Question #7: (13 marks) A 50V DC source with a 100 Ω internal impedance is connected via a switch (open for t<0) to a transmission line with characteristic impedance of 50 Ω. The transmission line is terminated with a 30 Ω load. The velocity of propagation is 200 m/μs and the line is 400 m long. The switch is closed at t=0 and remains closed. Draw and label a voltage reflection diagram and the voltage waveform ~~measured~~ half way along the transmission line for the first 8 μs. What is the steady-state value of voltage measured at this point in the transmission line?

voltage sums

$V_0 = 50V \quad Z_g = 100\Omega \quad Z_0 = 50\Omega \quad Z_L = 30\Omega$

$RTT = 2\,\dfrac{400m}{200m/\mu s} = 4\mu s$ (2 RTT in 8μs)

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{30-50}{30+50} = -\dfrac{1}{4} \qquad \Gamma_g = \dfrac{Z_g - Z_0}{Z_g + Z_0} = \dfrac{1}{3}$

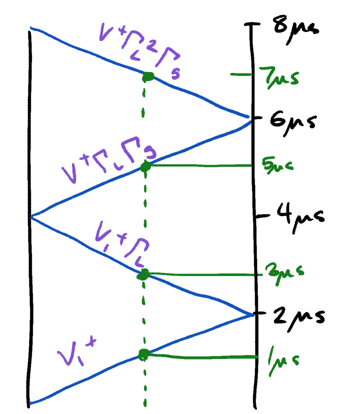
$V_1^+ = V_0 \dfrac{Z_0}{Z_g + Z_0} = 16.67V \qquad I_1^+ = \dfrac{V_1^+}{Z_0} = 333mA$

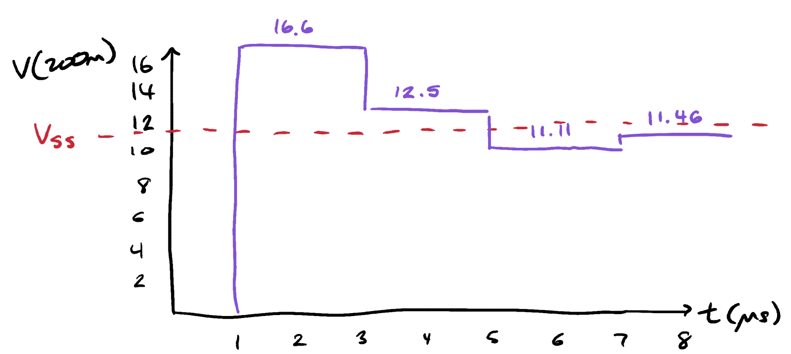
$V_1^- = \Gamma_L V_1^+ = -4.167V \qquad I_1^- = -\dfrac{V_1^-}{Z_0} = 83.34mA$

$V_2^+ = \Gamma_g V_1^- = -1.389V \qquad I_2^+ = -27.78mA$

$V_2^- = \Gamma_L V_2^+ = 347.3mV \qquad I_2^- = -\dfrac{V_2^-}{Z_0} = -6.945$

note when V swaps sign, I doesn't and vice-versa.





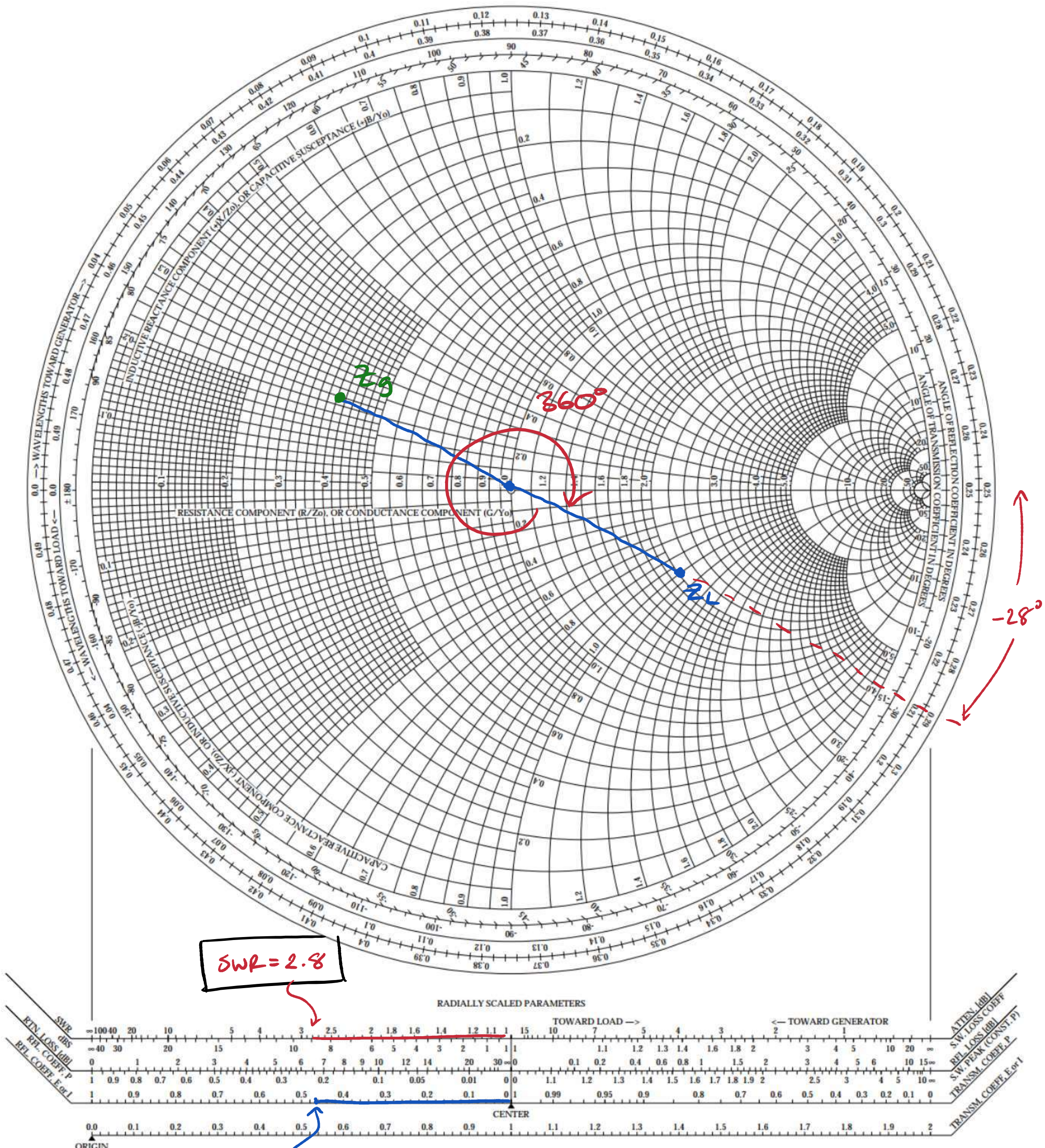
V(200m)

$V_{ss} = V_0 \dfrac{Z_L}{Z_g + Z_L} = 50 \dfrac{30}{100+30} = 11.54V$

Z_{in}

Question #8: (7 marks) Use the Smith chart to find the input impedance, Z_L, voltage reflection coefficient at the load, Γ_L, and voltage standing wave ratio (VSWR) of a 50 Ω transmission line that is 0.25λ long and terminated with a load of 100-j50 Ω. What is the input impedance if the length is doubled to 0.5λ? Show all your work on the Smith chart, and make sure you hand this in with your Quiz!

