Assignment 1 - Transmission Lines

Monday, January 25, 2016 8:21 AM

10.1 The parameters of a certain transmission line operating at $\omega = 6 \times 10^8$ rad/s are $L = 0.35\ \mu$H/m, $C = 40$ pF/m, $G = 75\ \mu$S/m, and $R = 17\ \Omega$/m. Find γ, α, β, λ, and Z_0.

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z = R + j\omega L = 17\frac{\Omega}{m} + j(6e8\tfrac{rad}{s})\,0.35\mu H = 17 + j210\ \frac{\Omega}{m}$$ net series impedance

$$Y = G + j\omega C = 75\frac{\mu S}{m} + j(6e8\tfrac{rad}{s})\,40\frac{pF}{m} = 75\mu + 24m\ \frac{S}{m}$$ net shunt admittance

$$\gamma = \sqrt{ZY} = \boxed{(94.305 \cdot 10^{-3},\ 2.2467)\ \frac{rad}{m}}$$ propagation constant

$$\alpha + j\beta = \sqrt{ZY}$$

$$\boxed{\alpha = 94.305\,m\ \frac{Np}{m}}$$

$$\boxed{\beta = 2.247\ \frac{rad}{m}}$$ phase constant

$$\beta = \frac{2\pi}{\lambda} \rightarrow \lambda = 2\pi/\beta = \boxed{2.7966\ m}$$ wavelength

$$Z_0 = \sqrt{\frac{Z}{Y}} \rightarrow Z_0 = \sqrt{\frac{(17, 210)}{(75\mu, 24m)}} = \boxed{93.62 - j\,3.637\ \frac{\Omega}{m}}$$ characteristic impedance

can't assume lossless

10.5 Two characteristics of a certain lossless transmission line are $Z_0 = 50\ \Omega$ and $\gamma = 0 + j0.2\pi\ \text{m}^{-1}$ at $f = 60$ MHz (*a*) find L and C for the line. (*b*) A load $Z_L = 60 + j80\ \Omega$ is located at $z = 0$. What is the shortest distance from the load to a point at which $Z_{in} = R_{in} + j0$?

$$\gamma = \alpha + j\beta \rightarrow \beta = 0.2\pi\ \frac{rad}{m}$$

$$\beta = \omega\sqrt{LC} \rightarrow C = \frac{\beta^2}{L\omega^2}$$

$$Z_0 \simeq \sqrt{L/C} \rightarrow C = L/Z_0^2$$

$$\frac{\beta^2}{L\omega^2} = \frac{L}{Z_0^2} \rightarrow \frac{\beta^2 Z_0^2}{\omega^2} = L^2 \rightarrow \frac{\beta Z_0}{\omega} = L$$

$$L = \frac{0.2\pi\,(50)}{2\pi\,60M} = \boxed{83.\bar{3}\ \frac{nH}{m}}$$

$$C = \frac{L}{Z_0^2} = \frac{83.\bar{3}n}{50^2} = \boxed{33.\bar{3}\ \frac{pF}{m}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(60,80) - 50}{(60,80) + 50} = (0.40541,\ 0.43243) \rightarrow \text{in polar form} \rightarrow 0.59275\angle\underbrace{46.848^\circ}_{\text{phase}}$$

$$z_{max} = \frac{-1}{2\beta}(\phi + 2n\pi) \rightarrow z_{max} = \frac{-1}{2(0.2\pi)}\left(46.848\frac{\pi}{180} + \emptyset\right) = -0.65067 \rightarrow \boxed{651\,mm}$$

$n = \emptyset$

10.6 A 50-Ω load is attached to a 50-m section of the transmission line of Problem 10.1, and a 100-W signal is fed to the input end of the line. (*a*)

10.6 A 50-Ω load is attached to a 50-m section of the transmission line of Problem 10.1, and a 100-W signal is fed to the input end of the line. (*a*) Evaluate the distributed line loss in dB/m. (*b*) Evaluate the reflection coefficient at the load. (*c*) Evaluate the power that is dissipated by the load resistor. (*d*) What power drop in dB does the dissipated power in the load represent when compared to the original input power? (*e*) On partial reflection from the load, how much power returns to the input and what dB drop does this represent when compared to the original 100-W input power?

from 10.1

Power loss (dB) = $8.69\,\alpha z \rightarrow 8.69\left(94.305\text{m}\,\frac{\text{Np}}{\text{m}}\right)(50\text{m}) = 40.976\text{dB}$

