Phasors and Physics

Friday, January 8, 2016 10:56 AM

342 TOOA PHYSICS REVIEW

LINEAR

$v = d/t$, $a = \dfrac{v_f - v_0}{t}$, $d = v_0 t + \frac{1}{2} a t^2$, $v_f^2 = v_0^2 + 2ad$, $\bar{F} = m\bar{a}$

ANGULAR

rpm $\rightarrow n$ 360° = 2π radians

rad/s $\rightarrow \omega$ 1 radian angle ≃ 60°

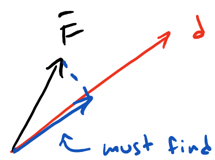
$\omega = \dfrac{\theta}{t}$, $\alpha = \dfrac{\omega_f - \omega_0}{t}$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $\omega_f^2 = \omega_0^2 + 2\alpha\theta$, $\bar{\tau} = \bar{R} \times \bar{F}$

$\bar{\tau} = I\bar{\alpha}$

I: angular mass (kg/m^2)

ENERGY

$\bar{W} = \bar{F} \cdot \bar{d}$



must find component of force in direction.

$W = \tau\theta$

Types:
1. Kinetic energy $\longrightarrow W_k = \frac{1}{2}mv^2$ / $W_k = \frac{1}{2}I\omega^2$
2. Potential energy
3. Electric energy
4. Magnetic energy
5. Heat

POWER

Do work in an interval, the rate at which we use energy

Watt $\equiv \dfrac{\text{Joules}}{\text{sec.}}$ 1HP ≡ 745.7W

LINEAR $P = \dfrac{W}{t} = Fv$ ANGULAR $P = \dfrac{W}{t} = \dfrac{T\theta}{t} = T\omega$

Ex) A car has a 300HP engine. It is 1705kg and can accelerate from ∅ to 100kPH in 5.6 seconds. Its tires have a diameter of 83.2 cm.

what is the efficiency of the gearbox?
what is the transmission system?

$a = \dfrac{100\,kP/h}{5.6\,s} = 4.96\,m/s^2$

$F = ma = (1705\,kg)(4.96\,\frac{m}{s^2}) = 8460\,N$

$W = F \cdot d = 8460\,N \cdot \dfrac{83.2\,cm}{2} = 3519\,J$

$$W = F \cdot d = 8460\,N \cdot \frac{83.2\,cm}{2} = 3519\,J$$

$$P = \frac{3519\,J}{5.6\,s} = 628\,W \rightarrow 0.8422\,hp$$

$$\text{efficiency, } \eta_m = \frac{P_{out}}{P_{in}} = \frac{0.8422\,hp}{300\,hp} = 0.28\,\%$$

342 TOOB PHASORS REVIEW

PHASOR 01

$3 + j4$

this is only a representation of a complex number

PHASOR 02

original ammeters measured average current through them so when measuring AC signals, they measured ØA because the average of a sinosoidal wave is zero.

So, to measure them we square it then take the average, but then we need to convert A^2 back to A, so we square-root it:

$$\sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

(root of the mean of the square of the current - RMS)

EFFECTIVE VALUE, I_{eff}

A DC current that transmits same amount of energy in the same amount of time as the original AC i(t).

TO RELATE i(t) & I_{eff}

$$R I_{DC}^2 = \frac{1}{T}\int_0^T R\, i^2(t)\,dt$$

power for DC current

average power for AC current

$$I_{DC} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

effective value of

a current
is its RMS
value

EFFECTIVE VALUE OF A SINOSOIDAL CURRENT (aka RMS value)

$$i(t) = I_{peak} \sin(\omega t + \theta)$$

$$I_{DC} = \sqrt{\frac{1}{T}\int_0^T I_{peak}^2 \sin^2(\omega t + \theta)\,dt} = \frac{I_{peak}}{\sqrt{2}}$$

AC: $i(t) = A\sin(\omega t + \theta)$

this has the same power as a DC current of $\frac{A}{\sqrt{2}}$

M is the effective value of current $i(t)$

$$M = \frac{A}{\sqrt{2}} \leftarrow \text{RMS value}$$

$$i(t) = \sqrt{2}\,M\sin(\omega t + \theta)$$

AC Ammeters / AC voltmeters measure the RMS value.