$Z_0 = 50\Omega \rightarrow z_L = 2 - j1$ $\qquad 0.25\lambda \rightarrow 180°, \quad 0.5\lambda \rightarrow 360°$



$\boxed{\Gamma_L = 0.48 \angle -28°}$

$z_g = 0.4 + j0.22$

$\boxed{\Gamma_L = 0.48 \angle -28^\circ}$

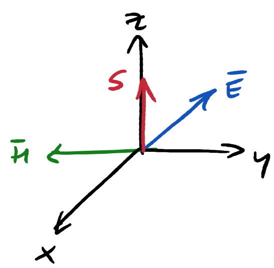
$z_g = 0.4 + j0.22$

$\hookrightarrow \boxed{Z_g = 20 + j11\,\Omega}$ 0.25λ

if $\ell = 0.5\lambda$, 360° $\therefore \boxed{Z_g = Z_L = 100 - j50\,\Omega}$ 0.5λ

Question #11 [10 marks]: In free space, $\vec{E}^+(z,t) = -10\,sin\,(\omega t - \beta z)\hat{a}_x$, [V/m]

A, [2 marks] find $\vec{H}^+(z,t)$
B, [2 marks] find the propagation constant β assuming $f = 300\text{MHz}$.
C, [2 marks] find the attenuation constant, α
D, [2 marks] find the Poynting vector, $\vec{S}$
E, [2 marks] assume that this wave is incident on a satellite dish antenna that is 20cm in diameter, what is the average power incident on the antenna.



From $-\beta z$ we kno S in $+z$

By RHR, H in $-a_y$

$|\vec{H}| = \frac{|\vec{E}|}{\eta} = \frac{10}{377}$, $\boxed{\vec{H} = -\frac{10}{377}\sin(\omega t - \beta z)\hat{a}_y \frac{V}{m}}$

b/c free space

Q $\beta = \omega\sqrt{\mu_o \varepsilon_o} = 2\pi 300(10^6)\,\frac{1}{c} = \boxed{6.283\,\frac{rad}{s}}$

free space $\therefore$ no loss, $\boxed{\alpha = \phi}$

$\vec{S} = E \times H^* = |E||H|\sin 90\,\hat{a}_z = \boxed{\frac{100}{377}\hat{a}_z}$

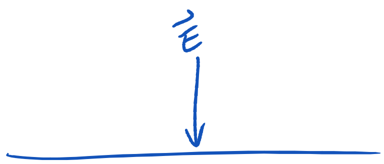
$\cancel{P_{dish} = SA} = \frac{100}{377}\pi r^2 = \frac{100}{377}\pi(10(10^{-2}))^2 = \cancel{\boxed{8.3\,mW}}$

$P_{dish} = \langle S \rangle A = \frac{1}{2}[E \times H^*]\pi r^2 = \frac{1}{2}\,\frac{100}{377}\pi(10 \cdot 10^{-2})^2$

$= \boxed{4.167\,mW}$

Question #12 [10 marks]: A normally incident EM wave with a frequency of 30MHz has an amplitude $|\vec{E}| = E^0_{free\ space}(z=0) = 1.0$ V/m in free space just outside of the surface of pure water. Water has general property that $\varepsilon'_r = 80, \mu_r = 1, \sigma = 0$S/m.

A, [3 marks] find the electric field intensity just inside the water but at the interface, i.e., $E^0_{water}(z=0)$ by considering the transmitted component across the interface.
B, [2 marks] at what angle of incidence would the incoming wave experience total reflection off of the water surface?
C, [2 marks] Assume we added salt to the water such that $\sigma = 0.2$ S/m, is this water considered a good dielectric or good conductor at the given EM frequency?
D, [2 marks] What is the attenuation coefficient of this wave inside the salted water, α.
E, [1 mark] Determine the penetration depth of the EM wave into this salted water.



$|\vec{E}| = 1 \ V/m$ $\quad \omega = 2\pi 10^6 \ Hz$

~~@ Boundary~~ $\cancel{\boxed{E_i = E_t = 1 \ V/m}}$

$\varepsilon'_{r_2} = 80$
$\mu_r = 1$
$\sigma = \phi$

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\sqrt{\frac{\mu_0}{80\varepsilon_0}}}{\sqrt{\frac{\mu_0}{\varepsilon_0}} + \sqrt{\frac{\mu_0}{80\varepsilon_0}}} = 0.2011$$

$$|E_t| = \tau |E_i| = \boxed{0.2011 \ V/m}$$

$\underbrace{\theta_i > \theta_c}_{\text{measured from normal}}$ total reflection

$$\theta_c = \operatorname{asin}\left(\frac{n_2}{n_1}\right) = \operatorname{asin}\left(\frac{\sqrt{\varepsilon'_{r_2}}}{\sqrt{\varepsilon_{r_1}}}\right) = \operatorname{asin}\left(\frac{\sqrt{80}}{1}\right) = \underline{\text{complex}} \quad \therefore \boxed{\text{no angle}}$$

$$\text{loss tangent} = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\varepsilon'\omega} = \frac{\sigma}{\varepsilon_0 \varepsilon'_{r_2} \omega} = \frac{0.2}{\varepsilon_0 80 \, 2\pi 30(10^6)} = 1.498$$

$$\frac{\varepsilon''}{\varepsilon'} \gg 1 \longrightarrow \boxed{\underline{\text{good conductor}}}$$

$$\hookrightarrow \therefore \alpha = \beta = \sqrt{\pi f \mu \sigma} = \sqrt{\pi 30(10^6) \mu_0 0.2}$$
$$= \boxed{4.867 \ Np/m}$$

$$\delta = \frac{1}{\alpha} = \boxed{205 \ mm}$$

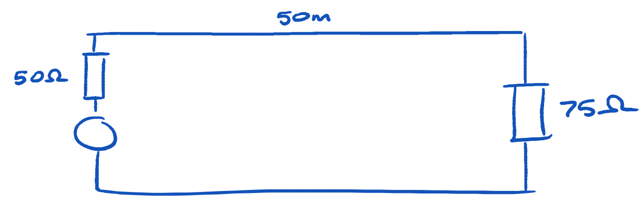
Question #21 [4 marks]: You are a FM radio system engineer and are asked to design a quarter-wavelength transmission line to connect and match an antenna that has an input impedance of 75 Ω to a power transmitter located 50m from the antenna and a generator impedance of 50 Ω. You want to use the shortest length of line that accomplishes full power transmission to the antenna. The operating frequency is 100MHz, the velocity factor is 0.66.

a, [2 marks] what is the characteristic impedance of the transmission line that you will need to use for quarter-wavelength matching the two systems?

b, [2 marks] what is the shortest length of transmission line that will both connect the transmitter to the antenna and achieve quarter wavelength matching?

$f = 100(10^6)\,Hz$

$v = \frac{2}{3}c$



for $\frac{1}{4}\lambda$ matching

$$Z_{02} = \sqrt{Z_{01} Z_{03}} = \sqrt{50 \cdot 75} = \boxed{61.24\,\Omega}$$

$$\lambda = \frac{2\pi}{\omega} v_p = \frac{\frac{2}{3}c}{100(10^6)\,Hz} = 1.979\,m \rightarrow \frac{1}{4}\lambda = 0.4947\,m$$

transmission line, $l = 50 + 0.4947 = \boxed{50.49\,m}$

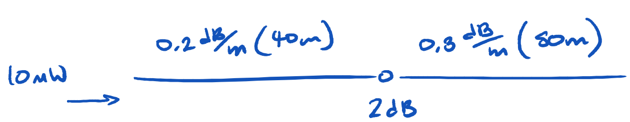
Question #22 [10 marks]: A transmitter and antenna are connected using a series connected pair of transmission lines. Line 1 has a measured loss of 0.2 dB/m, and line 2 is rated at 0.3 dB/m. The link is composed of 40 m of line 1 joined to 50 m of line 2. At the joint, a splice loss of 2 dB is measured.

a, [3 marks] If the transmitter power is 100 mW, what is the received power?

b, [3 marks] If 10% of the splice loss is due to contact resistance between line 1&2 and 90% is due to a mismatch in the two transmission lines, what is the magnitude of the reflection coefficient at the junction?

c, [2 marks] What is the attenuation coefficient for the first segment of transmission line?

