Problem 1

The probability of a test being positive p' is given by sensitivity $= P(T|F) = 0.90$, and specificity $= P(T^c|F^c) = 0.98$, where $P(F^c) = 1 - p$, $P(F) = p$.

$$\begin{aligned} p' &= P(test\ positive) \\ &= P(F)P(T|F) + P(F^c)P(T|F^c) \\ &= 0.01(0.90) + (1 - 0.01)(1 - 0.98) \\ &= 0.0288 \end{aligned}$$

So for this question, instead of using p, the probability an item is defective. We will be using p', the probability the test is positive. Originally I used p, and in case I am now misunderstanding the problem, here are my results for that:

m	**E(X)**	**Var(X)**
1000	5004.784	1079.235
500	2488.574	40795.699
200	871.020	116029.122
100	321.984	58013.167
50	103.748	14935.858
25	32.772	2700.239
20	23.209	1489.352
10	9.781	216.188
8	8.090	114.059
5	6.225	29.130

j	**k**	**m**	**E($\mathbf{T_k}$)**	**Var($\mathbf{T_k}$)**
1	10	1000	50047.841	10792.346
2	20	500	49771.476	815913.974
3	50	200	43551.016	5801456.079
4	100	100	32198.383	5801316.660
5	200	50	20749.697	2987171.573
6	400	25	13108.932	1080095.577
7	500	20	11604.653	744675.895
8	**1000**	**10**	**9780.896**	**216187.844**
9	1250	8	10112.765	142573.847
10	2000	5	12450.498	58259.969

Now continuing assuming we should be using p'.

(a) The probability that m additional tests will be taken is the probability that there is a faulty item in the pool. The probability that no items in m are faulty is $(1-p')^m$, so the probability that at least one item in the pool is faulty is $1-(1-p')^m$. Multiply this by $m+1$ to indicate the initial pool test and the subsequent m item tests.

If no items in the pool are faulty, $(1-p')^m$, then only one test will be taken so where $X_m =$ cost of testing one pool of size m:

$$X_m = \begin{cases} 5 & \text{with } P = (1-p')^m \\ 5(m+1) & \text{with } P = 1-(1-p')^m \end{cases}$$

$$E(X) = \sum_{i=1}^{2} X_i P_i$$

$$E(X^2) = \sum_{i=1}^{2} X_i^2 P_i$$

$$Var(X) = E(X^2) - E(X)^2$$

Table 1: Mean and Standard Deviation of X_m

m	**E(X)**	**Var(X)**
1000	5005	0
500	2504.999	2.820
200	1002.104	2887.190
100	478.095	12728.742
50	197.007	11135.028
25	69.796	3900.979
20	49.259	2467.043
10	17.670	472.974
8	13.339	264.013
5	8.399	73.413

(b) For k pools, $T_k = X_{m_1} + X_{m_2} + \cdots + X_{m_k}$:

$$E(T_k) = \sum_{i=1}^{k} E(X_{m_i}) = kE(X_m)$$

$$Var(T_k) = \sum_{i=1}^{k} Var(X_{m_i}) = kVar(X_m)$$

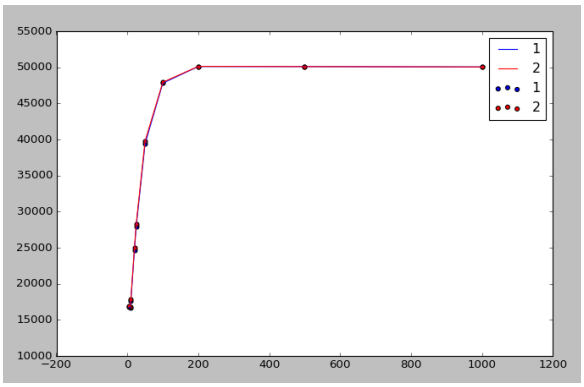
Table 2: Mean and Standard Deviation of T_j

j	**k**	**m**	**E($\mathbf{T_k}$)**	**Var($\mathbf{T_k}$)**
1	10	1000	50050	0
2	20	500	50099.977	56.396
3	50	200	50105.221	144359.508
4	100	100	47809.473	1272874.208
5	200	50	39401.450	2227005.660
6	400	25	27918.316	1560391.738
7	500	20	24629.582	1233521.403
8	1000	10	17670.107	472973.746
9	**1250**	**8**	**16673.317**	**330016.246**
10	2000	5	16797.053	146826.359

