## Assignment 2 (Ch. 2-3)

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2.48. For statements P and Q, show that  $(P \land (P \Rightarrow Q)) \Rightarrow Q$  is a tautology. Then state  $(P \land (P \Rightarrow Q)) \Rightarrow Q$  in words. (This is an important logical argument form, called **modus ponens**.)

- 2.52. Let P and Q be statements.
  - (a) Is  $\sim (P \vee Q)$  logically equivalent to  $(\sim P) \vee (\sim Q)$ ? Explain.
  - (b) What can you say about the biconditional  $\sim (P \vee Q) \Leftrightarrow ((\sim P) \vee (\sim Q))$ ?

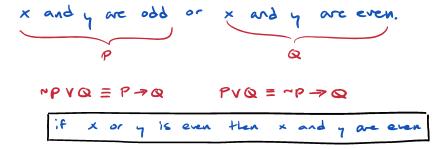
2.54. For statements P and Q, show that  $(\sim Q) \Rightarrow (P \land (\sim P))$  and Q are logically equivalent.

$$\sim Q \rightarrow (P \land \sim P) = \sim (\sim Q) \lor (P \land \sim P)$$
 by theorem 2.17  
= Q \ \( (P \lambda \cdot P) \) by double negation  
by tautology  
= Q \ \( \cdot P \)

= Q V (P \( NP \) by double regation
= Q V F by tautology
= Q by identity laws

- 2.60. Consider the implication: If x and y are even, then xy is even.
  - (a) State the implication using "only if."
  - (b) State the converse of the implication.
  - (c) State the implication as a disjunction (see Theorem 2.17).
  - (d) State the negation of the implication as a conjunction (see Theorem 2.21(a)).
  - a) "if P then Q" is equal to "Qonly if P"

    xy is even only if x and y are even
  - if xy is even then x and y are even
  - x or y is odd, or xy is even
  - d) Heorem 2.21 states  $\sim (P \rightarrow Q) \equiv P \land \sim Q$  $\Rightarrow$  and y are even, and  $\times y$  is odd
- 2.64. For which biconditional is its negation the following?  $n^3$  and 7n + 2 are odd or  $n^3$  and 7n + 2 are even.



- 2.68. State the negations of the following quantified statements:
  - (a) For every rational number r, the number 1/r is rational.
  - (b) There exists a rational number r such that  $r^2 = 2$ .

- a) there doesn't exists some rational number r such that /r is irrational
- b) for not all rational numbers r, r2 72

$$P(x, y, z): (x - 1)^2 + (y - 2)^2 + (z - 2)^2 > 0.$$

where the domain of each of the variables x, y and z is  $\mathbf{R}$ .

- (a) Express the quantified statement  $\forall x \in \mathbf{R}, \forall y \in \mathbf{R}, \forall z \in \mathbf{R}, P(x, y, z)$  in words.
- (b) Is the quantified statement in (a) true or false? Explain.
- (c) Express the negation of the quantified statement in (a) in symbols.
- (d) Express the negation of the quantified statement in (a) in words.
- (e) Is the negation of the quantified statement in (a) true or false? Explain.
- a) for all real numbers x, y, 2 (x-1)2+(y-2)2+(2-2)2>0
- b) it is false because if x=1, y=2, z=2 then it is not >0
- c)  $\sim (\exists x \in S \text{ s.t. } (x-1)^2 + (y-2)^2 + (2-2)^2 \leq 0)$
- d) there does not exist real numbers x,y,zsuch that  $(x-1)^2+(y-2)^2+(z-2)^2\leq 0$
- e) It is false because if x=1, y=2, and z=2then  $(x-1)^2 + (y-2)^2 + (z-2)^2 = 0$
- 3.14. Let  $S = \{1, 5, 9\}$ . Prove that if  $n \in S$  and  $\frac{n^2 + n 6}{2}$  is odd, then  $\frac{2n^3 + 3n^2 + n}{6}$  is even.

We will solve this using proof by cases for the cases where n=1, n=5, and n=a.

- CASE 1: if n=1, then  $\frac{n^2+n-6}{2} = \frac{1+1-6}{2} = -2$ , which is even. Since the antecedent of the implication is false, by definition, the implication is troe.
- CASE 2: if n=5, then  $\frac{n^2+n-6}{2} = \frac{5^2+5-6}{2} = 12$ , which is even. Since the antecedent of the implication is false, by definition, the implication is true.
- CASE 3: if n=9, then  $\frac{n^2+n-6}{2} = \frac{9^4+9-6}{2} = 42$ , which is even. Since the antecedent of the implication is false, by definition, the implication is troe.

As demonstrated, all possible cases for a result in the implication being true, so the statement is true.