Power loss $\left(\text{dB}/\text{m}\right) = 40.976\text{dB}/50\text{m} = \boxed{0.8195\ \text{dB}/\text{m}}$

from 10.1

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} \rightarrow \Gamma_L = \dfrac{50 - (93.62,\,-3.637)}{50 + (93.62,\,-3.637)} = \boxed{(-304 + j17.62)\,10^{-3}}$

POWER AT LOAD $\langle P(z)\rangle = \langle P(0)\rangle e^{-2\alpha z} \rightarrow \langle P(50)\rangle = 100\text{W}\, e^{-2(94.305\text{m})(50)} = 8.0239\text{mW}$

POWER DISSIPATED BY LOAD $\dfrac{\langle P_t\rangle}{\langle P_i\rangle} = 1 - |\Gamma|^2 \rightarrow \langle P_L\rangle = (1-|\Gamma|^2)\langle P_i\rangle = (1-|\Gamma|^2)\,8.0239\text{mW} = \boxed{7.28\text{ mW}}$

Power loss (dB) $= 10\log\dfrac{\langle P(0)\rangle}{\langle P(z)\rangle} \rightarrow P = 10\log\dfrac{100}{7.28\text{m}} = \boxed{41.38\text{ dB}}$

1 round trip

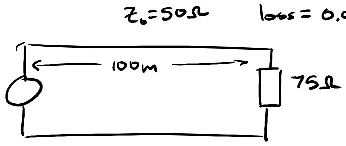
POWER RETURNED AFTER ROUND TRIP $\langle P(z)\rangle = \langle P(0)\rangle e^{-2\alpha z} \rightarrow \langle P(100)\rangle = 100 e^{-2(94.30\text{m})(50+50)} = 643.83\text{nW}$

POWER REFLECTED BY LOAD $\dfrac{\langle P_r\rangle}{\langle P_i\rangle} = |\Gamma|^2 \rightarrow P = |\Gamma|^2\, 643.83\text{nW} = \boxed{59.70\text{nW}}$

Decibel drop (dB) $= 10\log\dfrac{\langle P(0)\rangle}{\langle P(z)\rangle} \rightarrow P = 10\log\dfrac{100}{59.70\text{n}} = \boxed{94.24\text{dB}}$

10.8 An absolute measure of power is the dBm scale, in which power is specified in decibels relative to one milliwatt. Specifically, $P(\text{dBm}) = 10\log_{10}[P(\text{mW})/1\text{ mW}]$. Suppose that a receiver is rated as having a *sensitivity* of −20 dBm, indicating the *mimimum* power that it must receive in order to adequately interpret the transmitted electronic data. Suppose this receiver is at the load end of a 50-Ω transmission line having 100-m length and loss rating of 0.09 dB/m. The receiver impedance is 75 Ω, and so is not matched to the line. What is the minimum required input power to the line in (*a*) dBm, (*b*) mW?

$Z_0 = 50\Omega$ loss = $0.09\,\text{dB}/\text{m}$ min power = −20dBm



$\Gamma_L = \dfrac{75-50}{75+50} = \dfrac{1}{5}$

LOSS DUE TO TRANSMISSION

$L_1(\text{dB}) = 0.09\,\frac{\text{dB}}{\text{m}}(100\text{m}) = 9\text{dB}$

LOSS DUE TO MISMATCHED LOAD

$\dfrac{\langle P_t\rangle}{\langle P_{in}\rangle} = 1 - |\Gamma|^2$ & $L(\text{dB}) = 10\log\dfrac{\langle P_{in}\rangle}{\langle P_t\rangle}$

$\cancel{P_{in}}$ = 10log $\frac{1}{\ }$ = 0.17729 dB

$\frac{\langle P_t \rangle}{\langle P_{in} \rangle} = 1-|\Gamma|^2$ & $L(dB) = 10\log\frac{\langle P_{in}\rangle}{\langle P_t \rangle}$

$\hookrightarrow L_2(dB) = 10\log\frac{\cancel{P_{in}}}{\cancel{P_{in}}(1-|\Gamma|^2)} = 10\log\frac{1}{1-(\frac{1}{5})^2} = 0.17729\,dB$

TOTAL LOSS

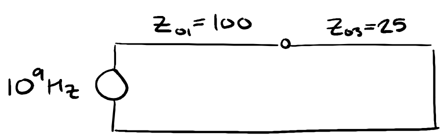
$L(dB) = 9\,dB + 0.17729\,dB = 9.17729\,dB$