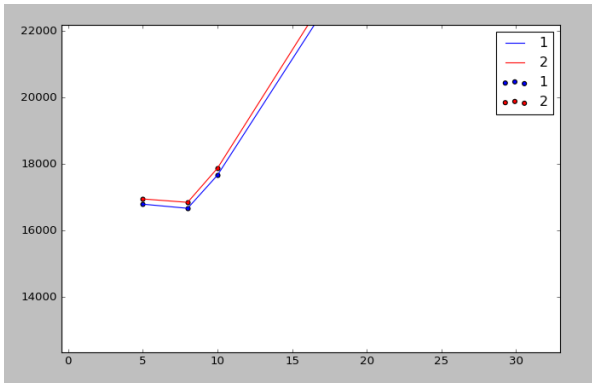
(c) The best strategy is where $m = 9$ and $k = 1250$.

Using the Python (code in Appendix A), I ran 1000 simulations for each pool size to confirm these results, comparing both the simulated and the calculated costs and graphing the two curves. In Figure 1, the *blue* curve[1] represents the costs as calculated using the above formula, and the *red* curve represents the cost as determined by averaging the results the simulations.

[1] If this has been printed in black and white, the observation is that the two curves are almost identical.



(a) domain $m = [0, 1000]$, range $E(T_k)$



(b) domain $m = [5, 30]$, range $E(T_k)$

Figure 1: Pooling size, m (x-axis) vs. Total cost, T (y-axis)

Problem 2

(a) *Proof.* In words, consider $P(T_j < S_i)$ as $P($ jth failure is before ith success $)$ for $i + j - 1$ trials. Observe that there will be at least j failures and at most $i - 1$ successes, therefore:

$$T_j \geq j$$

$$S_i \leq i - 1$$

So where we let $X = \#$ of successes in $i + j - 1$ trials, then

$$X \sim Bin(i + j - 1, p)$$

Therefore:

$$\begin{aligned} P(T_j < S_i) =& P(X \leq i - 1) \\ =& P(Bin(i + j - 1, p) \leq i - 1) \\ =& P(Bin(i + j - 1, p) \geq i) \end{aligned}$$

□

To further show the equality stands and validate this logic, I will show that these are equal through simulation. I wrote the Python code in Appendix B to simulate this scenario 5,000,000 times for each $(i,\ j)$.

In each simulation, I created random sequences of trials and determined the value of S_i and T_j to track how often $T_j > T_i$ was true. I also generated a random binomial and kept track how often $Bin(i+j-1, p) \geq i$. At the end I compared the values to compare the probabilities and they were very, very close.

Table 3: Results of Simulation

$(\mathbf{i},\mathbf{j})$	$\mathbf{P(T_j > S_i)}$	$\mathbf{P(Bin(i+j-1,p) \geq i)}$	**% Diff**
1,2	0.1900518	0.1900272	0.0129438
2,1	0.0099282	0.0099770	0.4915292
5,7	0.0027762	0.0027478	1.0229811
7,5	0.0000224	0.0000224	0

(b) The CDF of a binomial function is:

$$P(X \leq x) = \sum_{j=0}^{x} \binom{n}{x} p^x (1-p)^{n-x}$$

Where $n = i + j - 1$ and $x = i$.

$$P(Bin(i+j-1,p) < i)) = \sum_{j=0}^{i} \binom{i+j-1}{i} p^x (1-p)^{(i+j-1)-i}$$

Given $p = 0.10$:

