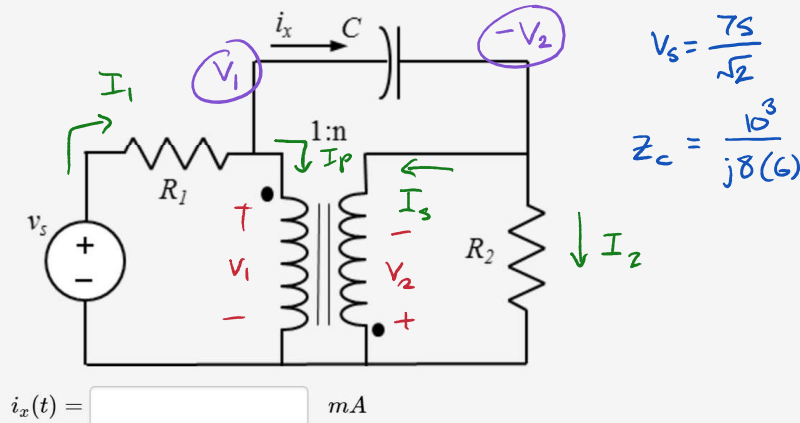


Assignment 5 - Transformers

Tuesday, March 1, 2016 12:38 PM

(13 pts)

Let $v_s(t) = 75\cos(8t)V$, $R_1 = 90\Omega$, $R_2 = 30\Omega$, $C = 6mF$ and $n = 3$. Find the current i_x (as a function of time). (**NB:** for your time-dependent expression to be accepted by WeBWork, your phase should be expressed in degrees, and with 5 significant figures; and ω in rad/s.)



Handwritten equations:

$$\frac{V_s - V_1}{R_1} = \frac{V_1 + V_2}{Z_c} + I_p$$

$$\frac{V_1 + V_2}{Z_c} = I_s - \frac{V_2}{R_2}$$

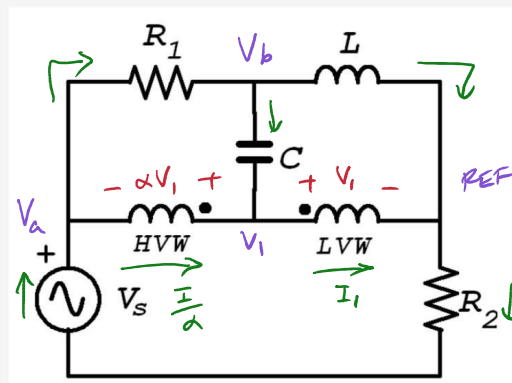
assume step-up transformer

$$V_2 = nV_1 \quad I_s = \frac{I_p}{n}$$

$$I_x = \frac{V_1 + V_2}{Z_c} = \boxed{136mA \angle 22^\circ}$$

(12 pts)

This shows a circuit that includes an ideal transformer of ratio $a : 1$. Let $v_s(t) = 7\cos(37t)kV$, $R_1 = 80\Omega$, $R_2 = 50\Omega$, $C = 4\mu F$, $L = 50mH$ and $\alpha = 8$. Compute the value that an AC voltmeter would read on the HV coil; and the current that an AC ammeter in series with the HV coil would read. (HVV = High Voltage Winding).



$V_{HVV} =$ V

$I_{HVV} =$ mA

$$\frac{0 - V_a + V_s}{50} = \frac{1}{8} I_1 + \frac{V_a - V_b}{80}$$

$$\frac{V_a - V_b}{80} = \frac{V_b - V_1}{\frac{10^6}{j37(4)}} + \frac{V_b}{j37(50)10^{-3}}$$

$$\frac{1}{8} I_1 + \frac{V_b - V_1}{\frac{10^6}{j37(4)}} = I_1$$

$$V_1 = V_a + 8V_b$$

$$V_1 = 435.2$$

$$I_1 = 74.87 \text{ mA}$$

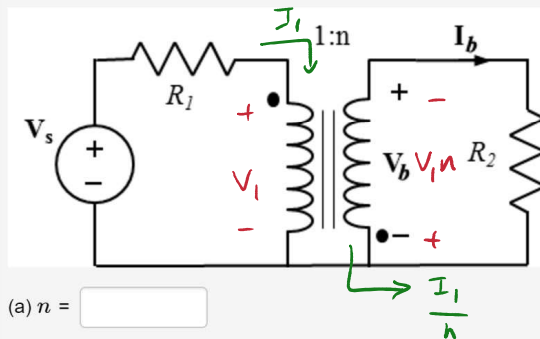
$$\hookrightarrow V_1 = 3.482 \text{ kV}$$

$$\hookrightarrow I_2 = 9.358 \text{ mA}$$

(12 pts)

In the circuits below, use $V_s = 50 \angle 0^\circ \text{ V}$, $R_1 = 55 \Omega$ and $R_2 = 495 \Omega$. (a) In the first figure, determine the value of n (the winding ratio, not necessarily an integer) that would result in maximum power transfer to the load, R_2 . (b) If $n = 7$ in the first figure, compute V_b and I_b .

