Ch. 9 - Functions

Saturday, July 9, 2016 1:01 PM

9.1 Relation from set A to set B

Then an element of A, x is related to no elements of B, or any number of elements of B

If every element of A is related to no elements of B, then the relation, R, is $\emptyset$

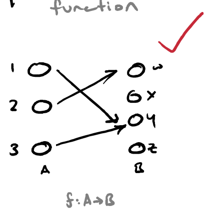
If every element of A is related to all elements of B, then $R = A \times B$

If every element of A is related to exactly one element of B, then R is a function

A function from set A to set B is written $f: A \to B$

↳ every element x in A is the first coordinate of exactly one ordered pair in f

in $A \to B$, mapping x value to y value, no repeated x values



the set A is the domain of f (x values)

the set B is the codomain of f (y values)

For $f: A \to B$, let $(a,b) \in f$ (ie. $b = f(a)$)
↖ image of a

↳ f contains only one ordered pair whose first coordinate is a, and b is the unique second coordinate of a, ie. if $(a,b) \in f$ and $(a,c) \in f$, then $b = c$.

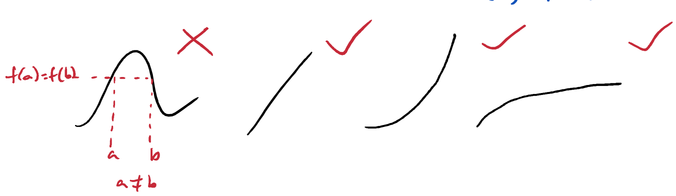
9.2 the set of all functions from A to B is represented as B^A

$$B^A = \{f : f : A \to B\}, \quad |B^A| = |B|^{|A|}$$

one-to-one / injective functions

↳ if every two distinct sets of A have distinct images in B

if $a, b \in A$ and $a \neq b$ then $f(a) \neq f(b)$



onto/surjective functions

↳ if every element of the codomain B is the image of some element of A

↳ every y value is the image of some x value input set = output set $A=\{1,2,3\}$ $B=\{x,y,z,w\}$ $f=\{(1,y),(2,w),(3,y)\}$

↳ not onto b/c x and z are not an image of A (i.e. missing from f ordered pairs)

Bijective

if $|A|\geq|B|$, f is surjective

if $|A|\leq|B|$, f is one-to-one

$|A|=|B|$, both onto and 1-to-1 ie, bijective

if $|A|=|B|=n$, then there are $n!$ bijective functions from A to B

Identity function (is bijective)

$i: A\to A$ is defined as $i(a)=a$ for each $a\in A$

ie if $A=\{1,2,3\}$ $i=\{(1,1),(2,2),(3,3)\}$

(recall identity matrix $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$)

Composition $f:A\to B$, $g:B\to C$

$(g\circ f)(a)=g(f(a))$

$A=\{1,2,3\}$ $B=\{x,y,z\}$

$f=\{(1,x)(2,z)(3,z)\}$ $g:\{(x,\alpha),(y,\alpha)(z,\beta)\}$

$g\circ f=g(f(a))$ for $a\in A$: $A\to C$

$f(1)=x$ $f(2)=z$ $f(3)=z$
$g(x)=\alpha$ $g(z)=\beta$ $g(z)=\beta$

$C=\{\alpha,\beta\}$

Inverse relation

$R^{-1}=\{(b,a):(a,b)\in R\}$

$R=\{(1,x),(2,y),(3,z)\}$ $R^{-1}=\{(x,1),(y,2),(z,3)\}$