$(\mathbf{i},\mathbf{j})$	$\mathbf{P(T_j < S_i)}$
(1,2)	0.82
(2,1)	0.98
(5,7)	0.9877
(7,5)	0.9998

(c) Again using the Python code in Appendix B to run a simulation, I determined the mean and standard deviation of the number of failures to be:

$$\mu \approx \boxed{179.98}$$

$$\sigma \approx \boxed{42.419}$$

Problem 3

Let Y represent the number of traffic accidents in a given time $[Y \approx P(\lambda)]$ where $\lambda = 5/\text{day} = 35/week = \frac{5}{24}/hour$

(a) For a Poisson distribution:

$$\mu = \sigma^2 = \lambda = 35/week$$

(b)

$$P(X > 40) = 1 - \sum_{i=0}^{40} P(X = i) = 1 - \sum_{i=0}^{40} \frac{e^{-35}(35^i)}{i!} = \boxed{0.17506}$$

(c) The probability of waiting less than four hours is the same as the probability that an accident happens in the $0th$, $1st$, $2nd$, or $3rd$ hour.

$$P(X < 4) = \int_0^4 \frac{5}{24} e^{-\frac{5}{24}t} dt = 1 - e^{-\frac{5}{6}} = \boxed{0.5654}$$

(d)

$$\frac{n}{\lambda} = \frac{4}{5/24} = \boxed{19.2 \text{ hours}}$$

Problem 4

(a) When you have independent measurements, then the sample mean estimates $E(X)$ and is:

$$\frac{1}{n} \sum_{i=1}^{n} x_i = \overline{x}$$

Therefore $E(X^2)$ is estimated by:

$$\frac{1}{n} \sum_{i=1}^{n} x_i^2 = \overline{x^2}$$

And since X is normally distributed:

$$X \sim N(\mu, \sigma^2)$$

$$E(X) = \mu$$

$$Var(X) = \sigma^2$$

Then since $Var(X) = E(X^2) - E(X)^2$:

$$E(X^2) = \sigma^2 + \mu^2$$

So $\boxed{\hat{\mu} = \overline{x} = 11.96}$

By the central limit theorem (CLT), if $n \geq 20$, $X_1 \dots X_n$.

$$\overline{X} \sim N(\mu, \sigma^2/n)$$

Because all measurements have the same expected value:

$$\mu = E(X_i)$$

$$\sigma^2 = Var(X_i)$$

And because the more measurements you get (n), the less is the standard area as it converges to the actual μ:

Then

$$n = \sqrt{15}$$

And so:

$$SE = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{15}}$$

(b) b1)

$$E(X) = \int x f_X(x) dx$$

$$M(X) = E(e^{tx}) = \int_0^\infty e^{tx} f_X(x) dx$$

Solve this for $E(X)$ for use in next part.

b2) Since X is a Gamma random variable with density:

$$f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \quad x > 0$$

Its expected value is:

$$E(X) = \frac{\alpha}{\lambda}$$

And the variance is:

$$Var(X) = \frac{\alpha}{\lambda^2}$$

Therefore (1):

$$\overline{X} = \frac{\hat{\alpha}}{\hat{\lambda}}$$

And (2):

$$\sigma^2 = \frac{\hat{\alpha}}{\hat{\lambda}^2}$$

Solve (1) and (2) for $\hat{a}$ and $\hat{\lambda}$.

Problem 5

Where U is a uniform random variable on the interval (0, 1) and $X = -\ln(1-U)/\lambda$

(a) *Proof.* Notice that the range of X is $(-\infty, 1)$, so for $x > 1$:

$$F_X(x) = 0$$

For $x \leq 1$:

$$\begin{aligned} F_X(x) &= P(X \leq x) \\ &= P(-\ln(1-U)/\lambda \leq x) \\ &= P(U \leq 1 - e^{-\lambda x}) \\ &= F_U(1 - e^{-\lambda x}) \end{aligned} \tag{1}$$

