Assignment 2 - Plane Waves

Friday, March 11, 2016 10:05 AM

11.2 A 10 GHz uniform plane wave propagates in a lossless medium for which $\epsilon_r = 8$ and $\mu_r = 2$. Find (*a*) v_p; (*b*) β; (*c*) λ; (*d*) $\mathbf{E}_s$; (*e*) $\mathbf{H}_s$; (*f*) $\langle \mathbf{S} \rangle$.

a) $v_p = \frac{c}{\sqrt{\epsilon_r}} \rightarrow \boxed{134(10^6)\ \text{m/s}}$

b) $\beta = \frac{\omega}{v_p} \rightarrow \frac{2\pi 10^8\ \text{rad/s}}{134(10^6)\ \text{m/s}} = \boxed{4.686\ \frac{1}{m}}$

c) $\lambda = \frac{2\pi}{\beta} \rightarrow \boxed{1.34\ m}$

d) $\overline{E}_s = (E_0 \measuredangle \beta z)\hat{a}_x \rightarrow \boxed{(E_0 \measuredangle 4.686z)\hat{a}_x\ \frac{V}{m}}$

e) $\eta = \frac{\eta_0}{\sqrt{\epsilon_r}}$

$\overline{H}_s = \frac{\overline{E}_s}{\eta}$

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377$

$\overline{H}_s = \left(\frac{E_0}{\eta} \measuredangle 4.686z\right)\hat{a}_y\ \frac{A}{m}$

$= \left(\frac{\sqrt{\epsilon_r}}{\eta_0} E_0 \measuredangle 4.686z\right)\hat{a}_y\ \frac{A}{m}$

$\boxed{= \left(5.93(10^{-3}) E_0 \measuredangle 4.686z\right)\hat{a}_y\ \frac{A}{m}}$

f) $\langle \overline{S} \rangle = \frac{1}{2} Re\{\overline{E}_s \times \overline{H}_s^*\} \rightarrow$

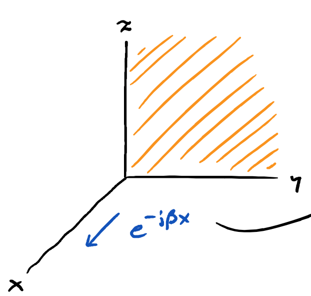
$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ E_0 & \emptyset & \emptyset \\ \emptyset & 5.93(10^{-3})E_0 & \emptyset \end{vmatrix}$$

$= \frac{1}{2} 5.93(10^{-3}) E_0^2\ \hat{a}_z\ \frac{VA}{m^2}$

$\boxed{= 2.9656(10^{-3}) E_0^2\, \hat{a}_z\ \frac{W}{m^2}}$

11.6 A uniform plane wave has electric field $\mathbf{E}_s = (E_{y0}\,\mathbf{a}_y - E_{z0}\,\mathbf{a}_z)\,e^{-\alpha x}\,e^{-j\beta x}$ V/m. The intrinsic impedance of the medium is given as $\eta = |\eta|\,e^{j\phi}$, where ϕ is a constant phase. (*a*) Describe the wave polarization and state the direction of propagation. (*b*) Find $\mathbf{H}_s$. (*c*) Find $\mathcal{E}(x, t)$ and $\mathcal{H}(x, t)$. (*d*) Find $< \mathbf{S} >$ in W/m^2. (*e*) Find the time-average power in watts that is intercepted by an antenna of rectangular cross-section, having width w and height h, suspended parallel to the yz plane, and at a distance d from the wave source.

a)



$E_{y0}\,\hat{a}_y - E_{z0}\,\hat{a}_z$

↳ polarized in y-z plane

→ propagating in +x

b) $\overline{H}_s = \frac{\overline{E}_s}{\eta} \rightarrow \frac{1}{|\eta| e^{j\phi}}(E_{y0}\hat{a}_y - E_{z0}\hat{a}_z)e^{-\alpha x}e^{-j\beta x}$

$\boxed{= \frac{1}{|\eta|}(E_{y0}\hat{a}_z - E_{z0}\hat{a}_y)e^{-\alpha x}e^{-j\beta x}e^{-j\phi}\ \frac{A}{m}}$

$$= \boxed{\frac{1}{|\eta|}\left(E_{y0}\hat{a}_z - E_{z0}\hat{a}_y\right) e^{-\alpha x} e^{-j\beta x} e^{-j\phi} \frac{A}{m}}$$

c) $\mathcal{E}(x,t) = Re\{\bar{E}_s e^{j\omega t}\} \rightarrow Re\{(E_{y0}\hat{a}_z - E_{z0}\hat{a}_y) e^{-\alpha x} e^{-j\beta x} e^{j\omega t}\}$