MINIMUM INPUT

$P_{in}(dB) = P_{min} + P_{loss} = -20\,dB + 9.17729\,dB = \boxed{-10.823\,dB}$

$P_{in}(W) = 1W\;10^{P(dB)/10} = 10^{\frac{-10.823}{10}} = \boxed{82.74\,mW}$

10.10 Two lossless transmission lines having different characteristic impedances are to be joined end to end. The impedances are $Z_{01} = 100\ \Omega$ and $Z_{03} = 25\ \Omega$. The operating frequency is 1 GHz. (*a*) Find the required characteristic impedance, Z_{02}, of a quarter-wave section to be inserted between the two, which will impedance-match the joint, thus allowing total power transmission through the three lines. (*b*) The capacitance per unit length of the intermediate line is found to be 100 pF/m. Find the shortest length in meters of this line that is needed to satisfy the impedance-matching condition. (*c*) With the three-segment setup as found in parts (*a*) and (*b*), the frequency is now doubled to 2 GHz. Find the input impedance at the line-1-to-line-2 junction, seen by waves incident from line 1. (*d*) Under the conditions of part (*c*), and with power incident from line 1, evaluate the standing wave ratio that will be measured in line 1, and the fraction of the incident power from line 1 that is reflected and propagates back to the line 1 input.



FROM EQN 103

$Z_{02} = \sqrt{Z_{01}Z_{03}} \rightarrow Z_{02} = \sqrt{100\cdot 25} = \boxed{50\,\Omega}$

$Z_0 = \sqrt{L/C} \rightarrow L = Z_0^2 C \rightarrow L_2 = Z_{02}^2 C_2 = 50^2(10^{-12}) = 2.5\,nH$

$\beta = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{\omega\sqrt{LC}} = \frac{2\pi}{10^9\cdot 2\pi\sqrt{2.5n(100p)}} = 2m$

in Hz

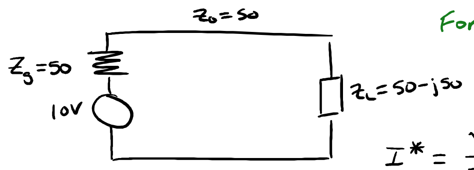
FROM EQN 100 CONDITIONS $\ell = \frac{1}{4}\lambda = \frac{1}{4}2m = \boxed{0.05m}$

Frequency x2 ∴ $\lambda/2$ instead of $\lambda/4$ ∴ $Z_{in} = Z_L$ EQN 99 $\boxed{= 25\,\Omega}$

$s = \frac{1+|\Gamma|}{1-|\Gamma|}$ ⊕ $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow \Gamma_{in} = \frac{25-100}{25+100} = -\frac{3}{5}$

$\rightarrow s = \boxed{4}$

10.14 A lossless transmission line having characteristic impedance $Z_0 = 50\ \Omega$ is driven by a source at the input end that consists of the series combination of a 10-V sinusoidal generator and a 50-Ω resistor. The line is one-quarter wavelength long. At the other end of the line, a load impedance, $Z_L = 50 - j50\ \Omega$ is attached. (*a*) Evaluate the input impedance to the line seen by the voltage source-resistor combination; (*b*) evaluate the power that is dissipated by the load; (*c*) evaluate the voltage amplitude that appears across the load.



For $\frac{\lambda}{4}$ $\quad Z_{in} = \frac{Z_{0\ell}^2}{Z_L} \longrightarrow Z_{in} = \frac{50^2}{(50,-50)} = \boxed{(25,25)}$

$$I^* = \frac{V^*}{Z^*}$$

$$V = V_0 \frac{R_L}{R_g + R_L}$$

$$I^* = \frac{V_{in}^*}{Z_{in}^*} = \frac{\left(V_0 \frac{Z_{in}}{Z_g + Z_{in}}\right)^*}{Z_{in}^*} = \frac{\left(10 \frac{(25,25)}{50 + (25,25)}\right)^*}{(25,25)^*}$$