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



(a) $n =$

(b) $I_b =$ mA

$V_b =$ V

$$V_s - R_1 I_1 = V_1$$

$$V_b - I_b R_2 = 0$$

$$V_b = -V_1 n$$

$$I_b = -\frac{I_1}{n}$$

$$P = V I = V \frac{V}{R_2} = I^2 R$$

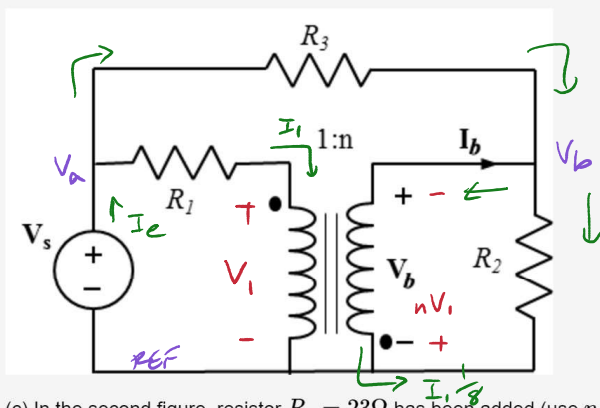
$$\boxed{I_b = -109.7 \text{ mA}}$$

$$\boxed{V_b = -54.31 \text{ V}}$$

maximum power transfer when loads matched

$$\frac{Z_p}{N_p^2} = \frac{Z_s}{N_s^2} \rightarrow N_s^2 = \frac{Z_s}{Z_p} N_p^2$$

$$N_p = 1 \quad N_s = ? \quad \hookrightarrow N_s = \sqrt{\frac{495}{55}} = \boxed{3}$$



(c) In the second figure, resistor $R_3 = 23\Omega$ has been added (use n from part (b)). Compute V_b and I_b .

$I_b =$ mA

$V_b =$ V

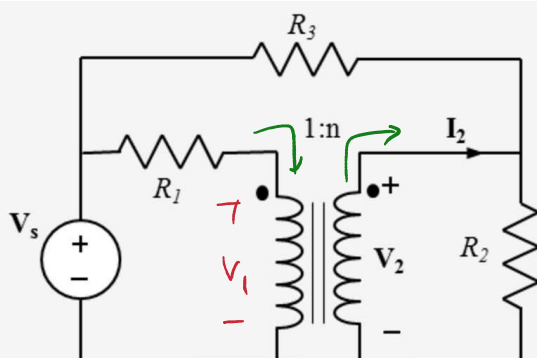
$n = 7$

$$I_b = \frac{-I_1}{7} \quad V_b = -7V_1 \quad I_1 = \frac{50 - V_1}{55}$$

$$I_s = I_1 + \frac{50 - V_b}{23}$$

$$\frac{50 - V_b}{23} = \frac{I_1}{7} + \frac{V_b}{495}$$

$$\begin{aligned} V_b &= 44.56V \\ I_b &= 1.025A \end{aligned}$$



(d) In the third figure, the polarity dots have been reversed. Compute V_2 and I_2 .

$I_2 =$ mA

$V_2 =$ V

$n = 7$

$$I_2 = \frac{I_1}{7} \quad V_2 = V_1/7 \quad I_1 = \frac{50 - V_1}{55}$$

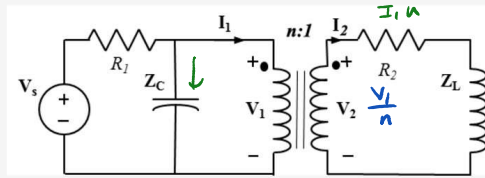
$$I_s = I_1 + \frac{50 - V_2}{23}$$

$$\frac{50 - V_2}{23} + I_2 = \frac{V_2}{495}$$

$$\begin{aligned} V_2 &= 50.22 \\ I_2 &= 111.2mA \end{aligned}$$

(12 pts)