$\rightarrow Re\{(E_{y0}\hat{a}_z - E_{z0}\hat{a}_y) e^{-\alpha x} e^{j\omega t - j\beta x}\}$

$e^{-j\phi} = \cos\phi - j\sin\phi \quad \rightarrow Re\{(E_{y0}\hat{a}_z - E_{z0}\hat{a}_y) e^{-\alpha x}(\cos(\omega t - \beta x) - j\sin(\omega t - \beta x))\}$

$$\boxed{\mathcal{E}(x,t) = (E_{y0}\hat{a}_z - E_{z0}\hat{a}_y) e^{-\alpha x}\cos(\omega t - \beta x)}$$

$\mathcal{H}(x,t) = Re\{\bar{H}_s e^{j\omega t}\} \rightarrow Re\left\{\frac{1}{|\eta| e^{j\phi}}(E_{y0}\hat{a}_y - E_{z0}\hat{a}_z) e^{-\alpha x} e^{j\omega t - j\beta x}\right\}$

$\rightarrow Re\left\{\frac{1}{|\eta|}(E_{y0}\hat{a}_y - E_{z0}\hat{a}_z) e^{-\alpha x} e^{j\omega t - j\beta x - j\phi}\right\}$

$$\boxed{\mathcal{H}(x,t) = \frac{1}{|\eta|}(E_{y0}\hat{a}_z - E_{z0}\hat{a}_y) e^{-\alpha x}\cos(\omega t - \beta x - \phi)}$$

d) $\langle S \rangle = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\}$

		$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\bar{E}_s$	$e^{-\alpha x} e^{-j\beta x}$	$\varnothing$	E_{y0}	$-E_{z0}$
$\bar{H}_s$	$e^{-\alpha x} e^{-j\phi} e^{-j\beta x}$	$\varnothing$	$-E_{z0}$	E_{y0}

$\langle S \rangle = \frac{1}{2} Re\{(-E_{y0}E_{y0} + E_{z0}E_{z0})$
$(e^{-2\alpha x} e^{-j2\beta x} e^{-j\phi})\hat{a}_x\}$

$$\boxed{\langle S \rangle = \frac{1}{2}(E_{y0}^2 + E_{z0}^2)(e^{-2\alpha x})\cos(2\beta x + \phi)\hat{a}_x \frac{W}{m^2}}$$

e) $\boxed{Q}$

11.14 A certain nonmagnetic material has the material constants $\epsilon_r' = 2$ and $\epsilon''/\epsilon' = 4 \times 10^{-4}$ at $\omega = 1.5$ Grad/s. Find the distance a uniform plane wave can propagate through the material before (*a*) it is attenuated by 1 Np; (*b*) the power level is reduced by one-half; (*c*) the phase shifts 360°.

$\frac{\varepsilon''}{\varepsilon'} = 4(10^{-4}) \rightarrow \varepsilon'' = 4(10^{-4})2\varepsilon_0 = 7.083(10^{-15})\ F/m$

$\varepsilon' = \varepsilon_0\varepsilon_r'$

assuming Grad $= 10^9$ rad, not grad

$\frac{\varepsilon''}{\varepsilon'} \ll 1 \therefore \alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}} \rightarrow \alpha \simeq \frac{\varepsilon''\omega}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \alpha \simeq \frac{\varepsilon''\omega}{2}\frac{\eta_0}{\sqrt{2}}$

$\sigma = \varepsilon''\omega$

$\eta = \sqrt{\frac{\mu}{\varepsilon'}} = \frac{\eta_0}{\sqrt{\varepsilon_r'}}$

$\alpha \simeq \frac{7.083(10^{-15})1.5(10^9)}{2}\frac{377}{\sqrt{2}}$

$\alpha = 1.4162(10^{-3})\ \frac{Np}{m}$

a) $z = \frac{1\,Np}{1.4162(10^{-3})\ Np} = \boxed{706.11\ m}$

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a) $z = \dfrac{1\,Np}{1.4162\,(10^{-3})\,\frac{Np}{m}} = \boxed{706.11\,m}$

b) $\frac{1}{2} = e^{-2\alpha z} \longrightarrow z = \frac{1}{2\alpha}\ln(2) = \dfrac{10^3}{2(1.4162)}\ln(2) = \boxed{244.7\,m}$

c) $z = 360^\circ = 1\lambda = \dfrac{2\pi}{\beta} \longrightarrow z = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{\omega\sqrt{\varepsilon_0\mu_0}\sqrt{\varepsilon_r'\mu_r}}$

