Quiz 2 Review

Wednesday, March 16, 2016 4:52 PM

CHAPTER 11 – Uniform Plane Wave

intrinsic impedance, $\eta\,(\Omega)$

↳ similar to transmission line characteristic impedance

$$Z_L = \sqrt{L/C} \qquad \boxed{\eta = \sqrt{\frac{\mu}{\varepsilon}}}$$

for free space, μ_0 & ε_0

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \boxed{377\,\Omega}$$

coefficient of reflection, Γ (unitless)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

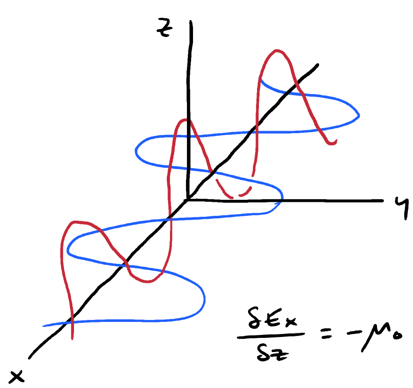
spacial frequency, $k\left(\frac{rad}{m}\right)$ → similar to β in transmission line

$$\boxed{k = \omega\sqrt{\varepsilon\mu} = \frac{2\pi}{\lambda}}$$

phase shift per unit distance

↳ also called propagation constant

Electric field and magnetic field are orthogonal to e/o



if $\vec{E}$ is along the x-direction, it is said to be polarized in the x-direction

GENERAL FORM $\boxed{\vec{E}_s = E_0 \cos(\omega t - kz)}$

$$\left.\begin{aligned} \frac{\delta E_x}{\delta z} &= -\mu_0 \frac{\delta H_y}{\delta t} \\ \frac{\delta H_y}{\delta z} &= -\varepsilon_0 \frac{\delta E_x}{\delta t} \end{aligned}\right\} \quad \overbrace{\frac{\delta^2 E_x}{\delta z^2} = \mu_0\varepsilon_0 \frac{\delta^2 E_x}{\delta t^2}}^{\text{wave equation}} = -k^2 E_x$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

Ex) Write $\mathcal{E}_y(z,t) = 100\cos(10^8 t - 0.5z + 30°)$ as a phasor

$$\mathcal{E}_y(z,t) = Re\{100 e^{j(10^8 t - 0.5z + 30°)}\}$$

$$\vec{E}_y = 100 e^{j(-0.5z + 30)}$$

complex permittivity, ε → there is also complex permeability, but it's usually insignificant and can be safely ignored

$$\boxed{\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0(\varepsilon_r' - j\varepsilon_r'')}$$

attenuation coefficient

$$\boxed{k = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\left(\frac{\varepsilon''}{\varepsilon'}\right)} = \alpha + j\beta}$$

phase constant

$\frac{\varepsilon''}{\varepsilon'}$, loss tangent

$\varepsilon'' = \emptyset \longrightarrow$ lossless

$$\boxed{\alpha = \emptyset \qquad \beta = \omega\sqrt{\mu\varepsilon'}}$$

$\frac{\varepsilon''}{\varepsilon'} \gg 1 \longrightarrow$ good conductor

↳ amplitude of $\vec{E}$ drops rapidly b/c electron acceleration ∴ resistive losses

$$\boxed{\sigma = \varepsilon''\omega \qquad \alpha = \beta = \sqrt{\pi f \mu \sigma} \qquad \eta = (1+j)\frac{\alpha}{\beta}}$$

$\frac{\varepsilon''}{\varepsilon'} \ll 1 \longrightarrow$ good dielectric

$$\boxed{\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}} \qquad \beta \simeq \omega\sqrt{\mu\varepsilon'} \qquad \eta \simeq \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\sigma}{2\omega\varepsilon'}\right)}$$

when $1 \ll \frac{\varepsilon''}{\varepsilon'} \ll 1 \rightarrow$ not particularly good conductor or dielectric

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2}}$$

attenuation coefficient ($\frac{Np}{m}$)

