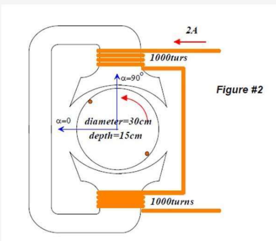
Assignment 2 - Magnetic Circuits

Wednesday, January 20, 2016 6:05 PM

(10 pts)

The cross section of the ferromagnetic circuit of a simplified DC machine with an infinite permeability alloy can be seen in the figure below. According to your experience in the first problem of this assignment, you know that the space distribution of flux (its "shape" if you wish) is squarish, but in this circuit the air gap has been tapered (it is not uniform, we know its minimum span δ_O = 0.5 cm) so that the spatial distribution of flux is sinusoidal instead along the gap, from $a = 0$ up to $a = 360$. That is $B_O(a)$ = $B_{\max}\sin(a)$. Observe the reference position for zero degrees.

Figure:



(a) What is the total flux, in webers, in this magnetic circuit.

(b) We install, in diametrally opposed slots on the surface of the cylinder in the magnetic circuit, a 100 turn coil and leave it open, and turn the cylinder around its axis at a rate of 1200 rpm. Assuming that the coil plane was horizontal at $t = 0$, what are the induced voltage at $t = 3ms$ and $t = 8ms$?

(a) Total flux = ______ mWb

(b) At $t = 3ms$, E= ______ V

at $t = 8ms$, E= ______ V

FIND B_{MAX}

$$\sum Hl = \sum NI$$

$$\hookrightarrow Hl = NI + NI \rightarrow H = \frac{NI + NI}{l}$$

H_{MAX} IS WHERE l IS SMALLEST

$$H_{MAX} = \frac{1000(2A) + 1000(2A)}{2(0.5cm)} = 400000 \frac{A}{m}$$

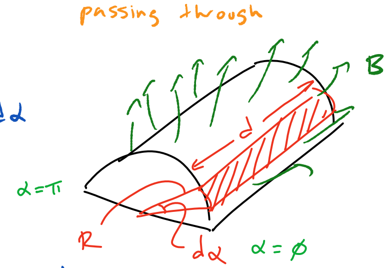
$$B = \mu H \rightarrow B_{MAX} = H_{MAX}\mu_0 = 0.503\,T$$

FIND TOTAL FLUX ACROSS SURFACE

$$B = \frac{\Phi}{A} \rightarrow \Phi = BA_c$$

$$\Phi_{TOTAL} = \int_S B(\alpha)\, \overset{\text{depth}}{d}\, \underbrace{R\, d\alpha}_{\text{width}}$$

over half of the cylinder's surface area because that's the flux passing through



$$= \int_S B_{MAX}\sin(\alpha) \cdot dR\, d\alpha$$

$$= B_{MAX}\, dR \int_0^{\pi} \sin\alpha\, d\alpha$$

$$= 0.502(15cm)(15cm)\int_0^{\pi} \sin\alpha\, d\alpha$$

$$\boxed{= 22.59\, mWb}$$

(10 pts)
A two-legged magnetic core with an air gap is shown in Figure 1. The depth of the core is 5 cm, the length of the air gap in the core is 0.06 cm, and the number of turns on the coil is 1000. The magnetization curve of the core material is shown in Figure 2. Assume a 5 percent increase in effective air-gap area to account for fringing.

(a) How much current is required to produce an air-gap flux density of 0.5 T?

(b) What is the total flux present in the air gap?

(c) What is the flux density in the top leg?

Figure 1:

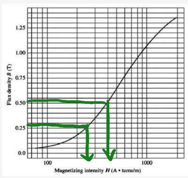
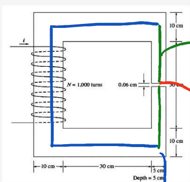


Figure 2:



40

0.06

35 + 5 + 35 + 40

(a) [ ] A

(b) [ ] wb

(c) [ ] T

$N = 1000 \qquad B = 0.5T \qquad A_g = (5cm \cdot 5cm) + 5\% = 2.625mm^2$

$\phi = BA \longrightarrow \phi = 0.5\,(2.625mm^2) = \boxed{1.313\,m\,Wb}$

$\phi = BA \longrightarrow B_T = \dfrac{\phi}{A_T} = \boxed{0.2625\,T} \longrightarrow 250\,\frac{A}{m}$

