Midterm Review

Sunday, May 29, 2016 11:56 AM

Quantitative Section

3 types of maximizing profits problems

- fixed input → maximize output
- fixed output → minimize input
- neither fixed → maximize difference

Q, production quantity
P, price per unit
C, cost per unit

Ex) A firm is planning to manufacture a new product. The sales department estimates that the quantity that can be sold depends on the selling price. As the selling price is increased, quantity sold decreases. They estimate: P=35-0.02Q. They also estimate that the average cost of manufacture will decrease as the quantity produced increases: C=4Q+8000. Management want to maximize profit; the difference between revenue (total income) and total cost. What quantity should be produced and what price should each unit be sold at?

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as $P\uparrow$ $Q\downarrow$, normal good, $P = 35 - 0.02Q$

as $Q\uparrow$ $C\downarrow$, economies of scale, $C = 4Q + 8000$

revenue = income − cost, income = PQ

$$\text{revenue} = PQ - C = (35 - 0.02Q)Q - 4Q - 8000$$
$$= -0.02Q^2 + 31Q - 8000$$

revenue is maximized where derivative = ∅

$$\emptyset = \frac{d}{dQ}(-0.02Q^2 - 31Q - 8000) = -0.04Q + 31$$

$\hookrightarrow \boxed{Q = 775}$

@ $Q = 775$, $P = 35 - 0.02(775) = \boxed{19.5}$

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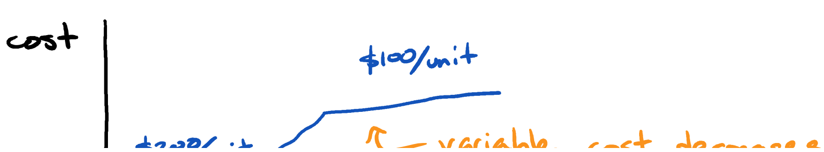
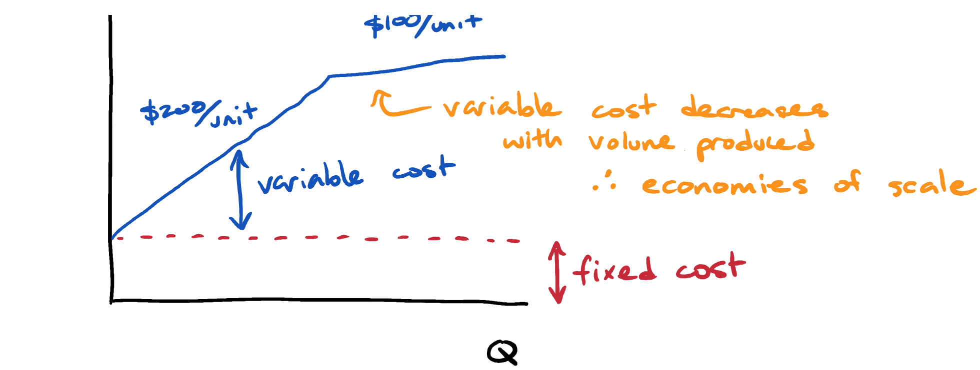
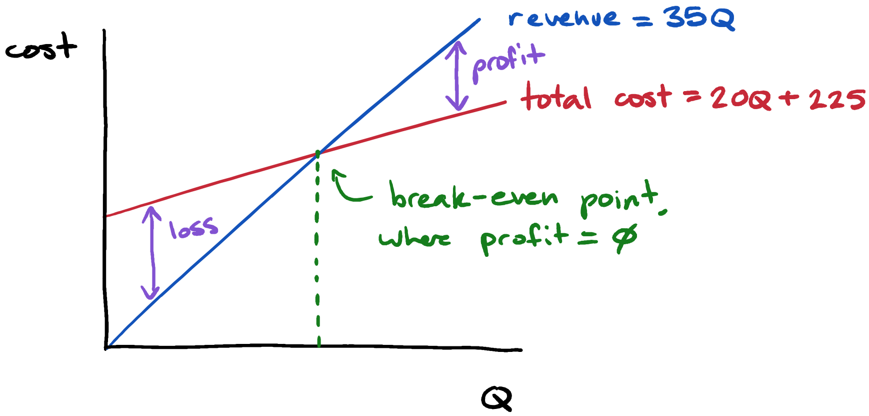
variable cost → cost per unit Q (depends on output level)
marginal cost → variable cost for one more unit produced







Ex)

A company operates a summer camp. The following cost data for a 12-week summer camp is as follows:

Price for camper	=$400/week
Variable cost per camper	=$220/week
Fixed costs	=$240,000 per summer season
Capacity per week	=200 campers

Determine the following

a) the total number of campers to breakeven for the season

b) the profit if the camp is operated at 90% capacity

c) the additional profit that can be made if a discount of $100 per week is given for another 10 campers