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2}}$$

phase constant ($\frac{rad}{m}$)

$$\boxed{\eta = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{\mu}{\varepsilon'}}\frac{1}{\sqrt{1 - j\,\varepsilon''/\varepsilon'}}}$$

intrinsic impedance (Ω)

FINDING $\vec{H}$ FROM $\vec{E}$

A) DIRECTION

1. determine direction of propagation of $\vec{E}$
   ↳ from β term of phase component

   ex. $E_s = E_{xo}\cos(\omega t - \beta z)$

   $-\beta$, therefore propagating in + direction

   z, therefore propagating along z axis

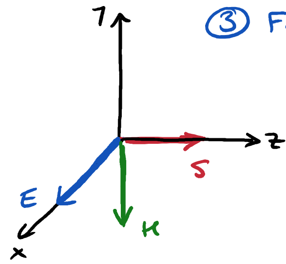
   ↳ propagating in $+z$ direction

   ↳ note, not same as polarity of $\vec{E}$ (x in this case)

2. Poynting vector in same direction as propagation

1A ③ From $\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\{\vec{E}_s \times \vec{H}_s^*\}$

② Poynting vector in same direction as propagation



③ From $\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}\{ \vec{E}_s \times \vec{H}_s^* \}$

↳ fingers E, palm H, thumb S

$\vec{H}$ in $-y$ in this example

B) MAGNITUDE

determine direction of $\vec{H}$

$$H_o = \frac{E_o}{\eta}$$

Ex) An electric field propagating in a lossless non-magnetic material is characterized as

$$\vec{E}(z,t) = 5\cos(8\pi 10^9 t - 400z) \ V/m$$

Find wave frequency, wavelength, relative permittivity of the media, and the magnitude of the magnetic field.

lossless material $\longrightarrow \varepsilon'' = \emptyset \longrightarrow \varepsilon' = \varepsilon$

non-magnetic material $\longrightarrow \mu_r = 1 \longrightarrow \mu = \mu_o$

$$f = \frac{\omega}{2\pi} = \frac{8\pi 10^9 \ \frac{rad}{s}}{2\pi} = \boxed{4(10)^9 \ Hz}$$

$$k = \frac{2\pi}{\lambda} \longrightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{400} = \boxed{1.57 \ cm}$$

$$\varepsilon = \varepsilon_o \varepsilon_r \longrightarrow \varepsilon_r = \frac{\varepsilon}{\varepsilon_o} = \frac{k^2/\omega^2\mu}{\varepsilon_o} = \frac{200^2}{(8\pi 10^9)^2 \mu_o \varepsilon_o} = \boxed{22.77}$$

$$k = \omega\sqrt{\varepsilon\mu}$$

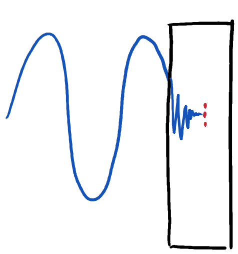
$$|\vec{H}| = \frac{|\vec{E}|}{\eta} \longrightarrow |\vec{H}| = \frac{|\vec{E}|}{\sqrt{\mu/\varepsilon}} = \frac{|\vec{E}|}{\sqrt{\frac{\mu_o}{\varepsilon_r \varepsilon_o}}} = \frac{5}{\sqrt{\frac{\mu_o}{22.77\varepsilon_o}}}$$

$$\eta = \sqrt{\mu/\varepsilon}$$

$$\boxed{= 63.33(10^{-3}) \ A/m}$$

SKIN DEPTH

↳ δ, denotes the depth of penetration of an electric field in conductive media



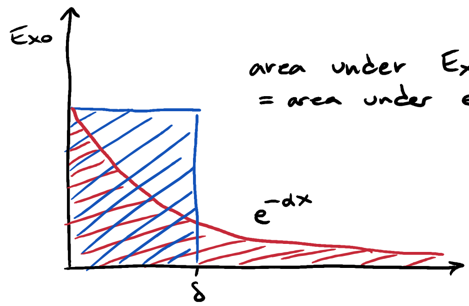
$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

note: as $f \uparrow$ $\delta \downarrow$

most energy is reflected except what is lost at skin-depth

$E_{xo} \uparrow$

skin-depth



area under $E_{xo} \times \delta$
$= $ area under $e^{-\alpha x}$

Ex) A uniform plane wave propagates in a lossless medium with a frequency of 10GHz.

