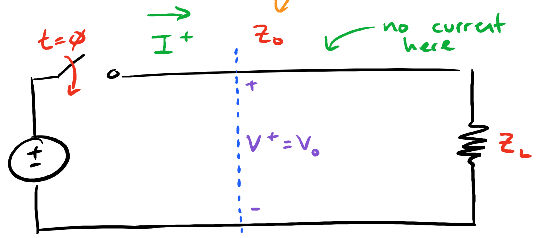
Quiz 1 Review

Thursday, February 4, 2016 7:11 PM

VOLTAGE/CURRENT WAVE

CHARACTERISTIC IMPEDANCE, Z_0

the amount of resistance the wire would have if it was infinitely long



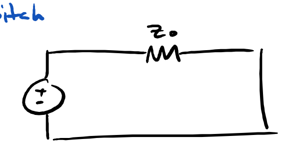
no current here

if Z_L is ∞, all of V^+ will be reflected

let's say it takes 1 second for current to reach load

@ $t=\emptyset$ we close switch

SOURCE SEES



current will start to flow based soley on the characteristic impedance, Z_0, of the transmission line

@ $t<1$ there is voltage at wave pulse V_0, some of the wire has current, some doesn't

@ $t=1$ V_0 reaches the load

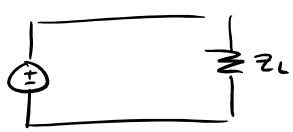
if $Z_L = Z_0$, current will flow through the same ← WE WANT TO MATCH IMPEDANCES TO PROVIDE MAXIMUM POWER TRANSFER AND MINIMIZE DISTORTION

if $Z_L \neq Z_0$, some voltage will be reflected

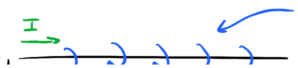
↳ then if internal impedance of the source is different from Z_0, it will again be reflected back

@ $t=\infty$ circuit is stabilized so transmission resistance is basically gone

SOURCE SEES

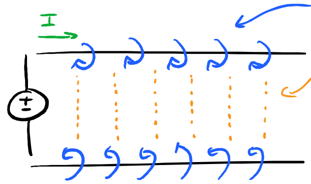


REALISTIC MODEL OF A TRANSMISSION LINE



magnetic field is created → creates inductance

and electric field is created → creates capacitance

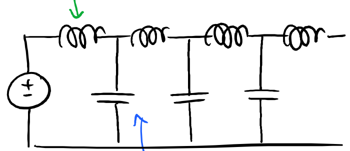


magnetic field is created → creates inductance

and electric field is created → creates capacitance

both of which slow down propagation of signal

inductor opposes current by generating voltage against it.
then eventually becomes s.c. after current stabilizes



an accurate representation of what is fundamentally happening in transmission line to delay propagation

"PULSE FORMING NETWORK"

then the capacitor gets charged and drains voltage while inductor stabilizes, then becomes an open-circuit which allows next inductor to get current, etc.

ASSUMING LOSSLESS TRANSMISSION

$$\boxed{Z_L = j\omega L}$$

$$\boxed{Z_C = \frac{1}{j\omega C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

↳ resistive, although composed of the inductance & capacitance of the wire

speed of the wave is proportional to how fast the inductors reach full current and how fast the capacitors can be charged

$\frac{1}{L}$

$\frac{1}{C}$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

← $\approx \frac{2}{3}c$ for coaxial cable

↳ also $\boxed{v = \lambda f}$

note: all these Ls & Gs & Rs and such are per unit length

LOSSLESS PROPAGATION

→ No power is dissipated in transmission line → two sources of loss are
→ All power at the input reaches output

1. LEAKAGE THROUGH DIELECTRIC
2. RESISTANCE OF THE CONDUCTOR

$R = G = \varnothing$

MODELING THE TRANSMISSION LINE USING MATH THIS TIME
(I'm going to skip the proofs & derivations though)