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$P = 12 \cdot 400Q, \quad C = 12 \cdot 220Q + 240000$

break-even where $P = C \rightarrow Q = 111.1 \rightarrow \boxed{Q = 112}$

at 90% capacity $Q = 200(0.9) = 180$

$\hookrightarrow$ profit $= 12 \cdot 400 \cdot 180 - 12 \cdot 220 \cdot 180 - 240000$
$= \boxed{148.8k}$

profit $= 12 \cdot 400 \cdot 180 + 12 \cdot 300 \cdot 10 - 12 \cdot 220 \cdot 190 - 240000$
$= 158.4k$

additional profit $= 158.4k - 148.8k = \boxed{9600}$

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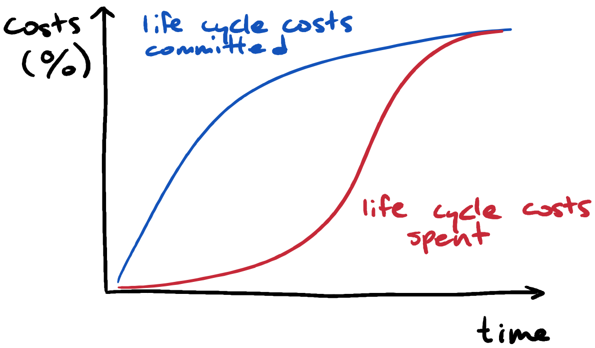
sunk costs $\rightarrow$ money already spent due to past decision

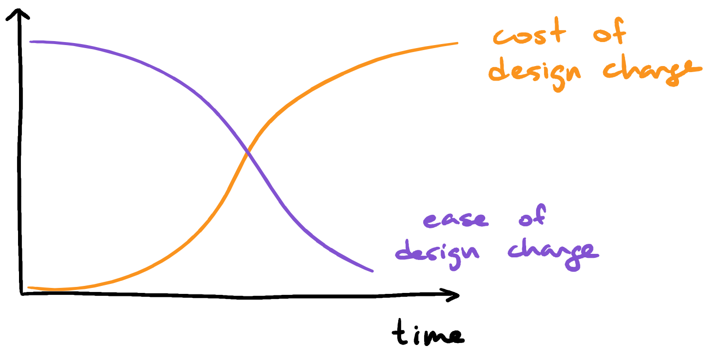
↳ SHOULD BE DISREGARDED WHEN MAKING ECONOMIC DECISIONS

opportunity costs → the cost associated with using a resource for something instead of something else

life-cycle costs → the later in the life cycle of a project a decision is made, the higher the cost

↳ EARLY DECISIONS LOCK-IN COSTS THAT ARE INCURRED LATER





ESTIMATING MODELS

Per-unit model → ex. cost per square foot (ratios)
Segmenting model → break into individual components and sum
Cost indexes → compare to historical costs (ratios)

$$\frac{\text{cost at time A}}{\text{cost at time B}} = \frac{\text{index value at time A}}{\text{index value at time B}}$$

Power sizing model → scale costs up or down

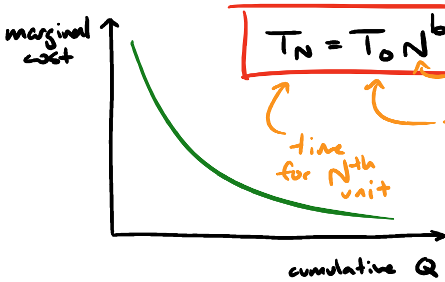
$$\boxed{\frac{\text{cost of A}}{\text{cost of B}} = \left(\frac{\text{capacity of A}}{\text{capacity of B}}\right)^x}$$

→ INCORPORATES ECONOMIES OF SCALE

$x = 1$ linear
$x < 1$ economies of scale
$x > 1$ diseconomies of scale

Triangulation → average different estimates/perspectives

Improvement & Learning Curve → as output ↑, production time ↓



$$\boxed{T_N = T_0 N^b}$$

learning curve exponent
\# completed units
time required for first unit of production
time for N^{th} unit

$$\boxed{b = \frac{\log(\%)}{\log 2}}$$

Ex

In a complex assembly operation, it is found the learning curve rate is 70%. A time of 3 minutes per assembly is reached after the 110th unit.

a) Calculate the time required for the very first unit.
b) Calculate the time required for the 200th unit.

