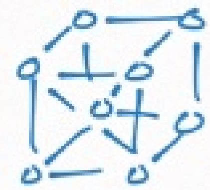
EECE 352 - MATERIALS & DEVICES REVIEW

ASSIGNMENT #1

1. draw FCC unit cell — " " BCC

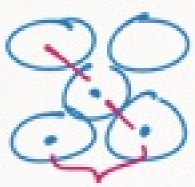
how many atoms in it?

$\frac{1}{8}\times 8 + \frac{1}{2}6 = 4$



$\frac{1}{8}\cdot 8 + 1 = 2$

assuming hard spheres & closely packed find a if $r_{Ni} = 1.24\text{Å}$, $r_{Fe} = 1.40\text{Å}$



a

$\sqrt{2}a = 2(1.24(10^{-10})m)\times 2$

$a = \frac{4(1.24(10^{-10}))m}{\sqrt{2}}$

$a = 350.72(10^{-12})m$

$\sqrt{3}a = 4(1.40(10^{-10}))m$

$a = 323(10^{-12})m$

what is the fraction of volume of unit cell occupied by atom

$$\frac{4\cdot\frac{4}{3}\pi r^3}{a^3} = 740.5(10^{-3}) \simeq 74\%$$

$$\frac{2\frac{4}{3}\pi r^3}{a^3} = 682(10^{-3}) \simeq 68\%$$

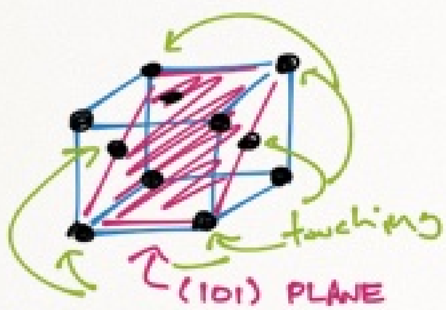
Knowing atomic weights $58.69\frac{g}{mol}$, $55.84\frac{g}{mol}$ calculate density, $N_A = 6.02(10^{23})\frac{atoms}{mol}$

$$\frac{4}{a^3}\cdot\frac{1}{N_A}\cdot 58.69\frac{g}{mol} = 9.04(10^6)\frac{g}{m^3} = 9.04\frac{g}{cm^3}$$

$$\frac{atom}{cm^3}\cdot\frac{mol}{atom}\cdot\frac{g}{mol} = \frac{g}{cm^3}$$

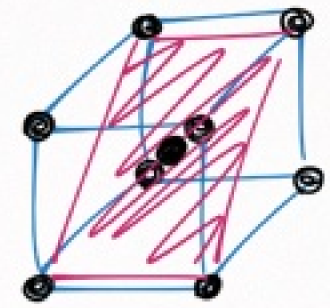
$$\frac{2}{a^3}\cdot\frac{1}{N_A}55.84\frac{g}{mol} = 5.505\frac{g}{mol}$$

show (101) plane & draw it & find planar concentration on plane



$\frac{1}{4}4 + \frac{1}{2}2 = 2 \frac{\text{atoms}}{\text{plane}}$

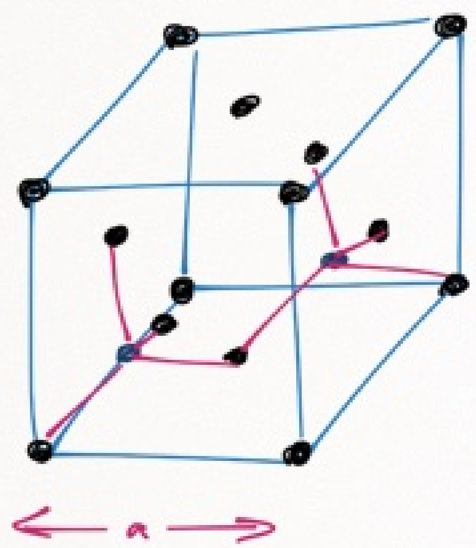
$\frac{2}{a\sqrt{2}a} = 1.15(10^{15}) \frac{\text{atoms}}{m^3}$