$B_B = B_L = B_T$ from graph

$B_R = \dfrac{\phi}{5cm \cdot 5cm} = 0.525T \longrightarrow 400\,\frac{A}{m}$

$B_g = 0.5T \longrightarrow \cancel{450\,\frac{A}{m}}$ free space, μ_0

$H = 0.5/\mu_0 = 398000\,\frac{A}{m}$

$NI = \sum H\ell$

$\hookrightarrow 1000I = 250(115cm) + 398k(0.06cm) + 400(40cm)$

$\boxed{I = 686mA}$

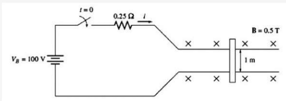
(15 pts)
The linear machine shown in Figure 1 has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25 Ohm, a bar length l = 1.0 m, and a battery voltage of 100 V.

(a) What is the initial force on the bar at starting? What is the initial current flow?

(b) What is the no-load steady-state speed of the bar?

(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady-state speed? What is the efficiency of the machine under these circumstances?

Figure 1:



(a) Starting Force = [ ] N

Starting Current = [ ] A

(b) No-Load Speed = [ ] $\frac{m}{s}$

(c) Speed = [ ] $\frac{m}{s}$

Efficiency = [ ]

$$F = BI\ell \longrightarrow F = 0.5\left(\frac{100V}{0.25}\right)1m = \boxed{200N}$$

$$I = \frac{100}{0.25} = \boxed{400A}$$

@ ss $a = \phi$ ∴ $F = \phi$ ∴ $I = \phi$ ∴ $E_a = V_t$

$$E_a = B\ell v \longrightarrow v = \frac{E_a}{B\ell} = \frac{100V}{0.5 \cdot 1m} = \boxed{200\,m/s}$$

$$v_f = \frac{V_t}{B\ell} - \frac{F_{load}R_a}{(B\ell)^2} \longrightarrow v_f = \frac{100}{0.5} - \frac{25(0.25)}{(0.5)^2} = \boxed{175\,m/s}$$

(15 pts)
A core with three legs is shown in Figure 1. Its depth is 8 cm, and there are 400 turns on the center leg. The remaining dimensions are shown in the figure. The core is composed of a steel having the magnetization curve shown in Figure 2. Answer the following questions about this core:

(a) What current is required to produce a flux density of 0.5 T in the central leg of the core?

(b) What current is required to produce a flux density of 1.0 T in the central leg of the core?

(c) What are the reluctances of the central and right legs of the core under the conditions in part **(a)**?

(d) What are the reluctances of the central and right legs of the core under the conditions in part **(b)**?

Figure 1:

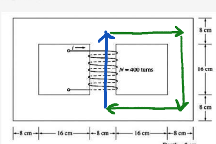
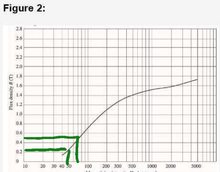


Figure 2:



(a) 0.13 and

(b) 0.22 A

(c) 5250 and

22500 and

(d) 6000 $\frac{At}{Wb}$

15750 $\frac{At}{Wb}$

$NI = H\ell \quad B_c = 0.5T \rightarrow H_c = 70\frac{A}{m}$

$$\hookrightarrow I = \frac{H\ell}{N} = \cancel{\frac{70(24cm)}{400} = \boxed{42mA}}$$

$\phi = BA \rightarrow \phi_c = 0.5(8cm)^2 = 3.2\,mWb$

$\phi_R = \phi_L = \frac{1}{2}\phi_c = 1.6\,mWb$

$B_R = B_L = \frac{1.6\,mWb}{(8cm)^2} = 0.25T \rightarrow 50\frac{A}{m}$

right loop

$NI = \sum Hl \rightarrow 400I = 50\frac{A}{m}3(24cm) + 70\frac{A}{m}24cm$ $\boxed{I = 130\,mA}$

$U = R\phi$, $U = Hl$ $\rightarrow$ $R = \frac{Hl}{\phi}$ $R_c = \frac{70(24cm)}{3.2mWb} = \boxed{5250\frac{A}{Wb}}$

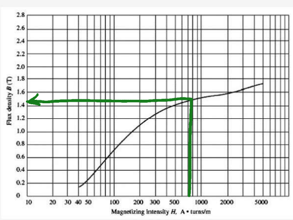
(10 pts)

A transformer core with an effective mean path length of 10 ∈ (cm) has a 300-turn coil wrapped around one leg. Its cross-sectional area is 0.25 $\in^2$ (cm), and its magnetization curve is shown in Figure 1. If current of 0.25 A is flowing in the coil:

(a) What is the total flux in the core?