d, [1 mark] Would you consider this to be a good transmission line?



line 1 $\quad L_1 = 10\log\frac{P_{in1}}{P_{out1}} \rightarrow 8dB = 10\log\frac{100mW}{P_{out1}}$

$P_{out1} = 15.85\,mW$

junction $\quad 2dB = 10\log\frac{15.85\,mW}{\quad} \rightarrow P_{out_i} = 10\,mW$

junction $\quad 2dB = 10\log \frac{15.85mW}{P_{out\,j}} \rightarrow P_{out\,j} = 10mW$

line 2 $\quad 15dB = 10\log \frac{10mW}{P_{out_2}} \rightarrow \boxed{P_{out_2} = 316\mu W}$

$\langle S_t \rangle = (1-|\Gamma|^2)\langle S_i \rangle$

$\cancel{\frac{\langle S_t \rangle}{\langle S_i \rangle} = 1-|\Gamma|^2 = 0.9} \rightarrow \cancel{\boxed{|\Gamma| = 0.316}}$

↳ 90% of 2dB is due to reflection

↳ $L_{dB} = 2dB(0.9) = 1.8dB = 10\log\frac{\langle S_i \rangle}{\langle S_t \rangle}$

$= 10\log\frac{1}{1-|\Gamma|^2} \rightarrow \boxed{\Gamma = 0.582}$

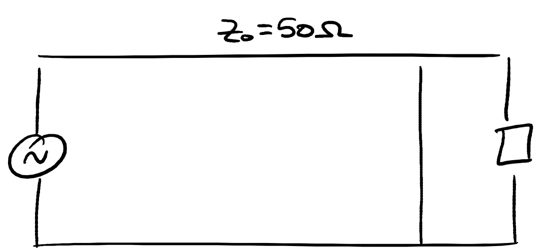
$L_{dB} = 8.69\alpha z \rightarrow \alpha_1 = \frac{8dB}{8.69(40m)} = \boxed{0.023 \frac{Np}{m}}$

$\boxed{\text{No. Almost all power is lost.}}$

Question #23 [6 marks]: A sinusoidal voltage source drives a parallel combination of an impedance, $Z_g = 50 - j50\ \Omega$, and a lossless transmission line of length, L, that is shorted at the load end. The line characteristic impedance of the transmission line is 50 Ω, and the signal wavelength is λ as measured on the line. Using a Smith chart, determine in units of wavelength, the shortest length of the transmission line that will result in the voltage source driving a total parallel impedance of 100+j0 Ω. [Show your work on the Smith chart]

$\boxed{Q}$

$\rightarrow z_p = 2\Omega$

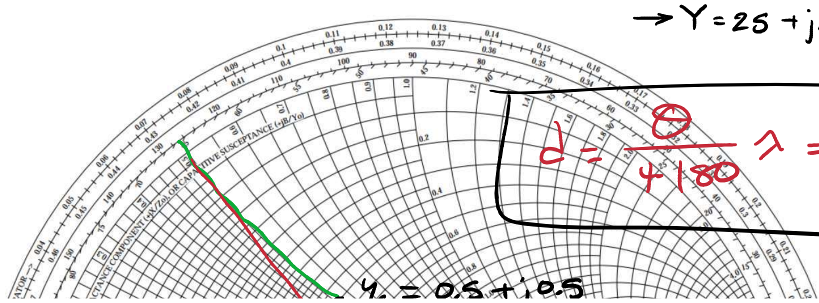


in parallel, so easier to use admittance.

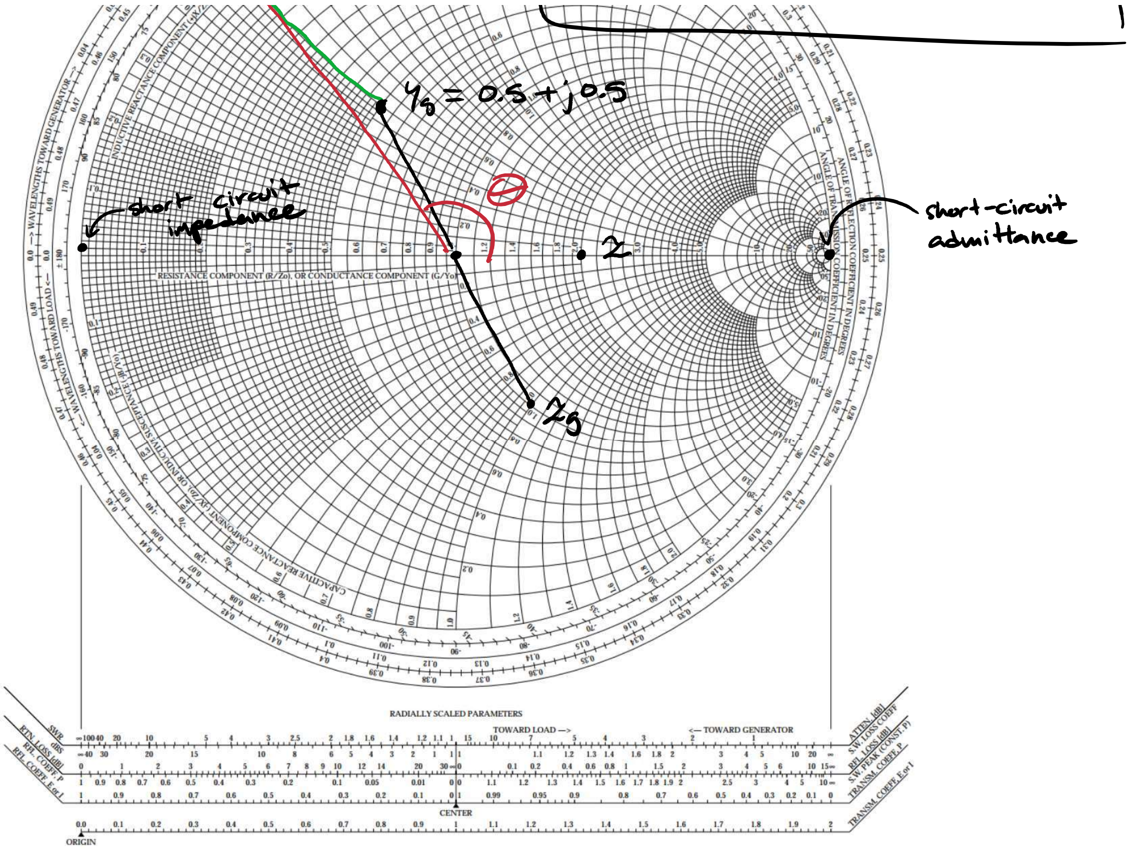
↳ $z_g = 1-j1\ \Omega$

from S.C. $\rightarrow y_g = 0.5+j0.5$

$\rightarrow Y = 25 + j25\ \mho$



$\boxed{d = \frac{\theta}{4 \cdot 180}\lambda = 0.177\lambda}$



Question #24 [10 marks]: An engineer is able to determine that the load of his transmission line results in reflection coefficient of $\Gamma = 0.5\angle 10°$.

(a) [4 marks] Using a Smith chart, find the normalized load impedance and normalized load admittance.

(b) [6 marks] Using the single stub method to match his load; i.e., find d_{stub} (the location of the stub) and d (the length of the stub) using a short circuit stub, provide the distances in units of wavelengths. [Show your work on the Smith chart]

vs. oc?