$\frac{1}{4}4 + 1 = 2 \frac{\text{atoms}}{\text{plane}}$

$\frac{2}{\sqrt{2}a^2} = 1.36(10^{15}) \frac{\text{atoms}}{m^3}$

2. bond length b/w C in diamond crystal is 110pm, find a



$\text{bond length} = \underbrace{\frac{\sqrt{3}}{4}}_{\text{forgot}} a = 110(10^{-12})m \rightarrow a = \frac{4}{\sqrt{3}} 110(10^{-12})m = 254(10^{-12})m$

atomic weight for carbon = $12.01 \frac{g}{mol}$ find density, if it has $10^{18} cm^{-3}$ defects find density

$\frac{\text{atoms}}{cm^3} \times \frac{mol}{atom} \times \frac{g}{mol} = \frac{g}{cm^3}$

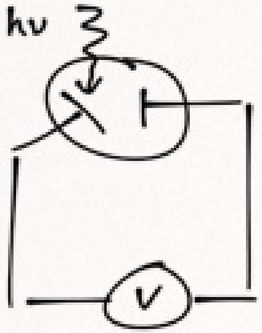
$\frac{8\times\frac{1}{8} + 4 + 6\times\frac{1}{2}}{a^3} \times \frac{1}{N_A} \times 12.01 \frac{g}{mol} = 9.74(10^6)\frac{g}{m}$

$\frac{\text{vacant mass}}{cm^3} = \frac{9.74(10^6)\frac{g}{m^3} \times \frac{((10^{-2})m)^3}{10^{18}}}{cm^3} = 9.74(10^{-18})\frac{g}{cm^3}$

$\text{density w/ defects} = 9.74(10^6)\frac{g}{m} - \overbrace{\cancel{9.74(10^{-18})}\frac{g}{cm^3}}^{\approx 0}$

$\approx 9.74(10^6)\frac{g}{m}$

3.

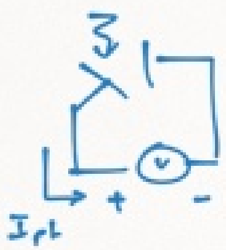


work function = 4.6eV = ϕ

draw I_{ph} & polarity of stopping voltage

the photons hit the side that sends e^- going in the opposite direction ∴ I_{ph} in same d as hν

to stop I_{ph} apply voltage against I in polarity shown



if $\lambda = 230nm$ or $\lambda = 480nm$, what is frequency & energy of light source?

$E = h\nu = h\frac{c}{\lambda} = 4.13(10^{-15})\ eV \cdot s \ \frac{3(10^8)\frac{m}{s}}{230(10^{-9})m} = 5.387eV$, $\underline{2.581\ eV}$

$KE_{max} = E_{photon} - \phi = 5.387eV - 4.6eV = 0.787eV$, $\underline{-2.019\ eV}$

$V_0 = 0.8V, -2.0V$

$\phi = 2.14eV$, $V_0 = ?$ for both wavelengths

$KE_{max} = 5.39\ eV - 2.14\ eV = 3.25eV$, $\underline{V_0 = 3.25V}$

$KE_{MAX} = 2.58\ eV - 2.14eV = 0.44eV$, $\underline{V_0 = 0.44V}$

ASSIGNMENT #2

1. using sep. of variables for 3D infinite well

$$V(x,y,z) = \begin{cases} \varnothing & (0<x<L_x) \,\&\, (0<y<L_y) \,\&\, (0<z<L_z) \\ \infty & \text{elsewhere} \end{cases}$$

show e^- in quantized form is $E = \frac{\hbar^2}{2m_0}(k_x^2 + k_y^2 + k_z^2)$, $k_x = \frac{n_x\pi}{L_x}$, $k_y = \frac{n_y\pi}{L_y}$, $k_z = \frac{n_z\pi}{L_z}$

