL ROOTS
d) FIND THE SETTLING TIME WHEN ROOTS EQUAL

CE: $1 + \underbrace{K\,G(s)H(s)}_{}$

$= 1\angle 180°$

ZEROS @ $s = \emptyset$, $s = -4$
POLES @ $s = -1 \pm i$



R.L. EXISTS TO LEFT OF ALL ODD # Z & P

$$\frac{d}{ds}\ \frac{s^2+2s+2}{s(s+4)} = \emptyset$$

$$\frac{2s^2-4s-8}{s^4+8s^3+16s^2} = \emptyset$$

$s = 1-\sqrt{5}$ U $\cancel{s = 1+\sqrt{5}}$

$s = -1.2361$

b) $$1 + \frac{k \cdot s(s+4)}{s^2+2s+2} = \emptyset$$

$$s = -\frac{2k+1 \pm \sqrt{4k^2+2k-1}}{k+1}$$

WHEN ROOTS ARE EQUAL

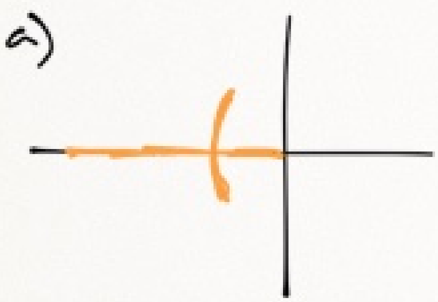
$\sqrt{4k^2+2k-1} = \emptyset$

$\therefore k = \dfrac{-1 \pm \sqrt{5}}{4} = \boxed{0.309}$

or -0.809

Why not?

a)



c) when $k = 0.309$

$$\text{roots} = s_{1,2} = -\frac{2k+1}{k+1} = \boxed{-1.2361}$$

d) $$T_s = \frac{\ln 50 - \ln \beta}{\zeta \omega_n} = \frac{\ln(50) - \ln\sqrt{1-\zeta^2}}{\zeta \omega_n}$$

@ $k = 0.309$

CE: $$1 + \frac{0.309 s(s+4)}{s^2+2s+2} = \emptyset$$

$$s^2 + 2s + 2 + 0.309 s^2 + 4 \cdot 0.309 s = \emptyset$$

$$1.309 s^2 + 3.236 s + 2 = \emptyset$$

$$(s + 1.2361)^2 = \emptyset$$

$\therefore \omega_n = 1.2361$

$\zeta = 1$ CRITICALLY DAMPED

$$T_s \simeq \frac{\ln(50)}{1.2361} = 3.2\,s$$

E7.2 $H(s) = 1$

$$G_c(s)G(s) = \frac{k}{s(s+2)(s^2+4s+5)}$$

a) sketch root locus & show dominant roots are $s = -0.35 \pm j0.80$ when $k = 6.5$

b) For dominant roots, calc. T_s & overshoot for step input

$$TF = \frac{KG}{1+KGH}$$

$$CE: \; 1 + KGH = \emptyset = 1 + \frac{6.5}{s(s+2)(s^2+4s+5)}$$

$$s(s+2)(s^2+4s+5) = -6.5$$

$s = (-0.35, \pm 0.8)$ ← this is dominant b/c much closer to y-axis (8x)

$s = (-2.65, \pm 1.228)$

$$T_s = \frac{\ln 50 - \ln\sqrt{\zeta^2 - 1}}{\zeta\omega_n}$$

take $(s+a)(s+a) = \emptyset$

$$(s + 0.35 + j0.8)(s + 0.35 - j0.8) = \emptyset$$

$$s^2 + 0.7s + 0.7625 = \emptyset$$

$$\omega_n = |(0.35, 0.8)| = 0.873$$

$$\zeta = \frac{0.7}{2(0.873)} = 0.4$$

$$T_s \simeq \frac{4}{\zeta\omega_n} \simeq \boxed{11.46\,s}$$

$$\%\ \text{overshoot} = e^{\frac{-\zeta\pi}{\beta}} = e^{\frac{-0.4\pi}{\sqrt{1-0.4^2}}}$$

$$= \boxed{25.38\%}$$

E7.3 $G_c(s)G(s) = \dfrac{k(s^2+4s+8)}{s^2(s+4)}$

HAS NEGATIVE UNITY FEEDBACK
SHOW $K=7.35$ FOR
$s=-1.3 \pm j2.2$, $\zeta=0.5$

$H=1$

$$1+KGH=\emptyset = 1+7.35\,\frac{s^2+4s+8}{s^2(s+4)}$$

$s=(-1.3, \pm 2.244)$

$s=-8.7$

E7.4 $G_cG = \dfrac{K(s+1)}{s^2+4s+5}$

FIND $\measuredangle$ OF DEPARTURE
FIND ENTRY POINT OF R.L.
ON REAL AXIS

UNITY

sum of angles to POI from poles & zeros = 180°

$$1+KGH=\emptyset = 1+\frac{K(s+1)}{s^2+4s+5}$$

$$\frac{d}{ds}\frac{1}{GH}=\emptyset=\frac{d}{ds}\frac{s^2+4s+5}{s+1} = \frac{s^2+2s-1}{(s+1)^2}$$

$\rightarrow s=-1\pm\sqrt{2}$

$\rightarrow s=\boxed{-2.4}$

EXTRA PROBLEMS

CE: $s^3 + (k-4)s^2 + 10ks + 29k = \emptyset$

find k for which system is stable

draw root locus

ROUTH HURWITZ

1	10k
k-4	29k
m_1	

$$m_1 = \frac{-1}{b}\begin{vmatrix} a & c \\ b & d \end{vmatrix} = \frac{10k^2 - 69k}{k-4} > \emptyset$$

$\hookrightarrow k > 0 \; \& \; k < 4 \quad || \quad k > 6.9$

E7.5 $$G_c(s)G(s) = \frac{s^2+2s+10}{s^4+38s^3+515s^2+2950s+6000}$$

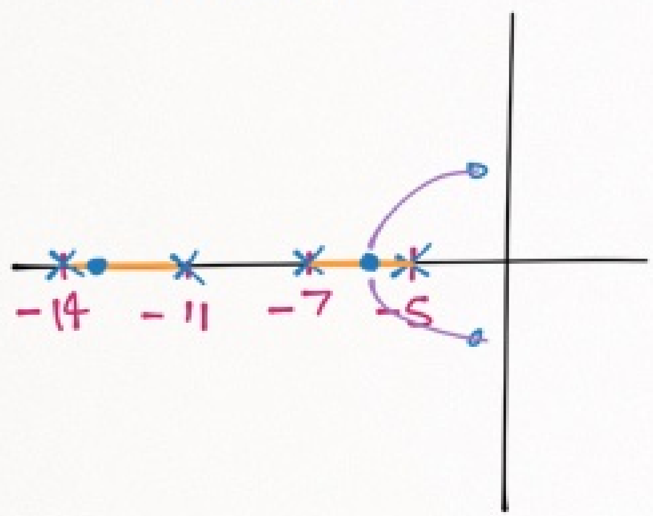
FIND BREAKAWAY POINTS ON REAL AXIS
FIND ASYMPTOTE CENTROID
FIND VALUES OF K AT BREAKAWAY POINTS

$\rightarrow$ ZEROS @ $(-1 \pm 3j)$

CE: $1+KGH=\emptyset$ $\rightarrow$ POLES @ $-5.2, -7.05, -14.25, -11.5$

$\frac{d}{ds}\frac{1}{GH}=\emptyset$ $\rightarrow$ BREAK POINTS @ -5.89, $\cancel{-8.93}$, -13.0

DOESN'T FIT ODD/EVEN REQUIREMENTS



E7.6 $G_cG = \frac{20K}{s(s^2+20s+100)}$

SKETCH ROOT LOCUS

FIND VALUE RESULTING IN UNSTABLE SYSTEM

$1 + KGH = \emptyset$

$\frac{d}{ds}\frac{1}{GH} = \emptyset \rightarrow s = -3.3, -10$

NOV 5, 2014

NYQUIST CRITERIA

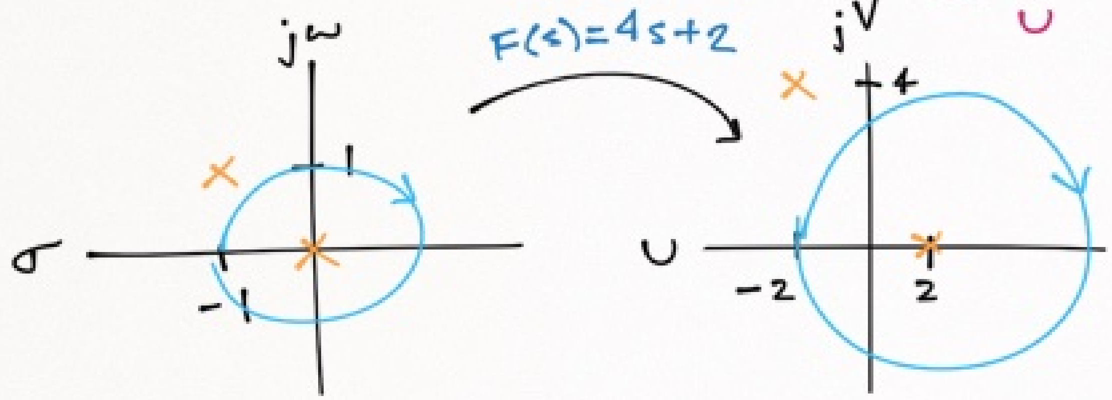
↳ MEASURE OF INSTABILITY

MATHEMATICAL BACKGROUND

Anytime you have function of s $(\sigma + j\omega)$, F(s) is also a complex # $(u + jv)$

$$F(s) = 4s + 2 = 4(\sigma + j\omega) + 2 = \underbrace{4\sigma + 2}_{u} + j\underbrace{4\omega}_{v}$$

scales by 4
& shifts right by 2



S - PLANE

UV - PLANE

NOV 7, 2014

EXAMPLE OUT OF TEXTBOOK

B = $1 + j\emptyset$

A = $1 + j1$

H = $\emptyset + j1$

G = $-1 + j1$

F = $-1 + j\emptyset$

$$F(s) = \frac{s}{s+2}$$

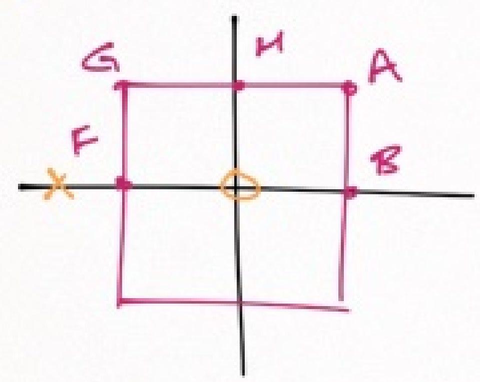
B → 1/3

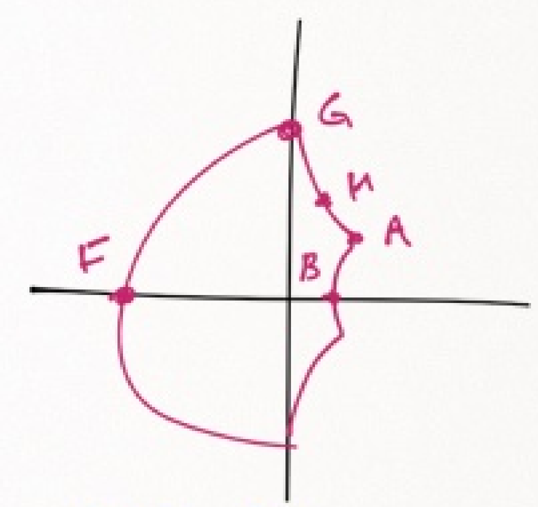
A → 0.4 + j0.2

H → 0.2 + j0.4

G → j

F → −1

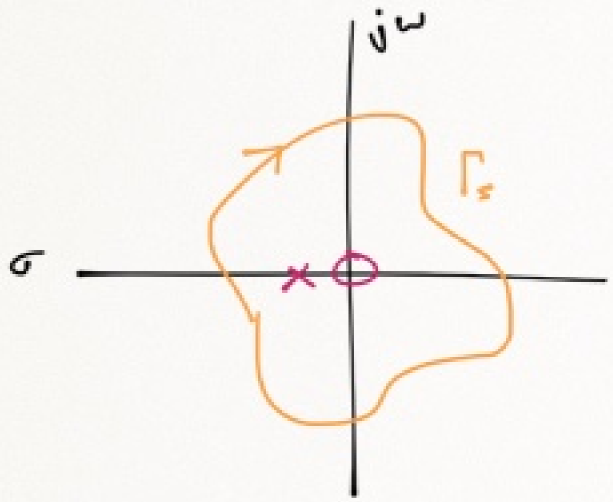




pick a few points, transform the points, then connect the dots

CAUCHY'S THEOREM

S-PLANE



IF S-PLANE CONTOUR

① CLOSED

② CLOCKWISE

③ DOES NOT INTERSECT +P/Z OF F(S)

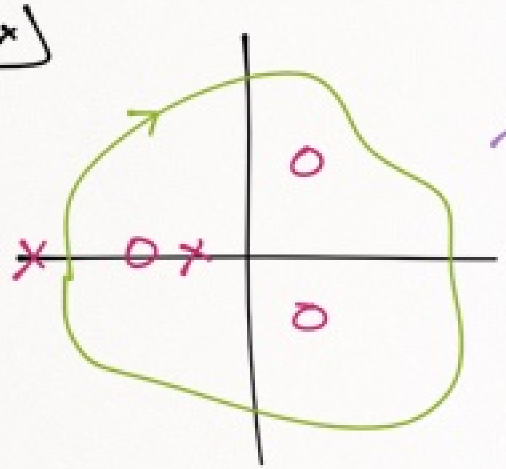
THEN U-V PLANE CONTOUR, Γ_s ← uppercase gamma

① Z = # ZEROS INSIDE Γ_s

② P = # POLES INSIDE Γ_s

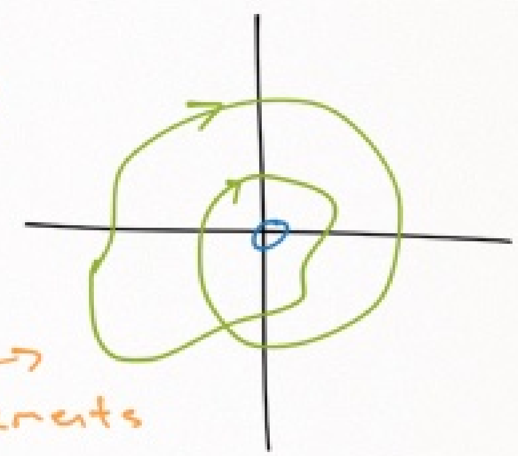
③ $N = Z - P$ = # CLOCKWISE ENCIRCLEMENTS OF ORIGION

Ex)



$$\frac{(s+2)(s^2+4s+8)}{(s+1)(s+5)}$$

$N = 3 - 2 = 2$

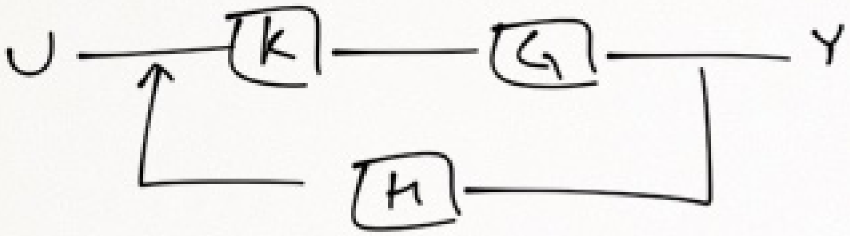


2 encirclements

INSTABILITY HAPPENS WHEN CLOSED-LOOP POLES IN RHP

↳ OPEN-LOOP POLES ARE CLOSED-LOOP POLES WHEN $K = \emptyset$

THIS THEOREM LETS US SEE IF THERE ARE ANY P/Z IN THE ENTIRE RHP IF WE CHOOSE A GIANT CONTOUR.



$$\frac{Y}{U} = \frac{KG}{1+KGH} = \frac{KG_N H_D}{KG_N H_N + G_D H_D}$$

$$KGH = \frac{KG_N H_N}{G_D H_D}$$

$$1 + KGH = \frac{G_D H_D + KG_N H_N}{G_D H_D}$$

IF YOU HAVE POLES IN THE RHP, IT'S GOING TO BE PRETTY UNRESOLVABLE.

IF ONLY ONE, YOU MIGHT BE ABLE TO ↑K UNTIL IT'S IN THE LHP.

ZEROS OF 1+KGH = POLES OF $\overbrace{\text{CLOSED-LOOP TF}}^{Y/U}$

POLES OF 1+KGH = POLES OF $\underbrace{\text{OPEN-LOOP TF}}_{KGH}$

IF ALL O-L POLES IN LHP:

N = # C-L IN RHP

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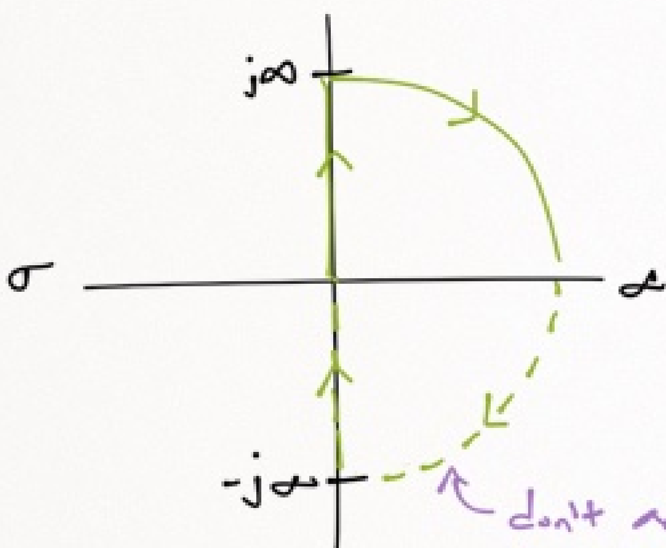
// SPRING VS. GRAVITY