$\beta \simeq \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu_0\mu_r\varepsilon_r'\varepsilon_0}$

$= \dfrac{2\pi}{\frac{1.5(10^9)}{3(10^8)}\sqrt{2}} = \boxed{0.889\,m}$

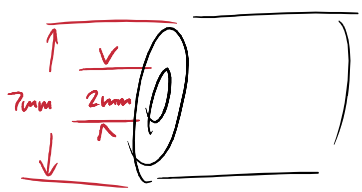
11.22 The inner and outer dimensions of a coaxial copper transmission line are 2 and 7 mm, respectively. Both conductors have thicknesses much greater than δ. The dielectric is lossless and the operating frequency is 400 MHz. Calculate the resistance per meter length of the (*a*) inner conductor; (*b*) outer conductor; (*c*) transmission line.

$R_{inner} = \dfrac{L}{\sigma S} = \dfrac{L}{2\pi a\sigma\delta}$

length

radius

$\delta = \dfrac{1}{\alpha} = \dfrac{1}{\beta} = \dfrac{1}{\sqrt{\pi f\mu\sigma}}$



$\hookrightarrow R_{inner} = \dfrac{L\sqrt{\pi f\mu_0\sigma}}{2\pi a\sigma}$, copper $\therefore$ $\sigma = 5.8\,(10^7)\,\frac{S}{m}$

a=2mm

and take $L = 1$

$= \dfrac{\sqrt{\pi\,400(10^6)\,4\pi\cancel{(10^{-7})}\,5.8\cancel{(10^7)}}}{2\pi\,2(10^{-3})\,5.8(10^7)} = \boxed{415\,\frac{m\Omega}{m}}$ a)

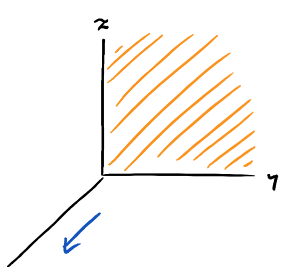
$\hookrightarrow R_{outer} = \dfrac{\sqrt{\pi\,400(10^6)\,4\pi\cancel{(10^{-7})}\,5.8\cancel{(10^7)}}}{2\pi\,7(10^{-3})\,5.8(10^7)} = \boxed{118\,\frac{m\Omega}{m}}$ b)

a=7mm

$R_{inner} + R_{outer} = \boxed{534\,\frac{m\Omega}{m}}$ c)

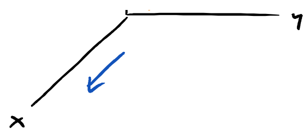
11.28 A uniform plane wave in free space has electric field vector given by $\mathbf{E}_s = 10e^{-j\beta x}\mathbf{a}_z + 15e^{-j\beta x}\mathbf{a}_y$ V/m. (*a*) Describe the wave polarization. (*b*) Find $\mathbf{H}_s$. (*c*) Determine the average power density in the wave in W/m^2.

a) $\tan\phi = \dfrac{E\hat{a}_z}{E\hat{a}_y} \longrightarrow \phi = \arctan\dfrac{10}{15} = \boxed{33.69^\circ}$ from y-axis



$\vec{E}_s = e^{-j\beta x}(10\hat{a}_z + 15\hat{a}_y)$

$\hookrightarrow$ polarized in y-z plane



b) $\bar{H}_s = \dfrac{\bar{E}_s}{\eta} \rightarrow \boxed{\bar{H}_s = \dfrac{1}{377}\left(10 e^{-j\beta x}\,\hat{a}_y + 15^{-j\beta x}\,\hat{a}_z\right)}$

$\eta_0 = 337$

c) $\langle S \rangle = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\}$

		$\hat{a}_y$	$\hat{a}_z$
E_s	$e^{-j\beta x}$	15	10
H_s	$\dfrac{e^{-j\beta x}}{377}$	10	15

$\langle S \rangle = \frac{1}{2} Re\left\{\left(\frac{15^2}{377} + \frac{10^2}{377}\right) e^{-j2\beta x}\,\hat{a}_x\right\} = \boxed{0.431\hat{a}_x \frac{W}{m^2}}$

12.4 A 10 MHz uniform plane wave having an initial average power density of 5 W/m^2 is normally incident from free space onto the surface of a lossy material in which $\epsilon_2''/\epsilon_2' = 0.05$, $\epsilon_{r2}' = 5$, and $\mu_2 = \mu_0$. Calculate the distance into the lossy medium at which the transmitted wave power density is down by 10 dB from the initial 5 W/m^2.