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Psi(x,y,z) - E\,\Psi(x,y,z) = \varnothing$$

from Schrödingers time independent

$V(x,y,z) = \varnothing$ inside wire

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)X(x)Y(y)Z(z) = E\,X(x)Y(y)Z(z)$$

$$\frac{-\hbar^2}{2m}\left(X''YZ + XY''Z + XYZ''\right) = EXYZ, \qquad \frac{-\hbar^2}{2m}\left(\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z}\right) = E$$

$$\frac{X''}{X} = -k_x^2 \qquad \frac{Y''}{Y} = -k_y^2 \qquad \frac{Z''}{Z} = -k_z^2, \qquad \boxed{\frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right) = E}$$

$$X'' + Xk_x^2 = \varnothing$$

$$X(x) = Ae^{jk_x x} + Be^{-jk_x x}$$

BOUNDARY CONDITIONS

$X(0) = A + B = \varnothing \longrightarrow X(x) = A\left(e^{jk_x x} - e^{-jk_x x}\right)$

$A = -B$

$= A\sin k_x x$

$X(L_x) = \varnothing \longrightarrow X(L_x) = A\sin k_x L_x = \varnothing \longrightarrow k_x L_x = n_x\pi$

$$\boxed{\longrightarrow k_x = \frac{n_x\pi}{L_x}}$$

REPEAT PROCEDURE FOR Y & Z

$$k_y = \frac{n_y\pi}{L_y} \qquad k_z = \frac{n_z\pi}{L_z}$$

a nanowire $L_x = L_z = 0.5nm$, $L_y = 20nm$ modelled as infinite potential well
find ~~six~~ lowest E levels of e^- in nanowire

$$\frac{\hbar^2}{2m}\left(\frac{n_x\pi}{L_x}^2 + \frac{n_y\pi}{L_y}^2 + \frac{n_z\pi}{L_z}^2\right) = E$$

lowest level
$n_x = n_y = n_z = 1$

$$\frac{\hbar^2\pi^2}{2m}\left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right) = \frac{\left(\frac{h(J)}{2\pi}\right)^2 \pi^2}{2m_e}\left(\frac{1}{0.5(10^{-9})^2} + \frac{1}{0.5(10^{-9})^2} + \frac{1}{20(10^{-9})^2}\right) \equiv J$$

$$= 482.66(10^{-21})J = 3.0166\,eV$$

write wave function for (111)

$$\Psi_{111}(x,y,z,t) = X(x)Y(y)Z(z)\,T(t)$$
$$= A\sin\left(\frac{n_x\pi}{L_x}x\right)\cdot B\sin\left(\frac{n_y\pi}{L_y}y\right)\cdot C\sin\left(\frac{n_z\pi}{L_z}z\right)\cdot e^{-j\frac{E_{111}}{\hbar}t}$$

find wavelengths emitted from transitions $\Psi_{1,1,2} \rightarrow \Psi_{1,1,1}$, $\Psi_{1,2,1} \rightarrow \Psi_{1,1,1}$

$E_{ph} = E_{112} - E_{111}$

$= 2.827\,meV$

$E_{ph} = E_{121} - E_{111}$

$= 4.5236\,meV$

$\lambda = \frac{c}{\nu}$ $\nu = \frac{E}{h}$

$$\lambda = \frac{ch}{E} = \frac{3(10^8)h(eV)}{2.827(10^{-3})eV}$$

$$\boxed{= 438\mu m}$$

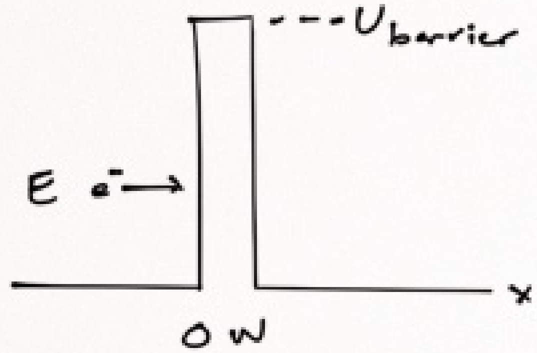
$$\lambda = \frac{ch}{E} = \frac{ch}{4.5236(10^{-3})} = \boxed{273.9\mu m}$$