↳ GRAVITY FORCE DOES NOT PUSH HARDER WHEN STRETCHED FURTHER

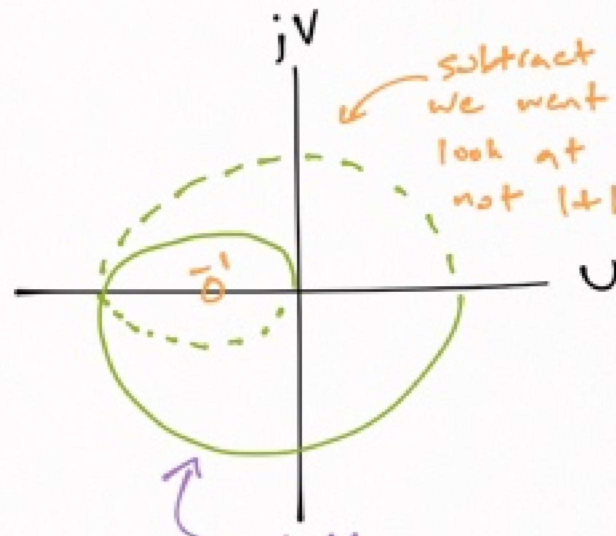
as long as # poles of KGH in RHP = ∅



don't actually have to do this half b/c mirror image

KGH

NYQUIST CRITERIA



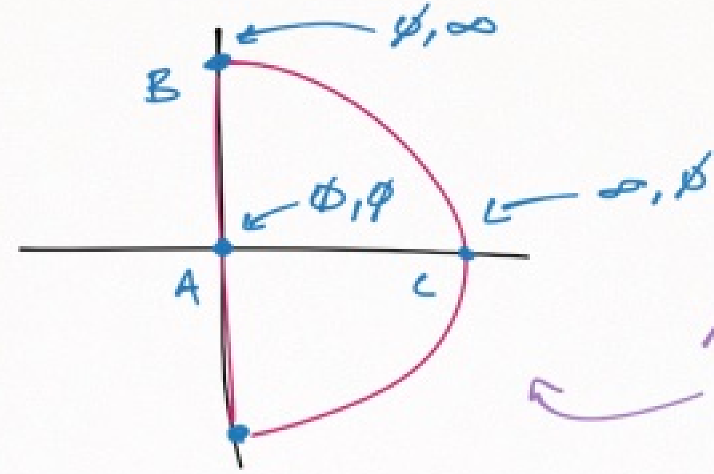
subtract 1 we want to look at KGH not 1+KGH

unstable

PI GAIN COUNTERS SPRING, BUT OUR HELICOPTER DOESN'T HAVE SPRING

Ex) $KGH = \dfrac{1000}{(s+1)(s+2)}$

DO NYQUIST CRITERIA



this is a combination of both magnitude & phase bode plots

s	A	B	C
real	100	$-\Delta$	Δ
imag.	∅	∅	∅
mag.	100	Δ	Δ
phase	∅	180°	∅

polar co-ords

$A = \dfrac{1000}{1(10)} = 100$

$B = \dfrac{1000}{(1+j\infty)(10+j\infty)} = \dfrac{1000}{-\infty^2} = -\Delta$

negative tiny

NOTE: tiny not zero b/c tiny has phase

$C = \dfrac{1000}{\infty^2} = \Delta$

you want to plot in dB

Bell

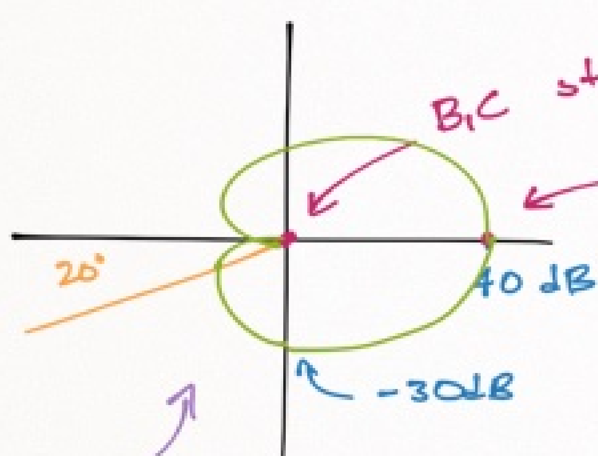
↳ power ratio

deci

$20 \log 100 = 40\,dB$

B, C stay at zero, but it rotates 180°

A = 20 log(100)



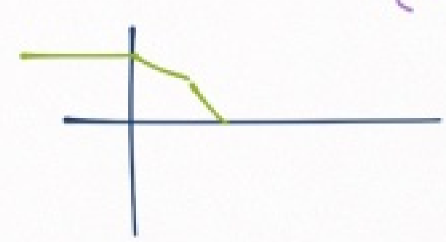
stable

FROM A to B $s = j\omega$

no σ ← envelope

↖ oscillation

$KGH = K_{DC} \dfrac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\cdots}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\cdots} = 100 \dfrac{1}{\left(1+\frac{s}{1}\right)\left(1+\frac{s}{10}\right)}$



-20 @ ∅ & -10

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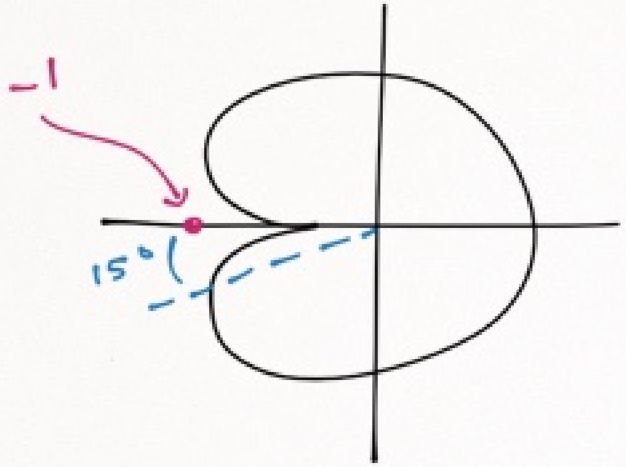


GAIN MARGIN

↳ gain at phase = 180°

PHASE MARGIN

↳ phase at gain 20dB

IF MAG COMES IN AT $20\frac{dB}{dec}$

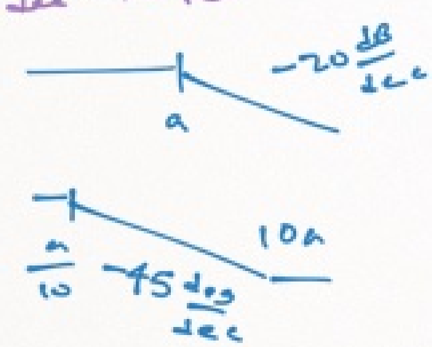
PHASE COMES IN AT 90°

" " $40\frac{dB}{dec} \to 180°$

" " $-20\frac{dB}{dec} \to -90°$

" " $-40\frac{dB}{dec} \to -180°$

MAG



root locus
gain you can
add before
unstable

$$KGH = \frac{100}{(1+s)(1+\frac{s}{10})}$$

INFORMATION FROM NYQUIST CRITERIA

→ gain margin ← this is also what the root locus gives you
→ phase margin
→ crossover margin

PHASE MARGIN

→ how much added phase before instability
↳ rotation of Nyquist

GAIN MARGIN

→ how much added gain before instability

CROSSOVER MARGIN

→ frequency at which the magnitude goes to one

YOU CAN USUALLY GET THIS INFORMATION FROM BODE PLOT

phase @ 1 ← phase margin

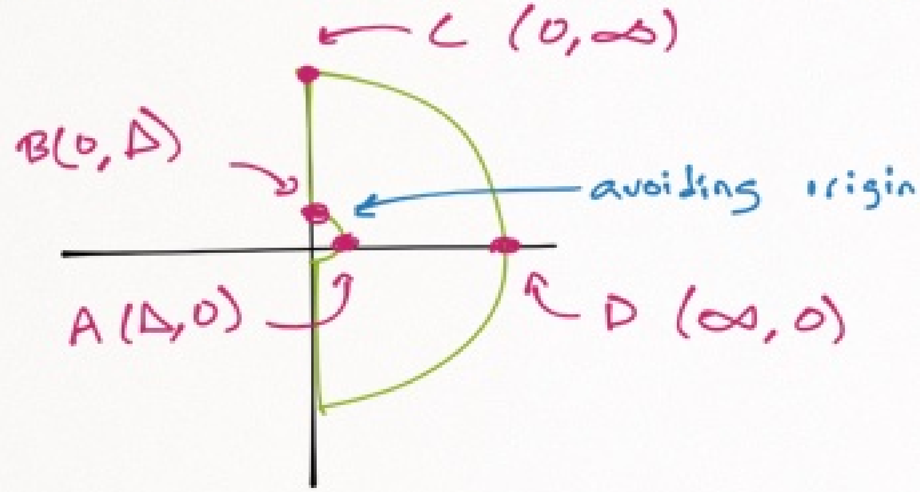
gain @ phase 180° ← ASYMPTOTICALLY SO MAYBE ∞
↳ how far below x-axis gain is

WHAT IF POLE AT ZERO?

Ex) $KGH = \dfrac{100}{s\left(1+\frac{s}{100}\right)}$

one criteria of Nyquist was that it doesn't touch any poles or zeros.

so define a new curve that detours around pole at zero

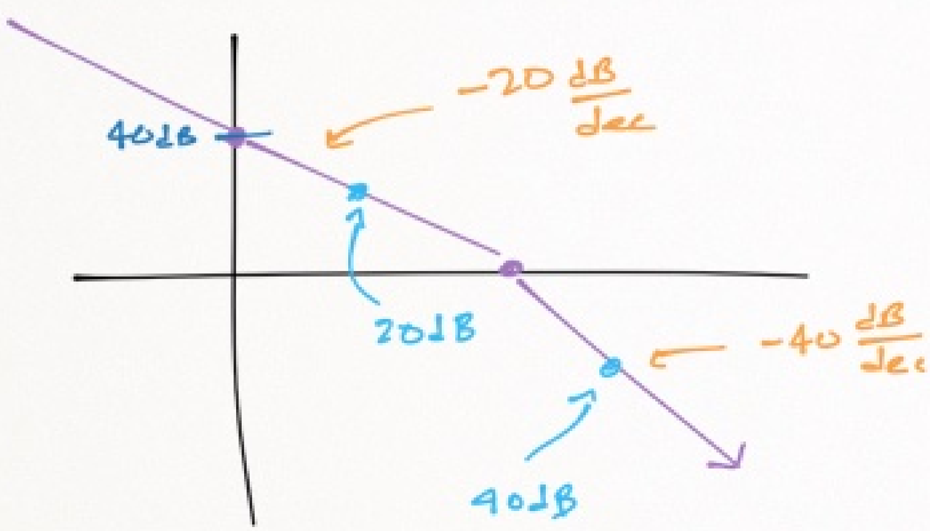


	s	KGH	$\lvert KGH \rvert$	$\measuredangle KGH$
A	Δ	$+\infty$	∞	$\emptyset$
B	$j\Delta$	$\frac{100}{j\Delta} = \frac{100}{\Delta\angle 90} = \underbrace{\infty\angle -90}_{-j\infty}$	∞	$-90°$
C	$j\infty$	$\frac{100}{-\infty^2} = -\Delta$	Δ	$-180°$
D	∞	Δ	Δ	$0°$

DC GAIN $= KGH(s=0) = \frac{1}{\emptyset} = \infty$

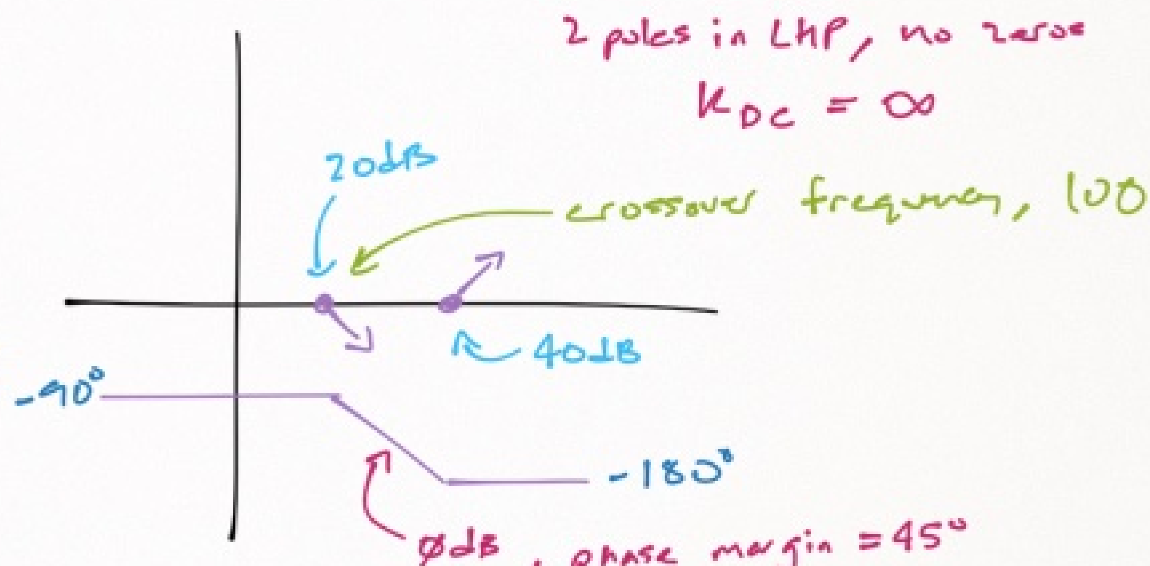
$\cancel{K_{DC}}$ $K_{\omega \simeq 1} = 10^{\circ}$

GET BODE PLOTS

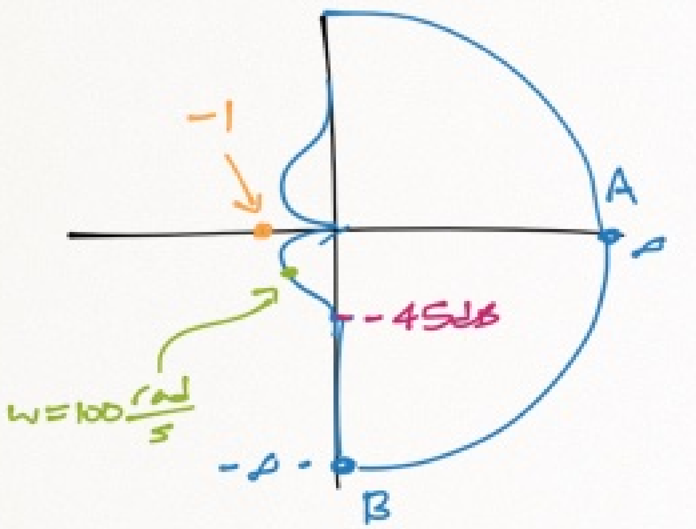


2 poles in LHP, no zeros

$K_{DC} = \infty$



Ex) CONT'D   DRAW NYQUIST

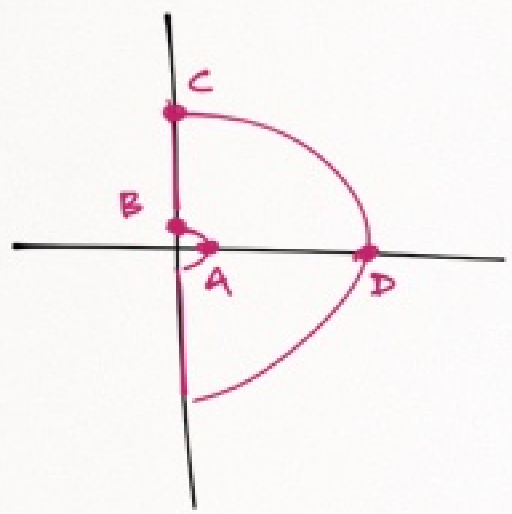


20 dB @ -90°

-40 dB @ ≃ -180°

Ex) $KGH = \dfrac{1000}{s(s+1)(s+100)}$ NYQUIST PLOT

EXCESS OF THREE POLES ∴ THERE WILL BE A GAIN MARGIN

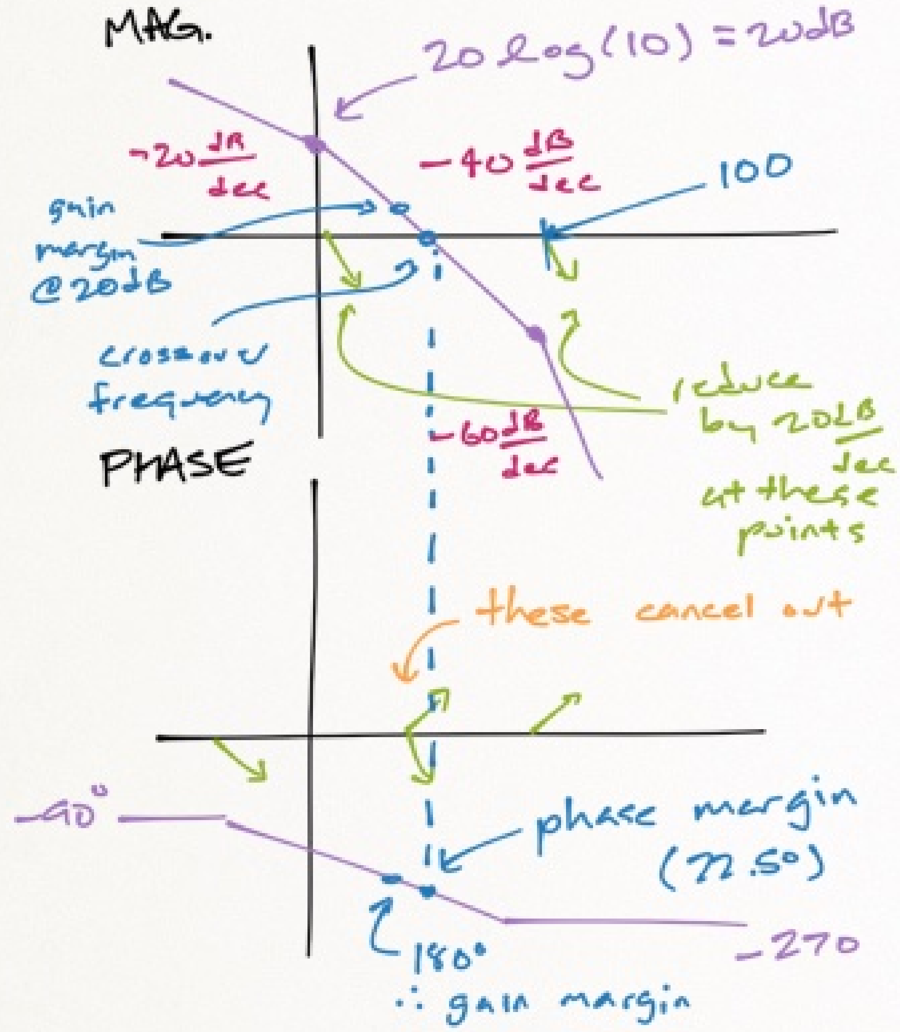


$$KGH = 10\,\frac{1}{s\left(1+\frac{s}{1}\right)\left(1+\frac{s}{100}\right)}$$

$K_{\omega=1}$ MAGNITUDE OF 10 @ $\omega=1$

A: $\dfrac{10}{\Delta(1)(1)} = \dfrac{10}{\Delta} = \infty \measuredangle \emptyset^\circ$

D: $\dfrac{10}{\infty\infty\infty} = \dfrac{10}{\infty^3} = \Delta \measuredangle \emptyset^\circ$



B:

don't bother doing this

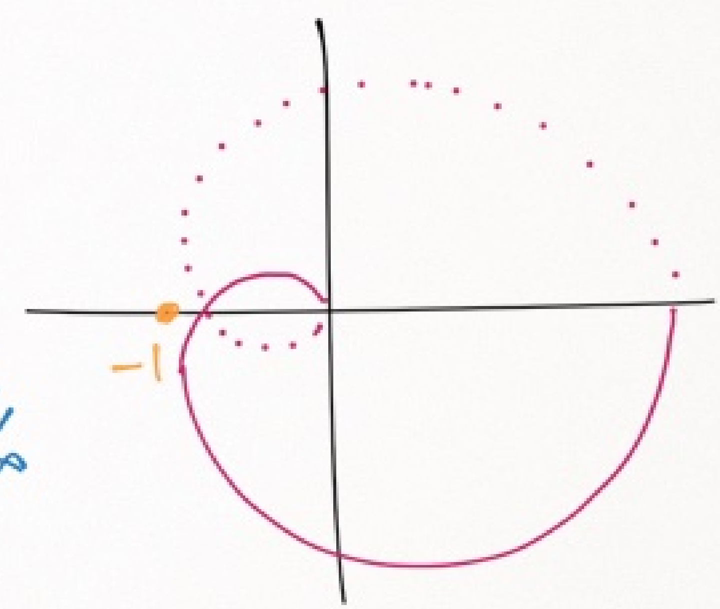
you can just read these off the bode plot which you're going to need to get anyway

C:

$B = \infty \measuredangle -90^\circ$

$C = \Delta \measuredangle 90^\circ$

an extremely small number, not negative ∞



Ex] $\dfrac{10^{14}(s+10^3)^2}{s(s+10^2)^2(s+10^5)^2}$

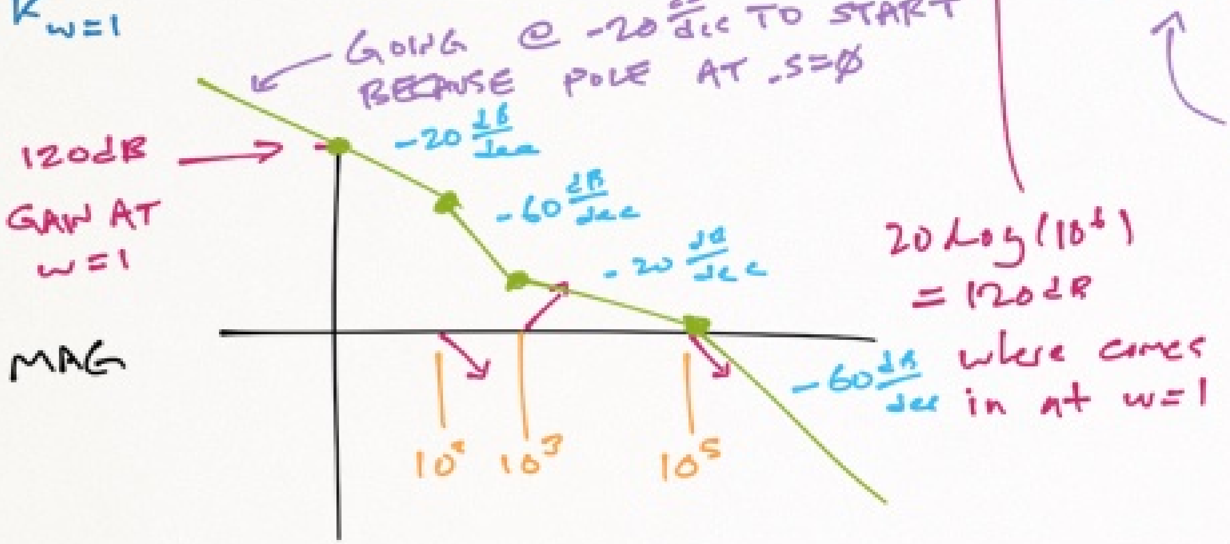
$= 10^6 \dfrac{\left(1+\frac{s}{10^3}\right)^2}{s\left(1+\frac{s}{100}\right)^2\left(1+\frac{s}{10^5}\right)^2}$ ← 2 ZEROS @ 10^3

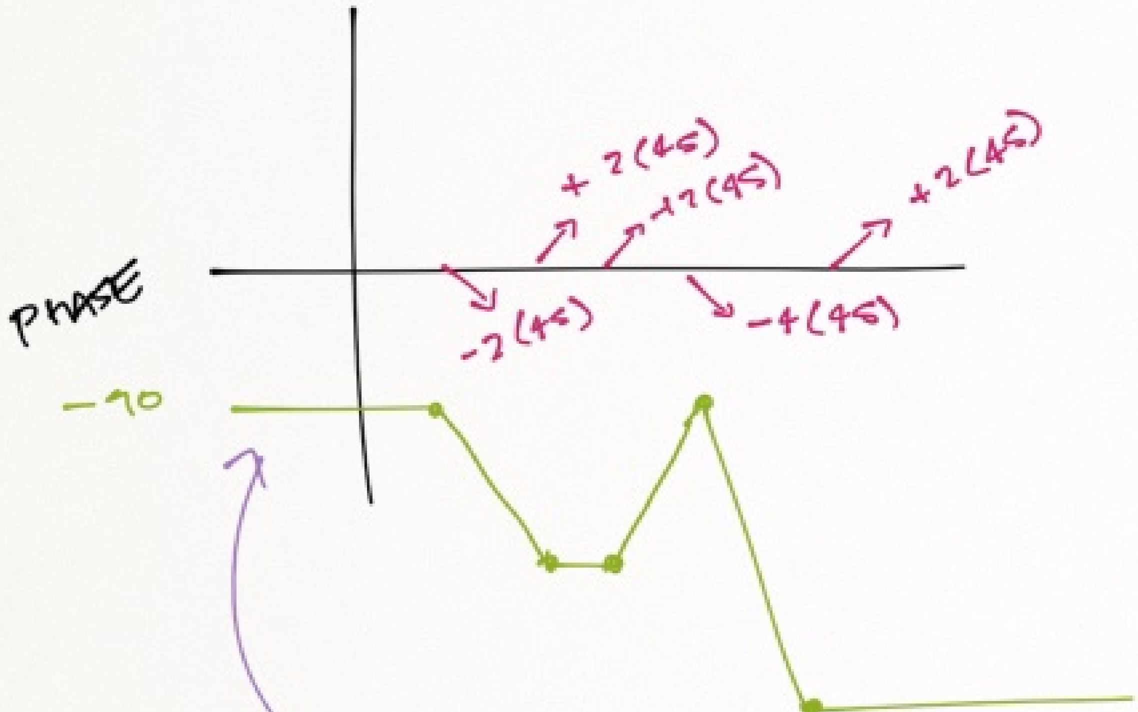
$\cancel{K_{DC}} = 10^{14+6-4-10} = 10^6$ →

$K_{w=1}$

2 poles @ 10^2 & 10^5

∴ −40dB, not −20dB



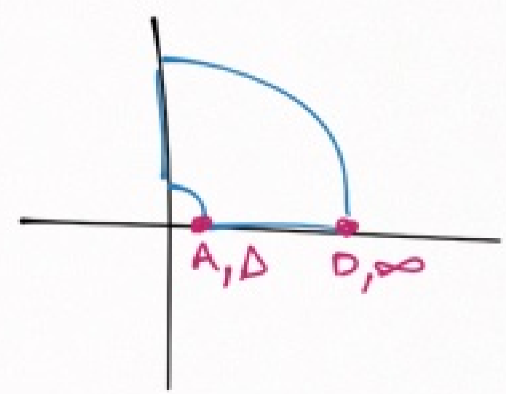


MUST BEGIN AND END LEVEL. IF IT DOESN'T YOU SCREWED UP.

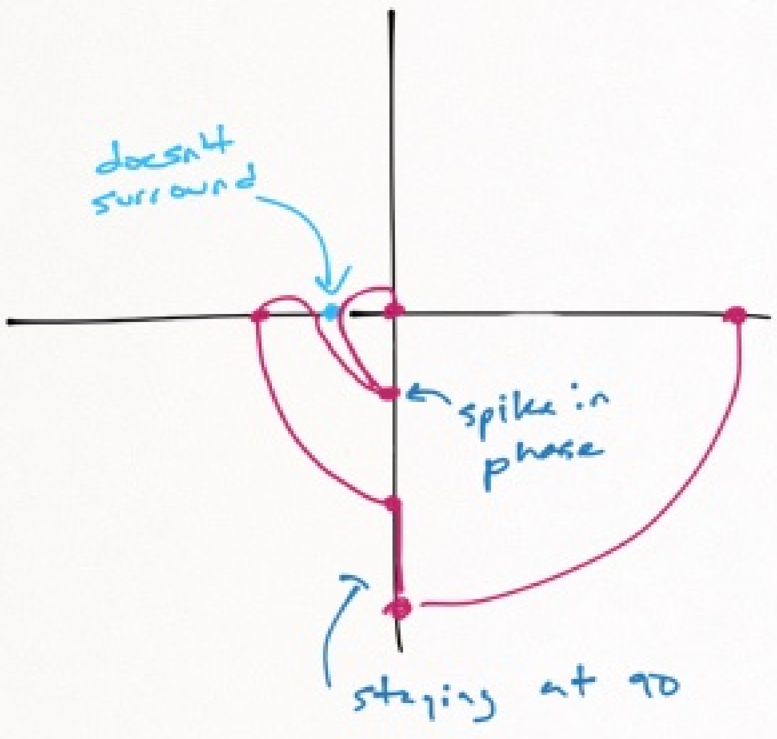
NYQUIST

A POINT Δ: $\dfrac{10^6}{\Delta} = \infty \angle 0^\circ$

D POINT ∞: $\dfrac{1}{\infty} = 0 \angle 0^\circ$



Ex] CONT'D



IMPORTANT POINTS

@ 180 IT GOES FROM
80 TO 20dB

IF YOU AMPLIFY OR ATTENUATE BY 20dB,
YOU WILL SURROUND THE ZERO.

gain marching: $\pm \sim 20$dB

phase margin: 70°

$w_c = 10^4 \frac{rad}{s}$

Ex] $KGH = \dfrac{10^4}{(s+100)(s^2+14s+98)}$

$=$

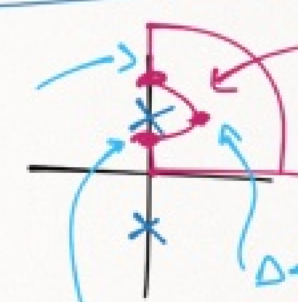
$$\rightarrow 10^2 \frac{1}{(1+\frac{s}{100})\underbrace{(s+7+j7)(s+7-j7)}_{(s+10\angle 45^\circ)(s+10\angle -45^\circ)}}$$

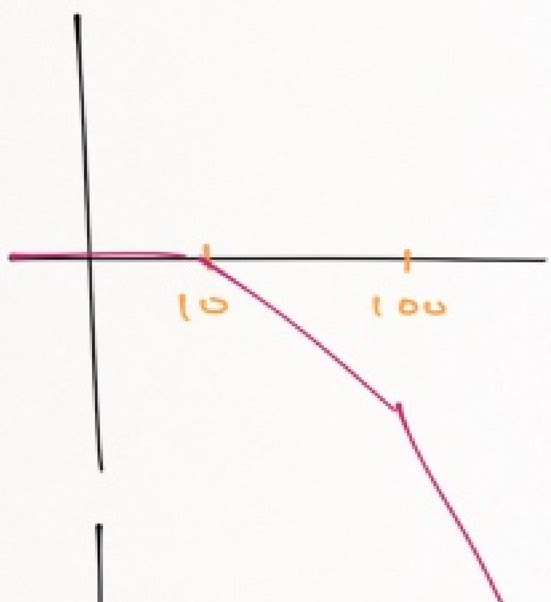
for bode plot, you can treat this as $(s+10)^2$

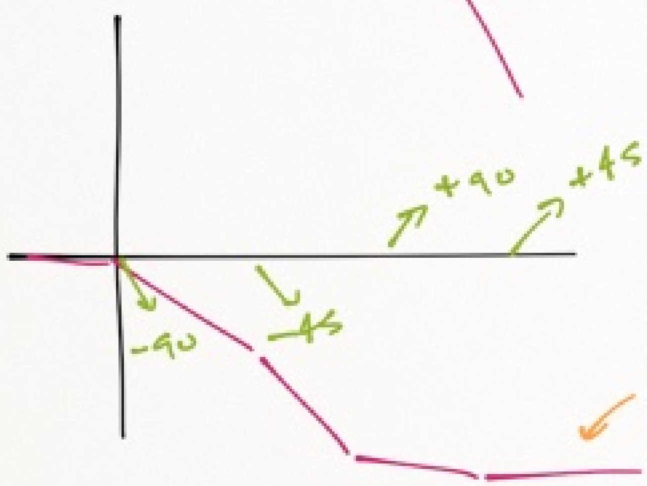
$$\rightarrow 1 \frac{1}{(1+\frac{s}{100})(1+\frac{s}{10\angle 45})(1+\frac{s}{10\angle -45})}$$

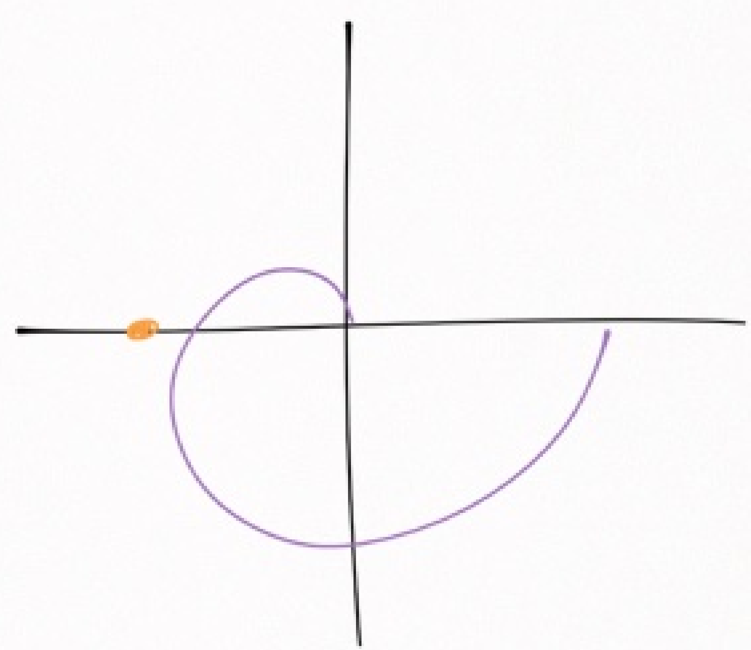
NOTE POSSIBILITY $\dfrac{1}{s^2+1}$

purely imaginary









GAIN MARGIN ~ 30dB

PHASE MARGIN ~ 180°

QUIZ # 4 REVIEW - NYQUIST

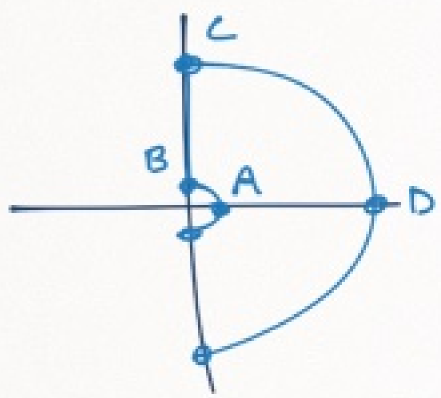
Ex) $KGH = \dfrac{1000}{s(s+1)(s+100)}$

DRAW IN CORRECT FORM FOR BODE PLOT

$$\frac{\frac{1000}{100}}{\underbrace{\frac{s}{100}}_{s}\underbrace{\left(\frac{s}{100}+\frac{1}{100}\right)}_{s+1}\left(\frac{s}{100}+1\right)} = 10\,\frac{1}{s(s+1)\left(\frac{s}{100}+1\right)}$$

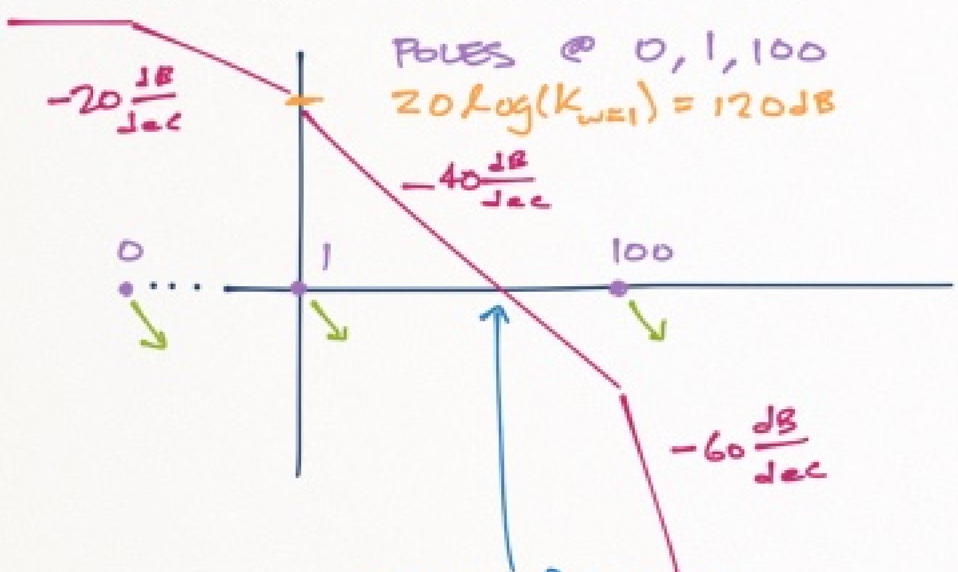
k @ w=1

DRAW THE ASSOCIATED NYQUIST CONTOUR

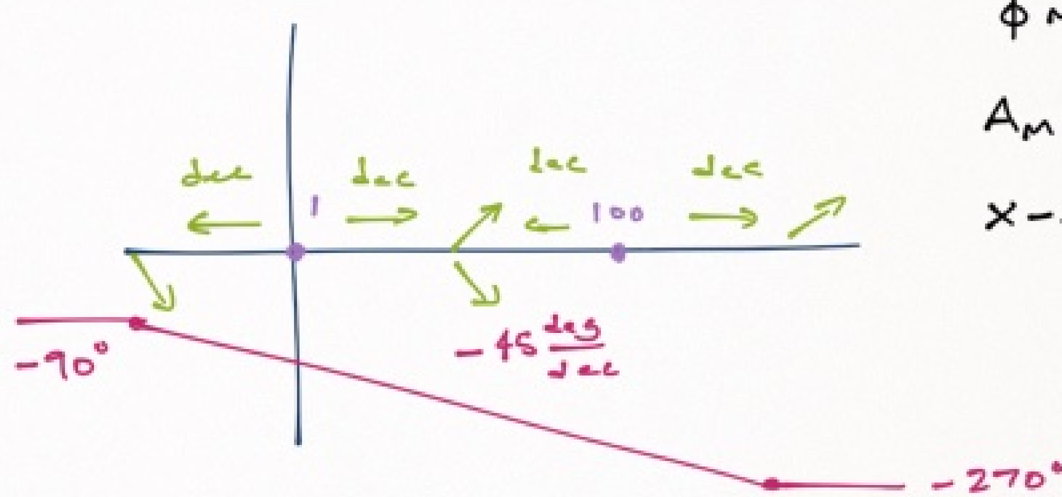


	s	$\lvert KGH \rvert$	$\measuredangle KGH$
$A = (\Delta, \emptyset)$		∞	$\emptyset$
$B = (\emptyset, \Delta)$		∞	-90
$C = (\emptyset, \infty)$		Δ	90
$D = (\infty, \emptyset)$		Δ	$\emptyset$