Let $V_s = 20\angle 30^\circ V$, $R_1 = 9\Omega$, $R_2 = 6\Omega$, $Z_c = -j5\Omega$, $Z_L = j8\Omega$, and $n = 2$. Compute the impedance seen by the source, Z_T as well as the signals V_1 and the current I_2 .



assuming step down

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$$Z_T = \text{[]} \angle \text{[]}^\circ \Omega$$

$$V_1 = \text{[]} \angle \text{[]}^\circ V$$

$$I_2 = \text{[]} \angle \text{[]}^\circ A$$

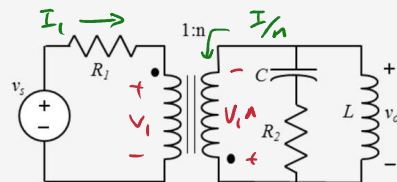
$$\left. \begin{aligned} \frac{V_s - V_1}{R_1} &= \frac{V_1}{Z_c} + I_1 \\ V_2 &= I_2 (R_2 + Z_L) \\ V_2 &= V_1 \frac{1}{n} \\ I_2 &= I_1 n \end{aligned} \right\} \begin{aligned} V_1 &= 10.11 \angle -24.98^\circ \\ I_2 &= 505.6 \text{ mA} \angle -78.11^\circ \end{aligned}$$

$$\frac{Z_p}{n_p^2} = \frac{Z_s}{n_s^2} \rightarrow Z_p = (R_2 + Z_L) \frac{n_p^2}{n_s^2} = (6 + j8) \frac{2^2}{1^2} = 24 + j32$$

$$Z_T = R_1 + Z_c \parallel Z_p = 9.46 - j6.517$$

(13 pts)

Let $v_s(t) = 29\cos(750t) \text{ kV}$, $R_1 = 335\Omega$, $R_2 = 730\Omega$, $C = 5\mu F$, $L = 400 \text{ mH}$, and $n = 4$. Compute $v_o(t)$ and the complex power delivered by the source. (**NB:** for your time-dependent expression to be accepted by WeBWork, your angle should be expressed in degrees, and with 5 significant figures.)



$$v_o(t) = \text{[]} \text{ kV}$$

$$S_S = \text{[]} \angle \text{[]}^\circ \text{ kVA}$$

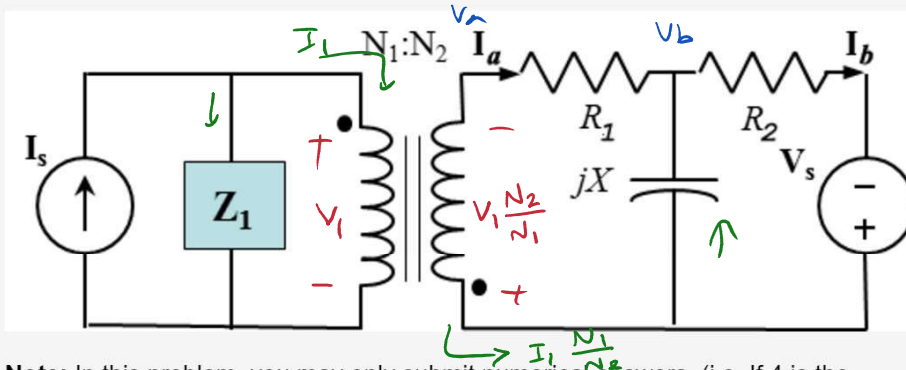
$$\left. \begin{aligned} \frac{V_s - V_1}{R_1} &= I_1 \\ v_o &= -V_1 n \\ V_1 n &= \frac{I}{n} (Z_c + R_2 \parallel Z_L) \end{aligned} \right\} \begin{aligned} V_o &= 4.766 \angle -115.8^\circ \text{ kV} \\ V_1 &= 1.192 \angle 64.25^\circ \\ I_1 &= 59.75 \text{ mA} \angle -3.073^\circ \end{aligned}$$

$$Z_T = R_1 + \frac{1}{\frac{1}{Z_c + R_2} + \frac{1}{Z_L}} = 342.7 + j18.4$$

$$S = VI = \frac{V_s^2}{Z_T} = 1225 \angle -3.07^\circ \text{ kVA}$$

(13 pts)

Let $I_s = 95 \angle 30^\circ \text{ A}$, $V_s = 80 \angle -20^\circ \text{ V}$, $Z_1 = 30 + j30 \Omega$, $R_1 = 65 \Omega$, $R_2 = 45 \Omega$, $X = -45 \Omega$, transformation ratio is $N_1 : N_2 = 4 : 7$. Compute the complex power received by Z_1 and the phasor currents I_a and I_b .