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$T_N = T_0 N^b$ $\quad b = \frac{\log 0.7}{\log 2} = -0.5146,$

↳ $3 = T_0 (110)^b \rightarrow \boxed{T_0 = 33.7 \text{ min}}$

$T_{200} = 33.7 (200)^b = \boxed{2.2 \text{ min}}$

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Ex

- Find the power sizing exponent of the VMIC equipment, given the following data:
- Capacity of VMIC100=100
- Capacity of VMIC50=50
- Cost VMIC100 today=$100,000
- Cost of VMIC50 five years ago=45,000
- VMIC Index value today=214
- VMIC Index value five years ago=151

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$\frac{\text{cost } 50}{\text{cost } 100} = \left(\frac{\text{cap } 50}{\text{cap } 100}\right)^x$

$\frac{\text{cost 100 today}}{\text{cost 100 5yrs}} = \frac{\text{cost 50 today}}{\text{cost 50 5yrs}} = \frac{\text{index today}}{\text{index 5 yrs}}$

$\frac{100k}{\text{cost 100 5yrs}} = \frac{\text{cost 50 today}}{45k} = \frac{214}{151}$

cost 100 5yrs ago = 70.56k
cost 50 today = 63.77k

$\frac{63.77k}{100k} = \left(\frac{50}{100}\right)^x \rightarrow \boxed{x = 0.649}$

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TIME VALUE OF MONEY

generally money has more value today than in the future
↳ b/c inflation, investment, and uncertainty

P, present amount of money $\quad$ i, interest rate
F, future amount of money $\quad$ n, # of periods

simple interest (mostly obsolete)

$F = P(1 + in)$ → interest only applied to original loan

compound interest

$$\boxed{F = P(1+i)^n}$$ → interest is applied to cumulative amount
↳ interest on interest

EQUIVALENCE

↳ the same sum of money has different values at different times

ex. \$1000 now is equivalent to \$1210 two years from now at 10% interest

→ useful b/c different engineering options have different cash flows that must be compared

notation

$$F = P\underbrace{(F/P, i, n)}_{(1+i)^n}$$ ← this shows we know P and want to determine F given the interest rate and # compounding periods

$$P = F\underbrace{(P/F, i, n)}_{(1+i)^{-n}}$$

we will use this notation due to different future cash flows we'll deal with

TYPES OF INTEREST

NOMINAL, r → nominal if compounding period doesn't equal interest period
→ don't use this in analysis

if interest is 12% annually, its only compounded once per year.

if interest is 12% nominal compounded monthly, it'll be 1% per year and you'll be paying more on interest.

EFFECTIVE, i_a → takes compounding into consideration
→ always convert to this if given nominal

$$\boxed{i = \left(1 + \frac{r}{n_c}\right)^n - 1}$$

n: # compounding periods in period of interest

n_c: compounding periods/yr

Ex) convert 12% per year nominal interest compounded monthly into effective monthly/yearly rate

$$i_{yearly} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = \boxed{12.68\%}$$

$$i_{yearly} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = \boxed{12.68\%}$$

$$i_{monthly} = \left(1 + \frac{0.12}{12}\right)^{1} - 1 = \boxed{1\%}$$

compounded 1 per month

Ex) If a money lender charges 8% interest rate compounded daily, calculate the monthly effective interest rate and the annual effective interest rate.

$$i_{monthly} = \left(1 + \frac{0.08}{365}\right)^{\sim 30} - 1 = \boxed{0.66\%}$$

~30 days/month

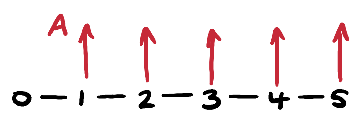
compounded 365 times per year

$$i_{yearly} = \left(1 + \frac{0.08}{365}\right)^{365} - 1 = \boxed{8.33\%}$$

note: for infinite compounding periods, $i_a = e^r - 1$

EQUIVALENCE FOR CASH FLOWS

UNIFORM SERIES, A → equal yearly payments starting at year 1 for n periods at interest rate i



can be converted to P or F with

$$P = A\underbrace{(P/A, i, n)}_{\frac{i(1+i)^n}{(1+i)^n - 1}} \quad \text{or} \quad F = A\underbrace{(F/A, i, n)}_{\frac{(1+i)^n - 1}{i}}$$

or vice-versa using reciprocals

Ex) Ravi wants to be able to purchase a dream car for about \$19,000 on 1 January 2004. Ravi has had a part time job and started making deposits of \$275 each month into an account that pays 9% compounded monthly beginning with the first deposit on 1 February 1999. The last deposit is to be made on 1 January 2004. Determine how much money he would have saved to buy the car. Will he be able to buy his dream car?

$n = 5(12) = 60$, $A = 275$, $i_{mo} = \left(1 + \frac{0.09}{12}\right)^{1} - 1 = 0.045$

$F = A(F/A, i, n) \rightarrow F = 275(F/A, 0.045, 60)$

$\boxed{= 20741}$

yes, he'll be able to afford it

Ex) You are repaying a debt of \$10,000 with equal payments made at the end of four equal periods. If the interest rate is 10% per period, how much of the original \$10,000 principal will be paid in the second payment?