how many states are there w/ $E \leq 3.8544\,eV$

$$\frac{\hbar^2\pi^2}{2m}\left(\frac{2}{(0.5nm)^2} + \frac{n^2}{(20nm)^2}\right) = 3.8544eV \longrightarrow n \leq 30$$

increases in x & y are negligible compared to z

2. e^- traveling in $+x$ direction towards barrier w/ $U_{barrier}$ & width W
e^- has $E < U_{barrier}$ & Ψ has amplitude A_I



write solutions for Schrodinger's equations in regions $x<0$, $0<x<W$, $x>W$

① $x<0$

$U(x)=0 \therefore E>U_b \therefore k_1 = \sqrt{\frac{2m(E-U_b)}{\hbar}} = \frac{\sqrt{2mE}}{\hbar}$

$\rightarrow \underline{\Psi_1(x) = A_1 e^{jk_1x} + B_1 e^{-jk_1x}}$

② $0<x<W \therefore U_b > E \therefore k_2 = \sqrt{\frac{2m(U_b-E)}{\hbar}}$

$U(x) = U_b$

$\rightarrow \underline{\Psi_2(x) = A_2 e^{k_2x} + B_2 e^{-k_2x}}$

③ $x>W \therefore E>U_b \therefore k_3 = \frac{\sqrt{2mE}}{\hbar}$

$U(x)=0$

same as ①

$\rightarrow \underline{\Psi_3(x) = A_3 e^{jk_3x} + B_3 e^{-jk_3x}}$

solving boundary conditions shows probability for e^- tunneling is $T = \frac{A_3^2}{A_1^2} = [1 + D\sinh^2(k_2W)]^{-1}$
assuming $W = 2nm$, $U_b = 9eV$, $E = 6eV$, find T

$D = \frac{U_b^2}{4E(U_b-E)}$

$D = \frac{(9q)^2}{4 \cdot 6q(9q-6q)} = 1.125 \qquad k_2 = \frac{\sqrt{2m(9q-6q)}}{1.05(10^{-34})\,Js} = 8.906(10^9)\frac{1}{m}$

$T = (1 + 1.125\sinh^2(8.906(10^9)\cdot 2(10^{-9})))^{-1} = \underline{1.2012(10^{-15})}$

ASSIGNMENT #3

1. @ $T=300K$ $E_g^{Si}=1.12eV$, $E_g^{Ge}=0.67eV$ density of intrinsic e^- of $Si = 10^{10}cm^{-3}$ & $Ge = 2.3(10^{10})cm^{-3}$
find density of intrinsic holes & electrons at 75°C assuming E_gs don't change

$n_i = \sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}}$ Si : $n_i(300K) = 10^{10}\frac{1}{cm^3} = A e^{\frac{-1.12q}{2\cdot 300\cdot k}} = \frac{10^{10}}{(10^{-2})^3}\frac{1}{m^3} = 10^{16}\frac{1}{m^3}$

$\hookrightarrow A = 25.07(10^{24})\frac{1}{m^3}$

$n_i(75+273.15) = 25.07(10^{24})\, e^{\frac{-1.12q}{2\cdot 348.15\cdot k}} = \underline{199.5(10^{15})\frac{1}{m^3}}$

Ge: same procedure $\underline{n_i = 1.38(10^{20})\frac{1}{m^3}}$

calculate conductivities of Si & Ge @ 300K & 348.15K if $\mu_n = 1350$, $\mu_p = 450\ \frac{cm^2}{Vs}$
for Si & $\mu_n = 3900$, $\mu_p = 1900\ \frac{cm^2}{Vs}$ for Ge