$$\vec{E} = 5e^{-j400\pi z}(-\hat{a}_y) \qquad \vec{H} = \frac{5}{60\pi} e^{-j400\pi z}(-\hat{a}_x)$$

Determine ε_r \& μ_r.

lossless $\rightarrow \varepsilon'' = \emptyset \rightarrow \varepsilon' = \varepsilon$

$\vec{E} = E_o e^{-j\beta z} \longrightarrow E_o = 5, \ \beta = 400\pi$

$\vec{H} = \frac{H_o}{\eta} e^{-j\beta z} \longrightarrow \eta = 60\pi$

$\beta = \omega\sqrt{\mu\varepsilon} \longrightarrow \beta = 2\pi 10^9 \sqrt{\mu_r \varepsilon_r}\sqrt{\mu_o \varepsilon_o} = 400\pi$

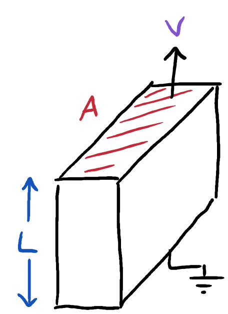
$\eta = \sqrt{\frac{\mu}{\varepsilon}} \longrightarrow \eta = \sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_o}{\varepsilon_o}} = 60\pi$

$$\boxed{\begin{aligned} \mu_r &= 3 \\ \varepsilon_r &= 11.98 \end{aligned}}$$

RESISTANCES

sheet resistance

$$R = \frac{L}{\sigma A}$$



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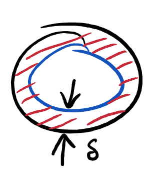
conductor resistance

$$\boxed{R = \frac{L}{\sigma A} = \frac{L}{2\pi a \sigma \delta} = \frac{L\sqrt{f\mu}}{2a\sqrt{\pi\sigma}}}$$

skin depth for conductor



$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

current is confined to outer skin of conductor so resistance is difference b/c only part of conductor is conducting

↳ doesn't matter w/ stranded wire

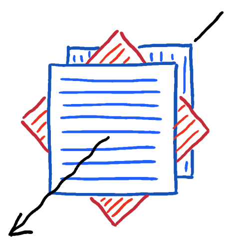
POLARIZATION

↳ defined as electric field orientation

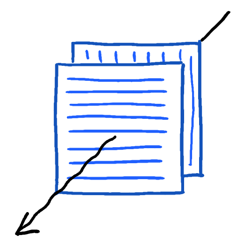
Linear Polarization

$$\boxed{\vec{E}_s = (E_{xo}\hat{a}_z + E_{yo}\hat{a}_y)e^{-\alpha z}e^{-j\beta z}}$$

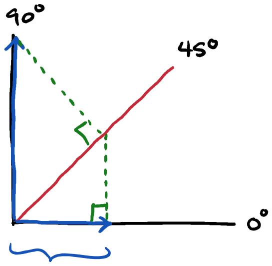
described in terms of perpendicular components of the electric field