$n = 4, \quad P = 10k, \quad i = 0.10$

$A = 10k(A/P, 0.1, 4) = 3155$

note, the payments are equal each period, but the percentage of those payments that's from interest is different

year 1: interest on unpaid debt $= 10\%(10000) = 1000$

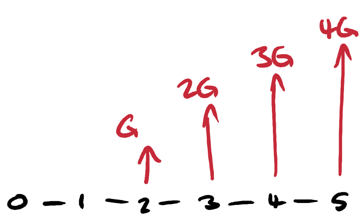
$\therefore$ debt paid off $= 3155 - 1000 = 2155$

year 2: interest on unpaid debt $= 10\%(10000 - 2155) = 784.50$

$\therefore$ debt paid off $= 3155 - 784.50 = \boxed{2370.50}$

ARITHMETIC GRADIENT, G (uniformly increasing)

↳ cash flow of an increasing amount G per period starting in year 2.



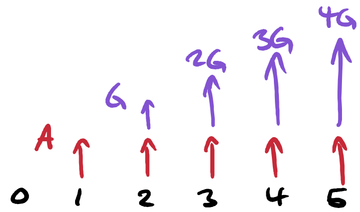
can be converted to P or G or A and vice-versa

$$P = G(P/G, i, n) \quad \text{or} \quad A = G(A/G, i, n)$$

$$(P/G, i, n) = \frac{(1+i)^n - in - 1}{i^2(1+i)^n} \qquad (A/G, i, n) = \frac{1}{i} - \frac{n}{(1+i)^n - 1}$$

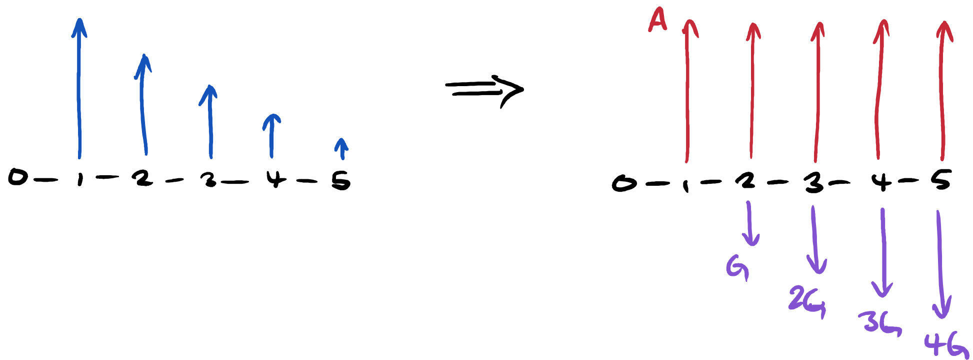
A \$ G together

because A starts in year 1 \$ G starts in year 2, often you will see a gradient starting in year 1 as a sum of an A \$ G series.



$$P = A(P/A, i, n) + G(P/G, i, n)$$

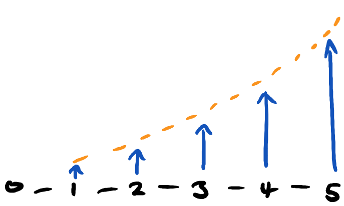
you will also see a decreasing gradient, which is conveniently expressed as a sum of an A and a negative G.



GEOMETRIC GRADIENT (no table for this)

→ like arithmetic gradient except percentage increase (exponential) instead of uniform amount (linear)

↳ uses a third variable, uniform rate g and initial amout A_1



$$P = A_1 \underbrace{(P/A, g, i, n)}$$

if $i = g$, $n(1+i)^{-1}$

else $\dfrac{1-(1+g)^n(1+i)^{-n}}{i-g}$

Ex) A set of cash flows begins at \$50,000 the first year, with an increase each year until n=15 years. If the interest rate is 7% what is the present value when

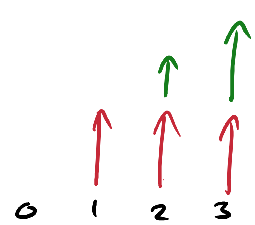
(a) The annual increase is \$5,000

(b) The annual increase is 10%

a) $P = A(P/A, i, n) + G(P/G, i, n)$

$= 50k(P/A, 0.07, 15) + 5k(P/G, 0.07, 15)$

$= \boxed{717.6k}$



$= \boxed{717.6k}$　　　　　　　　　　0　1　2　3

b) $P = A_1(P/A, g, i, n)$

$= 50k(P/A, 0.1, 0.07, 15)$

$= \boxed{856.7k}$

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PRESENT WORTH ANALYSIS

↳ convert cash flows to equivalent points in time to compare directly (usually to P)

→ assumptions: all cash flow amounts are incurred at the end of each period

no taxes, inflation/deflation

→ REQUIRES EQUAL PROJECT LIFE TO COMPARE USING PRESENT WORTH ANALYSIS (PWA)

↳ otherwise, use lowest common multiple or another method

Net PW = PW benefits − PW costs

↑ option with highest Net PW is best

Ex) Two outdoor facilities are being considered for an upcoming Olympic ice hockey event in three years. The ticket price is fixed for the event at \$150/person payable in the event year. Facility A requires a non-refundable deposit of \$250,000 and will hold 15,000 people for the event. Facility B does not require a deposit but holds only 13,000 people. If the event sells out in either facility, which facility should be chosen based on a present worth analysis, if the interest rate is 10%?

