# Ch. 1 Sets

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**CHAPTER 1**

***1.1 Describing a Set***

Order of elements in a set doesn't matter, {1,2,3} = {2,3,1}

A set can have different types of elements, {1, "a", do, {1,2} }

If a set has no elements it's called the **empty set**, {}

Can define sets based on some condition p(x), S = {x : p(x)}

For example, the set can be the solutions to an equation:

S = { x : (x-1)(x+2) = 0 }

Common sets:

| | |
|---|---|
| Z | Integers |
| N | Natural numbers |
| R | Real numbers |
| Q | Rational numbers |
| I | Irrational numbers |
| C | Complex numbers |

The **cardinality** of a set, |S|, is the number of elements in the set, | {1, 2, {1,2}, {} | = 4

**Example 1.3** *Let* $D = \{n \in \mathbf{N} : n \le 9\}$, $E = \{x \in \mathbf{Q} : x \le 9\}$, $H = \{x \in \mathbf{R} : x^2 - 2 = 0\}$ *and* $J = \{x \in \mathbf{Q} : x^2 - 2 = 0\}$.

(a) Describe the set $D$ by listing its elements.
(b) Give an example of three elements that belong to $E$ but do not belong to $D$.
(c) Describe the set $H$ by listing its elements.
(d) Describe the set $J$ in another manner.
(e) Determine the cardinality of each set $D$, $H$ and $J$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

A. D = {1,2,3,4,5,6,7,8,9}
B. -1.5, 1.5, 0
C. H = { -sqrt(2), sqrt(2) }
D. { } // nothing because there are no rational numbers that satisfy that
E. |D| = 9, |H| = 2, |J| = 0

**Example 1.4** *In which of the following sets is the integer* $-2$ *an element?*
$S_1 = \{-1, -2, \{-1\}, \{-2\}, \{-1, -2\}\}$, $S_2 = \{x \in \mathbf{N} : -x \in \mathbf{N}\}$,
$S_3 = \{x \in \mathbf{Z} : x^2 = 2^x\}$, $S_4 = \{x \in \mathbf{Z} : |x| = -x\}$,
$S_5 = \{\{-1, -2\}, \{-2, -3\}, \{-1, -3\}\}$.

S1, S4

***1.2 Subsets***

A is a subset of B if every element of A is also in B. This includes when the set A and B are the same, in which case A is a subset of B and B is a subset of A.

Let's say B = {

1.2 SUBSETS

A is a subset of B is all elements of a also belong to B, $A \subseteq B$

→ including when A and B are the same, except "proper subset", $A \subset B$

↳ note: $\boxed{\text{if } A \subseteq B \text{ and } B \subseteq A,\ A = B}$

→ the empty set is a subset of all sets → $\boxed{\phi \in A}$ for all sets A

**Example 1.5** *Find two sets A and B such that A is both an element of and a subset of B.*

$B = \{1, 2, \{1, 2\}\}$, $A = \{1, 2\}$, $A \in B$, $A \subseteq B$

**Example 1.6** *Two sets A and B have the property that each is a subset of* $\{1, 2, 3, 4, 5\}$ *and* $|A| = |B| = 3$. *Furthermore,*

(a) 1 *belongs to A but not to B.*
(b) 2 *belongs to B but not to A.*
(c) 3 *belongs to exactly one of A and B.*
(d) 4 *belongs to exactly one of A and B.*
(e) 5 *belongs to at least one of A and B.*

*What are the possibilities for the set A?*

A is 3 elements, $1 \in A$, $2 \in B$, $3 \in (A \vee B)$, $4 \in (A \vee B)$

$A = \{1, 3, 5\}$ $B = \{2, 4, 5\}$

$A = \{1, 4, 5\}$ $B = \{2, 3, 5\}$

If there are any elements of A not in B, $A \nsubseteq B$

**Example 1.7** *Let* $S = \{1, \{2\}, \{1, 2\}\}$.

(a) Determine which of the following are elements of $S$:
$1, \{1\}, 2, \{2\}, \{1, 2\}, \{\{1, 2\}\}$.

(b) Determine which of the following are subsets of $S$:
$\{1\}, \{2\}, \{1, 2\}, \{\{1\}, 2\}, \{1, \{2\}\}, \{\{1\}, \{2\}\}, \{\{1, 2\}\}$.

a) $1$, $\{2\}$, $\{1, 2\}$

b) $\{1\}$, $\{1, \{2\}\}$, $\{\{1, 2\}\}$

Frequent subsets of $\mathbb{R}$ are intervals

$(a,b) = \{ x \in \mathbb{R} : a < x < b \}$ open interval

$[a \quad = \{ x \in \mathbb{R} : a \leq x \leq b \}$ closed interval

POWER SET the set of all subsets, $\mathcal{P}(A)$

$\hookrightarrow$ $\boxed{\text{all power sets include } \phi}$ $\boxed{|\mathcal{P}(A)| = 2^{|A|}}$

**Example 1.8** *For each set A below, determine $\mathcal{P}(A)$. In each case, determine $|A|$ and $|\mathcal{P}(A)|$.*

(a) $A = \emptyset$, (b) $A = \{a, b\}$, (c) $A = \{1, 2, 3\}$.