VOLTAGE EQUATION

$$\frac{\delta V}{\delta z} = -\left(RI + L\frac{\delta I}{\delta t}\right)$$

CURRENT EQUATION

$$\frac{\delta I}{\delta z} = -\left(GV + C\frac{\delta V}{\delta t}\right)$$

Differential Equations

Differential Equations

$$\frac{\delta V}{\delta z} = -\left(RI + L\frac{\delta I}{\delta t}\right)$$

$$\frac{\delta I}{\delta z} = -\left(GV + C\frac{\delta V}{\delta t}\right)$$

where we took Δz to mean a short wire element of length Δz

NOW SOLVING FOR A LITTLE MORE

GENERAL VOLTAGE EQUATION

$$\frac{\delta^2 V}{\delta z^2} = LC\frac{\delta^2 V}{\delta t^2} + (LG+RC)\frac{\delta V}{\delta t} + RGV$$

GENERAL CURRENT EQUATION

$$\frac{\delta^2 I}{\delta z^2} = LC\frac{\delta^2 I}{\delta t^2} + (LG+RC)\frac{\delta I}{\delta t} + RGI$$

Wave Equations

these are both in the form of the wave equation → $\frac{\delta^2 f(z,t)}{\delta z^2} = \frac{1}{v^2}\frac{\delta^2 f(z,t)}{\delta t^2}$

note that because $v = \frac{1}{\sqrt{LC}}$, $\frac{1}{v^2} = LC$ which you can see in our equation!

which has the solution → $f(z,t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right)$

forward moving component

backward moving component

OUR SOLUTION → $\boxed{V = V^+ + V^-}$ $\boxed{V^+ = Z_0 I^+}$ $\boxed{V^- = -Z_0 I^-}$

love dat engineering

the negative sign is to counteract the negative direction of current, the voltage isn't necessarily negative

positive current is defined by convention as moving in the clockwise direction (+)

NOW LET'S TALK ABOUT THOSE PESKY REFLECTIONS

RECAP: if the load impedance doesn't match the characteristic impedance of the transmission line, there will be a reflection which reduces power transmission → bad

VOLTAGE REFLECTION COEFFICIENT, Γ

↳ the fraction of the voltage of the incident wave that's reflected by the load resistance

$$\boxed{V^- = \Gamma V^+}$$

$$\frac{\text{net load voltage}}{\text{net load current}} = R_L = Z_0\frac{1+\Gamma}{1-\Gamma}$$

$$\Gamma = \frac{R_L - Z_0}{R_L + Z_0}$$

← we can solve for the reflection coefficient as a function of the load and characteristic impedance

if $R_L > Z_0$ → $\Gamma > \emptyset$ → voltage same sign, current sign swap

if $R_L < Z_0$ → $\Gamma < \emptyset$ → voltage sign swap, current same sign

AND DON'T FORGET, THE SOURCE ALSO HAS REFLECTIONS

↳ source has internal resistance R_g

→ we calculate amplitude of initial voltage wave as a voltage divider between R_g & Z_0

$$\boxed{V^+ = V_0 \frac{Z_0}{Z_0 + R_g}}$$

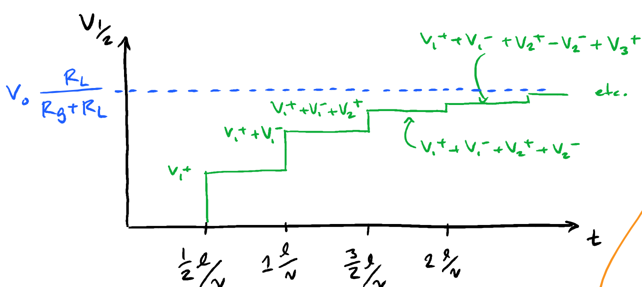
→ we calculate reflection coefficient same way as with load

$$\boxed{\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}}$$

VOLTAGE REFLECTION DIAGRAM (totally going to be on exam)

$$\boxed{\text{Final voltage} = V_0 \frac{R_L}{R_g + R_L}}$$

let's say we want to observe the voltage $1/2$ of the way down the transmission line



FIRST ROUND TRIP

$V_1^+ = V_0 \frac{Z_0}{Z_0 + R_g}$ $\qquad$ $V_1^- = \Gamma_L V_1^+$

SECOND ROUND TRIP

$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$ $\qquad$ $V_2^- = \Gamma_L V_2^+ = \Gamma_L^2 \Gamma_g V_1^+$

THIRD ROUND TRIP

$V_3^+ = \Gamma_g V_2^- = \Gamma_L^2 \Gamma_g^2 V_1^+$ $\qquad$ $V_3^- = \Gamma_L^3 \Gamma_g^2 V_1^+$

etc.