Since U is Unif$(0,1)$, then $F_U(u) = u,\ 0 \leq u \leq 1$, so:

$$\boxed{F_X(x) = 1 - e^{-\lambda x}}$$

$$f_X(x) = \frac{\partial}{\partial x} F_X(x) = \lambda e^{-\lambda x}$$

Mean:

$$\mu = E(X) = \int_0^\infty x f_X(x) dx = \int_0^\infty x \lambda e^{-\lambda x} dx$$
$$= -xe^{-\lambda x}\Big|_0^\infty + \frac{-1}{\lambda} e^{-\lambda x}\Big|_0^\infty = \boxed{\frac{1}{\lambda}}$$

Variance:

$$\sigma^2 = Var(X) = E(X^2) - E(X)^2$$
$$= \left(\int_0^\infty x^2 \lambda e^{-\lambda x} dx\right) - \left(\frac{1}{\lambda}\right)^2$$
$$= \left(\frac{2}{\lambda^2}\right) - \left(\frac{1}{\lambda}\right)^2 = \boxed{\frac{1}{\lambda^2}}$$

□

(b) Let:

$$Y = \left(X - \frac{1}{2}\right)^2$$

As before:

$$F_X(x) = 1 - e^{-\lambda x} > 0, \quad \text{for } x \geq 0$$

Therefore:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

For $y < 0$, $F_Y(y) = 0$. So the range is $\boxed{[0, \infty)}$

$$F_Y(y) = P(Y \leq y) = P\left(\left(X - \frac{1}{2}\right)^2 \leq y\right)$$
$$= P\left(-\sqrt{y} + \frac{1}{2} \leq x \leq \frac{1}{2} + \sqrt{y}\right)$$
$$= P\left(X \leq \sqrt{y} + \frac{1}{2}\right) - P\left(X \leq \frac{1}{2} - \sqrt{y}\right)$$
$$= F_X\left(\sqrt{y} + \frac{1}{2}\right) - F_X\left(-\sqrt{y} + \frac{1}{2}\right)$$

These are equivalent up until a certain point, then not anymore because function is cut-off at Y axis. The y-intercept is at $y = \frac{1}{4}$, so the above is only valid for $0 \leq y \leq \frac{1}{4}$.

$$\boxed{F_Y(y) = \begin{cases} F_X\left(\sqrt{y} + \frac{1}{2}\right) - F_X\left(-\sqrt{y} + \frac{1}{2}\right) & 0 \leq y \leq \frac{1}{4} \\ F_X\left(\sqrt{y} + \frac{1}{2}\right) & y > \frac{1}{4} \end{cases}}$$

Into order to differentiate this, we will leave it in the form:

$$F_Y(y) = \begin{cases} \left(1 - e^{-\lambda\left(\sqrt{y}+\frac{1}{2}\right)}\right) - \left(1 - e^{-\lambda\left(-\sqrt{y}+\frac{1}{2}\right)}\right) & 0 \leq y \leq \frac{1}{4} \\ 1 - e^{-\lambda\left(\sqrt{y}+\frac{1}{2}\right)} & y > \frac{1}{4} \end{cases}$$

So the PDF is:

$$\boxed{f_Y(y) = \frac{\partial}{\partial y} F_Y(y) = \begin{cases} \left(\frac{1}{2\sqrt{y}}\lambda e^{-\lambda\left(\sqrt{y}+\frac{1}{2}\right)}\right) - \left(\frac{1}{2\sqrt{y}}\lambda e^{-\lambda\left(\sqrt{y}-\frac{1}{2}\right)}\right) & 0 \leq y \leq \frac{1}{4} \\ \frac{1}{2\sqrt{y}}\lambda e^{-\lambda\left(\sqrt{y}+\frac{1}{2}\right)} & y > \frac{1}{4} \end{cases}}$$

To determine the mean, integrate this. This becomes too computationally extensive so I will leave it in the general form, and explain the process.