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Facility A　　0 — 1 — 2 — 3 (↓ 250k at 0; ↑ 150.15k at 3)

$NPW = 150.15k(P/F, 0.1, 3) - 250k = 1.44M$

Facility B　　0 — 1 — 2 — 3 (↑ 150.13k at 3)

$NPW = 150.13k(P/F, 0.1, 3) = \boxed{1.465M}$

Facility B $\quad 0 - 1 - 2 - 3$

$NPW = 150.13k(P/F, 0.1, 3) = \boxed{1.465M}$

more profitable

INFINITE ANALYSIS PERIOD

↳ the project lifetime of infrastructure like bridges and pipelines can be assumed to be infinite

capitalized cost → P value required to maintain project indefinitely at some i
→ money set aside at beginning to fund project forever

where $n=\infty$, $\boxed{\text{Capitalized Cost} = P = A/i}$

Ex) A municipal contractor has agreed to construct an electric power plant and to deposit sufficient money in a perpetual trust fund to pay a \$10,000/year operating cost and to perform a major renovation to the plant every 15 years at a cost of \$200,000. The plant itself will initially cost \$500,000 to construct. If the trust fund earns 10% interest per year (compounded annually), what is the capitalized cost to construct the plant, to make the future periodic renovations, and to pay the annual operating costs forever?

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annual cost $= A/i = \dfrac{10k}{0.1} = 100k$

to do infinite analysis on the renovation cost first, distribute the one payment every 15 years into a uniform payment for those 15 years, then do A/i with that (clever!)

renovation cost $= \dfrac{200k(A/F, 0.1, 15)}{0.1} = 62.95k$

Capital Cost, $P_0 = 500k + 100k + 62.95k = \boxed{662.9k}$

BOND PRICING

↳ usually have a fixed "face value" and a nominal interest rate

$$\boxed{NPW = F.V.(P/F, i, n) + r(F.V.)(P/A, i, n)}$$

Ex) A 6% coupon rate bond has a face value of $1000, pays interest semi-anually, and will mature in 10 years. If current market rate is 8% nominal interest compounded semi-annually, what is the bond's price?

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$n = 10\,yr \cdot 2/yr = 20$ $\quad i_a = \frac{8\%}{2} = 0.04$

↑ 2 compounding periods/year

$r = \frac{6\%}{2} = 0.03,$ $\quad A = 1000(0.03) = 30$

$$NPW = 1000(P/F, 0.04, 20) + 30(P/A, 0.04, 20)$$
$$= \boxed{864}$$

ANNUAL CASH FLOW ANALYSIS

↳ can use when projects of different project lifetimes
→ compare options based on annual cash flows
→ convert present $ one-time payments to uniform values

Ex) You are looking for new tires and you have identified the following alternatives:

	Tire Life	Price/Tire
A	12 Months	$30.95
B	24 Months	$44.95
C	36 Months	$53.95
D	48 Months	$59.95

If you figure that money is worth 12%, which tires should you choose based on EUAC?

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use annual cash flow analysis b/c different lifetimes

$30.95(A/P, 0.12, 1) = 34.66$

$44.95(A/P, 0.12, 2) = 26.60$

$53.95(A/P, 0.12, 3) = 22.46$

$59.95(A/P, 0.12, 4) = \boxed{19.74}$ ← tire D is the best deal

$53.95(A/P, 0.12, 3) = 22.46$

$59.95(A/P, 0.12, 4) = \boxed{19.74}$ ← tire D is the best deal for reducing annual costs

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annual cash flow analysis requires the assumption that projects may be repeated indefinitely

↳ may not be true in reality

Infinite analysis period, $n=\infty$, $A=Pi$

Ex)

The following data are available for three different alternatives:

	Alternative A	Alternative B	Alternative C
Initial Cost	$1000	$1500	$2000
Uniform Annual Benefits	$200	$276.20	$654.80
Useful Life in Years	Infinite	20	5
Interest Rate	15%	15%	15%

Alternatives B and C are replaced at the end of their useful lives with identical replacements. Using annual cash flow analysis find the most attractive alternative.