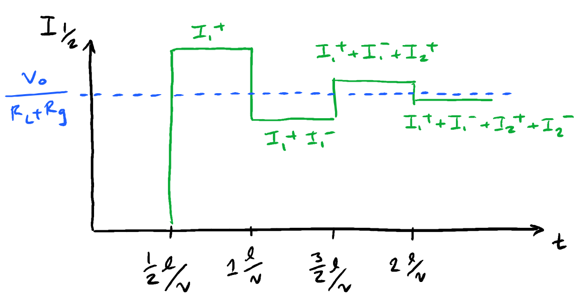
asymptotically approaches voltage divider value

↳ eventually will be treated as a wire

CURRENT REFLECTION DIAGRAM

$$\boxed{\text{Final current} = \frac{V_0}{R_L + R_g}}$$

use $V^+ = Z_0 I^+$ and $V^- = -Z_0 I^-$



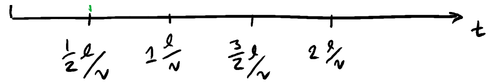
FIRST ROUND TRIP

$I_1^+ = V_1^+/Z_0$ $\qquad$ $I_1^- = -V_1^-/Z_0$

SECOND ROUND TRIP

$I_2^+ = V_2^+/Z_0$ $\qquad$ $I_2^- = -V_2^-/Z_0$

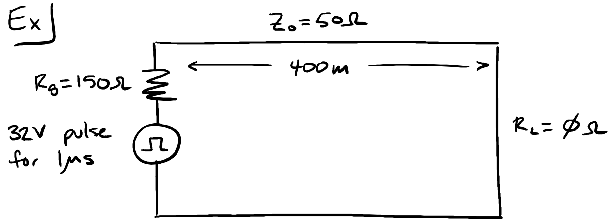
etc.



$I_2 = \;/Z_0 \qquad I_2 = \;/Z_0$

etc.

Ex)



velocity of wave $= 200\,m/\mu s$

PLOT REFLECTION DIAGRAMS AT SOURCE FOR AT LEAST 4 ROUND-TRIPS

DETERMINE REFLECTION COEFFICIENTS

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = -1 \qquad \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = 1/2$$

DETERMINE INITIAL WAVE PULSE & RTT

$$V_1^+ = V_0 \frac{Z_0}{R_g + Z_0} = 32 \cdot 1/4 = 8V \qquad T = 2\,d/v = 2\,\frac{400m}{200\,m/\mu s} = 4\mu s$$

DETERMINE REST OF PULSES

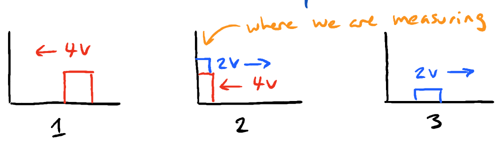
$V_1^+ = 8V$
$V_2^+ = \Gamma_g V_1^- = -4V$
$V_3^+ = 2V$
$V_4^+ = -1V$

$V_1^- = \Gamma_L V_1^+ = -1 \cdot 8 = -8V$
$V_2^- = 4V$
$V_3^- = -2V$
$V_4^- = 1V$

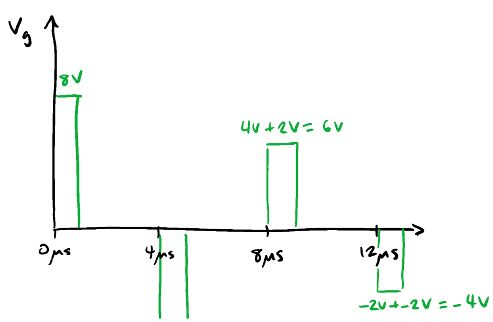
$I_1^+ = V_1^+/Z_0 = 8V/50\Omega = 160mA$
$I_2^+ = -80mA$
$I_3^+ = 40mA$
$I_4^+ = -20mA$

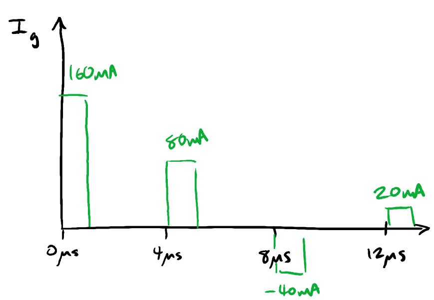
$I_1^- = -V^-/Z_0 = 160mA$
$I_2^- = -80mA$
$I_3^- = 40mA$
$I_4^- = -20mA$