$\sigma = q\mu_n n + q\mu_p p$ $n_i = n = p$

Si, 348.15K $= q(199.5(10^{15})\frac{1}{m^3})(1350+450)(10^{-2})^2\frac{m^2}{Vs} = 5.746(10^{-3})\frac{C}{mVs} = \underline{5.746\ \frac{mS}{m}}$

Si, 300K $\sigma = q10^{16}\frac{1}{m^3}(1800)10^{-4} = \underline{288\ \frac{\mu S}{m}}$

Ge, same procedure $\rightarrow$ $\underline{2.13\ \frac{S}{m},\ 12.8\ \frac{S}{m}}$

assume Si & Ge wafers are doped w/ donor density of $10^{16}cm^{-3}$. find densities & types of
minority & majority carriers

$n_o p_o = n_i^2$ $N_D = 10^{16}\frac{1}{(10^{-2})^3}\frac{1}{m^3} = 10^{22}\frac{1}{m^3}$ there's only donors, no acceptors $\therefore$ $\underline{n_o = N_D \text{ (majority)}}$

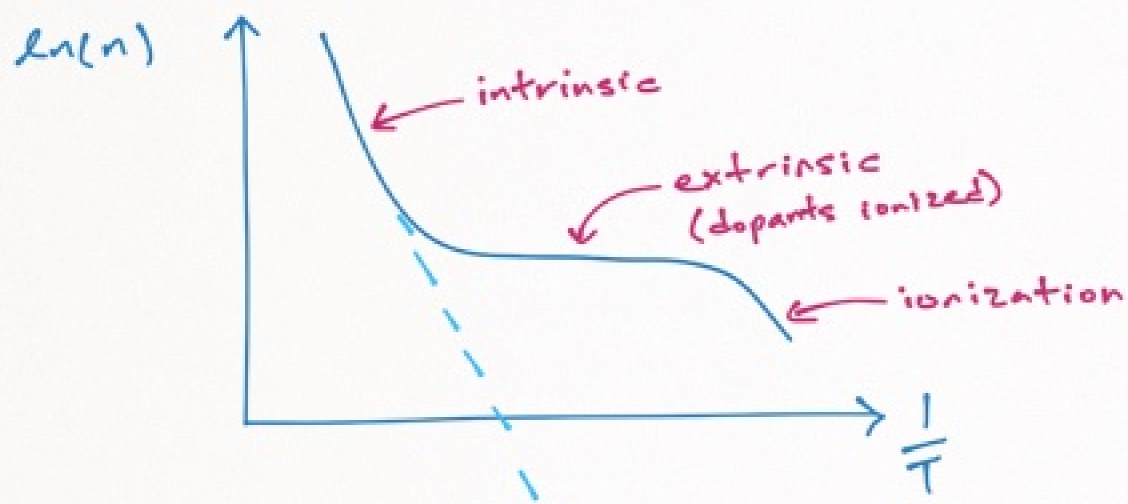
$p_o = \frac{n_i^2}{n_o} = \frac{(10^{16}\frac{1}{m^3})^2}{10^{22}\frac{1}{m^3}} = \underline{10^{10}\frac{1}{m^3} \text{ (minority)}}$

find conductivity of wafers @300K

$$\sigma = q(\mu_n n + \mu_p p)$$

Si: $\sigma = q(\mu_n n_0 + \mu_p p_0) = q(1350(10^{-4})\cdot 10^{22} + 450(10^{-4})\cdot 10^{10}) = 216\,\frac{S}{m}$

plot a graph showing changes in the maj. carrier densities in these as a function of $\frac{1}{T}$



undoped

in intrinsic semiconductors, density of n & p @ØK = Ø
n & p increase exponentially w/ temp.

the wafers are doped w/ an acceptor density of $9.9(10^{15})\frac{1}{cm^3}$, what are maj. & min. carrier densities & the conductivity @ 300K?