↳ in power systems, we are by default talking about RMS voltages & currents

PHASOR 03

elements in a circuit apply operations on a current or voltage:

$v(t) = R \cdot i(t)$ ← resistor multiplies by R

$v(t) = L \cdot \frac{di(t)}{dt}$ ← inductor differentiates

$v(t) = \frac{1}{C}\int i(t)\,dt$ ← capacitor integrates

$$v(t) = V_{MAX}\sin(\omega t + \theta)$$

amplitude (V_{MAX})

frequency in $\frac{rad}{s}$ (ω) $(2\pi f)$ ← Hz

phase shift in rad (θ)

PHASOR NOTATION (Steinmetz Notation)

$$v(t) = \sqrt{2}\,V_{RMS}\sin(\omega t + \theta)$$

$$\bar{V} = V_{RMS} \measuredangle \theta$$

30°

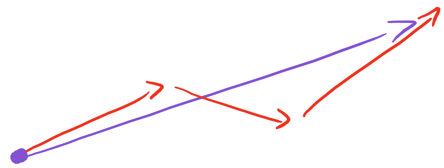
$v_1(t) = 100\sqrt{2}\sin\left(377t + \frac{30\pi}{180}\right) \longrightarrow \bar{V}_1 = (100 \measuredangle 30)$

$v_2(t) = 70\sqrt{2}\sin\left(377t - \frac{15\pi}{180}\right) \longrightarrow \bar{V}_2 = (70 \measuredangle -15)$

$v_3(t) = 90\sqrt{2}\sin\left(377t + \frac{45\pi}{180}\right) \longrightarrow \bar{V}_3 = (90 \measuredangle 45)$

added together $= (237.98 \measuredangle 23.68)$

?



$v_{sum}(t) = 237.9\sqrt{2}\sin\left(377t + \frac{23.7\pi}{180}\right)$

multiplying by $j\omega$ is the equivalent to differentiating in the time domain

$$v = L\frac{di(t)}{dt} \longrightarrow \boxed{\bar{V} = j\omega L \bar{I}}$$

dividing by $j\omega$ is equivalent to integrating in the time domain

$$v = \frac{1}{C}\int i(t)\,dt \rightarrow \boxed{\bar{V} = \frac{\bar{I}}{j\omega C}}$$

PHASOR DOMAIN

$$\bar{V} = R\bar{I}, \quad \bar{V} = j\omega L\bar{I}, \quad V = \frac{\bar{I}}{j\omega C} = -j\frac{\bar{I}}{\omega C}$$

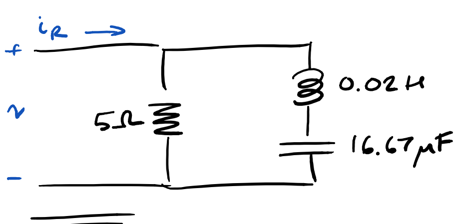
IMPEDANCES (in Ohms)

$$\bar{Z}_R = R, \quad \bar{Z}_L = j\omega L, \quad \bar{Z}_c = -j\frac{1}{\omega C}$$

using this generalized Ohms law $\bar{V} = \bar{I}\bar{Z}$, we can use traditional tools KVL, KCL, & MNA just like we used to.

↳ just represent elements by their impedances & voltages/currents by phasors

PHASOR 04



phase shift

$$v(t) = 50\sqrt{2}\sin\left(2000t - \frac{90\pi}{180}\right)\text{ V}$$

RMS value ↑ frequency ↑

what's $i_R(t)$?