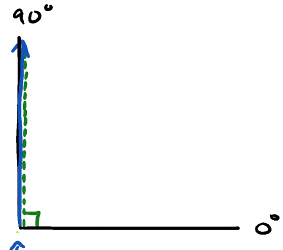
some light



no light



some light projected @ 0° b/c intermediate projection @ 45°

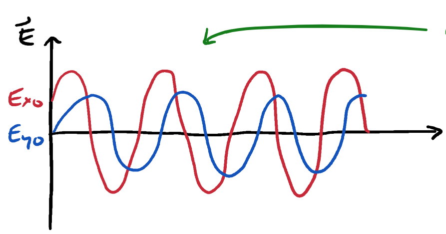


no light projected

Elliptical/Circular Polarization

↳ generated by materials w/ anisotropic permittivity

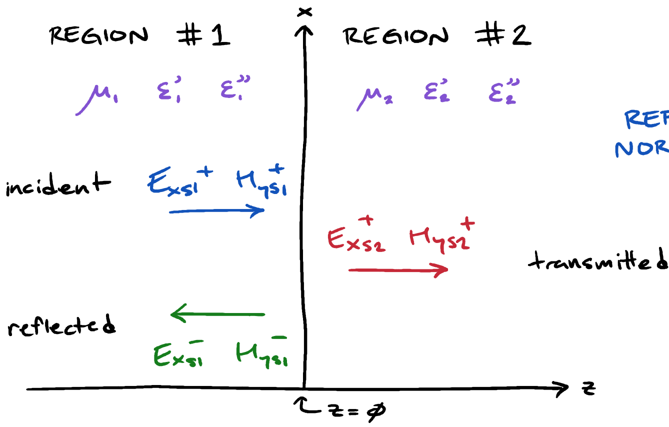
$$\boxed{\vec{E}_s = (E_{xo}\hat{a}_x + E_{yo}\hat{a}_y e^{j\phi})e^{-\alpha z}e^{-j\beta z}}$$



components maximize out of phase, if you look @ net, it looks like its rotating

CHAPTER 12 – PLANE WAVE REFLECTION AND DISPERSION

REFLECTION WITH NORMAL INCIDENCE



↳ @ BOUNDARY

$$E_{xs1}(z=\phi) = E_{xs2}(z=\phi)$$

$$H_{ys1}(z=\phi) = H_{ys2}(z=\phi)$$

reflection coefficient

$$\boxed{\Gamma = \frac{E_{x10}^-}{E_{x10}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad \boxed{\tau = 1 + \Gamma = \frac{2\eta_1}{\eta_2 + \eta_1}}$$

can be complex $|\Gamma| e^{j\phi}$

transmission coefficient

$$\langle S_i \rangle = \frac{1}{2} \mathrm{Re}\left\{ \frac{1}{\eta_1^*} \right\} |E_{x10}^+|^2$$

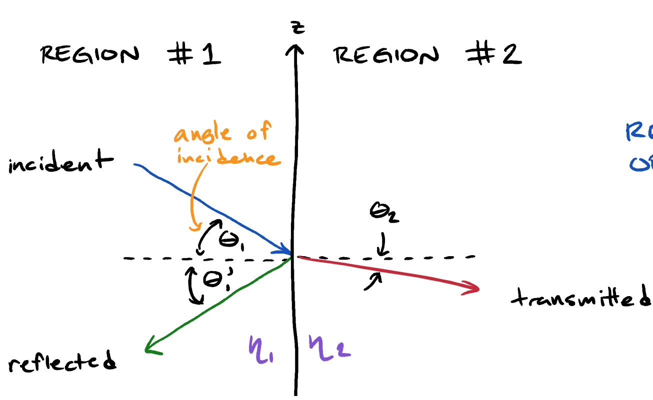
$$\boxed{\begin{aligned} \langle S_{reflected} \rangle &= |\Gamma|^2 \langle S_{incident} \rangle \\ \langle S_{transmitted} \rangle &= (1 - |\Gamma|^2) \langle S_{incident} \rangle \end{aligned}}$$

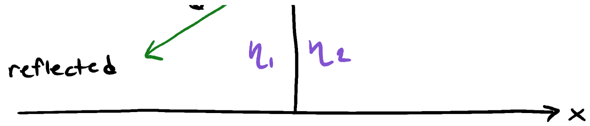
standing wave ratio

$$\boxed{SWR = s = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

note: if you know s, you can only determine magnitude of Γ

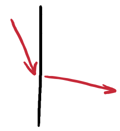
REFLECTION WITH OBLIQUE INCIDENCE



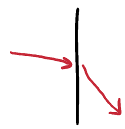


angles measured from normal of boundary

when wave passes into denser medium it bends towards normal



when wave passes into lighter medium it bends away from normal



SNELL'S LAW OF REFRACTION

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

relates angle of refraction to angle of incidence

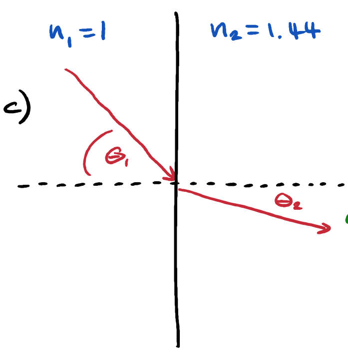
where n is index of refraction of material