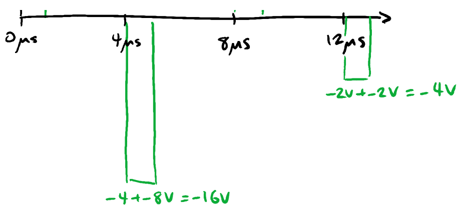
recall that we are measuring these values right at the source, this is where the interference between the incoming wave & the reflected wave is, so they will momentarily sum. for example:

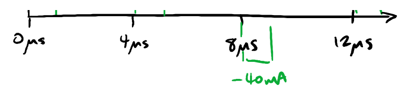


$\therefore V_{measured} = V_1^- + V_2^+,\ V_2^- + V_3^+,$ etc.









ALRIGHT, LET'S START TALKING ABOUT STEADY-STATE

PHASE CONSTANT, β $\frac{rad}{s}$ → represents "spatial frequency", phase-shift per unit distance

$$\boxed{\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v_p}}$$

$$V_s = |V_0| \cos\left[\omega\left(t - \frac{z}{v}\right) + \phi\right]$$
$$= V_0 \cos(\omega t - \beta z)$$
$$= V_0 \underbrace{e^{j\omega t}} e^{-j\beta z}$$

this component doesn't really effect impedance so we'll just drop it.

VOLTAGE PHASOR $\boxed{V_s = V_0 e^{-j\beta z}}$

In steady state realize there will be a forward wave and a net backward wave

↳ sum of all incident and reflected waves

WAVE IMPEDANCE $Z_w(z) = \dfrac{V^+ e^{-j\beta z} + V^- e^{+j\beta z}}{I^+ e^{-j\beta z} + I^- e^{+j\beta z}}$ using euler's identity & $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$\boxed{Z_w(z) = Z_0 \left[\frac{Z_L \cos\beta z - j Z_0 \sin\beta z}{Z_0 \cos\beta z - j Z_L \sin\beta z}\right]}$$

when the line is equal to half the signal wavelength

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{m\lambda}{2} = m\pi \quad (m = 0, 1, 2 \ldots)$$

$$\therefore \; Z_{in} = Z_w(l) = Z_0 \left[\frac{Z_L \cos m\pi + j Z_0 \sin m\pi}{Z_0 \cos m\pi + j Z_L \sin m\pi}\right]$$

$$Z_{in} = Z_{in}$$

when the line is equal to an odd multiple of a 1/4 wavelength

$$\beta l = \frac{2\pi}{\lambda}(2m+1)\frac{\lambda}{4}$$

$$\therefore \; Z_{in} = Z_0^2 /$$

$$\beta l = \frac{2\pi}{\lambda}(2m+1)\frac{\lambda}{4}$$

$$\therefore \; Z_{in} = Z_0^2 / Z_L$$

STANDING WAVES

happens when load not matched to transmission line

↳ the signal interferes constructively at some points & destructively at others

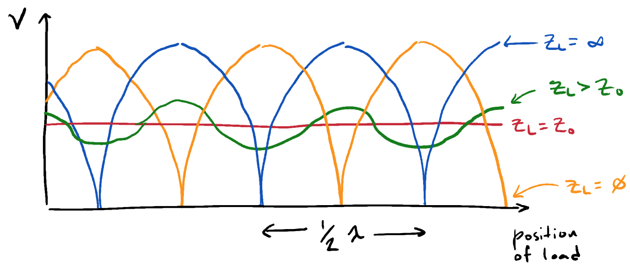
the load impedance can be calculated from the distance between the load and the minimum voltage and the voltage standing wave ratio