① write everything as a phasor

$$\bar{V} = (50\angle -90°)$$

② represent all elements by their impedances

$$\bar{Z}_R = 5\Omega$$

$$\bar{Z}_L = j\omega L = j2000 \cdot 0.02 = j40\Omega$$

$$\bar{Z}_c = -j\frac{1}{\omega C} = -j\frac{10^6}{2000 \cdot 16.67} = -j30\Omega$$

$$\bar{Z}_L + \bar{Z}_C = j40 - j30 = j10\Omega$$

$$\bar{Z}_{TOTAL} = \bar{Z}_R \,//\, (\bar{Z}_L + \bar{Z}_c) = 5 \,//\, j10 = 4 + j2$$

③ Find I_T phasor

$$\bar{I}_T = \frac{\bar{V}_T}{\bar{Z}_{TOTAL}} = \frac{(50 \angle -90^\circ)}{(4,2)} = (-5,-10) = 11.18 \angle -116.6^\circ$$

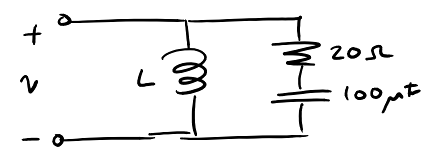
④ convert from phasor domain to time domain

$$i(t) = \sqrt{2}\,(11.18) \sin\left(2000t - \frac{116.6\pi}{180}\right)$$

$\sqrt{2}$ is to convert RMS to peak V

mind your sign, − means subtract, + means add

PHASOR 05



$v(t) = 100\sqrt{2} \sin(500t)$

$i(t) = 2.5\sqrt{2} \sin(500t)$

what's L?

① phasorify V & I

$$\bar{V} = (100 \angle 0^\circ) = 100V$$

$$\bar{I} = (2.5 \angle 0^\circ) = 2.5A$$

② impedancify R, L, C

$$\bar{Z}_R = 20, \quad \bar{Z}_C = -j\frac{1}{\omega C} = -j\frac{10^6}{500 \cdot 100} = -20j\,\Omega, \quad \bar{Z}_L = j\omega L = j500 \cdot L$$

$$\bar{Z}_{TOTAL} = \cancel{j500L \,/\!/\, (20-20j)}$$

don't need

③ Ohm's law

$$\bar{V} = \bar{I}\bar{R} \rightarrow \cancel{\frac{100 \angle 0^\circ}{2.5 \angle 0^\circ} = j500L \,/\!/\, (20-20j)}$$

$$\cancel{\frac{100}{2.5} = 40 = \frac{1}{\frac{1}{j500L} + \frac{1}{20-20j}}}$$

wrong strategy

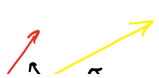
$$\bar{I}_{RC} = \frac{\bar{V}}{Z_R + Z_C} = \frac{100}{20-20j} = (2.5, 2.5) = (3.536 \angle 45^\circ)$$

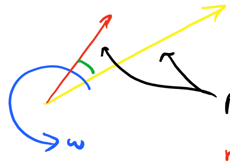
$$\bar{I}_L = \bar{I} - \bar{I}_{RC} = 2.5A - (3.536 \angle 45^\circ) = 2.5 \angle -90^\circ$$

$$\bar{Z}_L = \frac{\bar{V}}{\bar{I}_L} = \frac{100}{2.5 \angle -90^\circ} = 40 \angle 90^\circ \Omega$$

$$\bar{Z}_L = j\omega L \rightarrow L = \frac{\bar{Z}_L}{j\omega} = \frac{40 \angle 90^\circ}{j500} = 0.08 \angle 0^\circ = \boxed{0.08\,H}$$