DRAW THE MAGNITUDE PLOT



DRAW THE PHASE PLOT

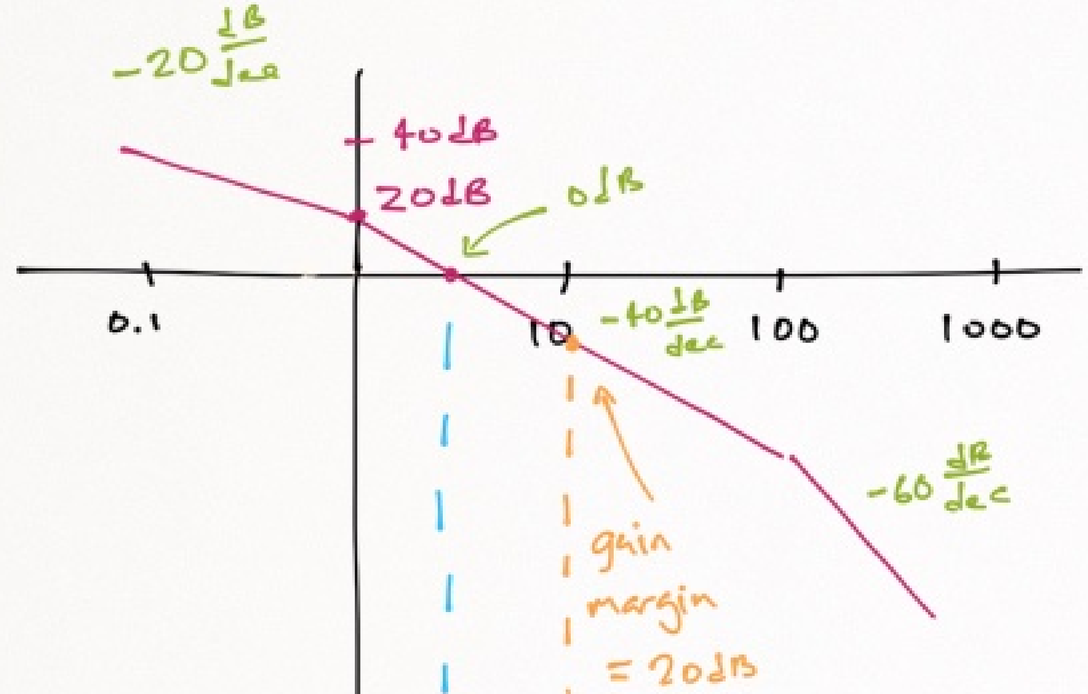


ϕ MARGIN: 122.5°

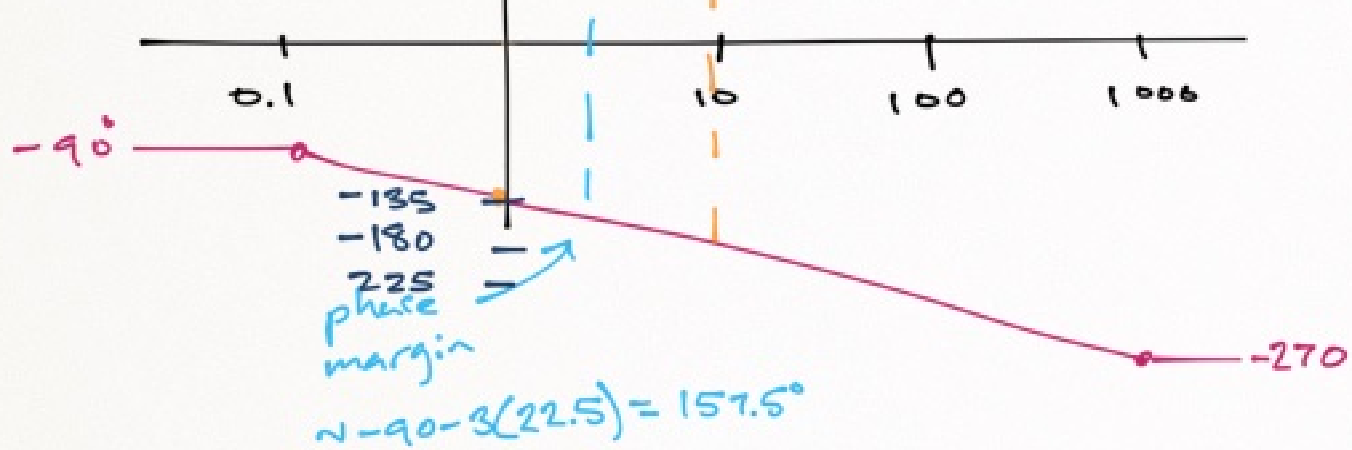
A_M MARGIN: 20 dB

X-FREQUENCY: 2-3 Hz

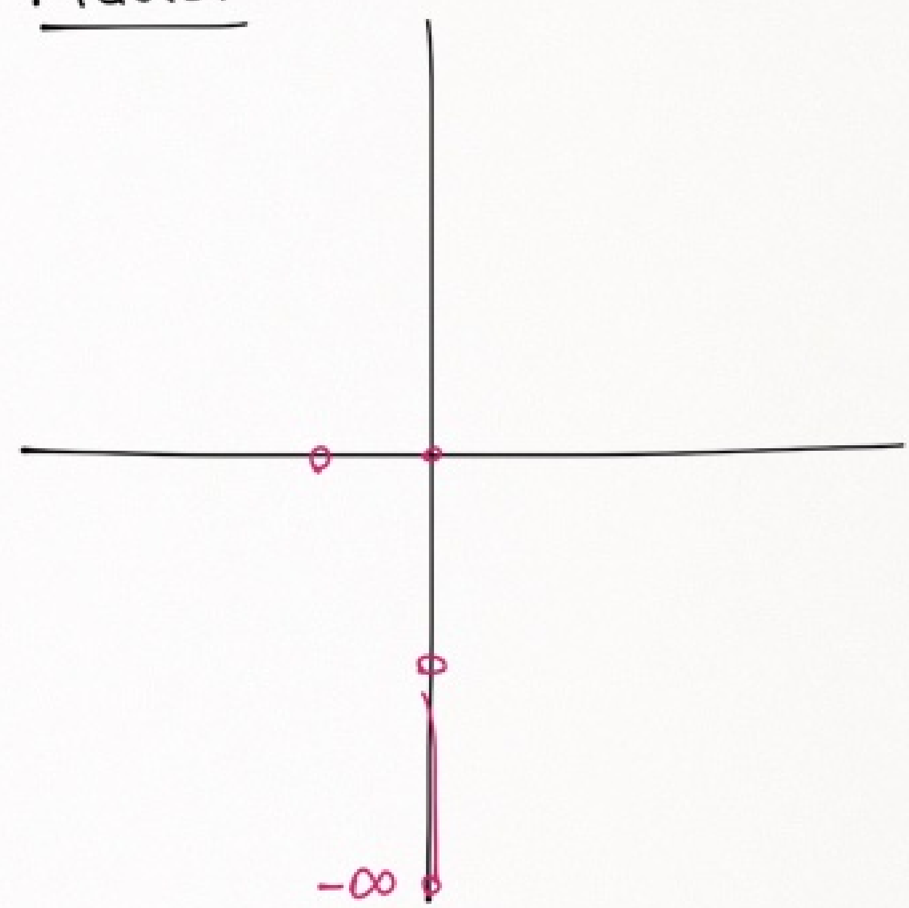
GAIN



PHASE



NYQUIST



Ex) $KGH = \dfrac{10^{14}(s+10^3)^2}{s(s+10^2)^2(s+10^5)^2}$

BODE PLOT FORM

$$10^6 \frac{\left(1+\frac{s}{10^3}\right)^2}{s\left(1+\frac{s}{10^2}\right)^2\left(1+\frac{s}{10^5}\right)^2}$$

$10^{14+2(3)-2(2)-2(5)} = 10^6 = K_{w=1}$

2 POLES @ 10^2 & 10^5, 2 ZEROS @ 10^3 ($40\frac{dB}{dec}$ changes w/ 2)

1 POLE @ 0

ASSOCIATED CONTOUR

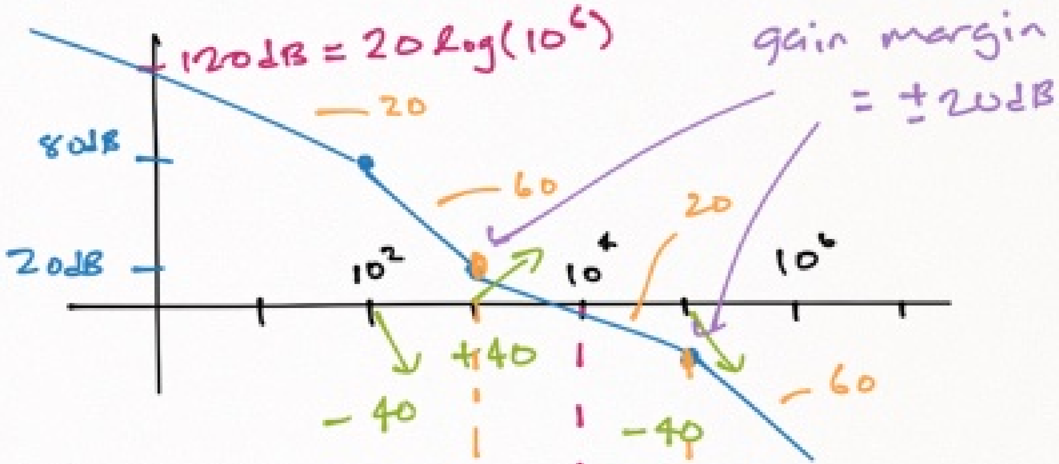
A $(\Delta, \emptyset) \longrightarrow \infty \angle 0$

B $(\emptyset, \Delta) \longrightarrow \infty \angle -90$

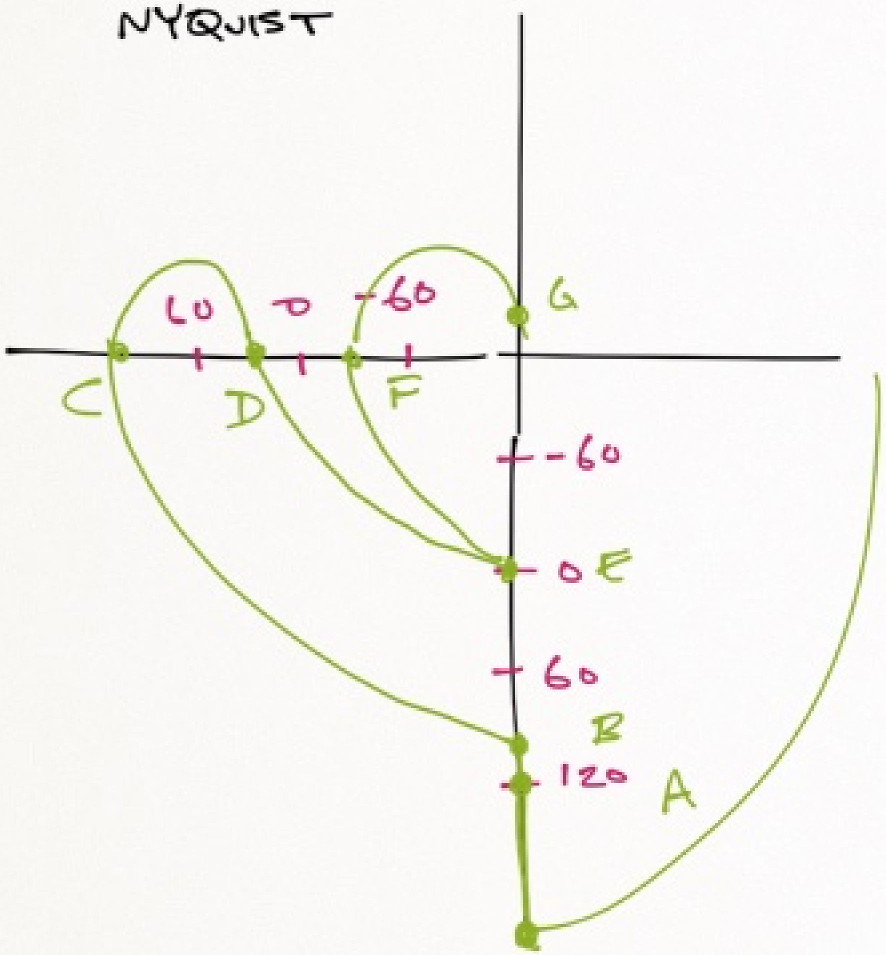
C $(\emptyset, \infty) \longrightarrow 0 \angle 90$

D $(\infty, \emptyset) \longrightarrow 0 \angle 0$

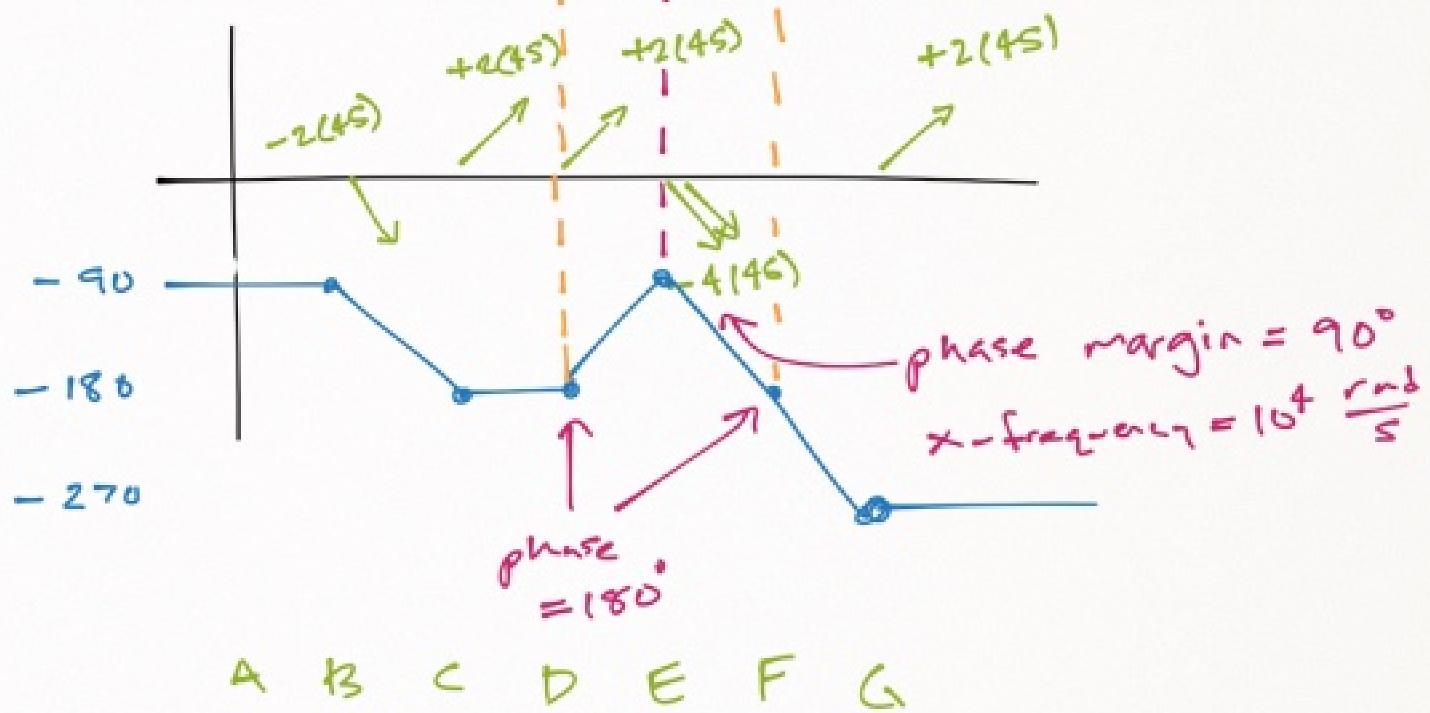
MAGNITUDE PLOT



NYQUIST



PHASE PLOT



Ex) $$\frac{10^9(s+10^3)}{(s+100)\underbrace{(s^2+14s+98)}_{\simeq (s+10\angle 45)(s+10\angle -45)}}$$

$=$

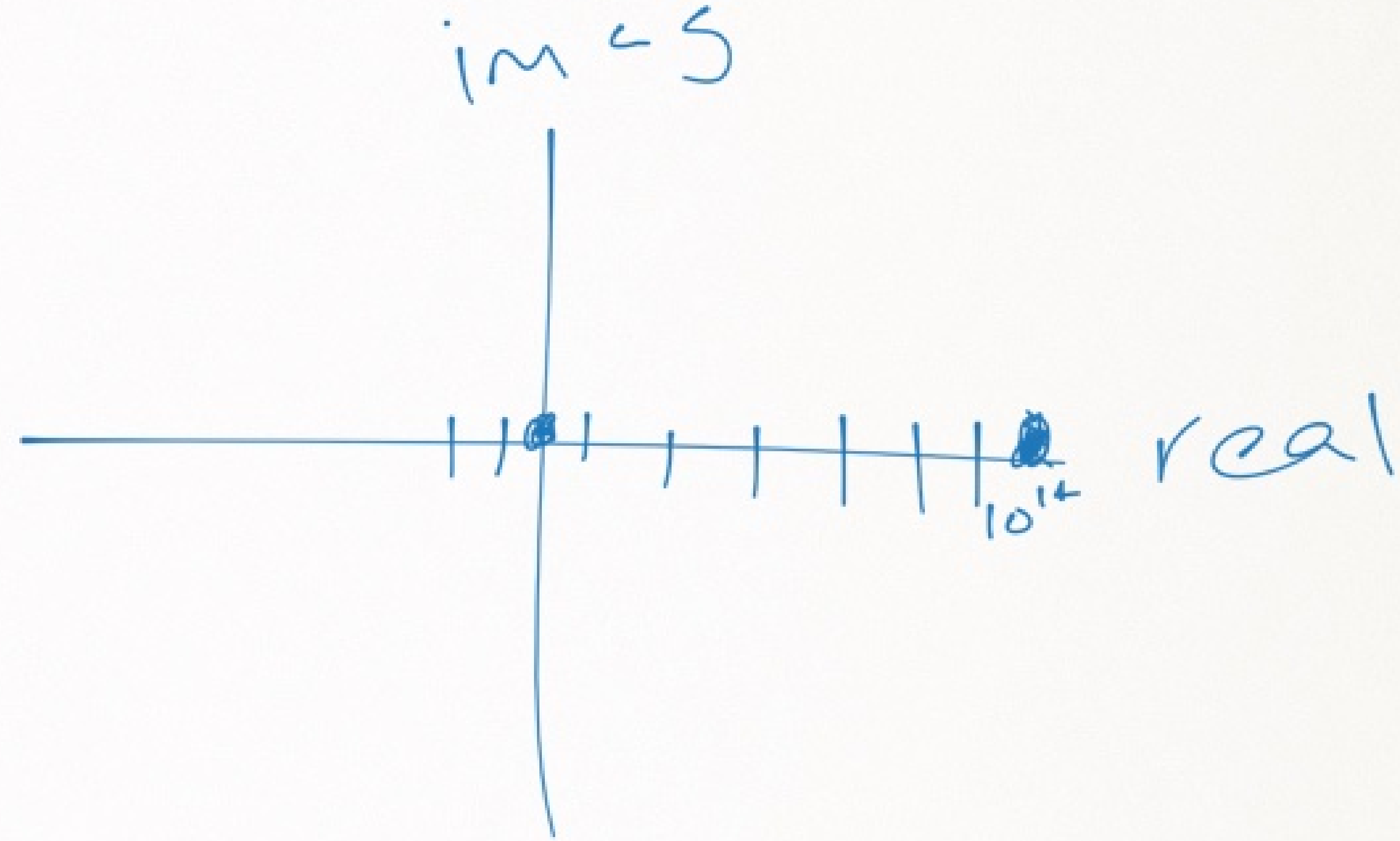
$$10^{9+3-2-2} = 10^8$$

$$10^6 \frac{\left(\frac{s}{10^3}+1\right)}{\left(\frac{s}{10^2}+1\right)\left(\frac{s}{10\angle 45}+1\right)\left(\frac{s}{10\angle -45}+1\right)}$$