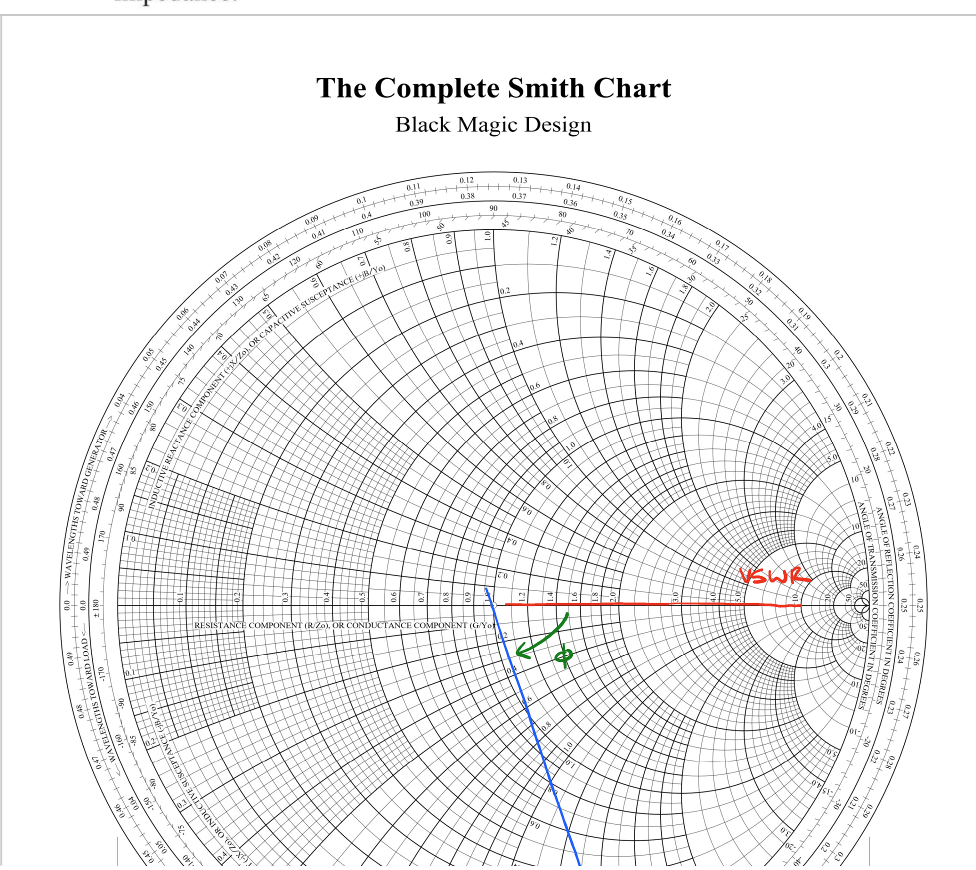
$$I^* = \frac{(4,2)^*}{(25,25)^*} = (0.12,\ 0.04)$$

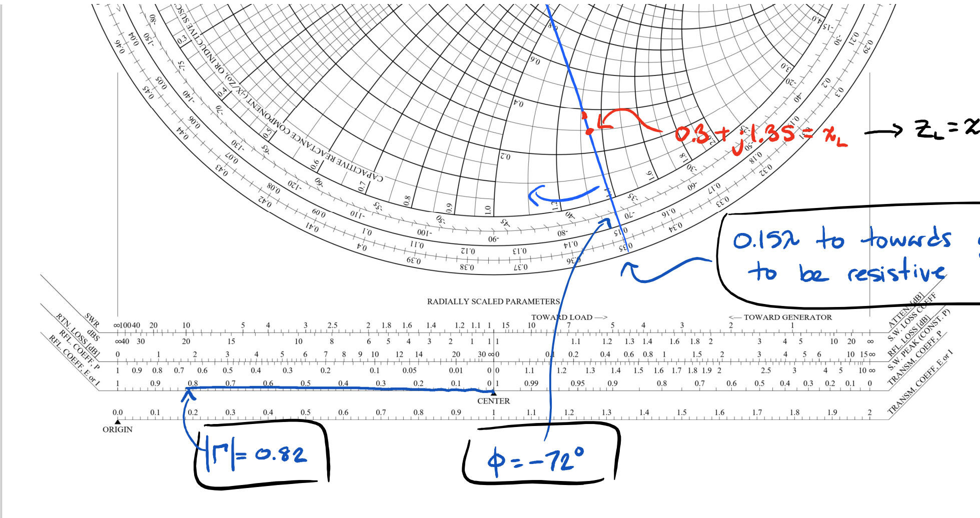
$$\langle P \rangle = \frac{1}{2} Re\{V I^*\} \longrightarrow \frac{1}{2} Re\{V_{in} I^*\} = \frac{1}{2} Re\{(4,2)(0.12,\ 0.04)\} = \boxed{0.2\,W}$$