PHASOR 06



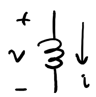


phasors rotating @ $\omega \frac{rad}{s}$ counter-clockwise

red is leading by θ
yellow is lagging by θ

convention is to use the smallest angle

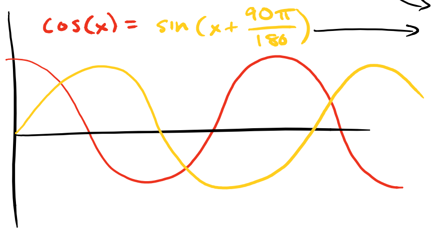
IN AN INDUCTOR



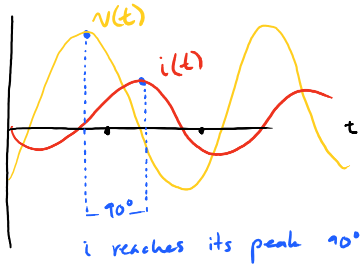
$$i(t) = I\sqrt{2}\sin(\omega t + \alpha)$$

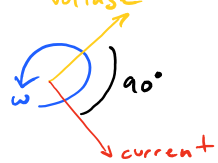
$$v(t) = L\frac{di(t)}{dt} = \omega L I\sqrt{2}\cos(\omega t + \alpha)$$

$$\cos(x) = \sin\left(x + \frac{90\pi}{180}\right) \longrightarrow v(t) = \omega L I\sqrt{2}\sin\left(\omega t + \alpha + \frac{90\pi}{180}\right)$$



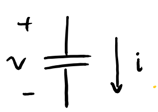
In an inductor, the current lags behind the voltage by 90°
↳ this makes sense b/c inductors slow change in current





i reaches its peak 90° after v reaches its peak

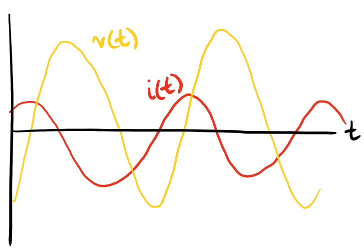
IN A CAPACITOR

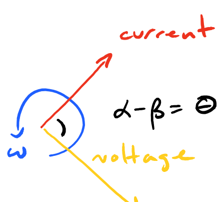


$$v(t) = V\sqrt{2}\sin(\omega t + \beta)$$

$$i(t) = C\frac{\delta v(t)}{\delta t} = \omega C V\sqrt{2}\cos(\omega t + \beta) = \omega C V\sqrt{2}\sin\left(\omega t + \beta + \frac{90\pi}{180}\right)$$

In a capacitor, the current leads the voltage by 90°.
↳ this makes sense b/c capacitors slow change in voltage.

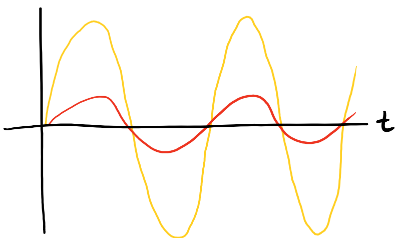


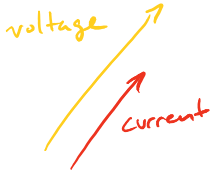


$\alpha - \beta = \theta = 90$

IN A RESISTOR

In a resistor, the voltage and current phase are the same





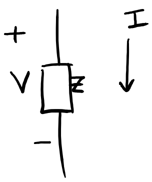
PHASOR 07

In a circuit AC steady state, all the voltages and currents have the same sinusoidal frequency, ω

The power also varies sinusoidally, but with frequency 2ω

↳ positive part of the time / negative part of the time

ie. absorbs power / delivers power in variation



$$v(t) = \sqrt{2}\, V \sin(\omega t + \alpha)$$
$$i(t) = \sqrt{2}\, I \sin(\omega t + \beta)$$

absorbed power: $p(t) = v(t)\, i(t)$

$$= 2VI \sin(\omega t + \alpha) \sin(\omega t + \beta)$$

using $\sin(x+y) = \sin x \cos y + \cos x \sin y$

then $\sin^2 x = \dfrac{1-\cos 2x}{2}$, $\cos^2 x = \dfrac{1+\cos 2x}{2}$, $\sin x \cos x = \dfrac{\sin 2x}{2}$