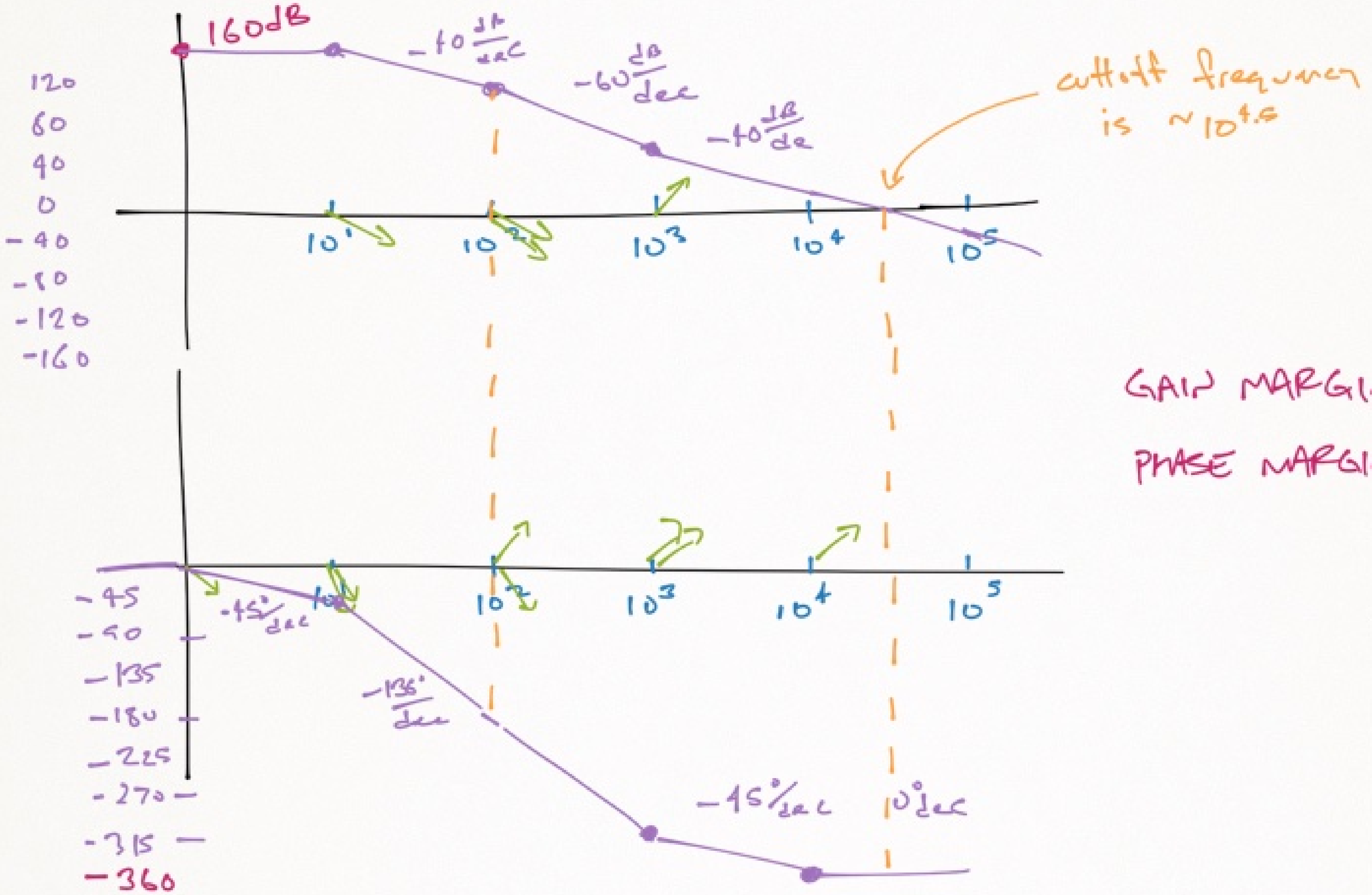
$K_{\omega=1} = 20\log(10^8) = 160\text{ dB}$

DRAW CONTOUR + DO ARC

	s	$\lvert KGH \rvert$	$\measuredangle KGH$
A	$(0,0)$	10^{14}	$\measuredangle \emptyset$
B	$(0,\infty)$	Δ	$\measuredangle 180$
C	$(\infty,0)$	Δ	$\emptyset$

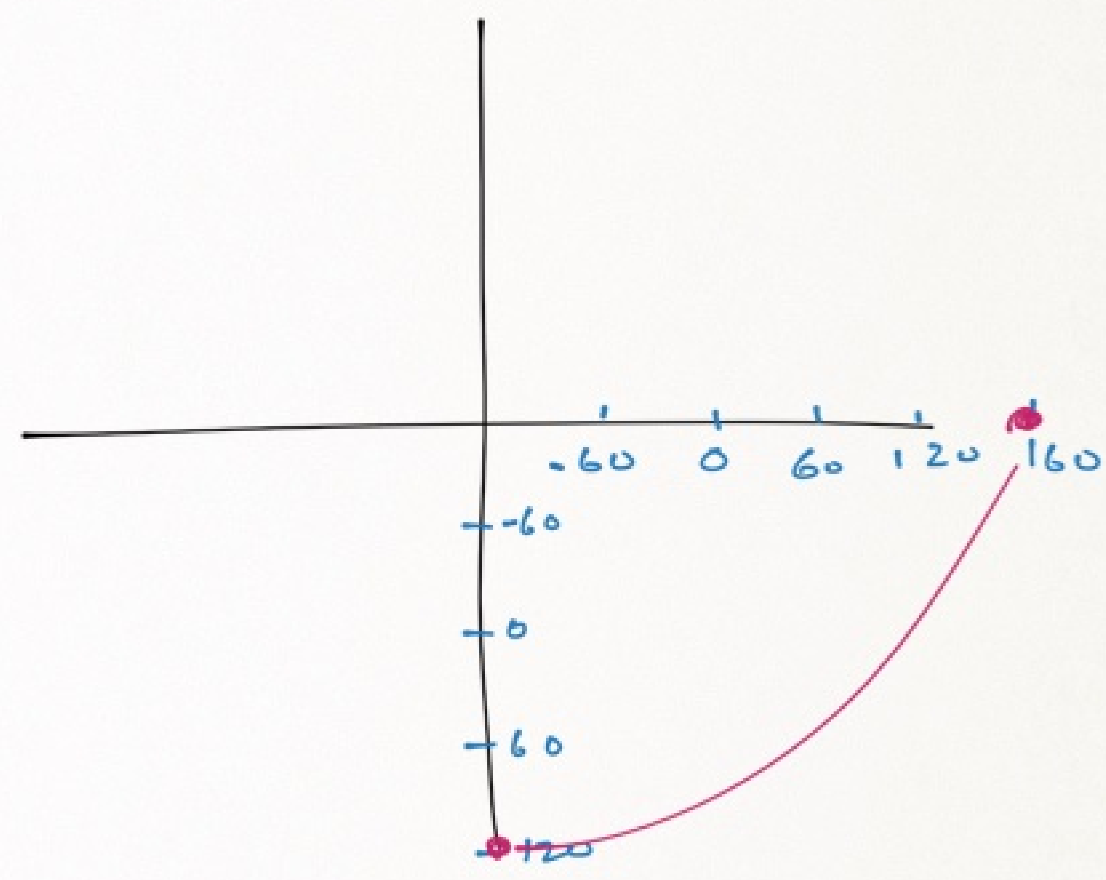


MAG. PLOT



GAIN MARGIN 120dB

PHASE MARGIN 360° = 0°?



STATE SPACE

↳ "SUPER DUPER CIRCUIT ANALYSIS"

↳ SOLVING EVERYTHING AT ONCE

NOV 17 / 2014

Cap: $\frac{1}{2}CV^2$

Ind: $\frac{1}{2}LI^2$

Work: Fd

Mass: $Mda = M\int v\,dt \, \frac{d}{dt}v = \frac{1}{2}mv^2$

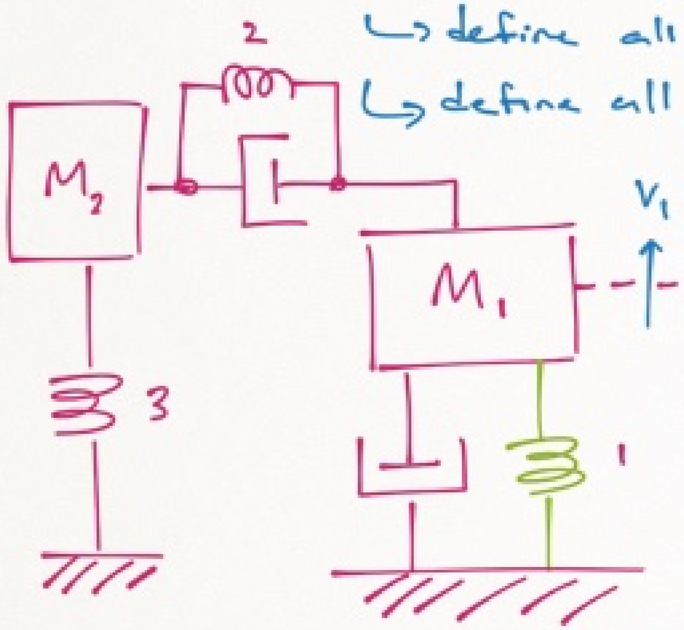
Spring: $Kdd = k\int d\,dt = \frac{1}{2}kd^2$

SYSTEM STATES → DEFINES HOW MUCH ENERGY A SYSTEM HAS AT A TIME

↳ all must be independent values

↳ define all energy in system

↳ define all possible initial conditions



STATES

$E_{M1} = \frac{1}{2}M_1 v_1^2$

$\underbrace{v_1, d_1}$ → defines energy in system

$E_{K1} = \frac{1}{2}k_1 d^2$

both independent, cannot calculate one from the other

$E_{M2} = \frac{1}{2}M_2 v_2^2$

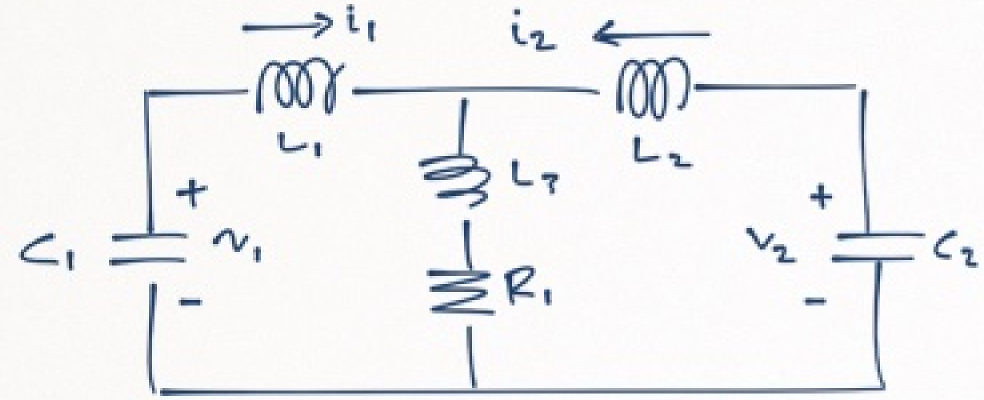
v_2, d_2

4 states

$E_{K2} = \frac{1}{2}k_2(d_2 - d_1)^2$

$E_{K3} = \frac{1}{2}k_3 d_3^2$ ← not it's own state.

ELECTRIC EXAMPLE STATE SPACE



STATES

$E_{C1} = \frac{1}{2} C_1 v_1^2$

$E_{L1} = \frac{1}{2} L_1 i_1^2$

$v_1,\ i_1$

$E_{C2} = \frac{1}{2} C_2 v_2^2$

$E_{L2} = \frac{1}{2} L_2 i_2^2$

$v_2,\ i_2$

$i_{L3} = i_1 + i_2 \longrightarrow i_1 + i_2$ is not its own state

STATE EQUATION

1. IDENTIFY ALL STATES
2. TREAT ALL STATES LIKE OUTPUT
3. WRITE EQUATION FOR EACH.
4. COMBINE IN MATRIX FORM
5. REARRANGE INTO:

$$\bar{\dot{x}} = \bar{\bar{A}}\,\bar{x} + \bar{\bar{B}}\,\bar{u}$$

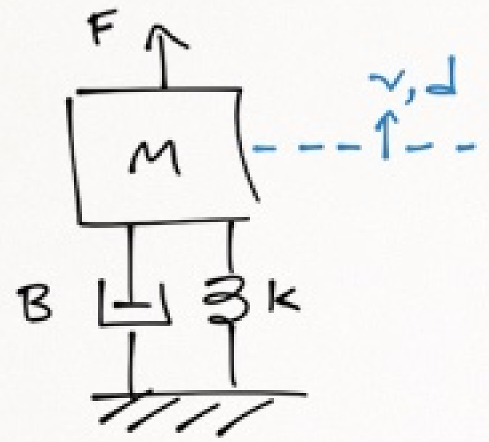
vector of inputs

vector of state derivatives

matrix

vector of states

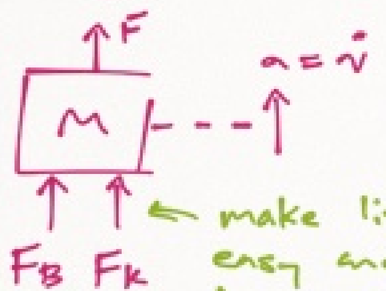
Ex)



$=$

STATES v, d

DO F.B.D.



make life easy and chose forces in same direction

$F_B + F_K = Ma = M\dot{v}$

F is input

$-Bv$ $-kd$

state   state   state derivative

Q WHY NEGATIVE? Q

$\bar{\dot{x}} = \begin{bmatrix} \dot{d} \\ \dot{v} \end{bmatrix}$ $\bar{x} = \begin{bmatrix} d \\ v \end{bmatrix}$ $\longrightarrow$

$\bar{\dot{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$

$\begin{bmatrix} \dot{d} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} ? \end{bmatrix} \begin{bmatrix} d \\ v \end{bmatrix} + \bar{\bar{B}}\bar{u}$

$\begin{bmatrix} \dot{d} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k}{M} & \frac{-B}{M} \end{bmatrix} \begin{bmatrix} d \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} F$

$\dot{v} = \frac{-k}{M} d - \frac{B}{M} v$

$\dot{d} = \cancel{0} d + \boxed{1} v$

usually get this one for free

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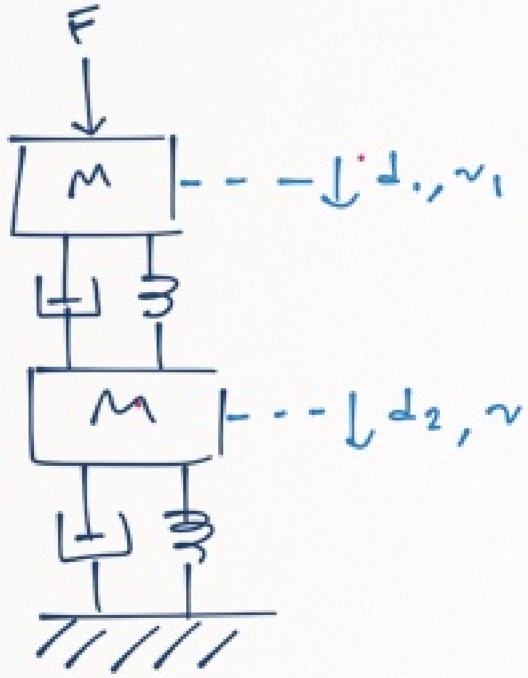
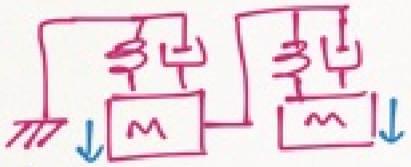
$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$





can also think of it like this

define vectors

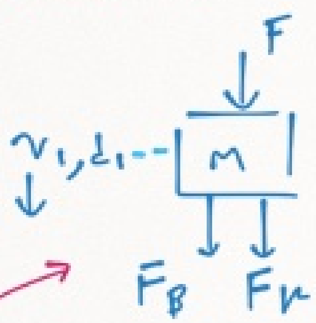
$$\bar{x} = \begin{bmatrix} d_1 \\ d_2 \\ v_1 \\ v_2 \end{bmatrix} \qquad \dot{\bar{x}} = \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} \qquad \bar{u} = F$$

states   inputs

define matrices

$$\begin{vmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{vmatrix} = \begin{vmatrix} \emptyset & \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 \\ -\frac{K}{M} & \frac{K}{M} & \frac{-B}{M} & \frac{B}{M} \\ \frac{K}{M} & \frac{-2K}{M} & \frac{B}{M} & \frac{-2B}{M} \end{vmatrix} \begin{vmatrix} d_1 \\ d_2 \\ v_1 \\ v_2 \end{vmatrix} + \begin{vmatrix} \emptyset \\ \emptyset \\ 1/M \\ \emptyset \end{vmatrix} F$$

KCL in mechanical system



$F + F_B + F_K = M\dot{v}_1$

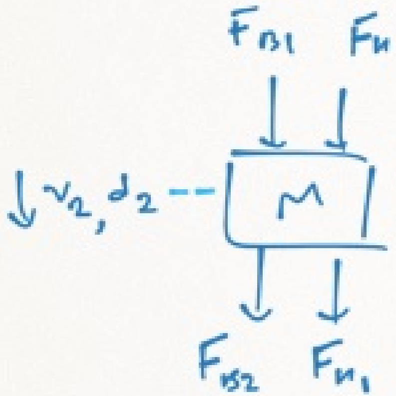
$\frac{F}{M} + \frac{B}{M}(v_2 - v_1) + k(d_2 - d_1) = \dot{v}_1$

$F_K = K(d_2 - d_1)$

watch sign

our drawing says it's stretching $\therefore v_2 > v_1 \therefore$ use $V_2 - V_1$ to keep positive

$F_B = F(v_2 - v_1)$



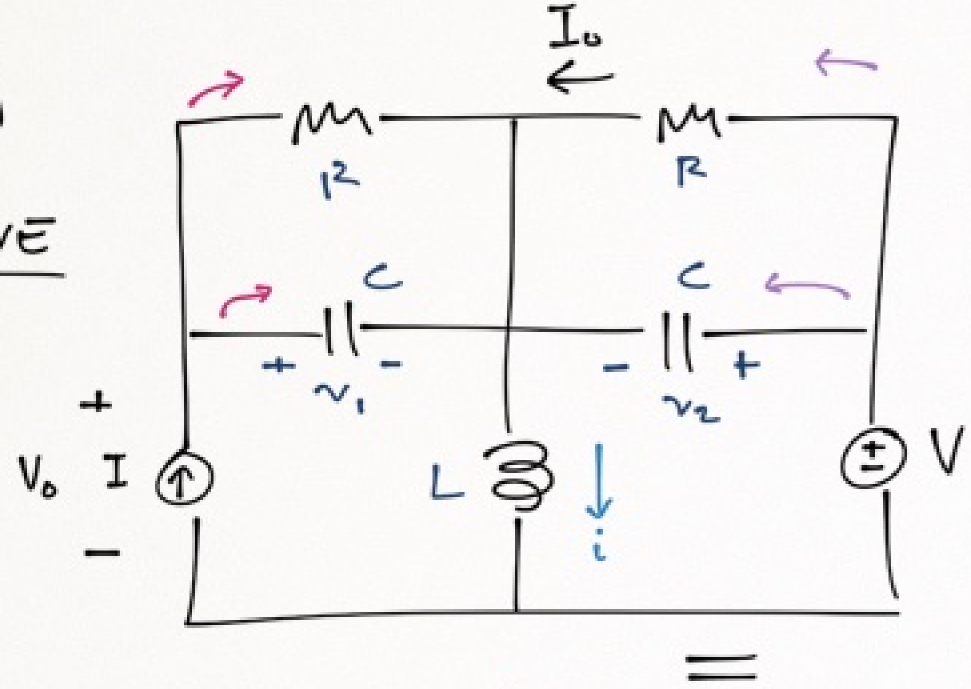
our drawing says it's compressing $\therefore v_1 > V_2 \therefore$ use $v_2 - v_1$ to keep positive

$F_{B1} + F_{K1} + F_{B2} + F_{K2} = \dot{v}_2 m$

$B(v_1 - v_2) + K(d_1 - d_2) - Bv_2 - Kd_1 = \dot{v}_2 M$

Ex)