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$$S_1 = \boxed{} \angle \boxed{}^\circ \text{ kVA}$$

$$I_a = \boxed{} \angle \boxed{}^\circ \text{ A}$$

$$I_b = \boxed{} \angle \boxed{}^\circ \text{ A}$$

$$I_a = -I_1 \frac{N_1}{N_2} \quad I_s = \frac{V_1}{Z_1} + I_1 \quad V_a = -V_1 \frac{N_2}{N_1}$$

$$\frac{0 - V_b}{jX} = \frac{V_b - V_a}{R_1} + \frac{V_b + V_s}{R_2} \quad \frac{V_b - V_a}{R_1} = I_1 \frac{N_1}{N_2}$$

$$I_a = 36.97 \angle -126.2^\circ$$

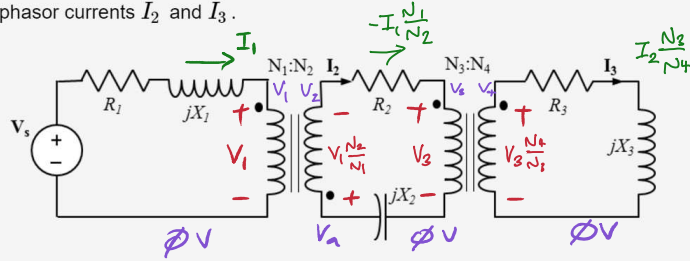
$$I_b = \frac{V_b + V_s}{R_2} = 25.06 \angle -169.8^\circ$$

$$S = \frac{V_1^2}{Z_1} = 83.35 \angle 32.8^\circ \text{ kVA}$$

$$S = V_1 I^* = V_1 (I_s - I_1)^* = 83.35 \angle 45^\circ \text{ kVA}$$

(12 pts)

Let $V_s = 80\angle 0^\circ$, $R_1 = 50\Omega$, $R_2 = 80\Omega$, $R_3 = 40\Omega$, $X_1 = 80\Omega$, $X_2 = -45\Omega$, $X_3 = 85\Omega$, $N_1 = 6$, $N_2 = 5$, $N_3 = 1$, and $N_4 = 2$. Compute the complex power supplied by the source and the phasor currents I_2 and I_3 .



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$$S_s = \boxed{} \angle \boxed{}^\circ \text{VA}$$

$$I_2 = \boxed{} \angle \boxed{}^\circ \text{A}$$

$$I_3 = \boxed{} \angle \boxed{}^\circ \text{A}$$

$$I_1 = \frac{V_s - V_1}{R_1 + jX_1}$$

$$I_2 = \frac{V_2 - V_3}{R_2}$$

$$I_3 = \frac{V_4}{R_3 + jX_3}$$

$$\frac{-V_4}{jX_2} = I_2$$

$$I_2 = -I_1 \frac{N_1}{N_2}$$

$$I_3 = I_2 \frac{N_2}{N_4}$$

$$V_4 = V_3 \frac{N_4}{N_3}$$

$$V_2 + V_1 \frac{N_2}{N_1} = V_4$$

$$I_2 = 517.9 \angle 165.7^\circ \text{ mA}$$

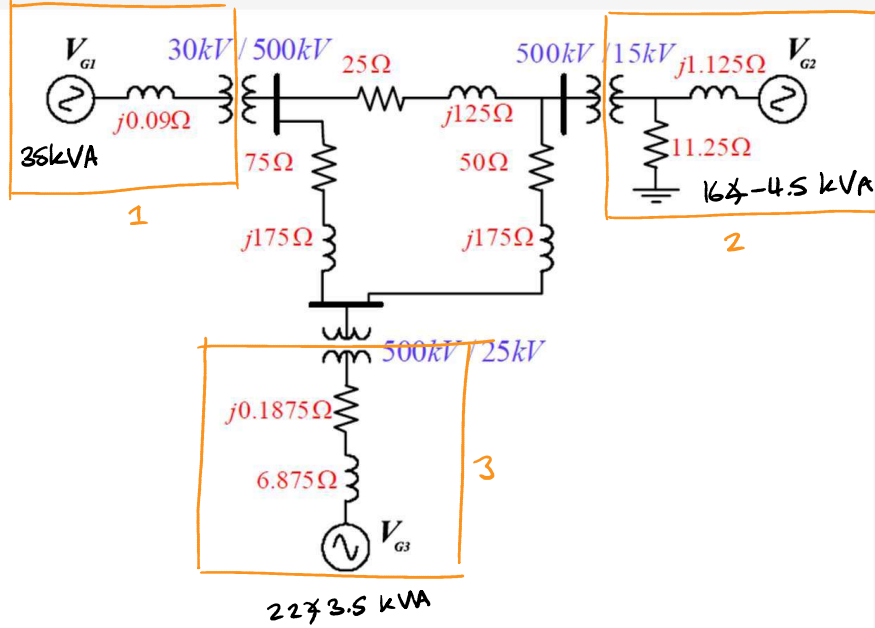
$$I_3 = 259 \angle 165.7^\circ \text{ mA}$$