instantaneous power

both RMS values

$$p(t) = \underbrace{VI\cos(\alpha-\beta)}_{\text{constant}} - \underbrace{VI\cos(2\omega t + \alpha + \beta)}_{\text{function of time}}$$

current phase shift
voltage phase shift

average of a cosine w/ respect to time = ∅

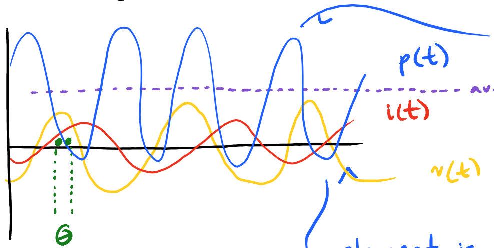
average power

$$\boxed{P = VI\cos(\alpha - \beta)}$$

ACTIVE POWER

angle of the voltage
− angle of the current

frequency is doubled



element is absorbing power most of the time, when positive
ACTIVE POWER, P

average power absorbed

element is delivering power when negative
REACTIVE POWER, S

↳ difference b/w voltage phase and current phase

APPARENT POWER, S

$S = VI$ (a real number)

units Volt-Amperes

ACTIVE POWER, P

$P = VI\cos\theta$ (a percentage of the apparent power)

This is the average power, units Watts

The percentage of apparent power that becomes active power is the power-factor

$PF = \cos\theta$

REACTIVE POWER, Q

$Q = VI\sin\theta$ (power not used)

units Volt-Ampere Reactive (VARs)

PHASOR 07b

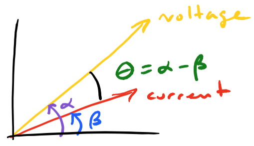
$v(t) = \sqrt{2}V\sin(\omega t + \alpha)$, $i(t) = \sqrt{2}I\sin(\omega t + \beta)$

$p(t) = VI\cos(\alpha - \beta) - VI\cos(2\omega t + \alpha + \beta)$

average

$P = VI\cos(\alpha - \beta)$

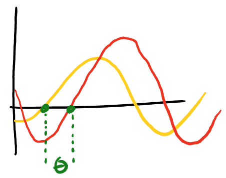
RMS values



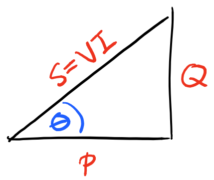
$S = V_{RMS} \cdot I_{RMS} \equiv VA$

$P = V_{RMS} I_{RMS} \cos(\alpha - \beta) \equiv W$

$Q = V_{RMS} I_{RMS} \sin(\alpha - \beta) \equiv VAR$



POWER TRIANGLE



$\bar{S} = P + jQ$

PHASORS: $\bar{V} = V_{RMS} \angle \alpha$, $\bar{I} = I_{RMS} \angle \beta$

COMPLEX POWER IS THE PRODUCT OF THE V PHASOR WITH THE CONJUGATE OF I.

$\bar{S} = \bar{V}\bar{I} = V_{RMS} I_{RMS} \angle(\alpha - \beta)$

FOR RESISTORS: $\boxed{P = RI_{RMS}^2 = \frac{V_{RMS}^2}{R}, \quad Q = \varnothing}$

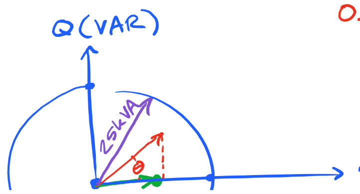
FOR INDUCTORS/CAPACITORS: $X = \omega L \qquad X = -\frac{1}{\omega C}$

reactance

$\boxed{P = \varnothing, \quad Q = XI_{RMS}^2 = \frac{V_{RMS}^2}{X}}$

PHASOR 09

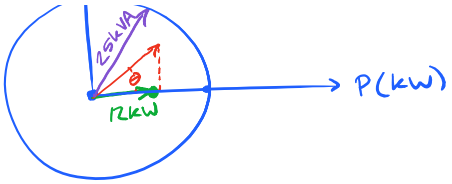
Ex) A 25 kVA transformer supplies a load of 12 kW at a PF of 0.6 lagging. Find the percentage of full load the transformer is carrying.