SOLVE



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|| HINT: state vect $= \begin{vmatrix} v_1 \\ v_2 \\ i \end{vmatrix}$ $\quad \bar{x} = \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix}$ $\quad \bar{u} = \begin{vmatrix} V \\ I \end{vmatrix}$

$i = C\dot{v}$

$v = L\dot{i}$

$$\underset{3\times1}{\dot{\bar{x}}} = \underset{3\times3}{\bar{\bar{A}}}\,\underset{3\times1}{\bar{x}} + \underset{3\times2}{\bar{\bar{B}}}\,\underset{2\times1}{\bar{u}}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{i} \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC} & \emptyset & \emptyset \\ \emptyset & \frac{-1}{RC} & \frac{1}{C} \\ \emptyset & \frac{-1}{L} & \emptyset \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i \end{bmatrix} + \begin{bmatrix} \emptyset & \frac{1}{C} \\ \emptyset & \frac{-1}{C} \\ \frac{1}{L} & \emptyset \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}$$

$$I = C\dot{v}_1 + \frac{1}{R}v_1$$

$$\rightarrow \dot{v}_1 = \frac{1}{C}I - \frac{1}{RC}v_1$$

$$= \frac{-1}{RC}v_1 + \frac{1}{C}I$$

$$V = v_2 + L\dot{i}$$

$$\rightarrow \dot{i} = \frac{-1}{L}v_2 + \frac{1}{L}V$$

supernode:
$I_{IN} = I_{OUT} = I$

$$i = I + C\dot{v}_2 + \frac{1}{R}v_2$$

$$\rightarrow \dot{v}_2 = \frac{-1}{RC}v_2 + \frac{1}{C}i - \frac{1}{C}I$$

OUTPUT EQUATION (V_o, I_o)

$$\underset{2\times1}{\bar{y}} = \underset{2\times3}{\bar{\bar{C}}}\,\underset{3\times1}{\bar{x}} + \underset{2\times2}{\bar{\bar{D}}}\,\underset{2\times1}{\bar{u}}$$

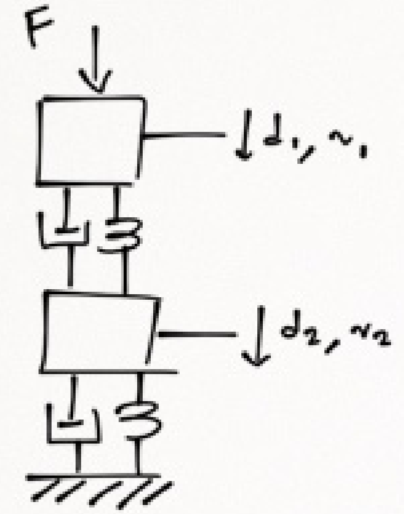
$$-v_1 + v_2 - V + V_o = \emptyset$$

$$\rightarrow V_o = v_1 - v_2 + V$$

$$I_o = \frac{1}{R}v_2$$

$$\begin{vmatrix} V_o \\ I_o \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{R} & 0 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \\ i \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} V \\ I \end{vmatrix}$$

Ex)



$$\Delta d = d_2 - d_1 \qquad \bar{y} = \bar{\bar{C}}\bar{x} + D\bar{u}$$
$$\Delta v = v_2 - v_1$$

(1) set up matrices

$$\begin{vmatrix} \Delta d \\ \Delta v \end{vmatrix} = \begin{vmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} \begin{vmatrix} d_1 \\ d_2 \\ v_1 \\ v_2 \end{vmatrix} + \begin{vmatrix} \emptyset \\ \emptyset \end{vmatrix} F$$

(2) transform into s-domain using laplace

subtracting initial conditions

$$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u} \xrightarrow{\mathcal{L}} s\bar{x} - \bar{x}_0 = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$$

$$s\bar{x} - \bar{\bar{A}}\bar{x} = \bar{x}_0 + \bar{\bar{B}}\bar{u}$$

$$s\bar{\bar{I}}\bar{x} - \bar{\bar{A}}\bar{x} = \bar{x}_0 + \bar{\bar{B}}\bar{u}$$

$$(s\bar{\bar{I}} - \bar{\bar{A}})\bar{x} = \bar{x}_0 + \bar{\bar{B}}\bar{u}$$

$$\bar{x} = \underbrace{(s\bar{\bar{I}} - \bar{\bar{A}})^{-1}}_{=\bar{\bar{\phi}}}\bar{x}_0 + \underbrace{(s\bar{\bar{I}} - \bar{\bar{A}})^{-1}}_{=\bar{\bar{\phi}}}\bar{\bar{B}}\bar{u}$$

total response

$$\bar{x} = \underbrace{\bar{\bar{\phi}}\bar{x}_0}_{\text{RESPONSE TO INITIAL CONDITIONS}} + \underbrace{\bar{\bar{\phi}}\bar{\bar{B}}\bar{u}}_{\text{RESPONSE TO FORCING FUNCTION}}$$

(natural resp.) RESPONSE TO INITIAL CONDITIONS

RESPONSE TO FORCING FUNCTION (forced resp.)

Ex CONT'D

Nov 24/2014

OUTPUT EQN.

$$\bar{y} = \bar{\bar{C}}\bar{x} + \bar{\bar{D}}\bar{u}$$

$$\bar{y} = \bar{\bar{C}}(\bar{\bar{\phi}}\bar{x}_0 + \bar{\bar{\phi}}\bar{\bar{B}}\bar{u}) + \bar{\bar{D}}\bar{u}$$

$$y = \underbrace{\bar{\bar{C}}\bar{\bar{\phi}}\bar{x}_0}_{\text{natural resp.}} + \underbrace{(\bar{\bar{C}}\bar{\bar{\phi}}\bar{\bar{B}} + \bar{\bar{D}})\bar{u}}_{\text{forced resp.}}$$

TF: $\bar{\bar{C}}\bar{\bar{\phi}}\bar{\bar{B}} + \bar{\bar{D}}$

wait for natural response to go away

① $\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$

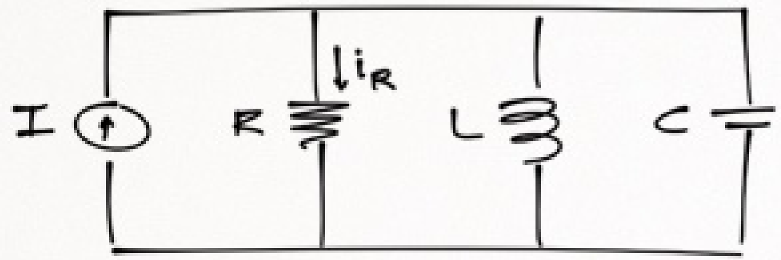
② $\bar{y} = \bar{\bar{C}}\bar{x} + \bar{\bar{D}}\bar{u}$

③ $\bar{\bar{\phi}} = [s\bar{\bar{I}} - \bar{\bar{A}}]^{-1}$

④ $\bar{x} = \bar{\bar{\phi}}\bar{x}_0 + \bar{\bar{\phi}}\bar{\bar{B}}\bar{u}$

⑤ $\bar{y} = \bar{\bar{C}}\bar{\bar{\phi}}\bar{x}_0 + \underbrace{(\bar{\bar{C}}\bar{\bar{\phi}}\bar{\bar{B}} + \bar{\bar{D}})}_{\text{TF}}\bar{u}$

Ex)



$y = i_R$

$u = I$

① identify energy

v_c, i_L

② validate, are any of these calculated by/dependent on eachother

$$\bar{x} = \begin{vmatrix} v \\ i \end{vmatrix} \qquad \dot{\bar{x}} = \underbrace{\begin{vmatrix} \frac{-1}{RC} & \frac{-1}{C} \\ \frac{1}{L} & \varnothing \end{vmatrix}}_{\bar{\bar{A}}} \bar{x} + \underbrace{\begin{vmatrix} \frac{1}{C} \\ \varnothing \end{vmatrix}}_{\bar{\bar{B}}} I$$

$v_c = i_c Z_c = i_c \frac{1}{sC}$

$i_c = sCv = Csv$

$v_L = i_L Z_L = i_L sL = Lsi$

$si = \frac{1}{L} v$

$I = i_R + i_L + Csv$

$I = \frac{1}{R} v + i_L + Csv$

$sv = \frac{-1}{RC} v - \frac{1}{C} i + \frac{1}{C} I$

$y = \frac{1}{R} v$

② $\bar{y} = \bar{\bar{C}} \bar{x} + \bar{\bar{D}} \bar{u}$

$$\bar{y} = \underbrace{\begin{bmatrix} \frac{1}{R} & \varnothing \end{bmatrix}}_{\bar{\bar{C}}} \bar{x} + \underbrace{\begin{bmatrix} \varnothing \end{bmatrix}}_{\bar{\bar{D}}} I$$

③ $\bar{\bar{\phi}} = [sI - A]^{-1}$

$$\bar{\bar{\phi}} = \left(\begin{bmatrix} s & \varnothing \\ \varnothing & s \end{bmatrix} - \begin{bmatrix} \frac{-1}{RC} & \frac{-1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \right)^{-1} = \begin{bmatrix} s + \frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & s \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2 + \frac{s}{RC} - \frac{1}{LC}} \begin{bmatrix} s & \frac{-1}{C} \\ \frac{1}{L} & s + \frac{1}{RC} \end{bmatrix}$$

$\frac{1}{det}$

$$\bar{\bar{C}} \bar{\bar{\phi}} = \frac{1}{det} \begin{bmatrix} \frac{1}{R} & \varnothing \end{bmatrix} \begin{bmatrix} s & \frac{-1}{C} \\ \frac{1}{L} & s + \frac{1}{RC} \end{bmatrix} = \frac{1}{det} \begin{bmatrix} \frac{s}{R} & \frac{-R}{C} \end{bmatrix}$$

$$\bar{\bar{C}} \bar{\bar{\phi}} \bar{\bar{B}} + \underset{\varnothing}{\bar{\bar{D}}} = \frac{1}{det} \begin{bmatrix} \frac{s}{R} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} \frac{1}{C} \\ \varnothing \end{bmatrix} = \frac{\frac{s}{RC}}{s^2 + \frac{1}{RC} s + \frac{1}{LC}} \quad \text{TF}$$

CE

Notice $TF = \bar{\bar{C}}\,\bar{\bar{\Phi}}\,\bar{\bar{B}} + \bar{\bar{D}}$

$$= \bar{\bar{C}}(s\bar{\bar{I}} - \bar{\bar{A}})^{-1}\bar{\bar{B}} + \bar{\bar{D}}$$

$$= \underbrace{\bar{\bar{C}}(s\bar{\bar{I}} - \bar{\bar{A}})^{-1}}\bar{\bar{B}} + \underbrace{(s\bar{\bar{I}} - \bar{\bar{A}})^{-1}(s\bar{\bar{I}} - \bar{\bar{A}})}_{\bar{\bar{I}}}D$$

$$\frac{1}{\det(s\bar{\bar{I}} - \bar{\bar{A}})}\left[\approx\right]$$

CE: $\det(s\bar{\bar{I}} - \bar{\bar{A}}) = \emptyset$

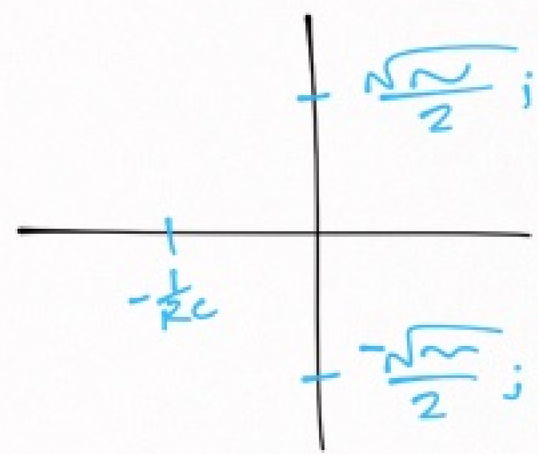
$\det(\bar{\bar{\Phi}}^{-1}) = \emptyset$

REMEMBER EIGENVALUES roots of CE : eigenvalues$(\bar{\bar{A}})$

$\det(\lambda\bar{\bar{I}} - \bar{\bar{A}}) = \emptyset$

CE: $s^2 + \frac{1}{RC}s + \frac{1}{LC} = \emptyset$

$$\frac{-\frac{1}{RC} \pm \sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}}{2}$$



NOV 26/2014

$$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u} \qquad \xrightarrow{\mathcal{L}} \qquad s\bar{x} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$$
$$\bar{y} = \bar{\bar{C}}\bar{x} + \bar{\bar{D}}\bar{u} \qquad\qquad\qquad \bar{y} = \bar{\bar{C}}\bar{x} + \bar{\bar{D}}\bar{u}$$

GRAPHICAL METHOD

1. DRAW INPUT NODES
2. DRAW ALL STATES & STATE DERIVS. AS NODES
3. DRAW ALL OUTPUT NODES
4. ADD CONNECTIONS (GAINS) → matrix values
5. REDRAW

ABOUT EXAM

SIGNAL FLOW

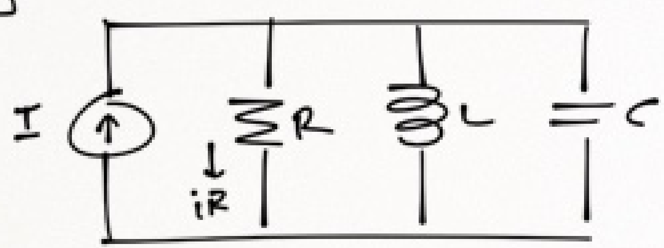
what val. crit damped
nat. frequency?
what's zeta?
what's settle time?

"DO YOUR WORK RIGHT"

"I'M TRAINING YOU TO BE AN ENGINEER & ENGINEERS DO WORK RIGHT"

"HALF-RIGHT ISN'T RIGHT"

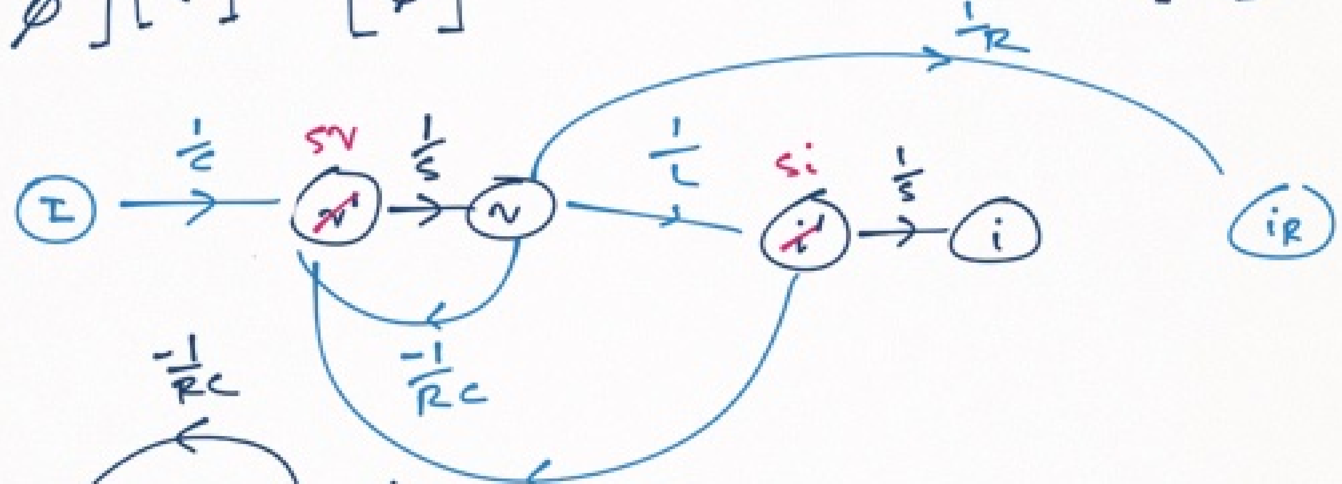
Ex|

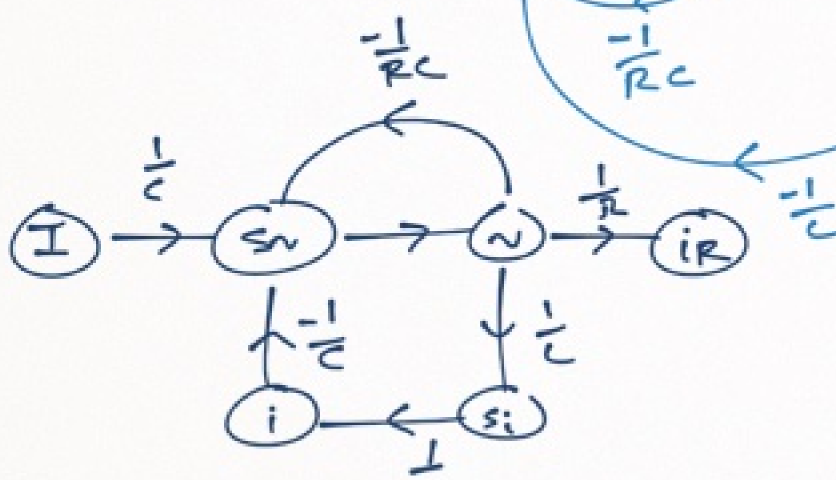


DRAW SIGNAL FLOW DIAGRAM

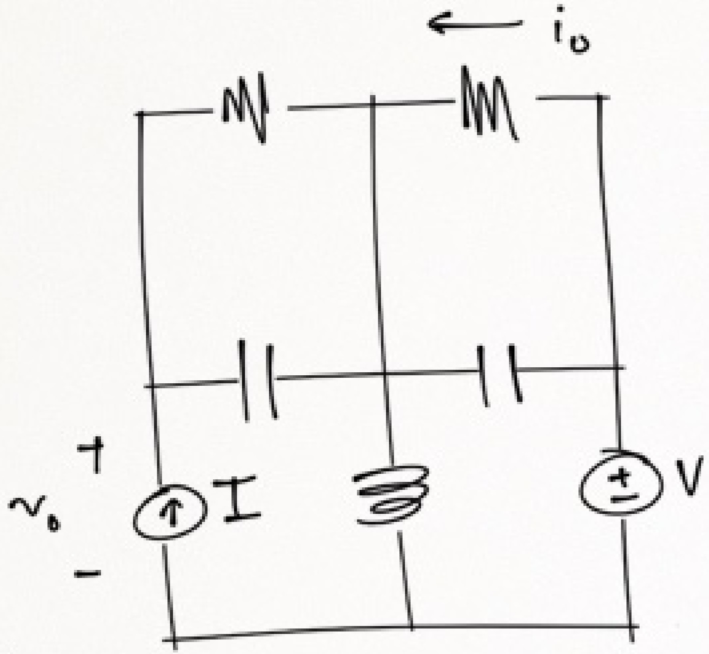
$$\begin{bmatrix} v' \\ i' \end{bmatrix} = \begin{bmatrix} \frac{-1}{RL} & \frac{-1}{C} \\ \frac{1}{L} & \emptyset \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \emptyset \end{bmatrix} I \qquad i_R = \begin{bmatrix} \frac{1}{R} & \emptyset \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \emptyset I$$

using primes for derivatives because this $\dot{i}$ shit is ridiculous.