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$\mathcal{P}(\phi) = \{\phi\}$ $|\mathcal{P}(\phi)| = 2^0 = 1$

$\mathcal{P}(\{a,b\}) = \{\{a\}, \{b\}, \{a,b\}, \phi\}$ $|\mathcal{P}(\{a,b\})| = 2^2 = 4$

$\mathcal{P}(\{1,2,3\}) = \{\{1\}, \{2\}, \{3\}, \{1,3\}, \{2,3\}, \{1,2\}, \{1,2,3\}, \phi\}$

$|\mathcal{P}(\{1,2,3\})| = 2^3 = 8$

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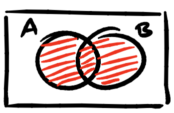
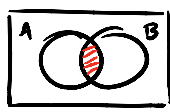
## 1.3 SET OPERATIONS

$\cup$, "union", "or" $\cap$, "intersect", "and"



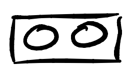

$A \cup B$



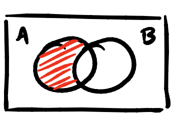

$A \cap B$

DISJOINT when A and B have no elements in common

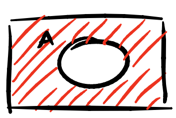
$\hookrightarrow$ A and B are $\boxed{\text{disjoint when } A \cap B = \phi}$ $\rightarrow$



$-$, "difference" $\bar{A}$, "complement"




$A - B$




$\bar{A}$

**Example 1.13** *For $A = \{x \in \mathbf{R} : |x| \leq 3\}$, $B = \{x \in \mathbf{R} : |x| > 2\}$ and $C = \{x \in \mathbf{R} : |x - 1| \leq 4\}$:*

*(a) Express A, B and C using interval notation.*

*(b) Determine $A \cap B$, $A - B$, $B \cap C$, $B \cup C$, $B - C$ and $C - B$.*

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a) $A = [-3, 3]$ $B = (-\infty, -2) \cup (2, \infty)$ $C = [-3, 5]$

b) $A \cap B = [-3,-2) \cup (2,3]$ $\quad A - B = [-2,2]$

$B \cap C = [-3,-2) \cup (2,5]$ $\quad B \cup C = (-\infty, \infty)$

$B - C = (-\infty,-3) \cup (5,\infty)$ $\quad C - B = [-2,2]$

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**Example 1.14** *Let* $U = \{1, 2, \ldots, 10\}$ *be the universal set,* $A = \{2, 3, 5, 7\}$ *and* $B = \{2, 4, 6, 8, 10\}$. *Determine each of the following:*

(a) $\overline{B}$, (b) $A - B$, (c) $A \cap \overline{B}$, (d) $\overline{\overline{B}}$.

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$\overline{B} = \{1,3,5,7,9\}$ $\quad A - B = \{3,5,7\}$

$A \cap \overline{B} = \{3,5,7\}$ $\quad \overline{\overline{B}} = \{2,4,6,8,10\}$

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## 1.4 INDEXED COLLECTION OF SETS

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cdots \cup A_n \qquad \bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cdots \cap A_n$$

## 1.5 PARTITIONS OF SETS

↳ collection of non-empty subsets such that all subsets are disjoint

↳ no common elements in two subsets

PARTION OF SET

→ union of partion is original

$$\bigcup_{x \in S} x = A$$

→ intersect of partion is $\phi$ $\quad$ Q

for every two sets $X, Y \in S$ $\quad X \cap Y = \phi$ or $X = Y$

→ no subset is empty

$X \neq \phi$ for every set $X \in S$

**Example 1.22** *Consider the following collections of subsets of the set* $A = \{1, 2, 3, 4, 5, 6\}$:

$$S_1 = \{\{1, 3, 6\}, \{2, 4\}, \{5\}\};$$
$$S_2 = \{\{1, 2, 3\}, \{4\}, \emptyset, \{5, 6\}\};$$
$$S_3 = \{\{1, 2\}, \{3, 4, 5\}, \{5, 6\}\};$$
$$S_4 = \{\{1, 4\}, \{3, 5\}, \{2\}\}.$$

*Determine which of these sets are partitions of* $A$.

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$S_1$ is partion b/c no overlap, contains all elements, and no $\phi$

$S_2$ is not partion b/c contains $\phi$

$S_3$ is not partion b/c duplicate 5

$S_4$ is not because doesn't contain any 6

$S_3$ is not partion b/c duplicate 5

$S_4$ is not because doesn't contain any 6

## 1.6 CARTESIAN PRODUCTS OF SETS

ORDERED PAIR

↳ ordered list of two elements $(a,b) \neq (b,a)$

CARTESIAN PRODUCT

→ $\boxed{A \times B = \{(a,b) : a \in A \text{ and } b \in B\}}$

→ distribute elements of A in-o elements of B

→ note: $\boxed{A \times B \neq B \times A}$ $\boxed{\text{if } A = \emptyset \text{ or } B = \emptyset \text{ then } A \times B = \emptyset}$

$\boxed{|A \times B| = |A||B|}$