(b) What is the flux density?

Figure 1:



(a) Wb

(b) T

$N = 300$, $l = 10cm$, $A = 25cm^2$, $I = 0.25A$

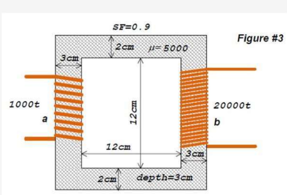
$NI = Hl \rightarrow H = \frac{300 \cdot 0.25A}{10cm} = 750\frac{A}{m} \rightarrow \boxed{1.5T}$

$\phi = BA \rightarrow \phi = 1.5T(25cm^2) = \boxed{375\,mWb}$

(10 pts)

The magnetic circuit below, built of sheets of ferromagnetic material with constant permeability of 5000 that of air's. The stacking factor is 0.9 (iron percentage of the cross section of the flux path). The coil on the right has 20000 turns. The coil on the left has 1000 turns. Assume that all the flux in the magnetic circuit is contained within the ferromagnetic material (that is, no magnetic leakage). We apply a sinusoidal voltage signal to the coil on the left, directly out of a BC-Hydro residential power outlet ($V = 120V$, $f = 60Hz$):

Figure:



(a) What is the RMS value of current in 1000 turns coil, if the 20000 turns coil is left open?

(b) What is the RMS value of voltage at the terminals of the open 20000 turns coil?

(c) If you connect a resistor of 500 $ohms$ to the 20000 turns coil, what is the RMS of current in 1000 turns coil?

(a) A

(b) V

(c) A

$NI = Hl \longrightarrow \ldots = Bl$ l, μ, A, N

(c) [ ] A

$NI = H\ell \qquad B=\mu H \longrightarrow NI = \dfrac{B\ell}{\mu}$ $\qquad \ell, \mu, A, N$

$\phi = BA \longrightarrow NI = \dfrac{\phi \ell}{A\mu}$ $\qquad A = wd\,0.9$

find potential drop over loop

$$NI = \phi\left(\frac{30cm}{3cm\cdot 2cm\cdot 5000\mu_0\, 0.9} + \frac{28cm}{3cm\cdot 3cm\cdot 5000\mu_0\, 0.9}\right)$$

$$I = \phi\, 143.4$$

$$N\phi = LI \longrightarrow L = \frac{N\phi}{I} = \frac{1000\,\cancel{\phi}}{143.4\,\cancel{\phi}} = 6.972\,H$$

$$Z_L = j\omega L = j2\pi 60(6.972H)$$

$$I_p = \frac{V}{Z_L} = \frac{120}{j2\pi 60\cdot 6.972} = \boxed{45.66\,mA}$$

from now on it's a transformer problem

$$V_s = \frac{20000}{1000}120 = \boxed{2400V}$$

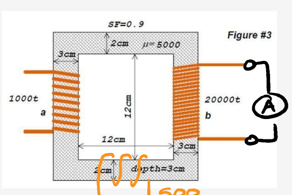
with 500Ω load

$$I_s = \frac{2400V}{500\Omega} = 4.8A \qquad I_p = \frac{20000}{1000}4.8A = \boxed{96A}$$

(10 pts)

A magnetic circuit is similar to that in Figure 3 below, but made out of “ideal iron”. The 20000 turn coil is shorted with an ideal AC ammeter.

Figure 3:



(a) If the current in the other coil is $i(t) = 237\sin(377)t$ **amps** (goes in at the top terminal of ‘a’), what is the reading in the ammeter?

(b) Additionally, on the bottom ‘leg’, a third coil, 500 turns, is wound in the same direction as coil ‘a’, with a current of $136\sin\left(377t - \frac{37\pi}{180}\right)$ **amps**, what is the reading of the ammeter in coil ‘b’? There are two possible answers, include both.