Ex)



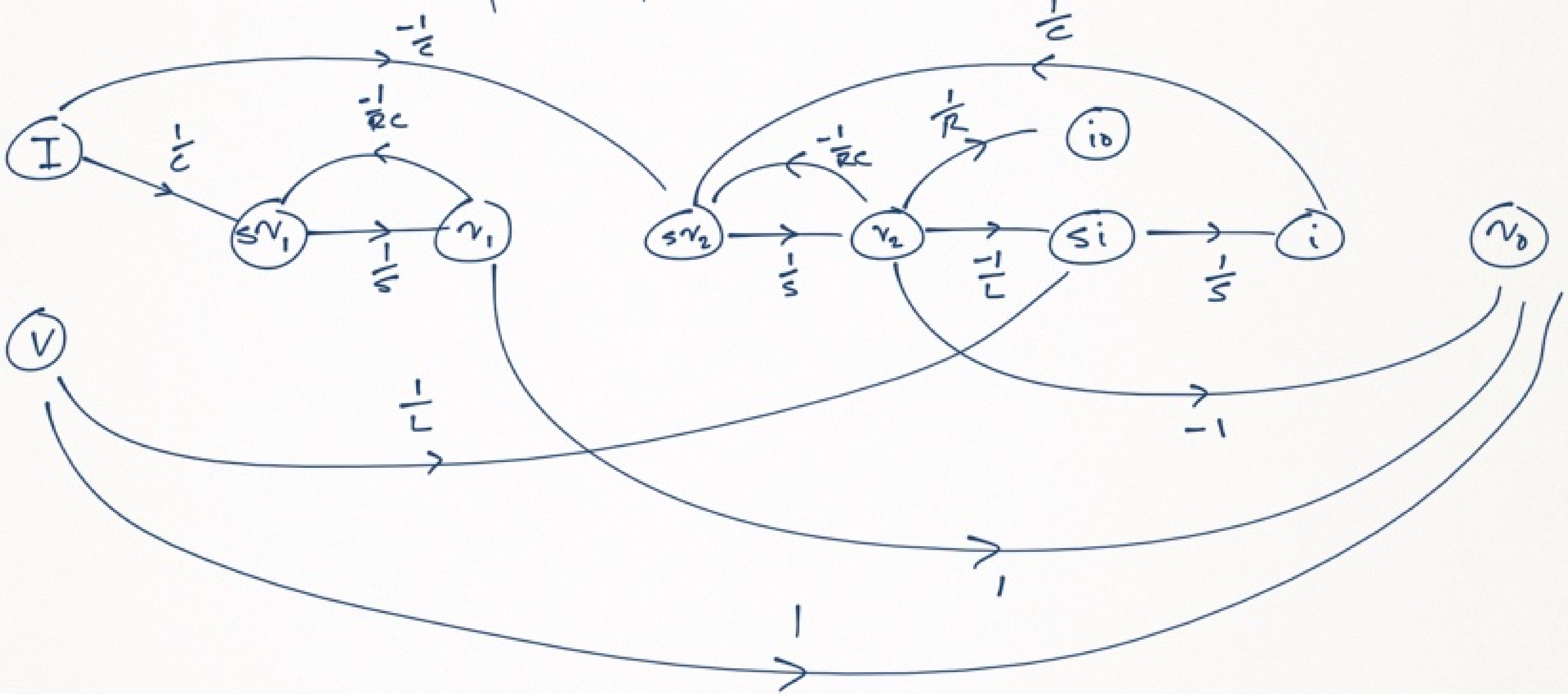
DRAW SIGNAL FLOW DIAGRAM

DETERMINE CE.

inputs V, I

outputs v_o, i_o

$$\begin{vmatrix} v_1' \\ v_2' \\ i' \end{vmatrix} = \begin{vmatrix} \frac{-1}{RC} & \emptyset & \emptyset \\ \emptyset & \frac{-1}{RC} & \frac{1}{C} \\ \emptyset & \frac{-1}{L} & \emptyset \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix} + \begin{vmatrix} \frac{1}{C} & \emptyset \\ \frac{-1}{C} & \emptyset \\ \emptyset & \frac{1}{L} \end{vmatrix} \begin{vmatrix} I \\ V \end{vmatrix} , \quad \begin{vmatrix} v_o \\ i_o \end{vmatrix} = \begin{vmatrix} 1 & -1 & \emptyset \\ \emptyset & \frac{1}{R} & \emptyset \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix} + \begin{vmatrix} \emptyset & 1 \\ \emptyset & \emptyset \end{vmatrix} \begin{vmatrix} I \\ V \end{vmatrix}$$



Ex) CONT'D

CE IS FROM eigs of $\bar{\bar{A}}$

$$\det(s\bar{\bar{I}} - \bar{\bar{A}}) = \emptyset$$

$$\det\begin{vmatrix} s+\frac{1}{RC} & \emptyset & \emptyset \\ \emptyset & s+\frac{1}{RC} & -\frac{1}{C} \\ \emptyset & \frac{1}{L} & s \end{vmatrix} = \left(s+\frac{1}{RC}\right)\left(s^2+\frac{s}{RC}+\frac{1}{LC}\right) = s^3 + s^2\frac{2}{RC} + s\left(\frac{1}{LC}+\frac{1}{RC^2}\right) + \frac{1}{RLC^2} = \emptyset$$

third order b/c 3 stored energy systems

ROUTH-HURWITZ

$$\begin{array}{ccccc} a & c & e & g & \emptyset \\ b & d & f & h & \emptyset \\ m_1 & m_2 & m_3 & \emptyset & \\ n_1 & n_2 & n_3 & \emptyset & \\ o_1 & o_2 & \emptyset & & \\ p_1 & p_2 & \emptyset & & \\ q_1 & \emptyset & & & \\ r_1 & \emptyset & & & \end{array}$$

$$m_1 = \frac{-1}{b}(ad - cb) \qquad n_1 = \frac{-1}{m_1}(bm_2 - dm_1)$$

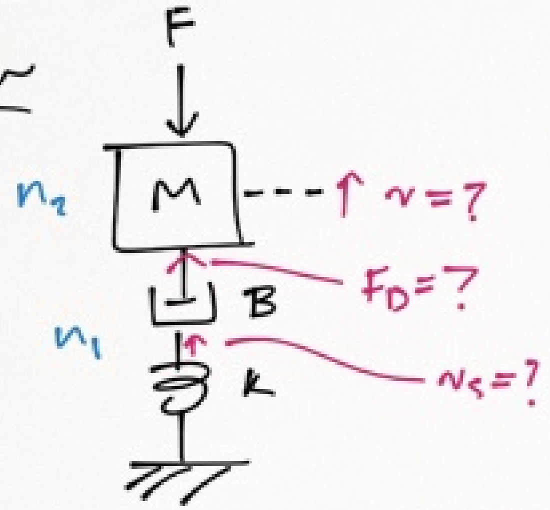
$$m_2 = \frac{-1}{b}(cf - be) \qquad n_2 = \frac{1}{-m_1}(bm_3 - fm_1)$$

$$m_3 = \frac{-1}{b}(ah - bg) \qquad n_3 = \frac{-1}{m_1}(b\emptyset - hm_1)$$

$$o_1 = \frac{-1}{n_1}(m_1n_2 - n_1m_2) \qquad p_1 = \frac{-1}{o_1}(n_1o_2 - o_1n_2)$$

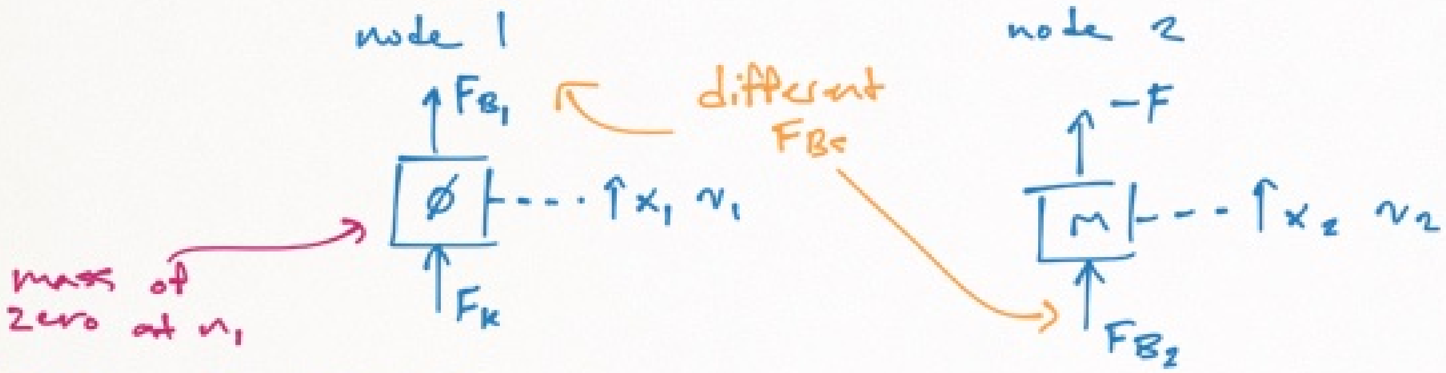
$$o_2 = \frac{-1}{n_1}(m_1n_3 - n_1m_3) \qquad p_2 = \frac{-1}{o_2}(n_1\emptyset - o_1n_3)$$

REVIEW



everything defined as upward even though force downward

1. F.B.D. at each node



$F_{B_1} = -F_{B_2}$

2. $\Sigma F = ma$ $\quad F_{B_1} + F_K = \emptyset$ $\quad$ ↖ $ma = \emptyset \dot{v}_1 = \emptyset$

$F_{B_2} - F = M\dot{v}_2$

$v_1 > v_2$

$B(v_2 - v_1) + K(-x_1) = \emptyset$

$\underline{B(v_1 - v_2) - F = M\dot{v}_2}$ $\longrightarrow B(\dot{x}_1 - v_2) - F = M\dot{v}_2$

↳ $\dot{v}_2 = \frac{B}{M}\dot{x}_1 - \frac{B}{M}v_2 - \frac{1}{M}F$

$\underline{Bv_2 - Bv_1 - Kx_1 = \emptyset}$ $\longrightarrow Bv_2 - B\dot{x}_1 - Kx_1 = \emptyset$

↘ $\dot{x}_1 = v_2 - \frac{K}{B}x_1$

↓

$\underbrace{\dot{v}_2 = \frac{B}{M}v_2 - \frac{B}{M}x_1 - \frac{B}{M}v_2 - \frac{1}{M}F}_{\text{for next page}}$

3. $\dot{x}_1 = v_1$

$\dot{x}_2 = v_2$

4. determine states $\longrightarrow$ mass & spring have energy determined by v_2 & x_1 $\quad \begin{vmatrix} x_1 \\ v_2 \end{vmatrix}$

5. get rid of non-states

6. $\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{v}_2 \end{vmatrix} = \begin{vmatrix} -\frac{k}{B} & 1 \\ -\frac{k}{M} & \emptyset \end{vmatrix} \begin{vmatrix} x_1 \\ v_2 \end{vmatrix} + \begin{vmatrix} \emptyset \\ \frac{-1}{M} \end{vmatrix} F$$

7. output equation

$$\begin{vmatrix} v_M \\ v_S \\ F_{B_2} \end{vmatrix} = \begin{vmatrix} \emptyset & 1 \\ -\frac{k}{B} & 1 \\ -k & \emptyset \end{vmatrix} \begin{vmatrix} x_1 \\ v_2 \end{vmatrix} + \begin{vmatrix} \emptyset \\ \emptyset \\ \emptyset \end{vmatrix} F$$

$v_S = \dot{x}_1$

$$\underbrace{F_B = M\dot{v}_2 + F}_{?} \underbrace{= -kx_1}$$

$$\bar{\bar{\Phi}} = [s\bar{\bar{I}} - \bar{\bar{A}}]^{-1} = \begin{bmatrix} s+\frac{k}{B} & -1 \\ \frac{k}{M} & s \end{bmatrix}^{-1} = \frac{1}{s^2 + \frac{k}{B}s + \frac{k}{M}} \begin{bmatrix} s & 1 \\ -\frac{k}{M} & s+\frac{k}{B} \end{bmatrix}$$

TF = ?
CE = ? $\leftarrow \det(s\bar{\bar{I}} - \bar{\bar{A}})$

do states
calc state matrix
C.E.?
poles etc.?

$2\zeta\omega_n$

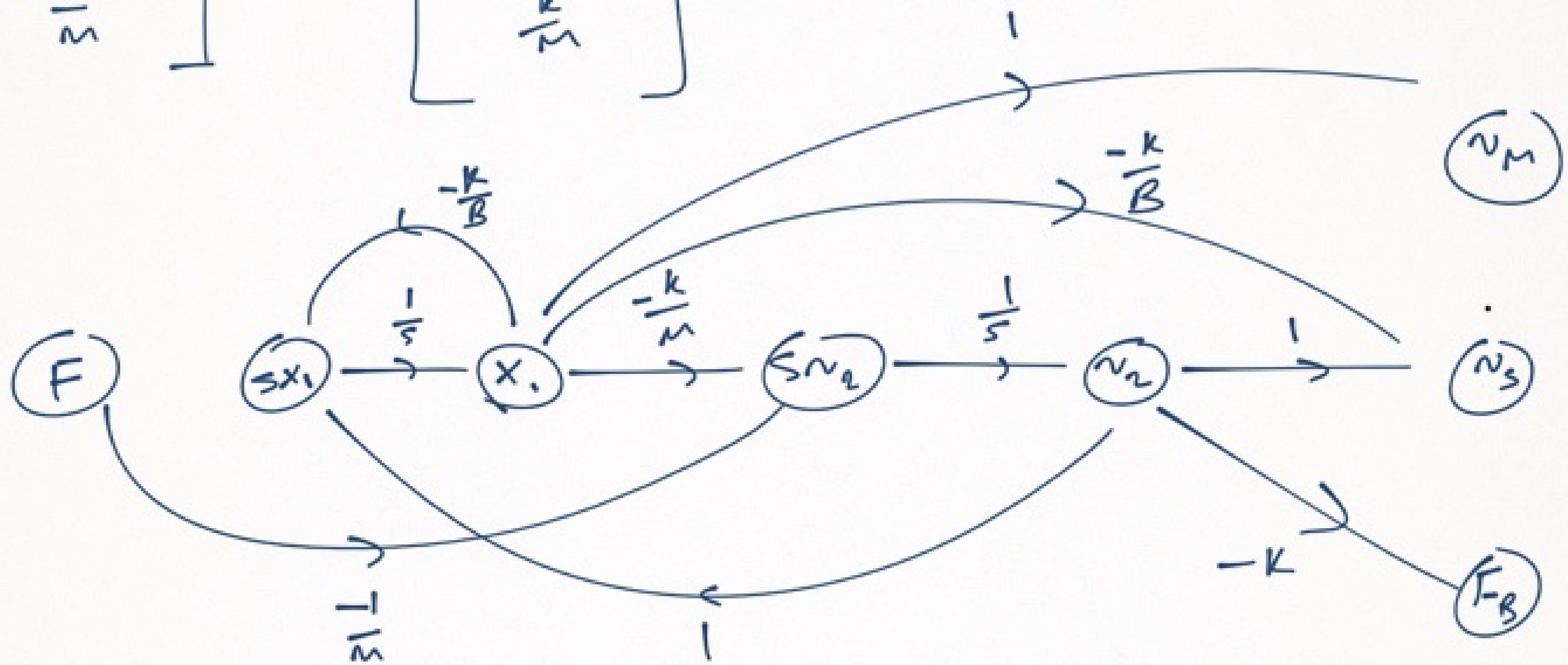
CE: $s^2 + \frac{k}{B}s + \frac{k}{M} = \emptyset$

$\omega_n = \sqrt{\frac{k}{M}}$

$$\zeta = \frac{k}{B(2\omega_n)} = \frac{k}{2B}\frac{1}{\sqrt{k/M}} = \frac{k}{2B}\frac{\sqrt{M}}{\sqrt{K}}$$

TF $\quad C\Phi B + D = \frac{1}{\det}\begin{bmatrix} 0 & 1 \\ -k/B & 1 \\ -k & 0 \end{bmatrix}\begin{bmatrix} s & 1 \\ -k/M & s+\frac{k}{B} \end{bmatrix}\begin{bmatrix} \emptyset \\ -\frac{1}{M} \end{bmatrix} = \frac{1}{\det}\begin{vmatrix} 0 & 1 \\ -k/B & 1 \\ -k & 0 \end{vmatrix}\begin{vmatrix} -\frac{1}{M} \\ -\frac{s}{M} - \frac{k}{BM} \end{vmatrix}$

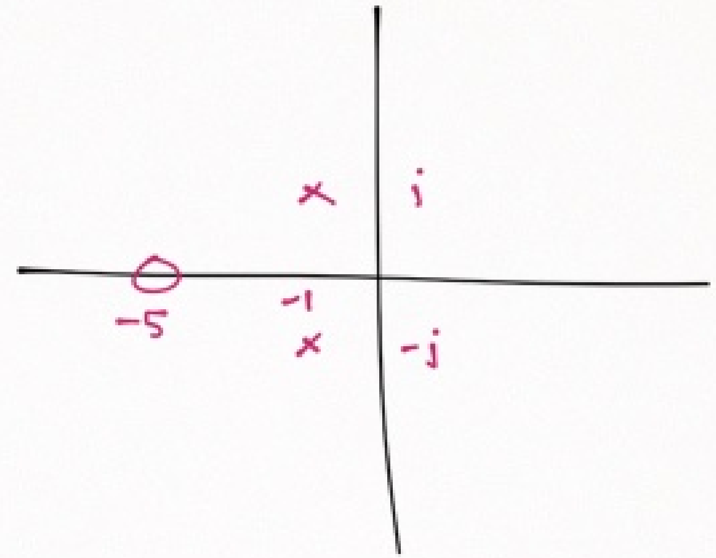
$$= \frac{1}{\det}\begin{bmatrix} -\frac{s}{M} - \frac{k}{BM} \\ \frac{k}{BM} - \frac{s}{M} - \frac{k}{BM} \\ -\frac{k}{M} \end{bmatrix} = \frac{1}{\det}\begin{bmatrix} -\frac{s}{M} - \frac{k}{BM} \\ -\frac{s}{M} \\ \frac{k}{M} \end{bmatrix}$$



NOV 28 2014

REVIEW #2

$$\frac{k(s+5)(s^2+2s+2)}{1+\frac{k(s+5)}{(s+1)(s^2+2s+2)}}$$



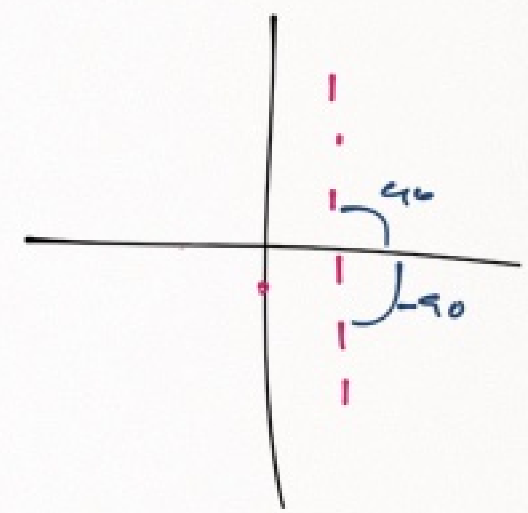
$$\frac{-2\pm\sqrt{4-8}}{2}$$

$XS = 3 - 1 = 2$

odd or even?

real axis is asymptote

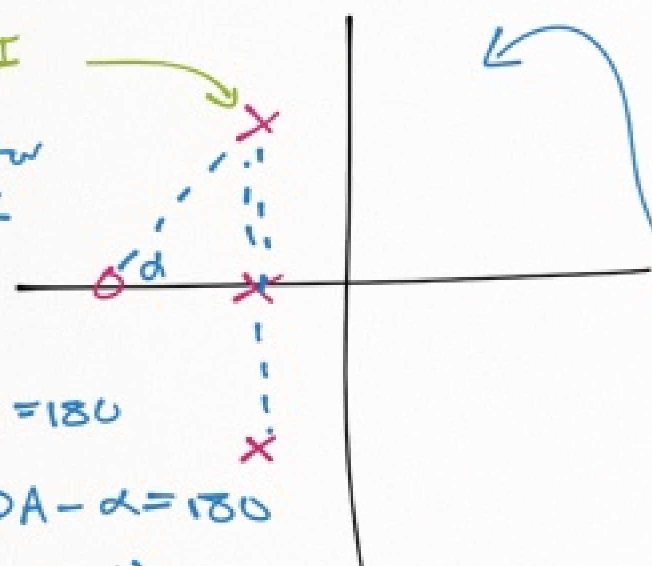
not a real axis & you can see 90°



Assym: ±90°

POI

draw vector from POI to each vector



$$AC = \frac{\Sigma P - \Sigma Z}{XS} = \frac{-3+5}{2} \to 1$$

departure angle?