$\dfrac{\varepsilon_2''}{\varepsilon_2'} \ll 1 \therefore$ low-loss

$\eta_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 377$

$\eta \simeq \sqrt{\dfrac{\mu}{\varepsilon'}}\left(1 + j\dfrac{\sigma}{2\omega\varepsilon'}\right) \rightarrow \eta_2 = \sqrt{\dfrac{\mu_0}{\varepsilon'}}\left(1 + j\dfrac{\varepsilon_2''}{2\varepsilon_2'}\right)$

$\tan\theta = \dfrac{\varepsilon''}{\varepsilon'} = \dfrac{\sigma}{\omega\varepsilon'}$

$\eta_2 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\dfrac{1}{\sqrt{\varepsilon_r'}}\left(1 + j\dfrac{\varepsilon_2''}{2\varepsilon_2'}\right)$

$\eta_2 = 377\dfrac{1}{\sqrt{5}}\left(1 + j\frac{1}{2}0.05\right)$

$\Gamma = \dfrac{\eta_2 - \eta_0}{\eta_2 + \eta_0} \rightarrow \Gamma = \dfrac{\eta_2 - 377}{\eta_2 + 377} = -0.38188 + j\,0.01067$

$\varepsilon_2'' = 0.5\varepsilon_2' = 0.5\varepsilon_{r_2}'\varepsilon_0$

$\alpha \simeq \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon'}} = \dfrac{\varepsilon''\omega}{2}\sqrt{\dfrac{\mu}{\varepsilon'}} = \dfrac{\varepsilon''\omega}{2}\sqrt{\dfrac{\mu_0}{\varepsilon_0}}\dfrac{1}{\sqrt{\varepsilon_r'}} = \omega\dfrac{\varepsilon''}{\varepsilon'}\varepsilon_{r_2}'\varepsilon_0\dfrac{\eta_0}{2\sqrt{\varepsilon_{r_2}'}}$

$\hookrightarrow \alpha \simeq 2\pi 10(10^6)(0.05)(5)\varepsilon_0\dfrac{377}{2\sqrt{5}} = 11.72(10^{-3})\ \dfrac{Np}{m}$

$\langle S_{transmitted}\rangle = (1 - |\Gamma|^2)\langle S_{incident}\rangle \qquad \langle S_{incident}\rangle = S_0 e^{-2\alpha z}$

$\hookrightarrow \langle S \rangle = (1 - |\Gamma|^2) S_0 e^{-2\alpha z} = \frac{1}{2}\frac{W}{m^2}$

$\hookrightarrow z = \dfrac{1}{2}\dfrac{\ln(10(1 - |\Gamma|^2))}{\alpha} = \boxed{91.5m}$

12.8 A wave starts at point a, propagates 1 m through a lossy dielectric rated at 0.1 dB/cm, reflects at normal incidence at a boundary at which $\Gamma = 0.3 + j0.4$, and then returns to point a. Calculate the ratio of the final power to the incident power after this round trip, and specify the overall loss in decibels.

$\boxed{Q}$

12.12 A 50-MHz uniform plane wave is normally incident from air onto the surface of a calm ocean. For seawater, $\sigma = 4$ S/m, and $\epsilon_r' = 78$. (a) Determine the fractions of the incident power that are reflected and transmitted. (b) Qualitatively, how (if at all) will these answers change as the frequency is increased?

$$\eta_s = \frac{(1+j)}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}\,(1+j) \rightarrow (7.025,\ 7.025)$$

$$\eta_0 = 377 \qquad \Gamma = \frac{\eta_s - \eta_0}{\eta_s + \eta_0} \rightarrow (-0.9628,\ 0.0359)$$

$$\frac{\langle S_{\text{reflected}} \rangle}{\langle S_{\text{incident}} \rangle} = |\Gamma|^2 = \boxed{0.9282}$$

$$\frac{\langle S_{\text{transmitted}} \rangle}{\langle S_{\text{incident}} \rangle} = 1 - |\Gamma|^2 = \boxed{0.07181}$$

12.18 A uniform plane wave is normally incident onto a slab of glass ($n = 1.45$) whose back surface is in contact with a perfect conductor. Determine the reflective phase shift at the front surface of the glass if the glass thickness is (a) $\lambda/2$; (b) $\lambda/4$; (c) $\lambda/8$.