$$n_0 + N_A = p_0 + N_D \qquad n_0 p_0 = n_i^2$$

if both N_D & N_A, $n_0 = N_D - N_A = 10^{22}\frac{1}{m^3} - 9.9(10^{21})\frac{1}{m^3} = 100(10^{18})\frac{1}{m^3} = 10^{20}\frac{1}{m^3}$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(10^{16}\frac{1}{m^3})^2}{10^{22}\frac{1}{m^3}} = 10^{10}\frac{1}{m^3}$$

$$\sigma = q(\mu_n n_0 + \mu_p p_0) = q(1350(10^{20}) + 450(10^{10}))(10^{-2})^2\frac{1}{m^3}\frac{m^2}{Vs} = 2.16\,\frac{S}{m}$$

2. Si PN junction w/ area of $1mm^2$, $N_D = 10^{16} cm^{-3}$, $N_A = 4(10^{15}) cm^{-3}$
lifetime of min. e^- = 1ns, lifetime of maj. holes = 0.2ns, $n_i = 10^{10} cm^{-3}$, $\mu_n = 1350$, $\mu_p = 450 \frac{cm^2}{Vs}$
thickness of n & p regions = 10μm

what are $\underbrace{\text{Fermi energies for p \& n}}_{E_{Fp}, E_{Fn}}$ regions $\underbrace{\text{W.R.T. middle of band gap}}_{E_i}$ @ 300K

$$E_{Fn} = kT \ln \frac{n}{n_i} + E_i$$

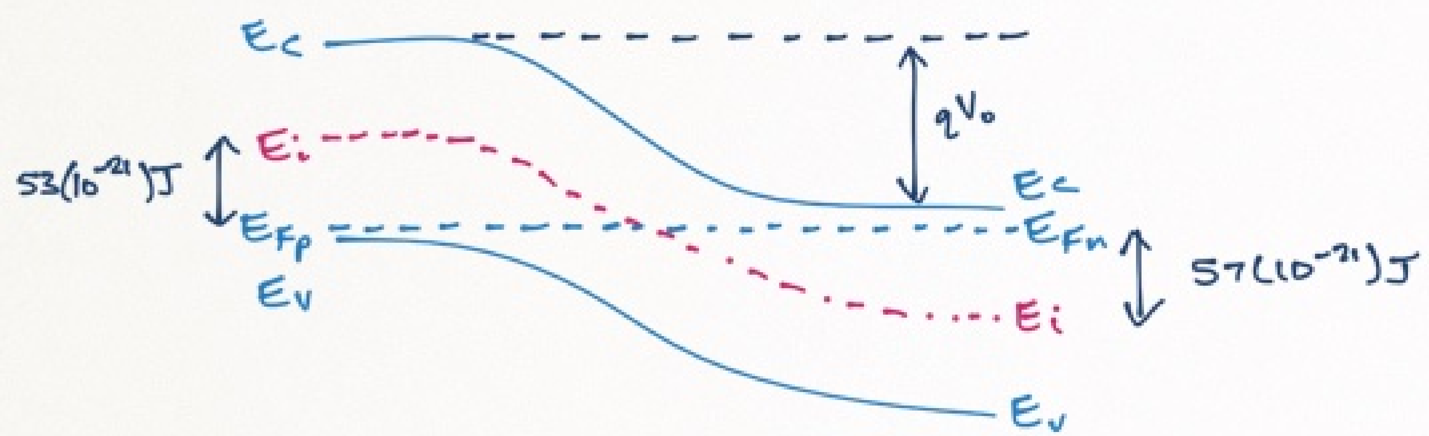
$$E_{Fp} = E_i - kT \ln \frac{p}{n_i}$$

b/c junction regions are separated and so you only have N_D & N_A, no n_o & p_o

$$E_{Fn} = kT \ln \frac{N_D}{n_i} + E_i = 300k \ln \frac{10^{16}}{10^{10}} + E_i = 57.196(10^{-21})J + E_i$$