↳ Z_{min}

$$\boxed{s = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

read from rotating Γ to the right on the horizontal line on Smith Chart

STANDING WAVES ON A TRANSMISSION LINE



$$\boxed{V_{max} = V_0(1+|\Gamma|)}$$

$$\boxed{V_{min} = V_0(1-|\Gamma|)}$$

$$\boxed{z_{max} = \frac{-1}{2\beta}(\phi + 2n\pi)}$$

$$\boxed{z_{min} = \frac{-1}{2\beta}(\phi + (2n+1)\pi)}$$

positions of max/min voltages along transmission line

$n = 0, 1, 2 \ldots$

SMITH CHART

allows us to solve problems by relating

- → input impedance
- → output impedance
- → line length

→ reflection coefficient

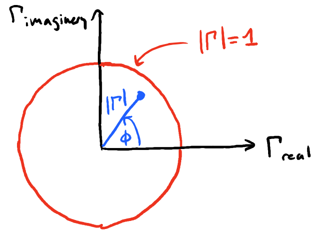
built inside the unit-circle, each point representing rectangular co-ordinates of the reflection coefficient, Γ

$\Gamma_{imaginary}$ ↑

$$\lambda = \frac{\nu}{f} = \frac{2/3\, c}{100 \cdot 10^6}$$

$$\frac{3}{2}$$

co-ordinates of the reflection coefficient, Γ

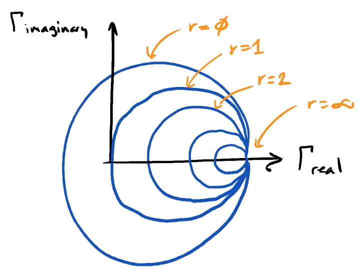


to use you must normalize your input impedance because the chart is calibrated in terms of per unit values of impedance & admittance

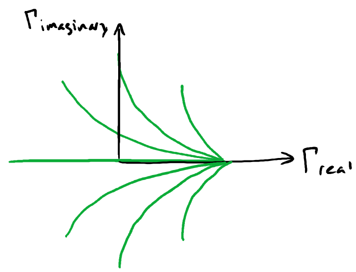
ie. divide your impedances by the characteristic impedance

$$\boxed{z = \frac{Z}{Z_0}} \qquad \boxed{\Gamma = \frac{z-1}{z+1}} \qquad \boxed{z = \frac{1+\Gamma}{1-\Gamma}}$$

smith chart is a graphical representation of these equations

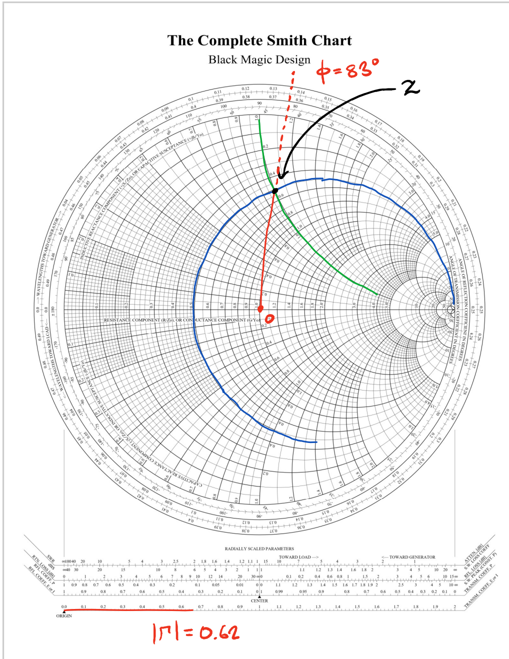


circles represent real part of impedance



curved lines represent imaginary part of impedance

Ex) Find reflection coefficient where load impedance is $25 + j50\,\Omega$



$Z_L = 25 + j50\,\Omega$, assume $Z_0 = 50\,\Omega$

$z = \dfrac{Z_L}{Z_0} = 0.5 + j1\,\Omega$ plot the normalized impedance on the smith chart

real part

imaginary part

measure distance from origin to z.
project that distance onto transmission coefficient scale