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@ $n=\infty$, $A=Pi$

option a) $200 - 1000(0.15) = 50$

option b) $276.20 - 1500(A/P, 0.15, 20) = 36.56$

option c) $654.80 - 2000(A/P, 0.15, 5) = \boxed{58.17}$

↳ option C has most annual cash flow

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MORTGAGES

→ long term loan for purchasing property
→ property is collateral on loan, can be seized
→ amortization is time to pay off loan (usually ~20 yrs)
↳ payments are consistent, A
↳ i does not change

→ equity is value of property – what hasn't been paid off
→ interest rate is usually nominal monthly

Ex)

Calculate the EUAC of the following machine:

Capital cost= $150,000

Annual O&M= $2,500

Useful life = 10 years

$i = 5\%$

Ex) Calculate the EUAC of the following machine:

Capital cost= $150,000

Annual O&M= $2,500 $i = 5\%$

Useful life = 10 years

Salvage value = $30,000

Major servicing = $20,000 after year 4 and $10,000 after year 8

Overhaul = $45,000 after year 5

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for future values, they must first be converted to present values from their year then after they're present values, they can be distributed into an A over 10 years

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\begin{aligned}
EUAC = \; & 150000(A/P, 0.05, 10) + 2.5k \\
& + 20k(P/F, 0.05, 4)(A/P, 0.05, 10) \\
& + 10k(P/F, 0.05, 8)(A/P, 0.05, 10) \\
& + 45k(P/F, 0.05, 5)(A/P, 0.05, 10) \\
& - 30k(A/F, 0.05, 10) \\
& = \boxed{27.11k}
\end{aligned}
$$

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Ex) Option 1: Capital cost $10,000, Uniform Annual Benefit (UAB)= $1600, Useful life= infinite

Option 2: Capital cost $15,000, UAB= $2400, Useful life= 20 years

$i = 10\%$

Using annual cash flow analysis, which option should be chosen?

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option 1) $1600 - (10k \cdot 0.1) = 600$

option 2) $2400 - 15k(A/P, 0.1, 20) = \boxed{638.1}$ more net benefit

Ex) A firm charges its credit customers interest 1.75% a month. What is the effective annual interest rate? What is the nominal interest rate?

Q

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$$i_a = \left(1 + \frac{r}{n_c}\right)^n - 1 = \left(1 + \frac{1.75\%}{1}\right)^{12} - 1 = \boxed{23.14\%}$$

$$r = 1.75\% \cdot 12 = \boxed{21\%}$$

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Ex)

- Compute the present value of the following cash flow:

Year 0: 0

Year 1: 0

Year 2: +50

Year 3: +60

Year 4: +70

Year 5: +80

i=15%

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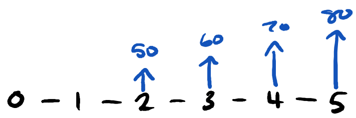

$$P = -40(P/F, 15\%, 1) + 40(P/A, 15\%, 5) + 10(P/G, 15\%, 5)$$
$$= -34.78 + 134.1 + 57.75$$
$$= \boxed{157.10}$$

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Ex) The market for a product is expected to increase at an annual rate of 8%. First-year sales are estimated at \$60,000, the horizon is 15 years, and the interest rate is 10%. What is the present value?

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$g = 8\%$, $i = 10\%$, $A_1 = 60k$, $n = 15$

$P = A_1(P/A, g, i, n) = 721.8k$

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Ex) Which equipment is preferred if the interest rate is 9%? In PW terms how great is the difference?

<u>Option A</u>: Installed cost=40000, Annual O&M=2900, Salvage value=0, Overhaul (after year 6) =10,000, Useful life=10yrs

<u>Option B</u>: Installed cost=35000, Annual O&M=4200, Salvage value=5000, Overhaul (after year 6) not required, Useful life=5yrs

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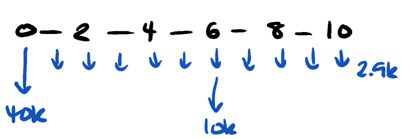
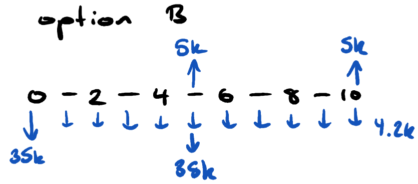
different lifes, use present worth with lowest common mult.

option A



option B



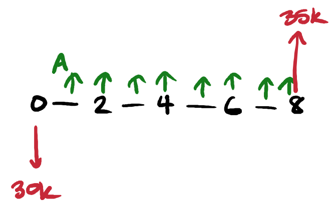
choose option A

$$\text{net} = -40k - 2.9k(P/A, 0.09, 10) - 10k(P/F, 0.09, 6) = \boxed{-64.57k}$$

$$\text{net} = -35k - 35k(P/F, 0.09, 5) - 4.2k(P/A, 0.09, 10) + 5k(P/F, 0.09, 5) + 5k(P/F, 0.09, 10)$$
$$= -79.34k$$

$$net = -35k - 35k(P/F, 0.09, 5) - 4.2k(P/A, 0.09, 10) + 5k(P/F, 0.09, 5) + 5k(P/F, 0.09, 10)$$
$$= -79.34k$$

