Assignment 3 - Waveguides

Monday, April 4, 2016 8:27 AM

13.11 A parallel-plate waveguide is known to have a cutoff wavelength for the $m = 1$ TE and TM modes of $\lambda_{c1} = 4.1$ mm. The guide is operated at wavelength $\lambda = 1.0$ mm. How many modes propagate?

$m = 1$, $\lambda_{c1} = 4.1\,mm$, $\lambda = 1mm$ modes?

$$\omega_{cm} = \frac{m\pi c}{nd} \qquad \lambda_{cm} = \frac{c}{f_{cm}} = \frac{c}{\omega_{cm}/2\pi}$$

$$\lambda_{cm} = \frac{\cancel{c}\,2\pi}{\frac{m\pi \cancel{c}}{nd}} = \frac{2nd}{m}$$

$$\lambda_{c1} = 4.1mm = \frac{2nd}{1}$$

$$\lambda = 1mm = \frac{2nd}{m}$$

assuming same n, $n = 1$

$m = 4.1 \quad \therefore \boxed{m = 4}$

13.12 A parallel-plate guide is to be constructed for operation in the TEM mode only over the frequency range $0 < f < 3$ GHz. The dielectric between plates is to be teflon ($\epsilon'_r = 2.1$). Determine the maximum allowable plate separation, d.

$n = \sqrt{\epsilon_r'} = \sqrt{2.1}$ $\qquad d_{max} = ?$

$\omega_{max} = 6\pi 10^9 \frac{rad}{s}$

$$\omega_{cm} = \frac{m\pi c}{nd}$$

TEM $\therefore$ no cutoff frequency

$m = 1$?

$$\hookrightarrow d = \frac{m\pi c}{n\omega_{cm}} = m\frac{\pi c}{\sqrt{2.1}\,6\pi 10^9} = 34.48\,mm$$

13.14 A $d = 1$ cm parallel-plate guide is made with glass ($n = 1.45$) between plates. If the operating frequency is 32 GHz, which modes will propagate?

$$m < \frac{2fnd}{c} \rightarrow m = \left\lfloor \frac{2(32(10^9))\,1.45\,(10^{-2})}{3(10^8)} \right\rfloor = 3, \quad \boxed{1\ 2\ 3}$$

13.18 In the guide of Figure 13.25, it is found that $m = 1$ modes propagating from left to right totally reflect at the interface, so that no power is transmitted into the region of dielectric constant ϵ'_{r2}. (*a*) Determine the range of frequencies over which this will occur. (*b*) Does your part (*a*) answer in any way relate to the cutoff frequency for $m = 1$ modes in either region? *Hint*: Remember the critical angle?

$\theta > \theta_c$ for total reflection

$\hookrightarrow \theta_i = 90 - \theta_m \rightarrow \theta_i \geq \theta_c$

$\theta_m \le 90 - \theta_c$

$\cos\theta = \frac{m\pi c}{\omega n_1 d} \rightarrow \arccos\theta_m\left(\frac{m\pi c}{\omega n_1 d}\right) \le 90 - \theta_c$

note sign change

$\frac{m\pi c}{\omega n_1 d} \ge \cos(90-\theta_c) = \sin(\theta_c)$

snell's law $\sin\theta_c = \frac{n_2}{\cancel{n_1}} = \frac{m\cancel{\pi}c}{2\cancel{\pi}f\cancel{n_1}d}$

$\hookrightarrow f_{max} = \frac{mc}{2fdn_2} = \frac{1\cdot 3(10^8)}{2(1(10^{-2}))\sqrt{2.1}} = 10.35(10^9)\,Hz$

a) $\boxed{0 < f \le 10.35(10^9)\,Hz}$

$\omega = \frac{m\pi c}{dn} \rightarrow f_c = \frac{1\cancel{\pi}c}{2\cancel{\pi}\sqrt{2.1}(10^{-2})} = 10.35(10^9)\,hz$

b) $\boxed{\text{they're the same}}$