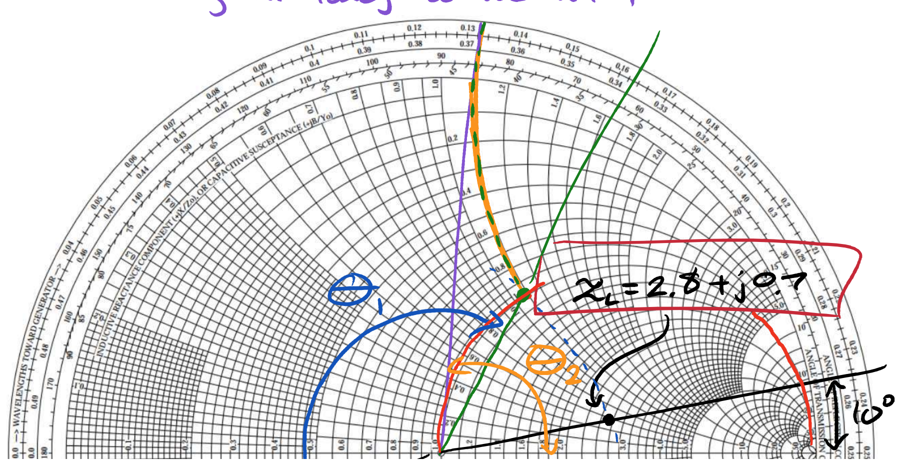
$\Gamma_L = 0.5\angle 10°$

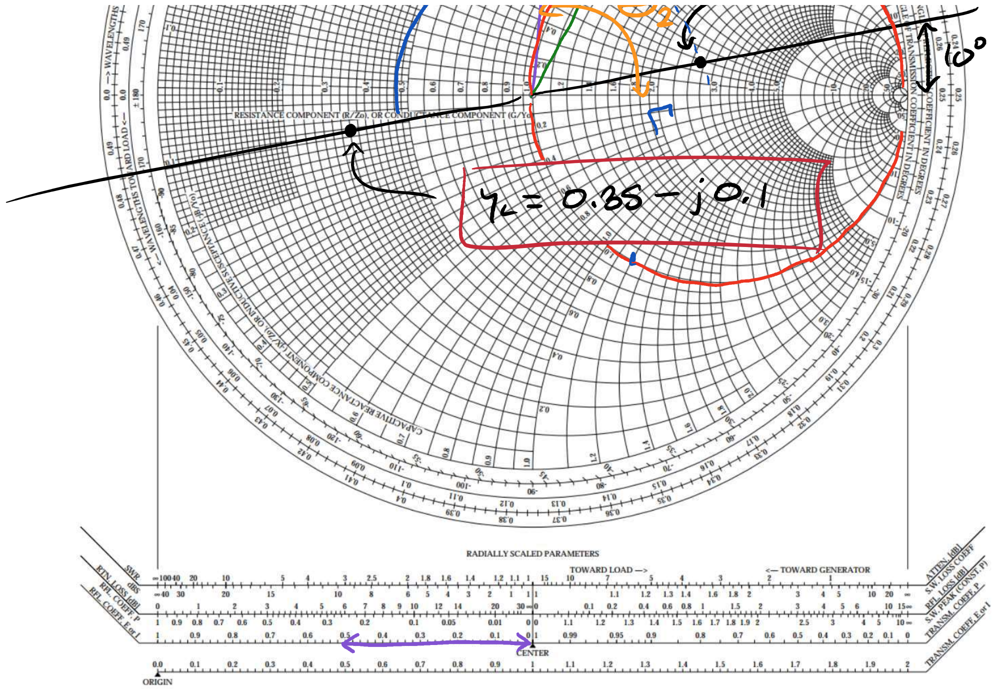
$$d_{stub} = \frac{\theta_1}{4\cdot 180°}\lambda = \frac{130°}{4\cdot 180°}\lambda = \boxed{0.180\lambda}$$

$$L_{stub} = \frac{\theta_2}{4\cdot 180°}\lambda = \frac{82}{4\cdot 180}\lambda = \boxed{0.113\lambda}$$

$$\frac{\theta}{4\cdot 180} = \lambda$$

we're looking at load, so use WTG



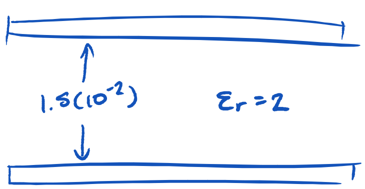


Q13, [8 marks] A parallel plate waveguide with $d = 1.5$ cm is filled with a dielectric that has a relative permittivity, $\varepsilon_r = 2$.

A. [3 marks] Determine the number of modes that will propagate (total modes including all TE, and TM) at f=50GHz?

B. [2 marks] In your own words, describe the difference between a transmission line and waveguide?

C. [3 marks] Can you name 3 different types of waveguides?



$TM_0 = TEM$ Q

$$\omega_{cm} = \frac{m\pi c}{nd} = \frac{m\pi c}{\sqrt{2}\, 1.5(10^{-2})}$$

↳ only propagates if $f > f_{cm}$

$$\frac{m\pi c}{\sqrt{2}\, 1.5\, 10^{-2}} = 2\pi\, 50GHz$$

↳ $m = 7.07$

propagates from modes $m = [0, 7]$

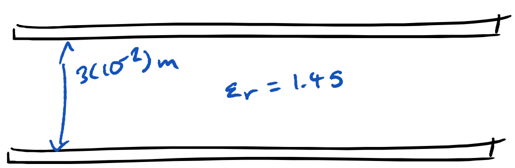
$TE_0 - TE_7$, $TM_0 - TM_7$, TEM → $\boxed{\text{15 total}}$

transmission line is a waveguide that always supports TEM modes

parallel plate, rectangular, cylindrical, optical fibre, photonic crystic, dielectric slab

Q14, [10 marks] A parallel plate waveguide with $d = 3$ cm is filled with glass, which has a relative permittivity, $\varepsilon_r = 1.45$.

A. [3 marks] Determine the wave angles, θ_m, for the first three modes ($m = 1,2,3$) when $f = 50$ GHz.
B. [1 mark] What is the wave angle at cut-off?
C. [1 mark] At cut-off is the wave evanescent or propagating?
D. [2 marks] What is the maximum frequency for which this waveguide will operate only in the TM_0 mode.
E. [2 marks] What is the group velocity of the TE_2 mode at $f = 50$ GHz?
F. [1 marks] What is the group velocity of the TE_2 mode as $f \to \infty$?



$$\theta_m = a\cos \frac{m\pi c}{\omega n d}$$

$$\theta_1 = a\cos \frac{\pi c}{2\pi 50(10^9)\sqrt{1.45}\, 3(10^{-2})}$$

$$\boxed{\begin{aligned}\theta_1 &= 85.24^\circ \\ \theta_2 &= 80.45^\circ \\ \theta_3 &= 75.58^\circ\end{aligned}}$$

$$\theta_{cm} = a\cos\left(\frac{m\pi \cancel{c}^{\,1}}{\cancel{\frac{m\pi c}{nd}}\, nd}\right) = \boxed{\phi^\circ}$$

@ cutoff wave is evanescent

$TM_0 = TEM \to$ no cutoff frequency

$$\to \omega_{c1} = \frac{1\pi c}{\sqrt{1.45}\, 3(10^{-2})} = 26.07(10^9)\frac{rad}{s} = \boxed{4.149\,GHz}$$

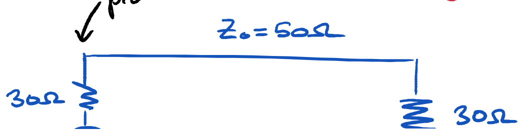
higher than this TE_1 & TM_1 will propagate

$$v_g = \frac{c}{n}\sin\theta_m \to v_g(f = 50GHz) = \frac{c}{\sqrt{1.45}}\sin\theta_2 = \boxed{245.5(10^6)\frac{m}{s}}$$

$$\to v_g(f = \infty) = \frac{c}{\sqrt{1.45}}\sin\theta_2 = \boxed{245.5(10^6)\frac{m}{s}}$$

Suppose that a square 50-V pulse (1μs long) generated at a source with 30-ohm internal resistance is incident on a 30-ohm load in a transmission line with a characteristic impedance of 50-ohms. The velocity of the pulse is 100m/μs and the length of the transmission line is 400m. Sketch the voltage and current reflection diagrams and the voltage and current waveforms at the input end of the transmission line for at least two reflections off the input end of the transmission line.