$$\mu = E(Y) = \int y f_Y(y) dy$$

Integrate the PDF piecewise, calling each integral I:

$$I_1(\lambda) = \int_{\frac{1}{4}}^{\infty} \frac{y}{2\sqrt{y}} \lambda e^{-\lambda\left(\sqrt{y}+\frac{1}{2}\right)} dy$$

$$I_2(\lambda) = \int_0^{\frac{1}{4}} \left(\frac{1}{2\sqrt{y}}\lambda e^{-\lambda\left(\sqrt{y}+\frac{1}{2}\right)}\right) - \left(\frac{1}{2\sqrt{y}}\lambda e^{-\lambda\left(\sqrt{y}-\frac{1}{2}\right)}\right) dy$$

And add the results to determine the mean:

$$\boxed{E(Y) = I_1(\lambda) + I_2(\lambda)}$$

Variance:

$$\boxed{Var(Y) = E(Y^2) - E(Y)^2 = \left(\int y^2 f_Y(y) dy\right) - (I_1(\lambda) + I_2(\lambda))^2}$$

Problem 6

Suppose the lifetime Y of a system has failure rate $h(y) = (y-5)^2$, $0 < y < 10$.

(a) The failure rate decreases for $0 < y < 5$, then increases for $5 < y < 10$. Assuming y represents time, then the system gets stronger at first, then gets weaker as it ages.

(b)

$$F_Y(x) = 1 - \exp\left(-\int_0^x h(y)\ dy\right) = 1 - \exp\left(-\int_0^x (y-5)^2\ dy\right) = \boxed{1 - e^{-\frac{x^3}{3}+5x^2-25x}}$$

$$f_Y(x) = \frac{\partial}{\partial x}\left(1 - e^{-\frac{x^3}{3}+5x^2-25x}\right) = \boxed{(x-5)^2 e^{-\frac{1}{3}x(x^2-15x+75)}}$$

(c)

$$F_Y(m) = \frac{1}{2} = 1 - e^{-\frac{m^3}{3}+5m^2-25m} \rightarrow m = 5 - \sqrt[3]{125 - 3\ln(2)}$$

$$\boxed{m \approx 0.027881}$$

Problem 7

$\mu = 70$, $\sigma = 10$, $X \sim N(\mu, \sigma^2)$

(a)

$$P(X > 80) = 1 - \Phi\left(\frac{80-\mu}{\sigma}\right) \approx \boxed{0.15866}$$

(b)

$$P(X \geq 60) = 1 - \Phi\left(\frac{60-\mu}{\sigma}\right) \approx \boxed{0.84134}$$

(c)

$$P(X < 60) = \Phi\left(\frac{60-\mu}{\sigma}\right) \approx \boxed{0.15866}$$

Problem 8

$$0.30 = P(X \geq 105) = 1 - \Phi\left(\frac{105-\mu}{\sigma}\right)$$
$$0.10 = P(X \geq 110) = 1 - \Phi\left(\frac{110-\mu}{\sigma}\right)$$

Doing a direct search with the following Python code for a mean and standard deviation that satisfy these equations, I found $\boxed{\mu \approx 101.5366}$ and $\sigma \approx 6.6042$, so $\boxed{\sigma^2 \approx 43.613}$.

```
def pnorm(a):
    return scipy.stats.norm.cdf(a)

flagged=False
step = 0.0000001
tolerance = 0.000001
```

```
for mean in numpy.arange(101.5365, 105, step):
    for std_dev in numpy.arange(6.6042, 10, step):
        c1 = 1 - pnorm((105-mean)/std_dev)
        c2 = 1 - pnorm((110-mean)/std_dev)
        if 0.3-tolerance < c1 < 0.3+tolerance and 0.1-tolerance < c2 < 0.1+tolerance:
            flagged = True
            print(mean, std_dev, c1, c2)

        if flagged: break
    if flagged: break
```