Ex) A firm that is considering whether to add an optional heat exchanger. A unit is available now for $30,000 and it is estimated that the exchanger will be worth $35,000 after 8 years (seemingly high salvage value is due to discount on purchase price). With a 15% required rate of return, what annual benefit is needed to justify the heat exchanger?



must convert F to P to then distribute to A over 8 years

use annual cash flow analysis to determine A such that break-even

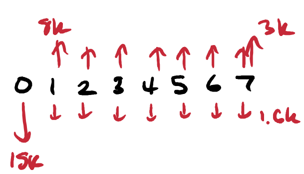
$$\emptyset = A + 35k(P/F, 0.15, 8)(A/P, 0.15, 8) - 30k(A/P, 0.15, 8) =$$
$$= A + 2.55k - 6.686k \longrightarrow \boxed{A = 4136}$$

Ex) Assuming an interest rate of 12%, use annual cash flow analysis to determine which of the following machines should be used:

<u>Option A</u>: Installed cost=15,000; Annual O&M=1,600; Salvage value=3,000; Annual benefit=8,000; Useful life=7yrs

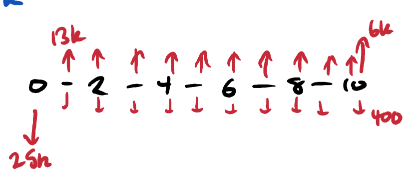
<u>Option B</u>: Installed cost=25,000, Annual O&M=400, Salvage value=6,000; Annual benefit=13,000; Useful life=10yrs

option A



$$EAUW = 8k + 3k(P/F, 0.12, 7)(A/P, 0.12, 7) - 15k(A/P, 0.12, 7) - 1.6k$$
$$= 3.411k$$

option B



$$EAUW = 13k + 6k(P/F, 0.12, 10)(A/P, 0.12, 10) - 25k(A/P, 0.12, 10) - 400$$
$$= \boxed{8.517k}$$

Suppose the bank changed their interest policy in Example 3-5 to "6% interest, compounded quarterly." For this situation, how much money would be in the account at the end of 3 years, assuming a $500 deposit now?

6% annual interest, compounded 4 times per year

$\therefore$ 4 periods per year, $3 \cdot 4 = 12$ periods total

$\therefore$ i per period $= \frac{6\%}{4} = 1.5\%$

$F = 500(F/P, 1.5\%, 12) = \boxed{598}$

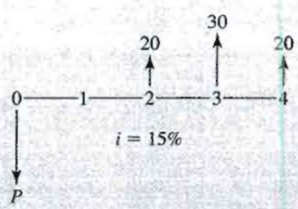
Jim Hayes read that out west, a parcel of land could be purchased for $1000 cash. Jim decided to save a uniform amount at the end of each month so that he would have the required $1000 at the end of one year. The local credit union pays 6% interest, compounded monthly. How much would Jim have to deposit each month?

6% annual interest, compounded 12 times per year

interest per period, $i = \frac{6\%}{12} = 0.5\%$

$A = 1000(A/F, 0.5\%, 12) = \boxed{81.07}$

Consider the following situation:



The diagram is not in a standard form, indicating that there will be a multiple-step solution. There are at least three different ways of computing the answer. (It is important that you understand how the three computations are made, so please study all three solutions.)

method 1, pure P/F

$P = 20(P/F, 0.15, 2) + 30(P/F, 0.15, 3) + 20(P/F, 0.15, 4)$

method 2, to F then to P

$P = [20 + 30(F/P, 0.15, 1) + 20(F/P, 0.15, 2)](P/F, 0.15, 4)$

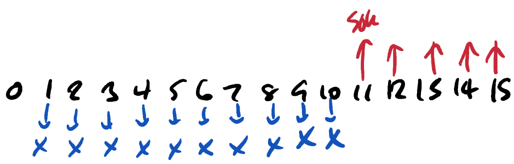
method 3, A+F, then to P

blah blah blah

Question 1: Tom is a 30-year old professional working full-time as a computer engineer. He is still thinking about years he had in college and wishes to pursue his dream and write a dissertation on approximation algorithms for NP problems in computer science. He knows he has to plan financially for this 5-year PhD program and provide the funds for the student fees (tuition, etc) and his forgone salary during that time in order to maintain a satisfactory lifestyle for himself. His plan is as follows:

- He has to save X dollars annually (paid into his account at the end of each working year) in the first 10 years.
- He will quit his job and start the PhD program after 10 years.
- He needs 50,000 $ (payable at end of each year of studying) for financing 5 years of the PhD program.
- His investment account pays 5 % interest annually on his savings.

a) Draw the cash flow for Tom's problem (5 points)
b) Find the value X (the amount of money he needs to invest in years 1-10 in his investment account for him to be able to complete his studies) (15 points)



using present worth, $\emptyset$ where break-even

$$P = \emptyset = X(P/A, 0.05, 10) - 50k(P/F, 0.05, 11) \ldots$$

↳ solve for X

Question 2: Leyla has just been approved for a 30-year mortgage plan by the bank. She can borrow 300,000 $ to buy an apartment in Burnaby for 6% nominal annual interest rate (compounded monthly) and equal monthly payments starting immediately (payable at the end of the month).