0.6 lagging therefore PF angle is going to be positive

$P = VI\cos\theta \rightarrow P = S(P.F.)$

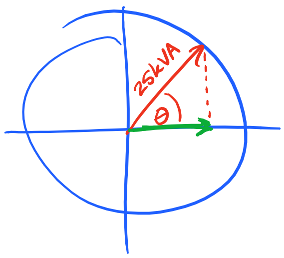
$S_{load} = \frac{P}{PF} = \frac{12kW}{0.6} = 20kVA$



$$S_{load} = \frac{P}{PF} = \frac{12kW}{0.6} = 20kVA$$

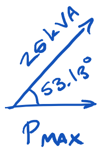
$$\%\ load = \frac{S_{load}}{S_{max}} = \frac{20kVA}{25kVA} = 80\%$$

Ex) cont'd How much more kW of resistive load can we add without overloading the transformer?



~~$0.6 = \cos\theta$~~

~~$\theta = \cos^{-1}(0.6) = 53.13°$~~



~~SOH CAH TOA~~

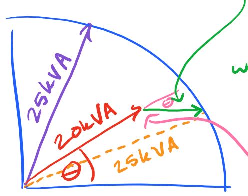
~~$\cos(53.13°) = \frac{P_{max}}{25kVA}$~~

~~$\rightarrow P_{max} = 0.6 \cdot 25kVA = 15kW$~~

~~$\rightarrow 15kW - 12kW = 3kW$~~

~~Could add 3kW more of power~~

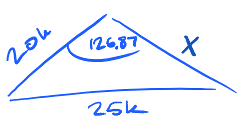
Because it is resistive load, it is only active power we are adding, so we only add horizontal



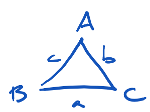
what is this length?

$\theta = \cos^{-1}(0.6) = 53.13°$

$180 - \theta° = 180 - 53.13 = 126.87°$



Cosine law: $a^2 = b^2 + c^2 - 2bc \cdot \cos A$



$$25^2 = x^2 + 20^2 - 2x \cdot 20 \cos(126.87)$$

$\downarrow$

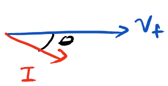
$x = \cancel{-31.2}$, $x = 7.209$ not what we're looking for

$$\boxed{\text{Could add 7.2kW more resistive load.}}$$

PHASOR 10

Flow of Active Power (P) & Reactive Power (Q)

Slightly Inductive (absorbs active and reactive power)

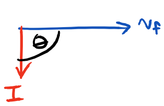


$\theta = \measuredangle V_f - \measuredangle I > \emptyset$

$\cos\theta > \emptyset \longrightarrow P > \emptyset$

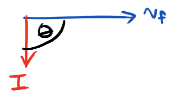
$\sin\theta > \emptyset \longrightarrow Q > \emptyset$

Purely Inductive (absorbs only reactive power)



$\theta = \measuredangle V_f - \measuredangle I = 90°$

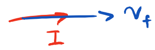
$\cos\theta = \emptyset \longrightarrow P = \emptyset$,



$\theta = \measuredangle V_f - \measuredangle I = 90°$

$\cos\theta = \emptyset \longrightarrow P = \emptyset$
$\sin\theta = 1 \longrightarrow Q = VI > \emptyset$

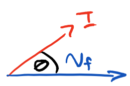
Resistive (absorbs only active power, no reactive power)



$\theta = \measuredangle V - \measuredangle I = \emptyset$

$\cos\theta = 1 \longrightarrow P = VI > \emptyset$
$\sin\theta = \emptyset \longrightarrow Q = \emptyset$

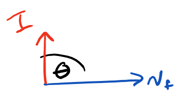
Capacitive (absorbs active power and produces reactive power)
↳ ie. absorbs negative Q