$$\boxed{n = \frac{c}{v_p} = \sqrt{\varepsilon_r}}$$

Ex) Light travels from air into optical fiber with index of refraction of 1.44.
a) which direction does light bend?
b) if angle of incidence is 22° what is angle of refraction inside fiber
c) sketch path of light as it changes media

$n_1 = 1$ b/c refraction index of air $= 1$

c)



a) bends towards normal b/c passing into denser medium

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\hookrightarrow \theta_2 = \arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$= \arcsin\left(\frac{1}{1.44}\sin 22\right)$$

b) $= \boxed{15.08°}$

Can relate reflection coefficient to angle of incidence and refraction

$$\boxed{\Gamma = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}}$$

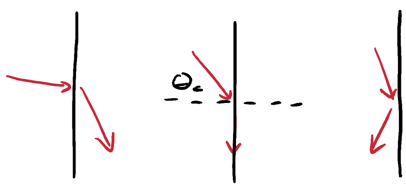
$$\boxed{\tau = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}}$$

TOTAL REFLECTION

↳ angle of incidence must be greater than critical angle, θ_c

↳ means $\theta_2 \geq 90^\circ$

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}}$$



total internal reflection

↳ $\boxed{\theta_i > \theta_c}$

Ex) Bibb Im Sw

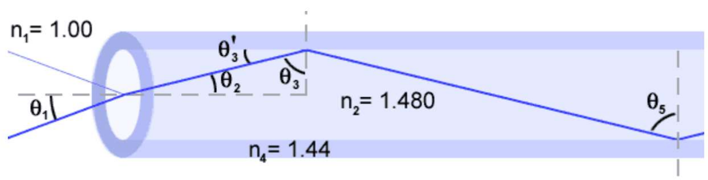
Consider the optical fiber in the figure below. The index of refraction of the inner core is 1.480 , and the index of refraction of the outer cladding is 1.44.

A. What is the critical angle for the core-cladding interface?

B. For what range of angles in the core at the entrance of the fiber (θ_2) will the light be completely internally reflected at the core-cladding interface?

C. What range of incidence angles in air does this correspond to?

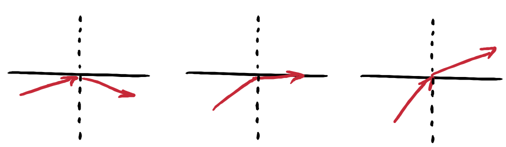
D. If light is totally internally reflected at the upper edge of the fiber, will it necessarily be totally internally reflected at the lower edge of the fiber (assuming edges are parallel)?



n of cladding

$$\sin\theta_c = \frac{n_2}{n_1} \rightarrow \theta_c^{\text{cladding}} = \arcsin\frac{1.44}{1.480} = \boxed{76.65^\circ} \quad a)$$

n of inner core



$$\underbrace{\theta_3 > \theta_c \qquad \theta_3 = \theta_c}_{} \qquad \theta_3 < \theta_c$$

$$\underbrace{\theta_3 \geq \theta_c \qquad \theta_3 = 90 - \theta_2}_{}$$

$$\theta_c \leq 90 - \theta_2$$

↳ $\theta_2 \leq 90 - \theta_c = 90 - 76.65 = 13.35$

b) $\boxed{\theta_2 \leq 13.35^\circ}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ core