prob ← on reflection edge, so overlap



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-20}{80} = \frac{-1}{4}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-20}{80} = \frac{-1}{4}$$

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{-1}{4}$$

$$RTT = 2\frac{400\,m}{100\,{}^{m}/_{10^{-6}s}} = 4\mu s$$

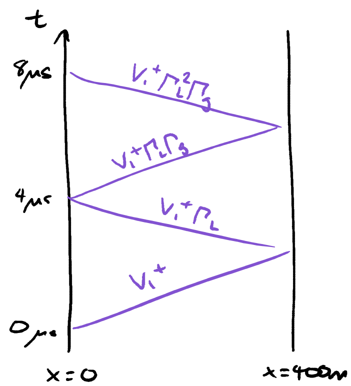
$$I_1^+ = \frac{V_1^+}{Z_0} = 625\,mA \qquad V_1^+ = V_0\frac{Z_0}{Z_0 + R_g} = 50\,\frac{50}{80} = 31.25V$$

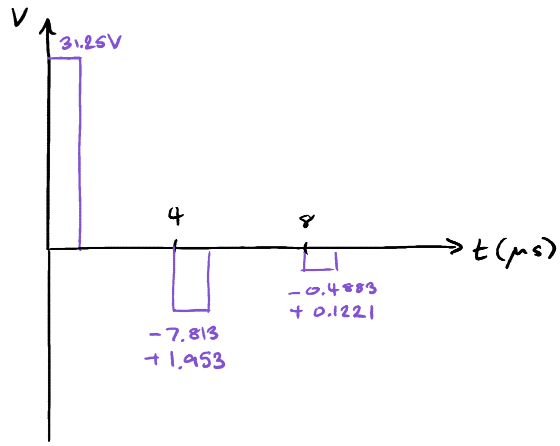
$$I_1^- = -\frac{V_1^-}{Z_0} = 156.3\,mA \qquad V_1^- = \Gamma_L V_1^+ = -7.813V$$

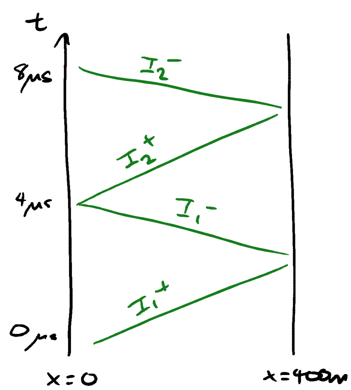
$$I_2^+ = 39.06\,mA \qquad V_2^+ = 1.953V$$

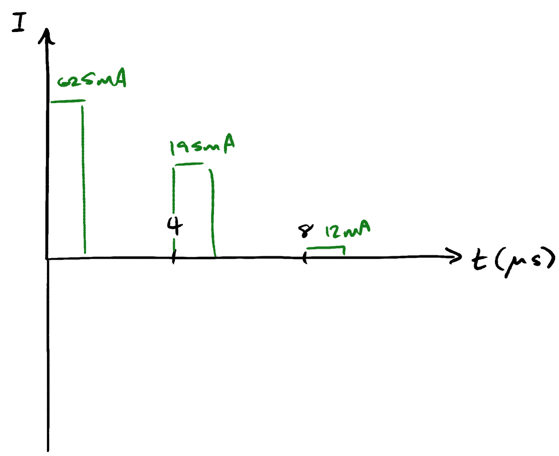
$$I_2^- = 9.766\,mA \qquad V_2^- = -0.4883V \qquad V_3^+ = 0.1221V$$

$$I_3^+ = 2.442\,mA$$









An electric field propagating in a lossless non-magnetic media is characterized by

$\hookrightarrow \sigma = 0$ $\hookrightarrow \mu_r = 1$

$$\vec{E}(z,t) = \hat{x}\, 5\cos(8\pi \times 10^9 t - 400z)\,\frac{V}{m}.$$

i. Find the wave frequency
ii. Find the wavelength
iii. Find the relative permittivity of the media
iv. Find the magnitude $|\vec{H}(z,t)|$

i) $f = \dfrac{8\pi 10^9 \frac{rad}{s}}{2\pi} = \boxed{4 GHz}$

ii) $\cancel{\lambda = \dfrac{c}{f} = \dfrac{3(10^8)\,m/s}{4(10^9)\,1/s} = \boxed{75 mm}}$ $\beta = \dfrac{2\pi}{\lambda} = 400 \rightarrow \lambda = \boxed{15.7 mm}$

iii) $k = \omega\sqrt{\mu\varepsilon} \longrightarrow 400 = 8\pi 10^9 \sqrt{\mu_o \varepsilon_o \varepsilon_r}$

$\hookrightarrow \varepsilon_r = \boxed{22.77}$

iv) $|\vec{H}| = \dfrac{|\vec{E}|}{\eta} = \dfrac{5}{\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_r}}} = \boxed{63.3\, m\frac{A}{m}}$

A uniform plane wave propagates in a lossless medium ($\varepsilon_r = ?$, $\mu_r = ?$) with a frequency of 10 GHz. The electric and magnetic field expressions are shown below. Determine ε_r and μ_r.

$$\underline{\vec{E}} = -\hat{y}\, 5e^{-j400\pi z} \qquad \underline{\vec{H}} = \hat{x}\,\frac{5}{60\pi} e^{-j400\pi z}$$

$\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_r \mu_o}{\varepsilon_r \varepsilon_o}} = 60\pi$

$\beta = 400\pi = \omega\sqrt{\mu_o \mu_r \varepsilon_o \varepsilon_r}$

$\hookrightarrow 2\pi 10(10^9)\frac{rad}{s}$

$\boxed{\mu_r = 3 \quad \varepsilon_r = 11.98}$

A plane wave propagating in a non-magnetic lossy medium is described below. Determine the wave frequency, attenuation constant, phase constant and the wavelength. Write an expression for the phasor electric field **E**(z). In what direction is the wave propagating?

$$\mathbf{E}(z,t) = 20.0\ e^{0.01z} \cos(\pi x 10^7 t + \pi z - \pi/4)\ \mathbf{a_x}\ \text{V/m}.$$

traveling $-z$ direction

$f = \frac{\pi 10^7}{2\pi} = \boxed{5 MHz}$

$E_{xs} = E_{xo} e^{-jk_o z}$

$\hookrightarrow jk_o z = 0.01z$

$\boxed{\beta = \pi} \rightarrow \boxed{\lambda = \frac{2\pi}{\pi} = 2}$

$\alpha = Re\{jk\} = \boxed{0.01\ \frac{Np}{m}}$

$\boxed{\text{propagating in } -z \text{ direction}}$

A 7.0 MHz plane wave travels in a non-magnetic lossy dielectric fluid.

The fluid electrical parameters are to be determined by measurement of the electric field of a linearly polarized plane wave travelling in the +z direction at two positions as given below (you can NOT assume a good conductor or low-loss dielectric ahead of time). Hint: determine and use k. Determine σ and ε_r.

$$\underline{\mathbf{E}}(z=0) = 5.0\ \angle 0°\ \mathbf{a_x}\ \text{V/m}.$$
$$\underline{\mathbf{E}}(z=3) = 3.7\ \angle{-150°}\ \mathbf{a_x}\ \text{V/m}.$$

$E_{xs} = E_{xo} e^{-\alpha z}$ $\rightarrow$ @ $z = \phi$, $E_{xs} = E_{xo} e^{0} = 5$

$\rightarrow$ @ $z = 3$, $E_{xs} = 5 e^{-\alpha 3} = 3.7$

$\hookrightarrow \alpha = 0.1004 \frac{Np}{m}$

Q

$\beta z = \theta \rightarrow \beta = \frac{150 \frac{\pi}{180}}{3m} = 0.872 \frac{rad}{m}$

$jk = \alpha + j\beta = 0.1004 + j0.872$

$\mu_r = 1$ b/c non-magnetic

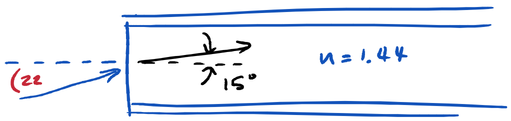
?