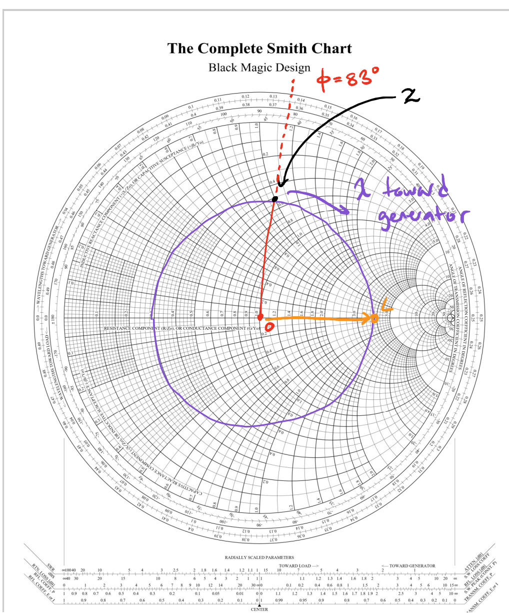
$|\Gamma| = 0.62$

measure angle from "angle of reflection coefficient" scale

$\phi = 83°$

Ex) Find the input impedance & VSWR from the load impedance

z_g will be on the circle with the radius of $|\Gamma|$, 0.62 in the previous example, rotated x wavelengths toward the generator as indicated on the chart



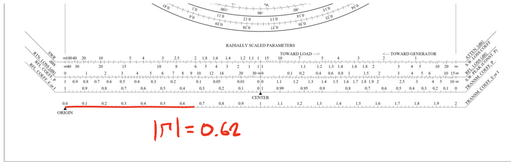
and vice-versa if we have the input impedance and we want the load impedance

$Z_L \rightarrow Z_g$ rotate toward generator

$Z_g \rightarrow Z_L$ rotate toward load

To find VSWR, rotate point along radius to the right until purely resistive (flat)

↳ there are two flat points with the radius, take the larger (rightmost on the graph)



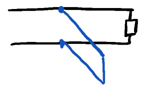
ST

WE'LL COME BACK TO SMITH CHARTS MOMENTARILY

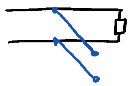
But first let's talk about matching transmission lines → RECAP: it minimizes losses & distortion

we can create a matched system by doing two things that seem insane in DC called SINGLE STUB MATCHING

1. short-circuit the load
   lower EM radiation leakage

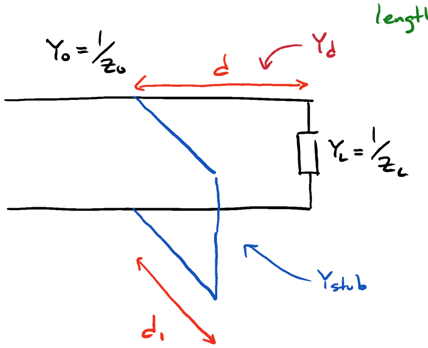


2. open-circuit the load
   more practical for microstrip circuits



let's talk about the first one

Two design parameters: distance of the stub from the load, d; length of the stub, d_1



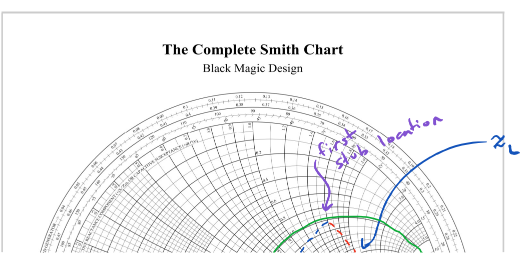
since it's in parallel, it'll be easier to work with admittance than impedance

IN ORDER TO MATCH, $Y_{stub} + Y_d = Y_0$

let's do an example for how to solve this using the Smith Chart

$\boxed{d_1 = \frac{\theta_1}{4\pi}\lambda}$ $\boxed{Y_d = Y_0 \pm j B_{stub}}$ $\boxed{Y_{stub} = \mp j B_{stub}}$

Ex) Design a circuit stub to match load $100 + j100\,\Omega$ to a $50\,\Omega$ transmission line



$Z_L = 100 + j100\,\Omega \rightarrow z = \frac{Z_L}{Z_0} = 2 + j2$

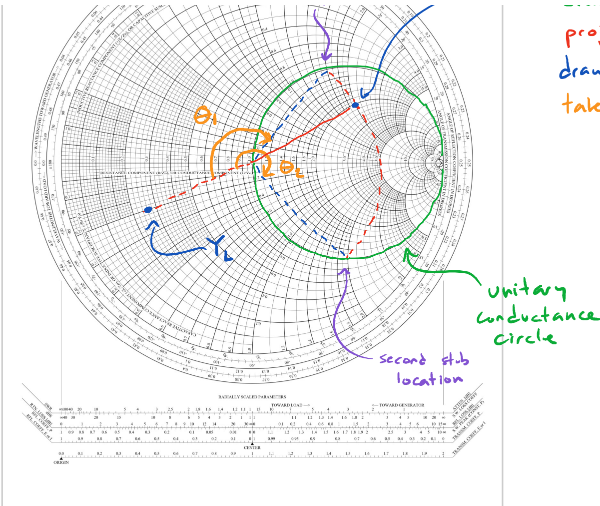
plot Z_L point

draw Γ line from origin to point

draw unit circle along $r = 1$ (unitary conductance circle)