Find the monthly payments Leyla has to pay (8 points)

$$P = 300k, \quad i_a = \left(1 + \frac{r}{n_c}\right)^n - 1 = \left(1 + \frac{6\%}{12}\right)^1 - 1 = 0.5\%$$

$$n = 12 \cdot 30 = 360$$

$$A = 300k(A/P, 0.5\%, 360) = \boxed{1799}$$

Question 3: A city plans building a pipeline to transport water from a distant area to the city. The pipeline will cost 1 million dollars and will have an expected life of 50 years. The city expects it will need to keep the water line in service indefinitely. Interest rate is 10% annually.

↳ rebuild after 50yr

a) Draw the cash flow diagram for this project (5 points)
b) Compute the EUAC (Equivalent Uniform Annual Cost) associated with this project. Cost payments start at the end of 1st year. (7 points)

0 2 4 6 8 10 ... 50 ... 100

$\downarrow$ 1M (at 0) $\qquad \downarrow$ 1M (at 50) $\qquad \downarrow$ 1M (at 100)

convert periodic F to A over that period

$A = 1M(\cancel{A/F}, 0.1, 50) = \boxed{\cancel{859}.2}$

$P = A/i = \cancel{\frac{859.2}{0.1}} = 8.592k \qquad A = 1M(A/P, 0.1, 50) = 100k$

Question 4: Sakura wants to raise 3000 $ in one year to buy a DSLR camera, a Video Microphone, and a tripod to join the circle of independent film-makers in Vancouver. She wants to deposit equal monthly payments in her investment account which earns 12% nominal interest annually. (Payments start at the end of 1st month)

a) How much should be her monthly savings if the account compounds the interest monthly? (8 points)
b) Suppose that the account compounds the interest continuously. How much will be the effective monthly interest rate? (7 points)

$F = 3k \qquad i_{MO} = \left(1+\frac{r}{nc}\right)^n - 1 = \left(1+\frac{0.12}{12}\right)^1 - 1 = 1\%$

$A = 3k(A/F, 1\%, 12) = \boxed{236.50}$

$i_{yr} = e^r - 1 = e^{0.12} - 1 = 12.75\%$

$i_{MO} = (1+i_{yr})^{1/12} - 1 = \boxed{1.005\%}$

Question 5: Prateek wants to estimate the annual labor cost for a new production line, as well as the cost of buying a 2000 hp compressor for this production line. He has gathered the following information:

Labor cost for a similar facility 10 years ago	10,000 $ ✓
Labor cost index 10 years ago	90 ✓
Labor cost index now	180 ✓
Cost of 200 hp compressor 5 years ago	20,000 $ ✓
Compressor cost index 5 years ago	130
Compressor cost index now	195
Power-sizing exponent value for compressors in this range	0.32 ✓

a) Estimate labor cost for this production line. (10 points)
b) Estimate cost of buying a 2000 hp compressor for this production line. (15 points)

$\frac{L\ cost\ t=0}{L\ cost\ t=-10} = \frac{L\ index\ t=0}{L\ index\ t=-10} = \frac{180}{90} = \frac{L\ cost\ t=\emptyset}{10k}$

↳ $L\ cost\ t=\emptyset = \boxed{20k}$

$\text{cost 2000hp}_{t=-5} = \left(\frac{2000}{\ }\right)^{0.32}$

cost 2000hp t=-5

$$\frac{\text{cost } 2000\text{hp}_{t=-5}}{\text{cost } 200\text{hp}_{t=-5}} = \left(\frac{2000}{200}\right)^{0.32} = \frac{\text{cost } 2000\text{hp } t=-5}{20k}$$

↳ L cost $t=\emptyset$ = $\boxed{20k}$

↳ cost 2000hp $t=-5$ = 41.79k

$$\frac{2000 \text{ hp cost } t=\emptyset}{2000 \text{ hp cost } t=-5} = \frac{\text{index } t=0}{\text{index } t=-5} = \frac{2000\text{hp } t=\emptyset}{41.79k} = \frac{195}{130}$$

↳ cost 2000hp $t=0$ = $\boxed{62.68k}$