$$S = VI^* = V_s I_1^* = 34.53 \angle 14.31^\circ$$

(13 pts)

The power system model below includes three ideal transformers. It connects three towns plus generators, represented by their Thevenin equivalents. Determine how much active and reactive power crosses each transformer going into the transmission system (the three transmission lines represented by simplified RL series circuits). The generator values are $V_{G1} = 35 \text{ kVA} \angle 0^\circ$; $V_{G2} = 16 \text{ kVA} \angle -4.5^\circ$; $V_{G3} = 22 \text{ kVA} \angle 3.5^\circ$. 582056 -3002.74 575749 -10841.3 575733 -10673.1

Power System with three towns and three transmission lines.



- (a) For the top left transformer, what is the complex power? + j MVA
- (b) For the top right transformer, what is the complex power? + j MVA
- (c) For the bottom transformer, what is the complex power? + j MVA

using per unit so transformers are transparent
take $S_b = 100 \text{ MVA}$ for entire system

$$R_1 \quad Z_b = \frac{V_b^2}{S_b} = \frac{(30 \text{ kV})^2}{100 \text{ MVA}} = 9 \quad Z = \frac{j0.09}{9} = j0.01$$

$$V_1 = \frac{33 \text{ kV}}{30 \text{ kV}} = 1.1 \angle 0^\circ$$

$$R_2 \quad Z_b = \frac{15 \text{ kV}^2}{100 \text{ M}} = 2.25 \quad Z = \frac{11.25}{2.25} = 5 \quad Z = \frac{j1.125}{2.25} = j0.5$$

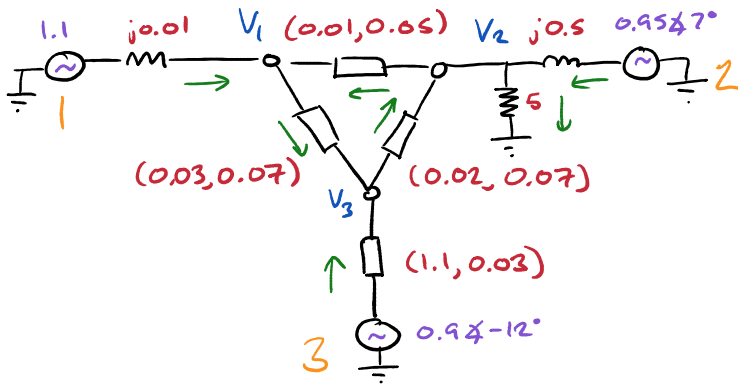
$$V_2 = \frac{14.25 \text{ kV} \angle -7^\circ}{15 \text{ kV}} = 0.95 \angle -7^\circ$$

$$R_3 \quad Z_b = \frac{25 \text{ kV}^2}{100 \text{ M}} = 6.25 \quad Z = \frac{6.875}{6.25} = 1.1 \quad Z = \frac{j0.1875}{6.25} = j0.03$$

$$V_3 = \frac{22.5 \text{ kV} \angle -12^\circ}{25 \text{ kV}} = 0.9 \angle -12^\circ$$

$$\text{inside} \quad Z_b = \frac{(500 \text{ kV})^2}{100 \text{ M}} = 2500$$

$$z_a = 0.01 + j0.05 \quad z_b = 0.03 + j0.07 \quad z_c = 0.02 + j0.07$$



from NNA

$$V_1 = 1.099 \angle -94.35 \cdot 10^{-3}$$

$$V_2 = 1.090 \angle -163.3 \cdot 10^{-3}$$

$$V_3 = 1.077 \angle -584.1 \cdot 10^{-3}$$

$$S_a = V_{G1} I_i^* = V_{G1} \left(\frac{V_{G1} - V_1}{j0.01} \right)^*$$