$$V_0^+ = \frac{V_{in}}{e^{j\beta\ell} + \Gamma_L e^{-j\beta\ell}}$$

$$V_L = V_0^+ (1 + \Gamma_L)$$

10.22 A lossless 75-Ω line is terminated by an unknown load impedance. A VSWR of 10 is measured, and the first voltage minimum occurs at 0.15 wavelengths in front of the load. Using the Smith chart, find (*a*) the load impedance; (*b*) the magnitude and phase of the reflection coefficient; (*c*) the shortest length of line necessary to achieve an entirely resistive input impedance.





$0.3 + j1.35 = z_L$ $\rightarrow Z_L = z_L Z_0 = (0.3 + j1.35)\,75\Omega$

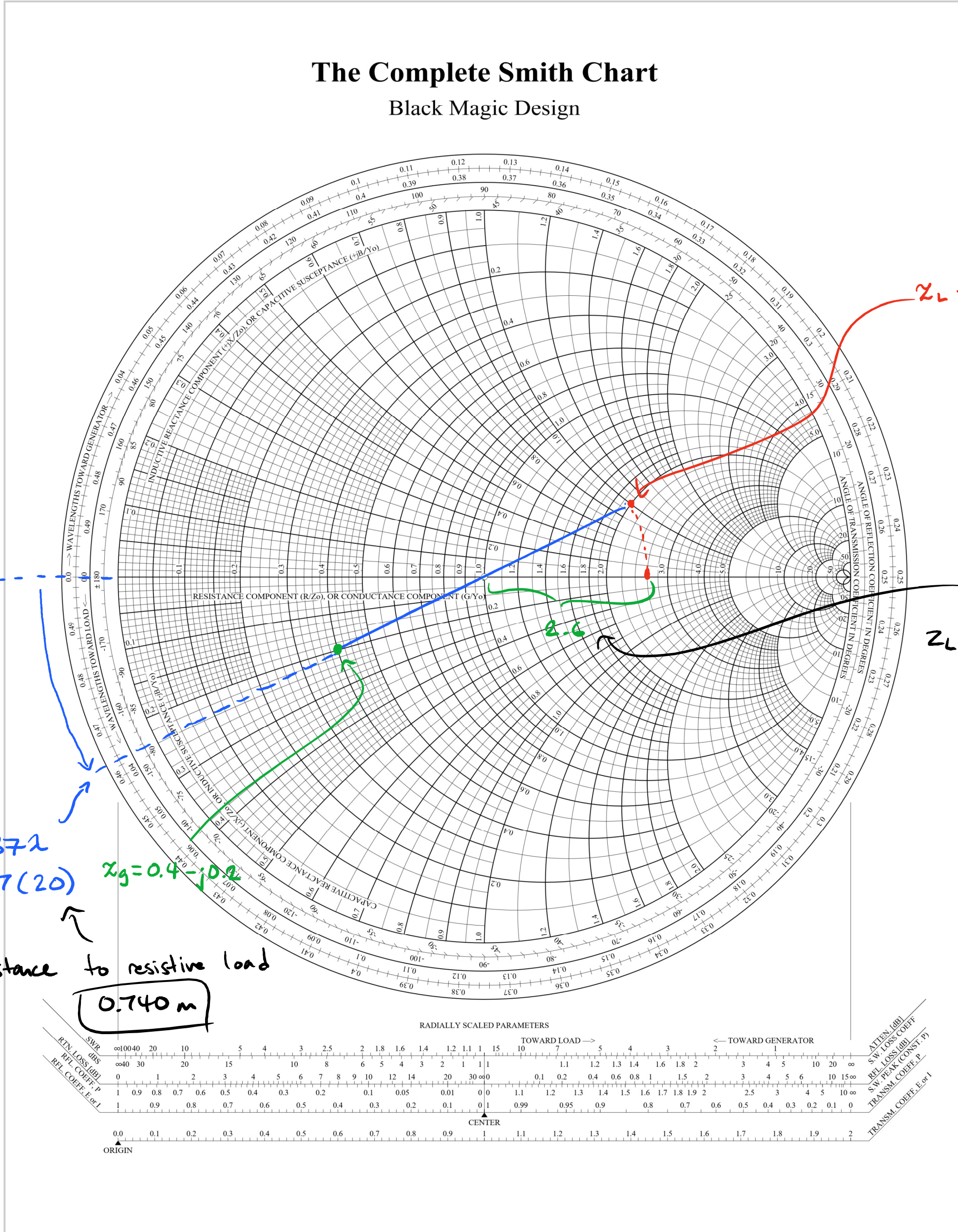
$= \boxed{22.5 + j101.25}$

0.15λ to towards generator to be resistive

$\boxed{|\Gamma| = 0.82}$

$\boxed{\phi = -72^\circ}$

10.23 The normalized load on a lossless transmission line is $2 + j1$. Let $\lambda = 20$ m and make use of the Smith chart to find (*a*) the shortest distance from the load to a point at which $z_{\text{in}} = r_{\text{in}} + j0$, where $r_{\text{in}} > 0$; (*b*) z_{in} at this point. (*c*) The line is cut at this point and the portion containing z_L is thrown away. A resistor $r = r_{\text{in}}$ of part (*a*) is connected across the line. What is s on the remainder of the line? (*d*) What is the shortest distance from this resistor to a point at which $z_{\text{in}} = 2 + j1$?



$z_L = 2 + j1$

z_L where purely resistive, $\boxed{2.6\Omega}$

2.6

0.037λ

$0.037(20)$

$z_g = 0.4 - j0.2$

distance to resistive load $\boxed{0.740\ \text{m}}$

10.32 In Figure 10.17, let $Z_L = 250\ \Omega$, $Z_0 = 50\ \Omega$, find the shortest attachment distance d and the shortest length d_1 of a short-circuited stub line that will provide a perfect match on the main line to the left of the stub. Express all answers in wavelengths.

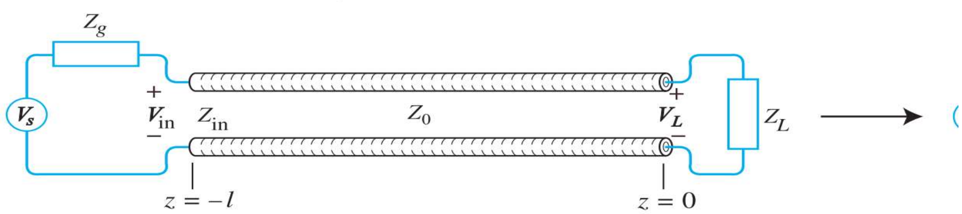
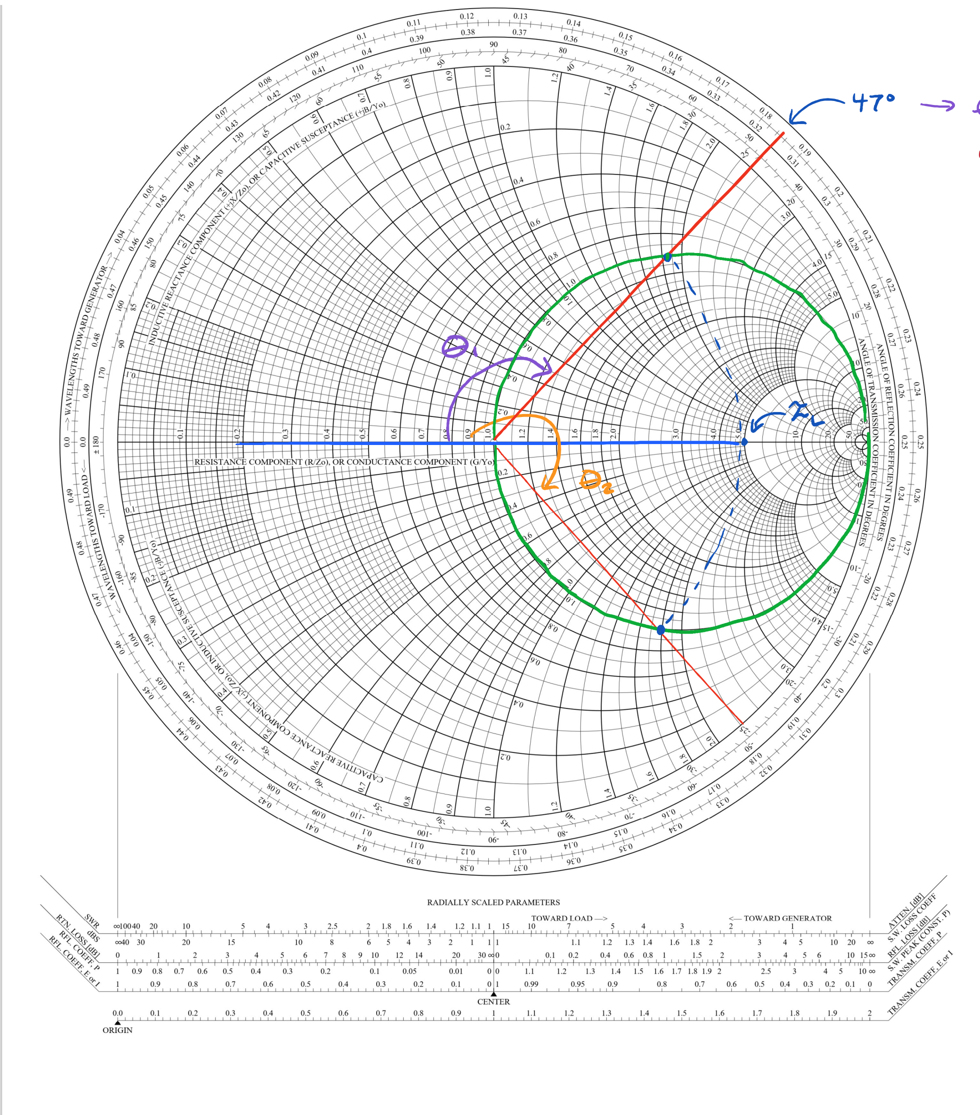