(a) [ ] A

(b) [ ] A

[ ] A

$$I_p = \frac{237}{\sqrt{2}} \qquad N_s I_s = N_p I_p$$

$$\hookrightarrow I_s = \frac{1000}{20000}\frac{237}{\sqrt{2}} = \boxed{8.379A}$$

$$I_{p2} = \frac{186}{\sqrt{2}} \qquad N_1 I_1 + N_2 I_2 = N_3 I_3$$

$$\hookrightarrow I_2 = \frac{1000\frac{237}{\sqrt{2}} + 500\frac{136}{\sqrt{2}}}{} = \boxed{1078\,A}$$

$$\hookrightarrow I_3 = \frac{1000\frac{237}{\sqrt{2}} + 500\frac{136}{\sqrt{2}}}{20000} = \boxed{10.78\,A}$$

if I_{p2} is negative $\boxed{I_3 = 5.975}$

(15 pts)

The magnetic circuit of an induction motor consists of a solid cylinder of ferromagnetic material (the actual one is not solid but laminated, but this is your first assignment) encased in a hollow cylinder of the same alloy, as shown in Fig. 1 below. The internal cylinder has a diameter of $75cm$ and a length of $40cm$. The surrounding hollow cylinder is separated from the solid one by an air gap of $2cm$. In two slots carved on the internal surface of the hollow cylinder we install a 200 turn coil. To plot the magnetic field, H, at any point in the air gap, consider H as positive if it is coming out of the rotor, and negative if it is entering the rotor.

Fig. 1:

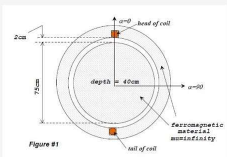


Figure #1

(a) If current in that coil is 28 A, what are the magnetic fields at $\alpha = 60^o$ and $\alpha = 220^o$?

$\alpha = 60^o$: $\frac{A}{m}$

$\alpha = 220^o$: $\frac{A}{m}$

(b) Now the current in the coil is a function of time $i(t) = 28\sin(377t)$ amps (where t is in seconds). What are the magnetic fields at $\alpha = 30^o$ and $\alpha = 310^o$ in $t = 2ms$?

$\alpha = 30^o$: $\frac{A}{m}$

$\alpha = 310^o$: $\frac{A}{m}$

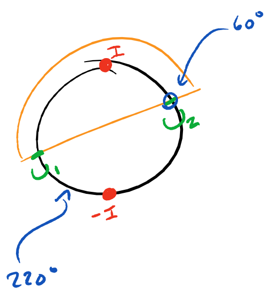
What are the magnetic fields at $\alpha = 190^o$ and $\alpha = 350^o$ in $t = 15ms$?

$\alpha = 190^o$: $\frac{A}{m}$

$\alpha = 350^o$: $\frac{A}{m}$

(c) In another pair of slots at $\alpha = 90^o$ and $\alpha = 270^o$ install another 200 turns coil. In that second coil, the current is $i(t) = 28\sin\left(377t - \left(\frac{\pi}{2}\right)\right)$ amps. What is the amplitude of magnetic fields at $\alpha = 60^o$ in $t = 3ms$?

Amplitude = $\frac{A}{m}$



$$NI = \sum Hl = U_1 + U_2 = Hl + Hl$$

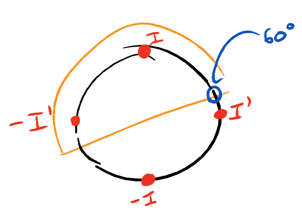
$$\hookrightarrow H = \frac{NI}{2l} = \frac{200\cdot 28A}{2(2cm)} = \boxed{140{,}000\,\frac{A}{m}}$$

$$H = \frac{-NI}{2l} = \boxed{-140000\,\frac{A}{m}}$$

$$H = \frac{NI}{2l} = \frac{200\cdot 28\sin(377\cdot 2ms)}{2(2cm)} = \boxed{95838\,\frac{A}{m}}$$

$$H = \boxed{-95838\,\frac{A}{m}}$$

$$H = \frac{-NI}{2l} = -\frac{200\cdot 28\sin(377\cdot 15ms)}{2\cdot 2cm} = \boxed{82275\,\frac{A}{m}} \times 2$$



$$H = \frac{NI - N'I'}{2l_0} = \frac{200\cdot 28\sin(377\cdot 15ms) - 200\cdot 28\sin(377 - \frac{\pi}{2})}{2(2cm)} = \boxed{186280\,\frac{A}{m}}$$