$k = \omega\sqrt{\mu\varepsilon} \rightarrow |0.1004 + j0.872| = 2\pi 7 (10^6) \sqrt{\mu_o \varepsilon_o \varepsilon_r}$

$\varepsilon_r = \varepsilon'_r + j\varepsilon''_r \boxed{= -34.91 + j8.141}$

$$\sigma = \varepsilon'' \omega = \varepsilon_0 \varepsilon''_r 2\pi f = \varepsilon_0 8.141 \; 2\pi \; 7(10^6)$$
$$\boxed{= 3.17(10^{-3})}$$

Light travels from air into an optical fiber with an index of refraction of 1.44. (a) In which direction does the light bend? (b) If the angle of incidence on the end of the fiber is 22o, what is the angle of refraction inside the fiber? (c) Sketch the path of light as it changes media.



when wave passes into a dense medium ie. $n_2 > n_1$, bends towards normal

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \longrightarrow \theta_2 = \arcsin\left(\frac{1}{1.44}\sin 22\right) = \boxed{15.08°}$$

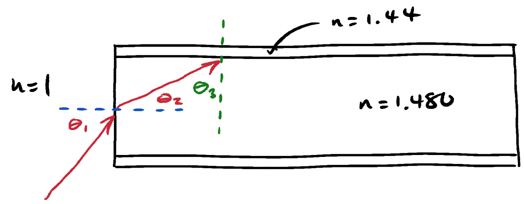
Problem 2 : Total Internal Reflection in Fiber Optics
Consider the optical fiber in the figure below. The index of refraction of the inner core is 1.480 , and the index of refraction of the outer cladding is 1.44.

A. What is the critical angle for the core-cladding interface?

B. For what range of angles in the core at the entrance of the fiber (θ_2) will the light be completely internally reflected at the core-cladding interface?

C. What range of incidence angles in air does this correspond to?

D. If light is totally internally reflected at the upper edge of the fiber, will it necessarily be totally internally reflected at the lower edge of the fiber (assuming edges are parallel)?



$$\theta_c = \arcsin\frac{n_2}{n_1} \longrightarrow \theta_c \text{ cladding} = \arcsin\frac{1.44}{1.48} = \boxed{76.65°}$$

total reflection if $\theta_i > \theta_c$

$\hookrightarrow \theta_3 > \theta_c \qquad \theta_3 = 90 - \theta_2$

$$90 - \theta_2 > \theta_c \longrightarrow \boxed{\theta_2 < 90 - 76.65 = 13.35°}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\hookrightarrow \theta_1 = \arcsin\frac{1.48}{1}\sin 13.35 = 19.98$$

to keep $\theta_2 < 13.35$, $\boxed{\theta_1 < 19.98}$

yes, because θ_3 is the same for both top and bottom reflections

Suppose we have $\vec{E}(z,t) = 1.0 \cos(2\pi \times 10^8 t - \beta z)\hat{a}_x$ $\left[\frac{V}{m}\right]$ propagating in air. We want to find the power normally incident on a 20-cm dish

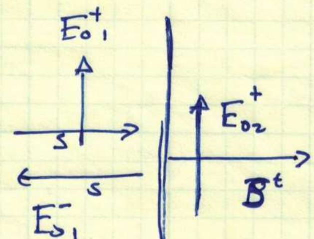
$\eta = 377\Omega$ b/c free space

$|\vec{H}| = \frac{|\vec{E}|}{\eta} = \frac{1}{377}$ $\qquad$ $\langle S \rangle = \frac{1}{2}[E \times H^*]$

↳ b/c ⊥ $\langle \vec{S} \rangle = \frac{1}{2}|\vec{E}||\vec{H}| = \frac{1}{754}$ W/m^2

$P_{dish} = \langle \vec{S} \rangle A = \frac{1}{754}\frac{W}{m^2}\,\pi\left(\frac{20(10^{-2})}{2}\right)^2 = \boxed{41.67\,\mu W}$

Determine the amplitudes of reflected and transmitted $\vec{E}$ and $\vec{H}$ at the interface if $E_{01}^+ = 1.5 \times 10^{-3}$ V/m



$\varepsilon_{r_1} = 8.5$ $\qquad$ $\varepsilon_{r_2} = 1$

$\mu_{r_1} = 1$ $\qquad$ $\mu_{r_2} = 1$

$\eta_1 = \sqrt{\frac{\mu_0}{8.5\varepsilon_0}} = 129.2$ $\qquad$ $\eta_2 = 376.7$ $\qquad$ $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.4892$

Incident $\qquad$ $E_{01}^+ = 1.5(10^{-3})\frac{V}{m}$ $\qquad$ $H_{01}^+ = \frac{E_{01}^+}{\eta_1} = 11.61\,\mu\frac{A}{m}$

$\langle \vec{S}_i \rangle = \frac{1}{2}E_{01}^+ H_{01}^+ = 8.706\,\frac{nV}{m^2}$

transmitted $\qquad$ $E_{02}^+ = (\Gamma + 1)E_{01}^+ = \boxed{2.234\,\frac{mV}{m} \qquad H_{02}^+ = 5.93\,\frac{\mu A}{m}}$

$\langle \vec{S}_t \rangle = \frac{1}{2}E_{02}^+ H_{02}^+ = 6.624\,\frac{nV}{m^2}$ $\qquad$ don't forget (-)ive

reflected $\qquad$ $E_{01}^- = \Gamma E_{01}^+ = \boxed{733.8\,\mu\frac{V}{m} \qquad H_{01}^- = \frac{-E_{01}^+}{\eta_1} = -5.68\,\frac{\mu A}{m}}$

$\langle \vec{S}_r \rangle = \frac{1}{2}E_{01}^- H_{01}^- = 2.084\,n\,\frac{V}{m^2}$

confirm with power densities

$\langle \vec{S}_i \rangle = \langle \vec{S}_r \rangle + \langle \vec{S}_t \rangle$ ✓ true

$\langle S_i \rangle = \langle S_r \rangle + \langle S_t \rangle$ ✓ true

$\langle S_i \rangle |\Gamma|^2$ $\qquad$ $\langle S_i \rangle (1 - |\Gamma|^2)$

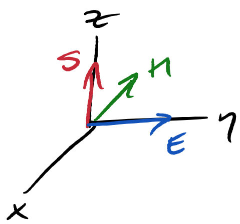
In free space, $\vec{E}(z,t) = 10^3 \sin(\omega t - \beta z)\, \hat{a}_y \left[\frac{V}{m}\right]$, find $\vec{H}$

$-\beta z$ $\therefore$ propagating in $+z$ direction

free space $\therefore$ $\eta = 377\,\Omega$

Using RHR to satisfy $\vec{S} = \vec{E} \times \vec{H}$

↳ H in $-x$ direction



$$\boxed{\vec{H}_x = \frac{-10^3}{377} \sin(\omega t - \beta z)\, \hat{a}_x \frac{A}{m}}$$

A 100 MHz uniform plane wave propagates in a lossless medium for which $\varepsilon_r = 5$ and $\mu_r = 1$

Find v_p, β, λ, $\vec{E}_s$, $\vec{H}_s$, $\langle S \rangle$

$\omega = 2\pi 100(10^6)\ \frac{rad}{s}$

alternative method $\quad v_p = \frac{1}{\sqrt{\varepsilon \mu}} \rightarrow \beta = \frac{\omega}{v_p}$

$k = \omega \sqrt{\varepsilon_o \varepsilon_r \mu_o} = 4.686\ \frac{rad}{m}$ $\qquad$ $\boxed{v_p = \frac{\omega}{\beta} = 134.1\,(10^6)\ \frac{m}{s}}$