$\theta = \measuredangle V - \measuredangle I < \emptyset$

$\cos\theta > \emptyset \longrightarrow P = VI\cos\theta > 0$
$\cos\theta < \emptyset \longrightarrow Q = VI\sin\theta < 0$

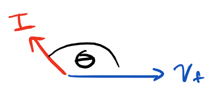
Purely Capacitive (absorbs no active power, produces reactive power)
↳ ie. absorbs negative Q



$\theta = \measuredangle V - \measuredangle I = -90°$

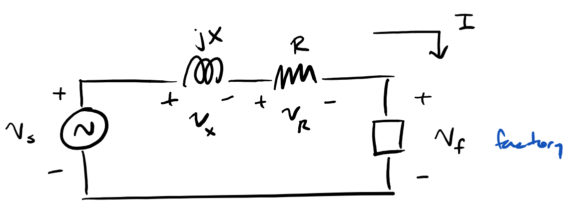
$\cos\theta = \emptyset \longrightarrow P = \emptyset$
$\sin\theta = -1 \longrightarrow Q < \emptyset$

Generator Load (produces active power, absorbs reactive power)



$\theta = \measuredangle V - \measuredangle I, \quad 180° < \theta < 90°$

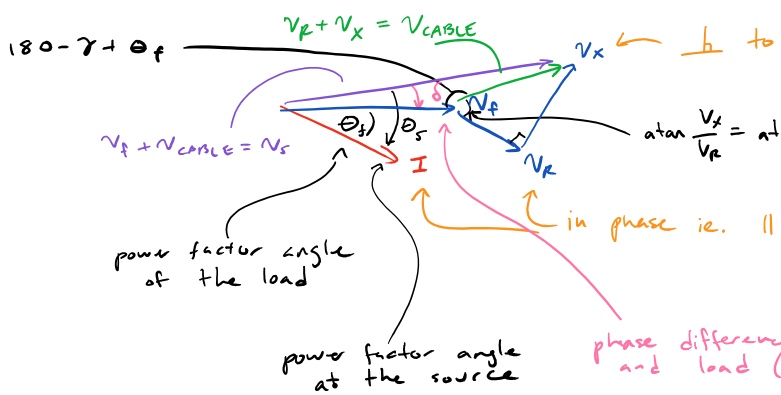
$\cos\theta < \emptyset \longrightarrow P < \emptyset$
$\sin\theta > \emptyset \longrightarrow Q > \emptyset$



$\bar{V}_R = R\bar{I}$ ∴ $\bar{V}_R$ is in phase w/ $\bar{I}$ (parallel)
$\bar{V}_x = jX\bar{I}$ ∴ $\bar{V}_x$ is out of phase w/ $\bar{I}$ (perpendicular)

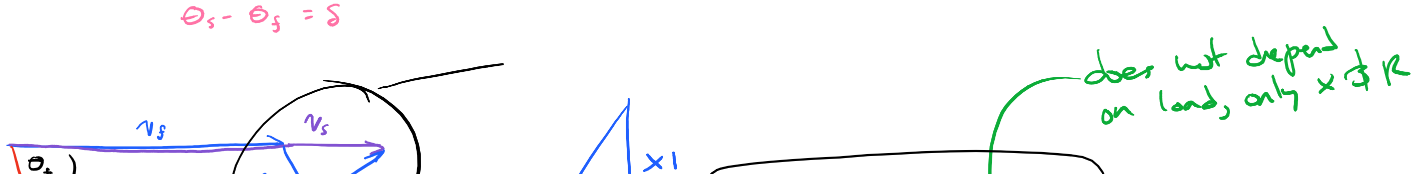
if not given any phases, you may choose one as reference arbitrarily.