$$E_{Fp} = E_i - kT \ln \frac{N_A}{n_i} = E_i - 300k \ln \frac{4(10^{15})}{10^{10}} = -53.403(10^{-21})J + E_i$$

draw the junction band diagram for PN junction @ equilibrium



find the built in voltage V_o, $\varepsilon = K\varepsilon_o = 11.7 \times 8.85 \times 10^{-14} \frac{F}{cm}$, also find depletion region W_o and $E_{o_{MAX}}$

$$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 691.24mV$$

$$E_o = -\frac{2V_o}{W} = -24.7(10^3)\frac{V}{cm}$$

$$W_o = \sqrt{V_o \frac{2\varepsilon}{q} \frac{N_A + N_D}{N_A N_D}} = 55.959 \mu cm$$
$$= 559.59 nm$$

find the forward I & the W $V_a = 0.6V$

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{\frac{qV}{kT}} - 1\right) \qquad D = \frac{kT}{q}\mu \qquad \mu = \frac{q\tau}{m^*} \qquad L = \sqrt{D\tau}$$

$$p_n = \frac{n_i^2}{N_D}, \quad n_p = \frac{n_i^2}{N_A}$$

$$I = qAn_i^2\left(\frac{D_p}{\sqrt{D_p\tau}\,N_A} + \frac{D_n}{\sqrt{D_n\tau}\,N_D}\right)\left(e^{\frac{qV}{kT}} - 1\right)$$

$$= q\left(1(10^{-3})^2 m^2\right)\left(10^{10}\frac{1}{(10^{-2})^3 m^3}\right)\left(\frac{\frac{kT}{q}1350(10^{-2})^2\frac{m}{Vs}}{\sqrt{\frac{kT}{q}1350(10^{-4})(10^{-9})s}} + \frac{\frac{kT}{q}450(10^{-4})}{\sqrt{\frac{kT}{q}450(10^{-4})0.2(10^{-9})s}}\right)\left(e^{\frac{0.6V}{25.8mV}} - 1\right)$$

$$= \underline{0.130A}$$

find total capacitance associated to neutral n & p regions at this bias voltage

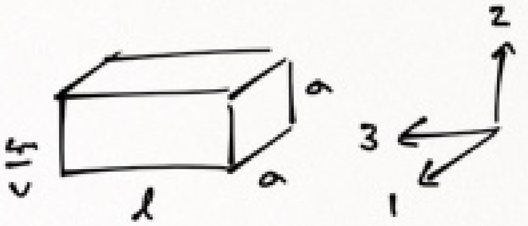
$$C_s = \frac{q}{kT}I(\tau_p + \tau_n) = \frac{1}{25.8mV}(0.130A)(1ns + 0.2ns) = 6.05nF$$

calculate the reverse I & W @ $V_a = -0.6V$ Find C

$I = -1.13(10^{-11})A$ $\qquad W = 765\,nm$ $\qquad C_j = \frac{\varepsilon A}{W} = \frac{\varepsilon 1(10^{-3})^2 m^2}{765(10^{-9})m} = 1.35(10^{-10})?$

ASSIGNMENT #4

1. build piezoelectric micropositioning system w/ max travel $5\mu m$
w/ $d_{13} = -2(10^{-8})$, $d_{33} = 5.3(10^{-8})$, $d_{24} = 3.7(10^{-8})$, $d_{25} = 4.1(10^{-8}) \frac{cm}{V}$



& dielectric strength $10^6 \frac{V}{cm}$ $\quad S_i = d_{ji} E_j$

write piezoelectric coefficients

how do we know we're getting it in this form?

$$d_{ij} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} \end{vmatrix}$$

$$d_{ji} = \begin{vmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ -2 & \emptyset & 5.3 \\ \emptyset & 3.7 & \emptyset \\ \emptyset & 4.1 & \emptyset \end{vmatrix} (10^{-8}) \frac{cm}{V}$$
$$\emptyset \quad \emptyset \quad \emptyset$$

these manage extension

these manage shear

how do we know there's another row here?