$\hookrightarrow \theta_1 = \arcsin\left(\frac{n_2}{n_1}\sin\theta_2\right)$ @ θ_c internal

air

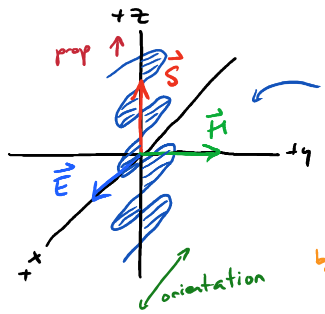
$$= \arcsin\left(\frac{1.480}{1}\sin 13.35\right) = 19.98^\circ$$

$$\boxed{\theta_1 < 19.98^\circ}$$

$$\boxed{\text{yes b/c } \theta_2 = \theta_s}$$

Ex) Suppose we have $\vec{E}(z,t) = 1.0\cos(2\pi 10^8 t - \beta z)\hat{a}_x \frac{V}{m}$ propagating in air. Find the power normally incident on a 20cm dish.

$\vec{E}$ is propagating in $+z$ direction b/c $-\beta z$ at an x-axis orientation



$\vec{H}$ in y-axis orientation

$$\vec{H}_y = \frac{E_{xo}}{\eta}\cos(\omega t - \beta z)$$

377 b/c air

same propagation direction

$$= \frac{1}{377}\cos(2\pi 10^8 t - \beta z)\ \hat{a}_y$$

$$\langle \vec{S} \rangle = \frac{1}{2}[\vec{E} \times \vec{H}^*]$$

$$\langle \vec{S} \rangle = \frac{1}{2}\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & \emptyset & \emptyset \\ \emptyset & 1/377 & \emptyset \end{bmatrix} = \frac{1}{2}\frac{1}{377}\hat{a}_z = 1.326(10^{-3})\frac{W}{m^2}$$

power density

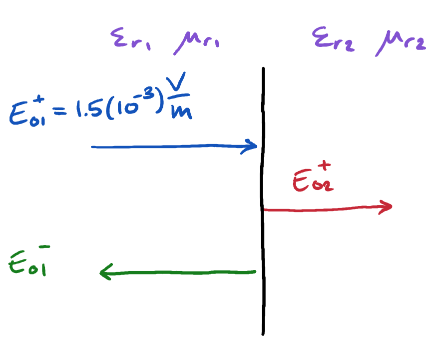
πr^2

$$P_{dish} = \langle S \rangle A = 1.326(10^{-3})\frac{W}{m^2}\ \pi\left(\frac{20(10^{-2})}{2}\right)^2 m^2 = \boxed{41.67\,\mu W}$$

Ex) Determine the amplitudes of reflected & transmitted $\vec{E}$ & $\vec{H}$ at the interface if $E_{01}^+ = 1.5(10^{-3})$ V/m

$\varepsilon_{r1} = 8.5$, $\mu_{r1} = 1$, $\varepsilon_{r2} = 1$, $\mu_{r2} = 1$

if no relative permeability given assume it's one



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \rightarrow \Gamma = \frac{\sqrt{\frac{\mu_0\mu_{r2}}{\varepsilon_0\varepsilon_{r2}}} - \sqrt{\frac{\mu_0\mu_{r1}}{\varepsilon_0\varepsilon_{r1}}}}{\sqrt{\frac{\mu_0\mu_{r2}}{\varepsilon_0\varepsilon_{r2}}} + \sqrt{\frac{\mu_0\mu_{r1}}{\varepsilon_0\varepsilon_{r1}}}} = 0.4892$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_r = E_i\Gamma \rightarrow \boxed{E_{01}^- = 0.7338(10^{-3})\frac{V}{m}}$$

$$E_t = E_i(1+\Gamma) \rightarrow \boxed{E_{02}^+ = 2.234(10^{-3})\frac{V}{m}}$$

note, amplitude can increase, but power conserved

Ex) verify your above answers using power densities

$$H_{01}^{+} = \frac{1.5(10^{-3})}{\eta_1} = 1.5(10^{-3})\frac{1}{\sqrt{\frac{\mu_0\mu_{r2}}{\varepsilon_0\varepsilon_{r2}}}} = 11.61(10^{-6})\,\frac{A}{m}$$

$$H_{02}^{+} = \frac{2.234(10^{-3})}{\eta_2} = 5.93(10^{-3})\,\frac{A}{m}$$

$$H_{01}^{-} = \frac{-E_{01}^{-}}{\eta_1}$$

this is negative b/c magnetic field must flip because $\vec{E}$ doesn't b/c $\Gamma > \emptyset$