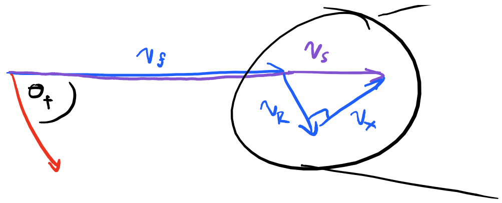
so choose V_f as $\measuredangle\emptyset$

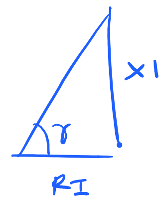


$\operatorname{atan}\frac{V_x}{V_R} = \operatorname{atan}\frac{X}{R} = \gamma$

$\theta_s - \theta_f = \delta$







on load, only X · I

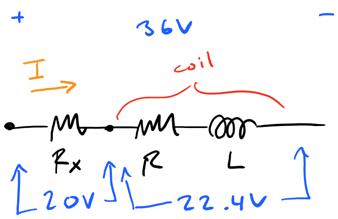
$$\boxed{\gamma = \operatorname{atan}\frac{XI}{RI} = \operatorname{atan}\frac{X}{R}}$$

PHASOR 12

A real coil has a resistance R and an inductance L. The coil is represented by a model consisting of its resistance R in series with its inductance L. To determine R and L, the coil is connected in series with an external resistor R_X = 10 ohms. Then we apply a sinusoidal voltage of 60Hz, 36V to the entire group "coil plus R_X." We measure the voltage in the coil V_{COIL}=22.4V, and the voltage in the external resistor V_{RX}=20V. Calculate the values of R and L, the resistance and inductance of the coil.

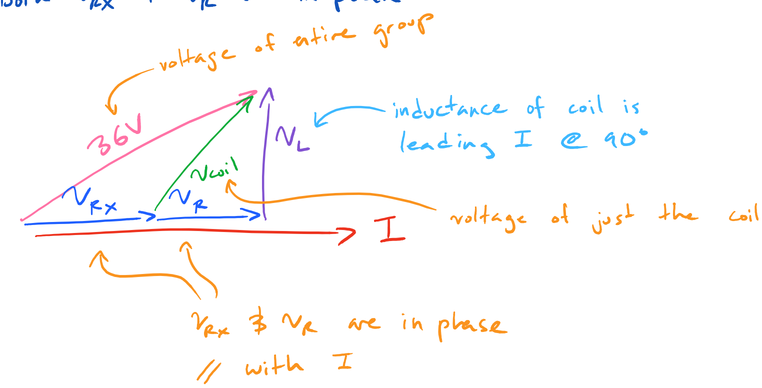
if $\theta_f = \gamma$ greatest difference b/w mag. voltage at factory and voltage at source

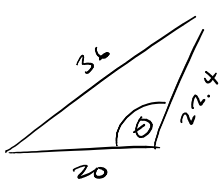
↳ max voltage drop



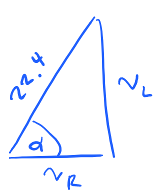
Choose phase reference I @ ∡0°

Both V_{RX} & V_R are in phase





$\theta = \operatorname{acos}\left[(36^2 - 20^2 - 22.4^2)/(-2 \cdot 20 \cdot 22.4)\right] = 116.10°$



$\alpha = 180° - \theta = 63.90°$

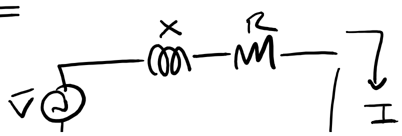
$V_R = 22.4 \cos\alpha = 9.86V$

$V_L = 22.4 \sin\alpha = 20.1V$

$R = \frac{9.86}{2A} = 4.93\,\Omega$

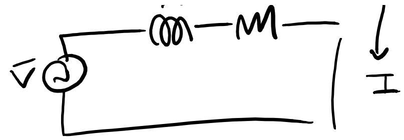
$V_L = 2X \rightarrow X = 10.06\,\Omega$

$X = 2\pi 60 L \rightarrow L = \frac{10.06}{2\pi 60} = 26.68\,mH$



$P = I^2 R$

$Q = I^2 X$



$$P = I^2 R$$

$$Q = I^2 X$$

$$S = I^2 Z$$