$$\Gamma = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0}$$

$\eta_3 = \emptyset$

$$\eta_{in} = \eta_2 \frac{\cancel{\eta_3 \cos\beta_2 l} + j\eta_2 \sin\beta_2 l}{\eta_2 \cos\beta_2 l + \cancel{j\eta_3 \sin\beta_2 l}} = j\eta_2 \frac{\sin\beta_2 l}{\cos\beta_2 l}$$

$$\hookrightarrow \Gamma = \frac{j\eta_2 \frac{\sin\beta l}{\cos\beta l} - \eta_0}{j\eta_2 \frac{\sin\beta l}{\cos\beta l} + \eta_0} = \Gamma_r + j\Gamma_i = \underbrace{\frac{\left(\eta_2^2 \frac{\sin^2\beta l}{\cos^2\beta l}\right) - \eta_0^2}{\left(\eta_2^2 \frac{\sin^2\beta l}{\cos^2\beta l}\right) + \eta_0^2}}_{\Gamma_r} + j\underbrace{\frac{2\eta_0\eta_2 \frac{\sin\beta l}{\cos\beta l}}{\left(\eta_2^2 \frac{\sin^2\beta l}{\cos^2\beta l}\right) + \eta_0^2}}_{\Gamma_i}$$

$$\phi = \arctan\left(\frac{\Gamma_i}{-}\right)$$

$\eta_0 = 1.45$, take $\eta_2 = 1$

Γ_r Γ_i

$\phi = \arctan\left(\frac{\Gamma_i}{\Gamma_r}\right)$ $\quad n_0 = 1.45$, take $n_2 = 1$

$$\hookrightarrow \phi = \arctan \frac{2 n_0 n_2 \frac{\sin\beta l}{\cos\beta l}}{\left(n_2^2 \frac{\sin^2\beta l}{\cos^2\beta l}\right) - n_0^2} = \arctan \frac{2(1.45)\frac{\sin\beta l}{\cos\beta l}}{\frac{\sin\beta l}{\cos\beta l} - 2.103}$$

@ $l = \lambda/2$, $\beta l = \beta\frac{\lambda}{2} = \pi$ $\longrightarrow \phi = \arctan \frac{\emptyset}{-2.103} = \boxed{\emptyset^\circ}$

@ $l = \lambda/4$, $\beta l = \beta\frac{\lambda}{4} = \pi/2$ $\longrightarrow \phi = \arctan \frac{2(1.45)\frac{\sin\pi/2}{\cos\pi/2}}{\frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} - 2.103} = \boxed{70.97^\circ}$

@ $l = \lambda/8$, $\beta l = \beta\frac{\lambda}{8} = \pi/4$ $\longrightarrow \phi = \arctan \frac{2(1.45)\frac{\sin\pi/4}{\cos\pi/4}}{\frac{\sin\pi/4}{\cos\pi/4} - 2.103} = \boxed{-69.18^\circ}$

12.22 A dielectric waveguide is shown in Figure 12.17 with refractive indices as labeled. Incident light enters the guide at angle ϕ from the front surface normal as shown. Once inside, the light totally reflects at the upper $n_1 - n_2$ interface, where $n_1 > n_2$. All subsequent reflections from the upper and lower boundaries will be total as well, and so the light is confined to the

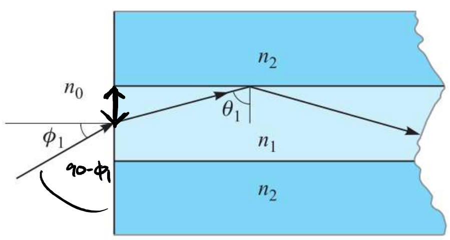


Figure 12.17 See Problems 12.22 and 12.23.

guide. Express, in terms of n_1 and n_2, the maximum value of ϕ such that total confinement will occur, with $n_0 = 1$. The quantity $\sin\phi$ is known as the *numerical aperture* of the guide.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\phi_1 = \arcsin\left(\frac{n_1}{n_0}\sin\phi_2\right)$$

$$\phi_1 = \arcsin\left(\frac{n_1}{n_0}\underbrace{\cos(90-\phi_2)}_{\sqrt{1-\left(\frac{n_2}{n_1}\right)^2}}\right)$$

$$\phi_1 = \arcsin\left(n_1\sqrt{1-\frac{n_2^2}{n_1^2}}\right)$$

$$\boxed{\phi_1 = \arcsin\left(\sqrt{n_1^2 - n_2^2}\right)}$$