$k = \beta?$ $\longrightarrow$ $\boxed{\beta = 4.686\ \frac{rad}{s}}$ $\qquad$ $\boxed{\lambda = 2\pi/\beta = 1.341\,m}$

$\eta = \sqrt{\frac{\mu_o}{5\varepsilon_o}} = 168.5$ $\qquad$ $\boxed{\vec{E}_s = E_o e^{-j4.686z}\, \hat{a}_x\ \frac{V}{m}}$ $\quad$ assume $+z$ propagation, x polarity

$$\boxed{\vec{H}_s = \frac{E_o}{168.5} e^{-j4.686z}\, \hat{a}_y\ \frac{A}{m}}$$

$$\boxed{\langle S \rangle = \frac{1}{2} \frac{E_o^2}{168.5}\, \hat{a}_z\ \frac{W}{m^2}}$$

A 2-GHz plane wave has amplitude $E_{yo} = 1.4 \times 10^3$ at $(0,0,0,\ t=0)$ and is propagating in the $\hat{a}_z$ direction in a medium where $\varepsilon'' = 1.6 \times 10^{-11}$, $\varepsilon' = 3.0 \times 10^{-11}$, and $\mu = 2.5\,\mu\ H/m$

Find: E_y & H_x @ P(0,0,1.8cm) @ 0.2ns

$\omega = 2\pi 2(10^9)\ \frac{rad}{s}$ $\qquad$ $\sigma = \varepsilon'' \omega = 0.2011$

$\frac{\varepsilon''}{\varepsilon'} = 0.533 \ll 1$ not particularly good dielectric

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon'}{2}} \left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right)^{\frac{1}{2}} = 28.10 \ \frac{Np}{m}$$

$$\beta = \sqrt{\frac{\mu\varepsilon'}{2}} \left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1 \right)^{\frac{1}{2}} = 112.4 \ \frac{rad}{m}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = 263.1 + j65.77$$

$\hookrightarrow |\eta| = 0.271$

$$E_y(z,t) = E_{y0} e^{-\alpha z} \cos(\omega t - \beta z)$$

$$= 1.4(10^3) e^{-28.1z} \cos(4\pi 10^9 t - 112.4z)$$

$\hookrightarrow \boxed{E_y(1.8cm, 0.2ns) = 744.9 \ V/m}$

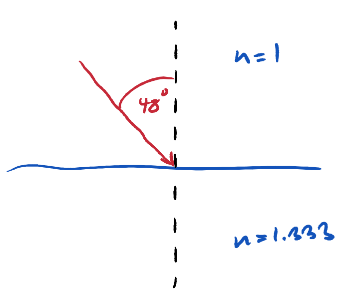
$$H_x(z,t) = \frac{1.4(10^3)}{0.271} e^{-28.1z} \cos(4\pi 10^9 t - 112.4z)(-\hat{a}_x)$$

$\hookrightarrow \boxed{H_x(1.8cm, 0.2ns) = -2.749(10^3) \ A/m}$

What is the speed of light in water? Find the angle of refraction of light incident on a water surface at an angle of 48° to the norm ($n = 1.333$)

$n_{water} = 1.333 = \sqrt{\varepsilon_r} \longrightarrow \varepsilon_{r\,water} = 1.777$

$$v = \frac{1}{\sqrt{\mu_0 1.777 \varepsilon_0}} = \boxed{224.9(10^6) \ m/s}$$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\hookrightarrow \theta_2 = a\sin\left(\frac{1}{1.333} \sin 48\right)$

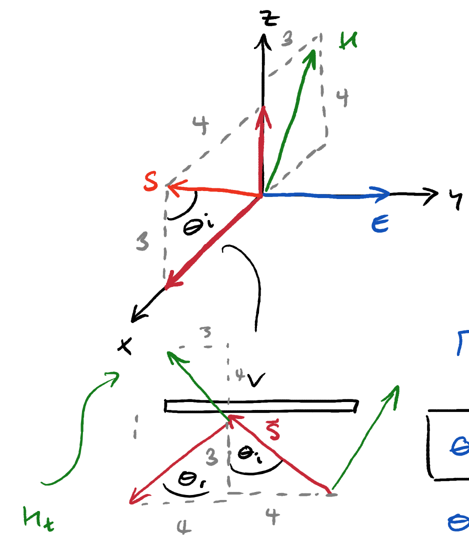
$\boxed{\theta_2 = 33.88°}$

A uniform plane wave in air with

$$\vec{E} = 8\cos(\omega t - 4x - 3z)\hat{a}_y \left[\frac{V}{m}\right]$$

is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1.0$ and $\varepsilon_r = 2.5$, $\sigma = 0$

(a) The polarization of the wave
(b) The angle of incidence
(c) The reflected $\vec{E}$
(d) The transmitted $\vec{H}$



$-4z$
$-3x$

$\sigma = \phi \rightarrow \alpha = \phi$

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)^2$$

$= 238.3$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.225$$

$$\boxed{\theta_i = \operatorname{atan}\left(\frac{4}{3}\right) = 53.13°}$$

$\theta_r = \theta_i = \underline{53.13°}$

$$\vec{E}_r = \Gamma \vec{E}_i = -0.225 \cdot 8\cos(\omega t + 4x + 3z)\hat{a}_y$$

$$\vec{E}_t = (1+\Gamma)\vec{E}_i = 6.2\cos(\omega t - 4x - 3z)\hat{a}_y$$

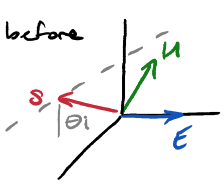
$$\vec{H}_t = -\frac{E_t}{|\eta|} = 35.56\cos(\omega t - 4x - 3z)(-3\hat{a}_x + 4\hat{a}_z)$$

note: after reflection, $\vec{E}$ points in same direction but $\vec{H}$ flips

↳ ∴ $\boxed{\text{S polarized}}$

↳ $$\Gamma = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}$$

before



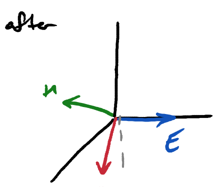
from snell's law,

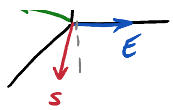
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_2 = \theta_{trans} = \arcsin \frac{n_1}{n_2} \sin\theta_1$

$= \arcsin \frac{\sqrt{1}}{} \sin 53.13$

after





$\theta_2 = \theta_{trans} = \arcsin \frac{n_1}{n_2} \sin\theta_1$

$= \arcsin \frac{\sqrt{1}}{\sqrt{2.5}} \sin 53.13$

$\theta_t = 30.4°$

$\Gamma = \dfrac{238.3 \cos 53.13 - 377 \cos 30.4}{238.3 \cos 53.13 + 377 \cos 30.4} = -0.3892$

$\boxed{E_r = \Gamma E_i = -0.3892 \cdot 8 \cos(\omega t - 4x + 3z)\hat{a}_y}$ + z b/c now going in opposite z direction

$H_t = \dfrac{E_i}{|\eta_2|}\cancel{(1+\Gamma)} = \dfrac{8(1-0.3892)}{(238.3)^2} \cos(\omega t - 4x + 3z)(3\hat{a}_x + 4\hat{a}_z)$