project circle of radius Γ from origin to intersections with unit circle

draw lines from origin to intersection



project circle of radius Γ from origin to intersections with unit circle

draw lines from origin to intersection

take angles between load admittance line & intersections

↳ you can put a stub at either of the corresponding locations

There will always be two solutions:

first has a positive susceptance, must be cancelled with capacitive stub

second has a negative susceptance, must be cancelled with inductive stub

$$\theta = \frac{4\pi}{\lambda} d_1 \longrightarrow d_1 = 160° \frac{1/4}{180} \lambda = 0.222\lambda$$

LOSS IN TRANSMISSION LINES

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

loss leads to distortion

$$\boxed{\alpha + j\beta = \sqrt{ZY}}$$

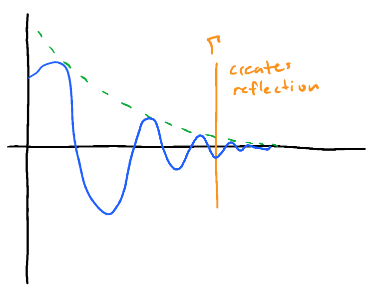
α: signal decay, $\frac{Np}{m}$ "attenuation coefficient"

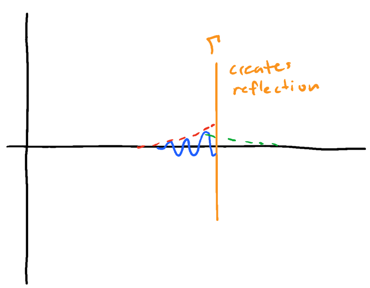
β: phase constant, $\frac{rad}{m}$

$$\boxed{V(z,t) = \underbrace{V^+ e^{-\alpha z}}_{\text{exponential dec forward}} \cos(\omega t + \beta z) + \underbrace{V^- e^{\alpha z}}_{\text{exponential decay backward}} \cos(\omega t + \beta z)}$$

incident term: $V^+ e^{-\alpha z} \cos(\omega t + \beta z)$

reflected term: $V^- e^{\alpha z} \cos(\omega t + \beta z)$





LOW-LOSS PROPAGATION

$R \ll \omega L$, $G \ll \omega C$ ← often true in practice

$$\boxed{\alpha \simeq \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)}$$

$$\boxed{\beta \simeq \omega\sqrt{LC}\left[1+\frac{1}{8}\left(\frac{G}{\omega C} - \frac{R}{\omega L}\right)^2\right]}$$

↳ with this value, can determine phase velocity that depends on frequency

$$\boxed{v_p = \frac{\omega}{\beta}}$$

In lossy system:

$Z_0 \neq \sqrt{L/C}$

$$\boxed{Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = |Z_0|e^{j\theta}}$$

$|Z_0| \measuredangle \theta$

HEAVISIDE CONDITION

↳ means the signal is not distorted if condition is met

$$\boxed{R/L = G/C}$$

to achieve this, you can modify any term but really only want to modify L as increasing R or G increases distortion

$R \neq \emptyset$ & $G \neq \emptyset$

loss increases w/ frequency b/c R increases w/ frequency

POWER TRANSMISSION

power decays 2x as fast as V or I

$$\boxed{\langle P \rangle = \frac{1}{2}\frac{|V_0|^2}{|Z_0|} e^{-2\alpha z} \cos\theta}$$

time-average power

POWER ATTENUATION

power at input

$$\boxed{L_{dB} = 10\log\left(\frac{\langle P(0)\rangle}{\langle P(z)\rangle}\right) = 10\log e^{2\alpha z} = 20\alpha z \log e = 8.69\alpha z}$$