$$= -5.679(10^{-6})\,\frac{A}{m}$$

$$\langle S_i \rangle = \frac{1}{2}\left[E_{01}^{+} \times H_{01}^{+*}\right] \rightarrow \frac{1}{2}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1.5(10^{-3}) & \emptyset & \emptyset \\ \emptyset & 11.61(10^{-6}) & \emptyset \end{vmatrix} = 8.708(10^{-9})\,\frac{W}{m^2}$$

$$\langle S_r \rangle = \langle S_i \rangle |\Gamma|^2 \longrightarrow 8.708(10^{-9})(0.4892)^2 = 2.084(10^{-9})\,\frac{W}{m^2}$$

$$\langle S_t \rangle = \langle S_i \rangle (1 - |\Gamma|^2) \longrightarrow 8.708(10^{-9})(1 - 0.4892^2) = 6.624(10^{-9})\,\frac{W}{m^2}$$

check $\langle S_i \rangle = \langle S_r \rangle + \langle S_t \rangle$ ✓

Ex) $E(z,t) = 10^3 \sin(\omega t - \beta z)\,\hat{a}_y\,\frac{V}{m}$ is in free space.
a) what direction is this wave propagating?
b) which direction is the magnetic field density?
c) find $\vec{H}$

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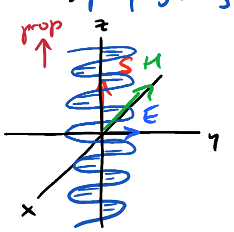
a) propagating in $+z$



b) $\vec{H}$ is oriented in $-x$ direction

c) $\vec{H}_x = \frac{-E_y}{\eta} = \frac{-10^3}{377}\sin(\omega t - \beta z)$

free space $\therefore \eta = 377$

Ex) A 100 MHz uniform plane wave propagates $+z$ in a lossless medium for which $\varepsilon_r = 5$, $\mu_r = 1$ @ an x polarization
a) find v_p, β, λ, $\bar{E}_s$, H_s, $\langle s \rangle$

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lossless $\rightarrow$ $\varepsilon'' = \emptyset$, $\alpha = \emptyset$, $\beta \simeq \omega\sqrt{\mu\varepsilon}$, $k = \beta$

$$\beta = 2\pi 100(10^6)\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} = \boxed{4.686\ \frac{rad}{s}}$$

$$v_p = \frac{1}{\sqrt{\varepsilon\mu}} \rightarrow \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}} = \boxed{134.1(10^6)\,\frac{m}{s}}$$

$$\lambda = \frac{2\pi}{k} \longrightarrow k = \beta \longrightarrow \lambda = \frac{2\pi}{4.686} = \boxed{1.341}$$

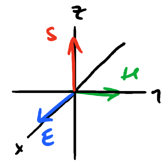
$$\bar{E}_s = E_0 \sin(\omega t - \beta z)$$

$$(a)\ \vec{E} = E_0 \sin\left(2\pi 100(10^6)t - 4.686z\right)\frac{V}{m}$$

$\bar{E}_s = E_0 \sin(\omega t - \beta z)$

$\hookrightarrow \boxed{\vec{E}_x = E_0 \sin\left(2\pi 100(10^6) t - 4.686 z\right) \frac{V}{m}}$

$|H_0| = \frac{|E_0|}{\eta}$



$\eta = \frac{\sqrt{\mu_0 \mu_r}}{\sqrt{\varepsilon_0 \varepsilon_r}} \rightarrow \eta = 168.5$

$\rightarrow \boxed{\vec{H}_y = \frac{E_0}{168.5} \sin\left(2\pi 100(10^6) t - 4.686 z\right) \frac{A}{m}}$

$\langle S \rangle = \frac{1}{2}\left[\vec{E}_s \times \vec{H}_s^*\right]$

$\hookrightarrow \boxed{\frac{1}{2} \frac{|\vec{E}_0|^2}{168.5} \hat{a}_z \frac{W}{m^2}}$