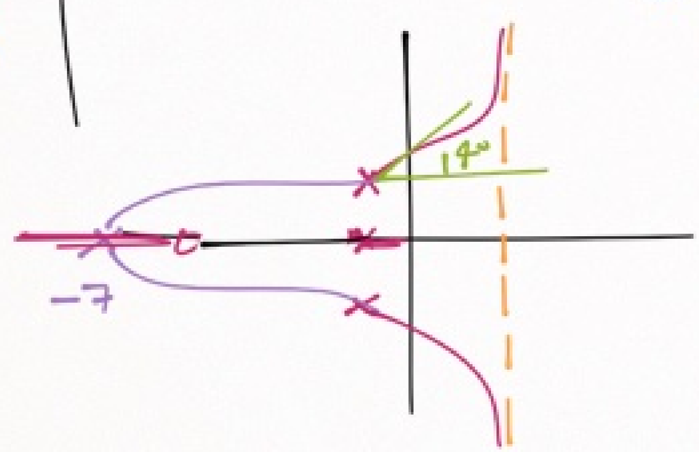
~~arrival angle?~~ N/A need complex zeros

$\Sigma P - \Sigma Z = 180$

$90 + 90 + DA - \alpha = 180$

$\alpha = \tan^{-1}\left(\frac{1}{4}\right)$

$\therefore DA = 14°$



break points

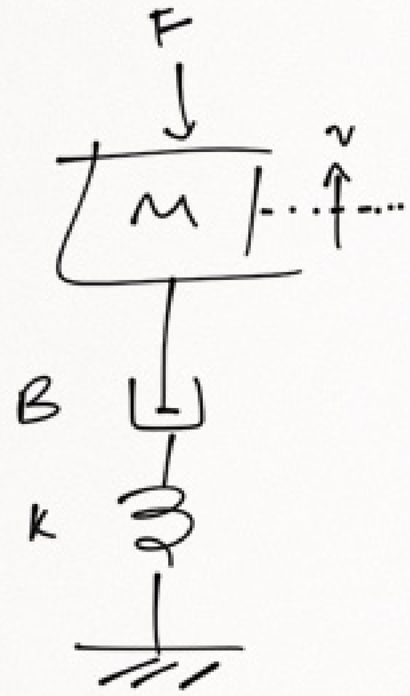
$$\frac{d}{ds}\frac{1}{GH} = \emptyset$$

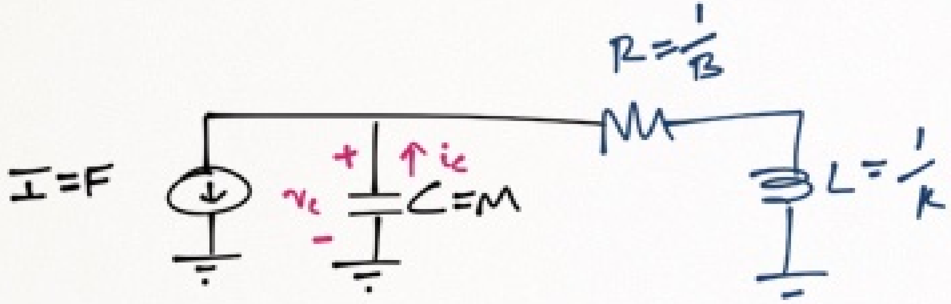
$(s+7)(\cancel{s + \ldots x^2} \sim)$

-7

we only care about breakpoints on real axis

$\frac{v}{F} = ?$





velocity of mass corresponds to voltage on capacitor

apply force, increase velocity down
∴ reduce velocity up

∴ current down

current divider

$Z_C = \frac{1}{sC}$ $\quad Z_{RL} = R + sL$

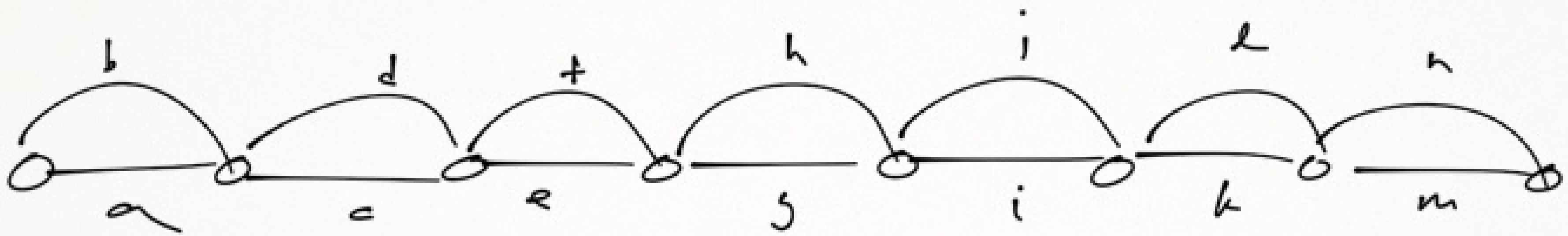
$Y_C = sC$ $\quad Y_{RL} = \frac{1}{R+sL}$

$$i_C = \frac{Y_C}{Y_{RL}+Y_C} I = \frac{sC}{sC+\frac{1}{R+sL}} I = \frac{sC(R+sL)}{s^2LC+sRC+1} I$$

$$v_C = i_C Z_C = \frac{-1}{sC}\cdot i_C = \frac{-(R+sL)}{s^2LC+sRC+1} I$$

$$\frac{v_C}{I} = \frac{-\frac{1}{C}s - \frac{R}{LC}}{s^2+s\frac{R}{L}+\frac{1}{LC}} = \frac{-s\frac{1}{M} - \frac{K}{BM}}{s^2+\frac{K}{B}s+\frac{K}{M}} \quad ✓$$

$$\frac{i_1}{I} = \frac{R_2}{R_1+R_2} = \frac{G_1}{G_1+G_2}$$



$P = acegikm$

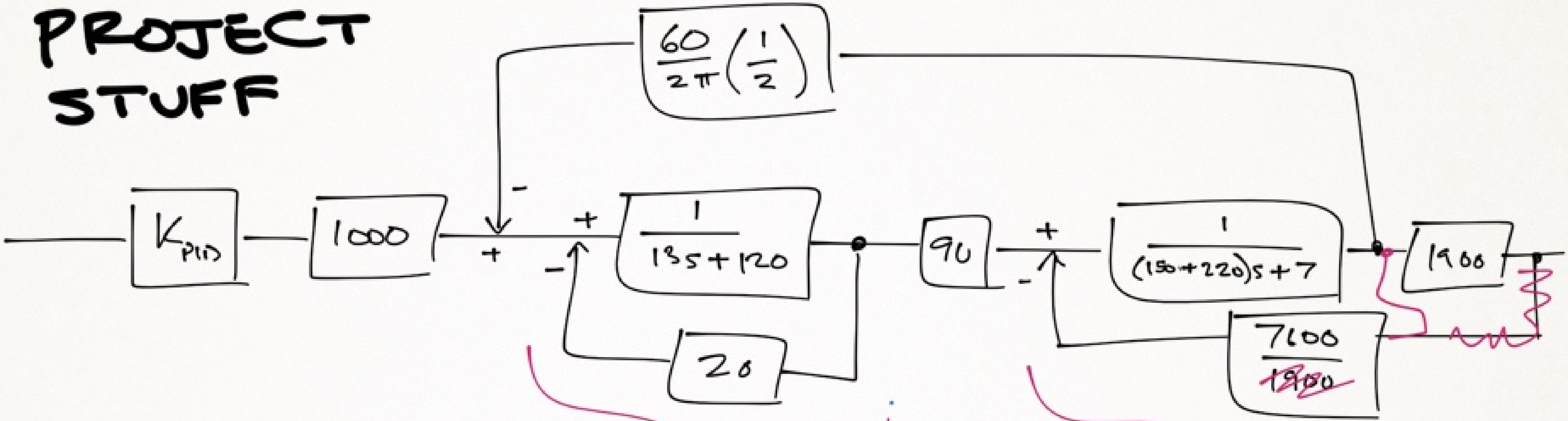
$L_1 = ab \quad L_2 = cd \quad L_3 = ef \quad \cdots$

$$\sum \pi_1 = L_1 + L_2 + L_3 + \cdots L_7$$

$$\begin{aligned}\sum \pi_2 = {} & L_1L_3 + L_1L_4 + L_1L_5 + L_1L_6 + L_1L_7 \\ & + L_2L_4 + L_2L_5 \cdots + L_2L_7 \\ & + L_3L_5 + L_3L_6 + L_3L_7 \\ & + L_4L_6 + L_4L_7 + L_5L_7\end{aligned}$$

$$\begin{aligned}\sum \pi_3 = {} & L_1L_3L_5 + L_1L_3L_6 + L_1L_3L_7 \\ & + L_1L_4L_6 + L_1L_4L_7 + L_1L_5L_7\end{aligned}$$

PROJECT STUFF



$$\frac{\frac{1}{13s+120}}{1+\frac{20}{13s+120}} \qquad \frac{\frac{1}{370s+7}}{1+\frac{7600}{370s+7}}$$

$$\underbrace{\frac{1}{6700s^2}\left[\left[1000\cdot\frac{\left(\frac{\frac{1}{13s+120}}{1+\frac{20}{13s+120}}\right)(90)\left(\frac{\frac{1}{370s+7}}{1+\frac{7600}{370s+7}}\right)}{1+\frac{60}{4\pi}\cdot 90\cdot\frac{\frac{1}{13s+120}}{1+\frac{20}{13s+120}}\cdot\frac{\frac{1}{370s+7}}{1+\frac{7600}{370s+7}}}\cdot 1900\right]-(6700\cdot 9.81)\right]}_{\text{OPEN LOOP TRANSFER FUNCTION}}$$

OPEN LOOP TRANSFER FUNCTION SIMPLIFIED

$$\frac{-9.81s^2 - 307.33s - 2167.598}{s^4 + 31.33s^3 + 221.5s^2} = G(s)$$

CLOSED LOOP

$$\frac{G(s)}{1 + G(s)}$$ ← characteristic eqn.

CHARACTERISTIC EQN

$$1 + KGH = K_{PID} \cdot G(s) + 1$$

~~ZEROS @ s=-20.605, s=-10.723~~

~~POLES @ s=0, s=0, -10.77836, -20.5503256627~~

zeros @ s=-3.1257, s=3.1245
s=-10.78346, s=-20.549

poles @ s=0, s=0
s=-10.77836, s=-20.55

ROUTH-HURWITZ

$$1s^4 + 31.33s^3 + 221.5s^2 + \emptyset s + k_c + 1$$

1	221.5	(k_c+1)
31.33	$\emptyset$	
221.47	(k_c+1)	

6.5594

721 522 624.6
0 622 1014

GRAY: (6.8)

722 5228 624.4
0 630 1010

MAW 721 522 624

TAIL 0 620 1012

6.57

6.55

MAW 724 518 624

TAIL 100 500 750

1 1414 35092.87 −480120.6 K_c+1

62.6573 13263.87 −135982.25 ∅

726 525 627
300 660 930

6.4901

65438

721 522 624

90 491 964

721 522 624
90 ~~499.5~~ 986
500

726 525 627
450 860 900

6.47

457 860 875
6.478

726 525 627
465 865 871
625 6.4699

6.5340

65433

721 522 624
90 493 964

721 522 624
90 501.5 986

6.5299

725 523
88 ~~510~~ 995
511
~~6.5196~~
65174.

6.5405

721 522 624
90 493 964

725 524 626
0 ~~650~~ ~~1015~~

63216

only main fuck.

6.5400

721 522 624
90 ~~495.5~~ 964
496

726 525 627
~~0~~ ~~625~~ ~~1039~~

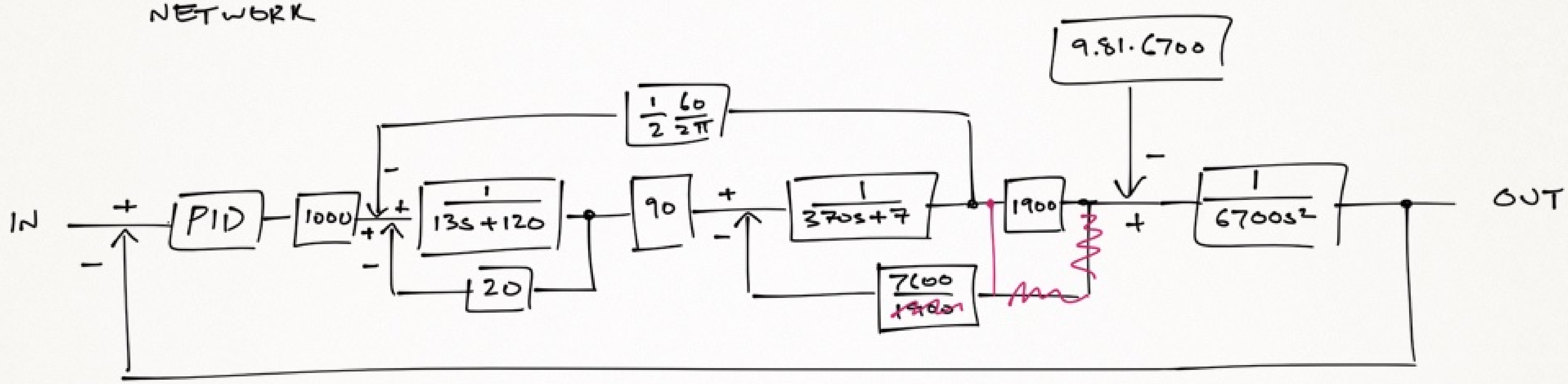
6.3191 240 746 ~~994~~

6.5633

726 525 627
250 750 950

6.5037

NETWORK



$$G_1(s) = 90 \cdot \frac{\frac{1}{13s+120}}{1+\frac{20}{13s+120}} \cdot \frac{\frac{1}{370s+7}}{1+\frac{7600}{370s+7}} = \frac{90}{4810s^2+150691s+1064980}$$

$$1000 \cdot \frac{G(s)}{1+\frac{60}{4\pi}G(s)} \cdot 1900 = \frac{171 \cdot 10^6 \pi}{(4810s^2+150691s+1064980)\pi+1350}$$

① OPEN LOOP TF EXCLUDING PLANT AND DISTURBANCES

$$\left[\frac{171 \cdot 10^6 \pi}{(4810s^2+150691s+1064980)\pi+1350} - 9.81(6700)\right]\frac{1}{6700s^2} \simeq \frac{-9.81s^2-307.33s+2167.6}{s^4+31.32s^3+221.4989s^2}$$

② OPEN LOOP TF INCLUDING DISTURBANCES

DOES NOT MATCH MODEL

$$\frac{171 \cdot 10^4 \pi}{(322270s^4+10096297s^3+71353660s^2)\pi+90450s^2}$$

③ OPEN LOOP TF EXCLUDING DISTURBANCES

MATCHES MODEL

ROUTH HURWITZ

OF CLOSED LOOP CE OF 1

$$4810\pi \qquad 1064980\pi + 1350 + k_u + 1$$

$$150691\pi \qquad \emptyset$$

$$\emptyset < \left[\emptyset - 150691\pi(1064980\pi + 1350 + k_u + 1) \right] \frac{-1}{150691\pi}$$

$$\emptyset < 1064980\pi + 1350 + k_u + 1$$

$$k_u > -3347084.34422$$

$$\therefore k_p = 0.45(k_u) \simeq -1.5062 \cdot 10^6$$ UNACCEPTABLE

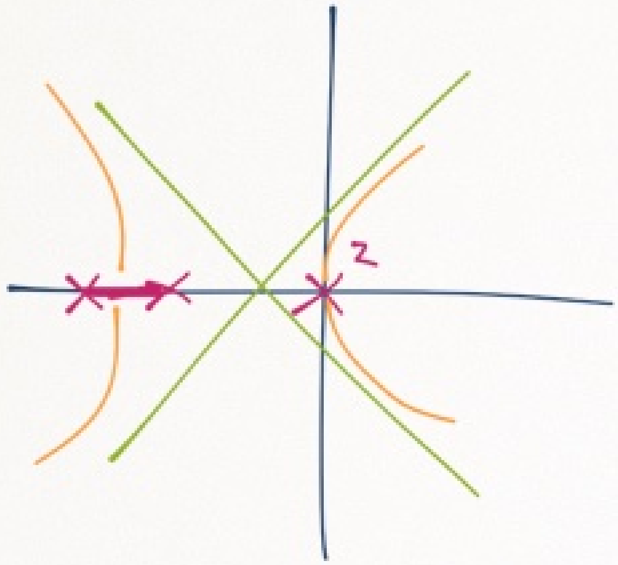
OF CLOSED LOOP CE OF 3

$$322270\pi \qquad 71353660\pi + 90450 \qquad 1 + k_u$$

$$10096297\pi \qquad \emptyset$$

NO VALUE OF k_u CAN PREVENT SIGN CHANGE

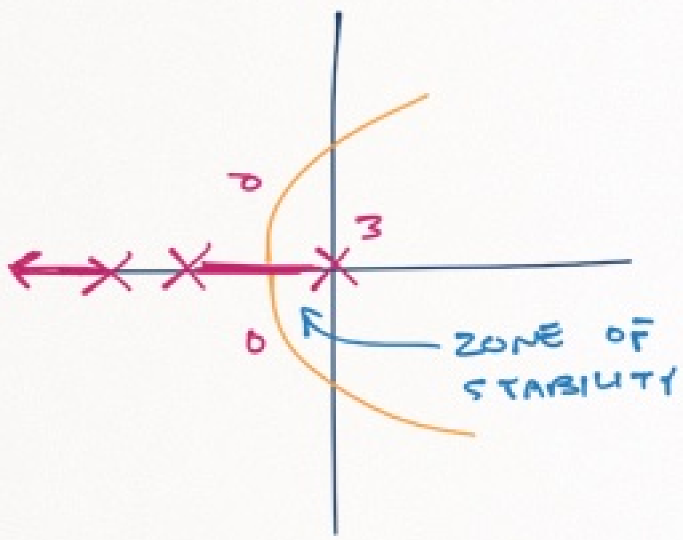
ZEIGLER NICHOLS IS NOT GOING TO WORK



ROOT LOCUS OF NON-PID SYSTEM

NO AMOUNT OF K_P WILL MAKE SYSTEM STABLE

XS = 4(?)



PID ADDS POLE @ Ø
& 2 ZEROS WHERE WE DEFINE

XS -- which changes asymptotes (?)

FIND ZEROS THAT MAKE STABLE BY REVERSE ENGINEERING PID FROM $\underbrace{k_p = 725,\ k_i = 623,\ k_d = 525}_{\text{TOP SECRET!}}$

$$s^2 + s\frac{k_p}{k_d} + \frac{k_i}{k_d} = \varnothing$$

$$s \simeq -0.690 \pm j\,0.842$$