$= 20 \log \frac{|V_0(0)|}{|V_0(z)|}$

power at point z

→ power at any point along transmission line

→ converted to decibels so that losses of connected lines can be added

Ex) A 20m length of transmission line has a loss of 2dB from end-to-end. a) what fraction of power reaches the output? b) what fraction of power reaches the midpoint? c) what $\underbrace{\text{attenuation coefficient}}_{\alpha}$ does this represent?

fraction of input power @ 20m $= \frac{\langle P(20)\rangle}{\langle P(0)\rangle}$

$$L_{dB} = 10\log\frac{\langle P(0)\rangle}{\langle P(z)\rangle} \longrightarrow 2\,dB = 10\log\frac{\langle P(0)\rangle}{\langle P(20)\rangle}$$

$$\frac{\langle P(0)\rangle}{\langle P(20)\rangle} = 10^{2/10}$$

$$\longrightarrow \frac{\langle P(20)\rangle}{\langle P(0)\rangle} = 10^{-2/10} = \boxed{0.63}$$ ← 63% of power reaches output

loss per distance $= \frac{2\,dB}{20\,m} = 0.1\,\frac{dB}{m}$

@ 10m, $10\,m\left(0.1\,\frac{dB}{m}\right) = 1\,dB$

$$1\,dB = 10\log\frac{\langle P(0)\rangle}{\langle P(10)\rangle}$$

$$\longrightarrow \frac{\langle P(10)\rangle}{\langle P(0)\rangle} = 10^{-1/10} = \boxed{0.79}$$ ← 79% of power reaches midpoint

$$\underbrace{10\log\left(\frac{\langle P(0)\rangle}{\langle P(z)\rangle}\right)}_{2\,dB} = 8.69\,\alpha z \longrightarrow \alpha = \frac{2\,dB}{8.69\cdot 20\,m} = \boxed{0.0115\,\frac{Np}{m}}$$

REFLECTED POWER

$$\boxed{\frac{\langle P_r\rangle}{\langle P_i\rangle} = |\Gamma|^2}$$ ← ratio of reflected power to incident power

TRANSMITTED POWER

$$\boxed{\frac{\langle P_t\rangle}{\langle P_i\rangle} = 1-|\Gamma|^2}$$

Ex) Two lossy lines are joined end-to-end. First line is 10m long w/ loss of 0.2 dB/m. Second line is 15m long w/ loss of 0.1 dB/m. Reflection coefficient between lines is 0.3 & power at input is 100mW. Determine total loss & determine transmitted power.

$$\frac{\langle P_t\rangle}{\langle P_i\rangle} = 1-|\Gamma|^2$$

$$L_{dB} = 10\log\frac{\langle P(0)\rangle}{\langle P(z)\rangle}$$

loss at junction

$$L_{j\,dB} = 10\log\frac{\cancel{\langle P_i\rangle}}{\cancel{\langle P_i\rangle}(1-|\Gamma|^2)} = 10\log\frac{1}{1-|\Gamma|^2}$$

$$= 10\log\frac{1}{1-0.3^2} = 0.4096\,dB$$

loss in line 1 $L_1 = 10m\left(0.2\frac{dB}{m}\right) = 2dB$

loss in line 2 $L_2 = 15m\left(0.1\frac{dB}{m}\right) = 1.5dB$

TOTAL LOSS $L_T = 2dB + 1.5dB + 0.41dB = \boxed{3.91\ dB}$

TRANSMITTED POWER

$$L_{dB} = 10\log\frac{\langle P(0)\rangle}{\langle P(\ell)\rangle} \longrightarrow 3.91\,dB = 10\log\frac{100\,mW}{P_t}$$

$$\boxed{P_t = 41.6\ mW}$$

@ two connected lines w/ different characteristic impedences

→ reflections occur at junction

$$\boxed{\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}}$$

power that propagates into second line: $\boxed{1 - |\Gamma|^2}$

SPECIAL CASES:

QUATER-WAVE MATCHING

↳ limited to where

$\ell \simeq (2m+1)\frac{\lambda}{4}$ (odd multiple)

line length is $\frac{1}{4}\lambda$ $\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$ $\boxed{Z_{02} = \sqrt{Z_{01}Z_{03}}}$

line length is $\frac{1}{2}\lambda$ $\boxed{Z_{in} = Z_L}$