Problem 9

$\mu = 1$, $\sigma = 0.1$, $X \sim N(\mu, \sigma^2)$

(a)

$$\begin{aligned} P(0.85 < X < 1.1) &= \Phi\left(\frac{1.1-\mu}{\sigma}\right) - \Phi\left(\frac{0.85-\mu}{\sigma}\right) \\ &= \texttt{pnorm((1.1-1)/0.1)-pnorm((0.85-1)/0.1)} \\ &\approx \boxed{0.7745375} \end{aligned}$$

(b) Based on the probability above, $p = 0.7745375$, there should be $p200 = 154.9075$ acceptable wafers. Since the half-way point between 140 and 160, $150 < 154.9075$, then the probability that between 140-160 wafers are acceptable is slightly less than p. This is supported by my following Python simulation:

```
import numpy, scipy.stats, math

mean = 1
std_dev = 0.1
n = 1000000

p = 0.77453754479968506
n_wafers = 200

between_counter = 0
for i in range(n):
    norm = numpy.random.normal(loc=mean, scale=std_dev, size=n_wafers)
    n_good = sum(1 for i in range(n_wafers) if 0.85 < norm[i] < 1.1)

    if 140 < n_good < 160:
        between_counter += 1

print(n_good, n_wafers, bet_counter/n)
```

With yielded a probability of $\boxed{0.770507}$.

Problem 10

(i) *Proof.*

$$\begin{aligned} P(F_X^{-1}(U) \leq x) &= P(U \leq (F_X^{-1})^{-1}(x)) \\ &= P(U \leq F_X(x)) \\ &= F_X(x) \end{aligned}$$

□

This is true for all x because $0 \leq u \leq 1$ and $F_X(x)$ is a CDF so it's a probability and thus for all x, $0 \leq F_X(x) \leq 1$.

Proof.

$$\begin{aligned} P(Y \leq y) &= P(F_X^{-1}(U) \leq y) \\ &= P(F_X(F_X^{-1}(U)) \leq F_X(y)) && \text{Taking } F_X \text{ of both sides} \\ &= P(U \leq F_X(y)) \\ &= F_X(y) \end{aligned}$$

□

(ii)

$$F(x) = 1 - \left(\frac{1}{x}\right)^5, x > 1$$

Density is inverse of CDF:

$$F^{-1}(u) = \frac{1}{(1-u)^{\frac{1}{5}}}$$

Appendix

A: Problem 1 Simulation Code

```
import matplotlib.pyplot as plt
import numpy, random

n = 1000
T = 5
p = 0.01
pool_sizes = [1000, 500, 200, 100, 50, 25, 20, 10, 8, 5]

def main():
    tests = []

    for m in pool_sizes:
        # calculate the total cost using probability
        k = n/m
        number_of_tests_per_pool = (m+1)*(1-(1-p)**m)+1*(1-p)**m
        cost_per_pool = number_of_tests_per_pool*T
        number_of_tests_total = number_of_tests_per_pool*k
        cost_total = number_of_tests_total*T

        # simulate 'n' tests with random sequences of approximately p
        simulation_costs = []
        for i in range(n):
            simulated_items = create_random_items(n, p)
            simulated_pools = split_into_sublists(simulated_items, m)
            simulation_costs.append(calculate_cost(simulated_pools))

        # save the results
        test_result = {
            "m": m,
            "calculated" : {
                "tests_per_pool":   number_of_tests_per_pool,
                "tests_total":      number_of_tests_total,
                "cost_per_pool":    cost_per_pool,
                "cost_total":       cost_total,
            },
            "simulated" : {
                "cost_total" : numpy.mean(simulation_costs),
                "cost_variance" : numpy.var(simulation_costs),
                "cost_std_dev" : numpy.std(simulation_costs)
            }
        }
        tests.append(test_result)

    # print the results
    for test_result in tests:
        #print(test_result["m"], "\t", test_result["calculated"]["cost_total"], "\t",
        test_result["simulated"]["cost_total"])
        print(test_result["m"], "\t", test_result["calculated"]["tests_per_pool"], "\t", test_result["simulated"]["cost_variance

    # plot the results
    #scatter([[t["m"], t["calculated"]["cost_total"], t["simulated"]["cost_total"]] for t in tests], connect_dots=True)

# create an array of booleans, approximately p of which are False (defective)
def create_random_items(n, p):
    arr = []
    for i in range(n):
        num = random.randint(1,int(1/p))
        arr.append( False if num == 1 else True)
    return arr

# convert a list into a list of lists segmented into chunks
```

```
def split_into_sublists(arr, size):
    a = [arr[x:x+size] for x in range(0, len(arr), size)]
    if not all(len(i) == len(a[0]) for i in a):
        return None
    return a

# check if all items in an list are True
def all_true(arr):
    return len(arr) == arr.count(True)

def calculate_cost(pools):
    cost = 0
    for pool in pools:
        # cost for initial test of pool
        cost += T
        # if all in pool are true, then only this one test needed to be conducted
        # otherwise, conduct tests again for all members of the pool
        if not all_true(pool): cost += len(pool)*T
    return cost

# scatter plot a 2d array
def scatter(arr, connect_dots=False):
    colors = ["b", "r", "g", "y"]
    x = [i[0] for i in arr]
    for series in range(1, len(arr[0])):
        y = [i[series] for i in arr]
        plt.scatter(x, y, label=str(series), c=colors[series-1])
        if connect_dots:
            plt.plot(x, y, label=str(series), c=colors[series-1])
    plt.legend()
    plt.show()

if __name__ == "__main__":
    print("Starting...")
    main()
    print("Done.")
```