For maximum power supply voltage 110V and safety parameter of 1.5 find minimum dimensions of the beam (a & l)

$$S = dE = d_{13}E_1 + d_{33}E_3$$

$$\frac{\Delta l}{l} = d_{13}\frac{V_1}{a} + d_{33}\frac{V_3}{l}$$

$$\Delta l = 5\mu m = \frac{d_{13}V_1 l}{a} + d_{33}V_3$$

$$\frac{l}{a} = (5\mu m - d_{33}V_3)\frac{1}{d_{13}V_1}$$
$$= \left(5\mu m - 5.8(10^{-8})(10^{-2})\frac{m}{V}\cdot 110V\right)\frac{1}{-2(10^{-8})(10^{-2})110V}$$
$$= -224.37$$

$$E_{BREAKDOWN} = \frac{10^6 \frac{V}{cm}}{1.5} = 66.67(10^3)\frac{V}{m}$$

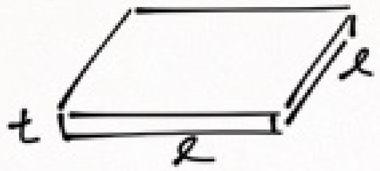
$$a_{min} = \frac{V}{E_{BREAK}} = \boxed{1.65\mu m}$$

$$|l| = a_{min}\left(\frac{l}{a}\right) = 1.65\mu m(224.37) = \boxed{370\mu m}$$

2. Transparent film $BaTiO_3$ poled in direction 3 under Curie temp. shows an anisotropic dielectric constant

$\varepsilon_r' = 4100 \perp$ to axis &

$\varepsilon_r' = 160 \parallel$ to axis



sides = $100\mu m$, thickness = $0.2\mu m$

Assume polarizability of each ion is $2(10^{-36})\ \mathrm{Fcm}^2$ and lattice parameter, $a = 0.4\,nm$

Calculate refractive index & speed of visible light in this film

refractive index, $n = \sqrt{\varepsilon_r}$

$$\varepsilon_r = \frac{1 + \frac{2N\alpha}{3\varepsilon_0}}{1 - \frac{N\alpha}{3\varepsilon_0}} \qquad \frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N}{3\varepsilon_0}\alpha \qquad N = \frac{5\ \frac{\text{atoms}}{\text{unit cell}}}{(0.4\,nm)^3} = 78.12(10^{27})\ \frac{\text{atoms}}{m^3}$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{2(10^{-36})(10^{-2})^2\ Fm}{3\varepsilon_0} \times 78.12(10^{27})\ \frac{\text{atoms}}{m^3}$$

$$\varepsilon_r = 5.2906$$

$$n = 2.3$$

$$n = \frac{c}{v} \longrightarrow v = \frac{c}{n} = \frac{3 \cdot 10^8\ \frac{m}{s}}{2.3} = 130(10^6)\ \frac{m}{s}$$

if we place 2 electrodes on top & bottom of film estimate capacitance 100kHz if $\tau = 1\mu s$

$$C = \varepsilon_0 \varepsilon_r'(\omega)\frac{A}{d} \qquad \cancel{\varepsilon_r' = 1 + \frac{N\alpha}{\varepsilon_0(1+\omega^2\tau^2)} = 1 + \frac{78.12(10^{27}) \cdot 2(10^{-36})(10^{-4})}{\varepsilon_0(1+(100k\,2\pi)^2(1(10^{-6}))^2} = 1.2657}$$

$$C = \varepsilon_0(4100)\frac{(100(10^{-6}))^2}{0.2(10^{-6})} = 1.8143\,nF$$

find loss tangent

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} \qquad \varepsilon_r'' = \frac{\omega\tau\alpha N}{\varepsilon_0(1+\omega^2\tau^2)} =$$