Figure 10.7 Finite-length transmission line configuration and its equivalent circuit

$z_L = \frac{250}{50} = 5\Omega$

$\theta = \frac{4\pi}{\lambda} d_1 \rightarrow d_1 = \frac{133° \cdot 1/4}{180}\lambda = \boxed{0.1847\lambda}$

stub length



$47°$ → $\theta_1 = 180 - 47 = 133°$

$\theta_2 = 180 + 47 = 227°$

10.40 In the charged line of Figure 10.25, the characteristic impedance is $Z_0 = 100\ \Omega$, and $R_g = 300\ \Omega$. The line is charged to initial voltage, $V_0 = 160$ V, and the switch is closed at $t = 0$. Determine and plot the voltage and current through the resistor for time $0 < t < 8l/\nu$ (four round-trips). This problem accompanies Example 10.12 as the other special case of the basic charged-line problem, in which now $R_g > Z_0$.

$I^+ = \frac{V^+}{Z_0}$, $I^- = -\frac{V^-}{Z_0}$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ ⟶ $\Gamma_g = \frac{300-100}{300+100} = \frac{1}{2}$

$\Gamma_L = \frac{\infty - 100}{\infty + 100} = 1$

$V^+ = \bar{V}_0 \frac{Z_0}{Z_0 + R_g}$ ⟶ $V_1^+ = \bar{1}60\,V \frac{100\,\Omega}{100\,\Omega + 300\,\Omega} = \bar{4}0\,V$

$V^-(0,t) = \Gamma V^+(0,t)$ → $V_1^- = \Gamma_L V_1^+ = 40V$

$V_2^+ = \Gamma_g V_1^- = 20V$, $V_2^- = \Gamma_L V_2^+ = 20V$

$V_3^+ = \Gamma_g V_2^- = 10V$, $V_3^- = \Gamma_L V_3^+ = 10V$

$V_4^+ = \Gamma_g V_4^- = 5V$, $V_4^- = \Gamma_L V_4^+ = 5V$

$V_{R_1} = V_0^- + V_1^+ = 120V$ $\qquad$ $I_1^+ = V_1^+/Z_0 = 40/100 = 0.4\,A$

$V_{R_2} = V_1^- + V_2^+ = 60V$ $\qquad$ $I_2^+ = V_2^+/Z_0 = 20/100 = 0.2\,A$

$V_{R_3} = V_2^- + V_3^+ = 30V$ $\qquad$ $I_3^+ = V_3^+/Z_0 = 10/100 = 0.1\,A$

$V_{R_4} = V_3^- + V_4^+ = 15V$ $\qquad$ $I_4^+ = V_4^+/Z_0 = 5/100 = 0{,}05\,A$

$V_{R_5} = 7.5V$ $\qquad$ $I_5^+ = V_5^+/Z_0 = 2.5/100 = 0.025\,A$