B: Problem 2 Simulation Code

```
import numpy
from math import factorial

p = 0.10
N = 50000

vals = [(1,2), (2,1), (5,7), (7,5)]

def main():
    for i, j in vals:
        n = i+j-1
        x = i
        print(1-sum([ choose(n, x)*(p**x)*(1-p)**(n-x) for i in range(x) ])  )

    b()
    c()

def b():
    for i, j in vals:

        left_true = 0
        right_true = 0
        for x in range(N):
            trials = numpy.random.binomial(1, p, 500)
```

```
            if T(trials, j) > S(trials, i):
                left_true += 1

            if numpy.random.binomial(i+j-1, p) >= i:
                right_true += 1

        print(tex(i, j, left_true/N, right_true/N, percent_difference(left_true/N, right_true/N)))

def c():
    failure_counts = []
    for i in range(N):
        trials = numpy.random.binomial(1, p, 500)
        successes = 0
        failures = 0
        for result in trials:
            if result == 1:
                successes += 1
            else:
                failures += 1

            # not sure if this would actually make a difference
            if successes >= 20:
                failure_counts.append(failures)
                break

    mu = numpy.mean(failure_counts)
    sigma = numpy.std(failure_counts)

    print(mu, sigma)

# find the position of the list at which
# there's the ith success (nonzero value)
def S(binary_list, i):
    successes = 0
    for position in range(len(binary_list)):
        if binary_list[position] == 1:
            successes += 1
        if successes == i:
            return position + 1
    raise Exception("Not correct position, list not long enough")

# find the position of the list at which
# there's the jth failure (zero value)
def T(binary_list, j):
    failures = 0
    for position in range(len(binary_list)):
        if binary_list[position] == 0:
            failures += 1
        if failures == j:
            return position + 1
    raise Exception("Not correct position, list not long enough")

def percent_difference(a, b):
    try:
       return 100*abs(a - b)/a
    except ZeroDivisionError:
        return 0

def choose(n, r):
    if n < r : return 0
    return factorial(n) // factorial(r) // factorial(n-r)

# print out to allow easy copy and paste into LaTeX table
def tex(*args, dec=7):
    formatting = "%." + str(dec) + "f"
    l = [formatting % i for i in args]
    s = "\t&\t".join(l) + "\t\\\\"
```

```
    s = s.replace("." + "0" * dec ,"")
    return s

if __name__ == "__main__":
    main()
```