$\tau \neq 1 + \Gamma$ in this case

$\tau = \dfrac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} = 0.611$

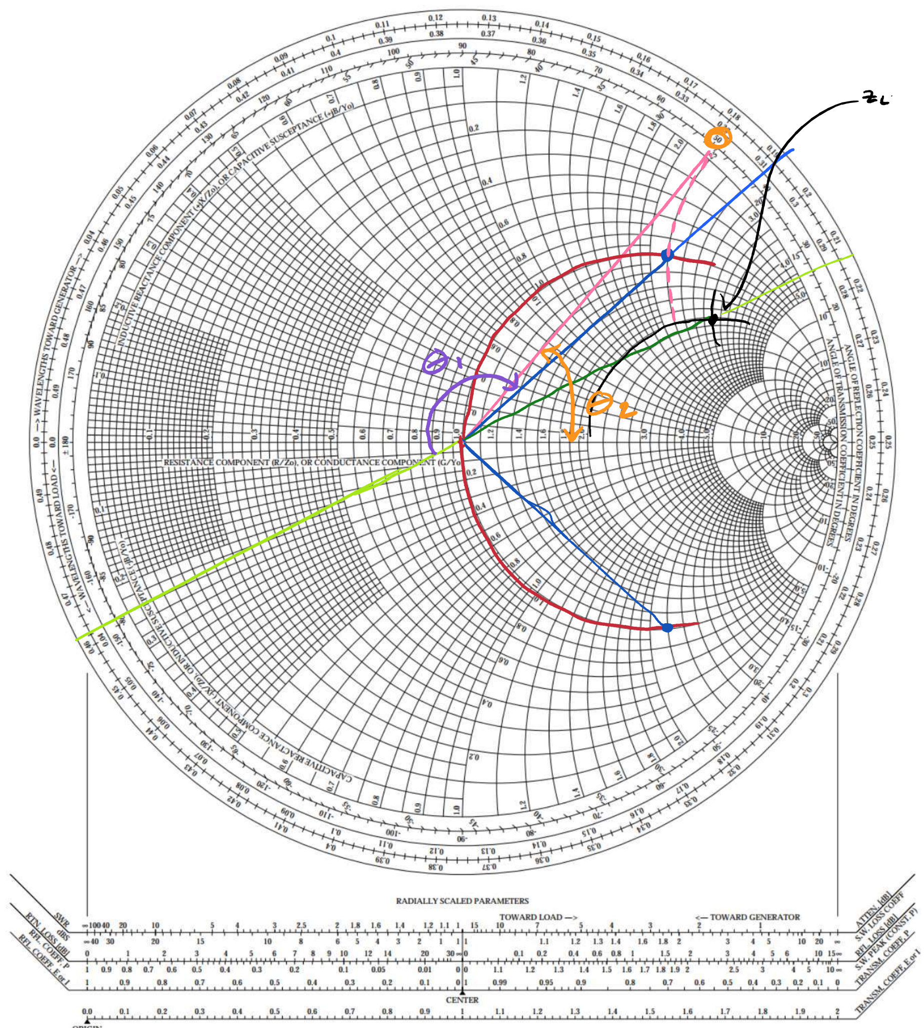
$\bar{H}_t = \dfrac{\vec{E}_i \tau}{|\eta_2|}$

Ex) single stub match $Z_L = 100 + j150$ where $Z_0 = 50\Omega$

$z_L = 2 + j3$

$d = \dfrac{\theta_1}{4 \cdot 180}\lambda = \dfrac{171}{4 \cdot 180}\lambda = 0.2375\lambda$

$L = \dfrac{\theta_2}{4 \cdot 180}\lambda = \dfrac{50}{4 \cdot 180}\lambda = 0.06944\lambda$



13.18 In the guide of Figure 13.25, it is found that $m = 1$ modes propagating from left to right totally reflect at the interface, so that no power is transmitted into the region of dielectric constant ϵ'_{r2}. (*a*) Determine the range of frequencies over which this will occur. (*b*) Does your part (*a*) answer in any way relate to the cutoff frequency for $m = 1$ modes in either region? *Hint*: Remember the critical angle?

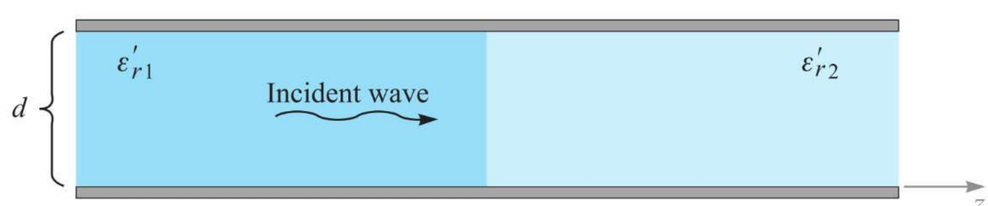
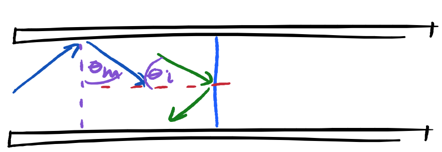


Figure 13.25 See Problems 13.17 and 13.18.

$$\theta_m = \text{acos}\,\frac{m\pi c}{\omega n d}$$



$$\theta_c = \text{asin}\,\frac{n_2}{n_1} = \text{asin}\,\frac{\sqrt{\varepsilon'_{r2}}}{\sqrt{\varepsilon'_{r1}}}$$

for total internal reflection

$\theta_i > \theta_c$ $\qquad$ $\theta_m + \theta_i + 90 = 180$

$\hookrightarrow \theta_i = 90 - \theta_m$

$90 - \theta_m > \theta_c$

$90 - \theta_c > \theta_m$

$$90 - \theta_c > \theta_m = \mathrm{acos}\, \frac{m\pi c}{\omega \sqrt{\varepsilon_{r,}} d}$$

$$\cos(90 - \theta_c) < \frac{m\pi c}{\omega \sqrt{\varepsilon_{r,}} d}$$

sign change

$$\sin\theta_c < \frac{m\pi c}{\omega \sqrt{\varepsilon_{r,}} d}$$

$$\sin\left(\mathrm{asin}\,\frac{\sqrt{\varepsilon'_{r_2}}}{\sqrt{\varepsilon'_{r_1}}}\right) < \frac{m\pi c}{\omega \sqrt{\varepsilon'_{r_1}} d}$$

$$\frac{\sqrt{\varepsilon'_{r_2}}}{\cancel{\sqrt{\varepsilon'_{r_1}}}} < \frac{m\pi c}{\omega \cancel{\sqrt{\varepsilon'_{r_1}}} d}$$

$$\sqrt{\varepsilon'_{r_2}} < \frac{m\pi c}{\omega d}$$

$$\boxed{\omega < \frac{m\pi c}{d\sqrt{\varepsilon'_{r_2}}}}$$

Ex) in an air filled waveguide w/ plate gap 1.25cm

a) find cutoff frequency for TE_0, TM_0, TE_1, TM_1, TM_2

b) find phase velocity of modes at 15GHz

c) find lowest order TE & TM that cannot propagate at 25GHz

$d = 1.25(10^{-2})$ m, $\quad$ air filled $\therefore$ $n = 1$

$$\omega_{cm} = \frac{m\pi c}{nd} \rightarrow \omega_{c1} = \frac{\pi c}{1.25} 10^2 = 75.35(10^9)\frac{\text{rad}}{s} = \boxed{12\ \text{GHz}} \quad TE_1, TM_1$$

$$\rightarrow \omega_{c2} = \frac{2\pi c}{1.25} 10^2 = \boxed{24\text{GHz}} \quad TM_2$$

Q

$TM_0 = TEM$?? $\quad \therefore \boxed{\omega_{cm} = \emptyset}$

TE_0

@15GHz, $\omega > \omega_{c1}$ but $\omega < \omega_{c2}$

so only mode 1 propagates.

$$v_p \text{ @ } 15\text{GHz} = \frac{c}{n \sin\theta_m}$$

so only mode 1 propagates.

$$v_p \text{ @ } 15\text{GHz} = \frac{c}{n\sin\theta_m}$$

$$\theta_m = a\cos\left(\frac{m\pi c}{\omega n d}\right) = a\cos\left(\frac{\pi c}{2\pi 15(10^9)\,1.25(10^{-2})}\right)$$

$$= 36.9°$$

$$\boxed{v_p = 499(10^6)\ \tfrac{m}{s}}$$

@ $m=3$, $\omega_{cm} = 36\text{GHz}$ $\therefore$ $\omega_{c3} > \omega$ $\therefore$ no propagation