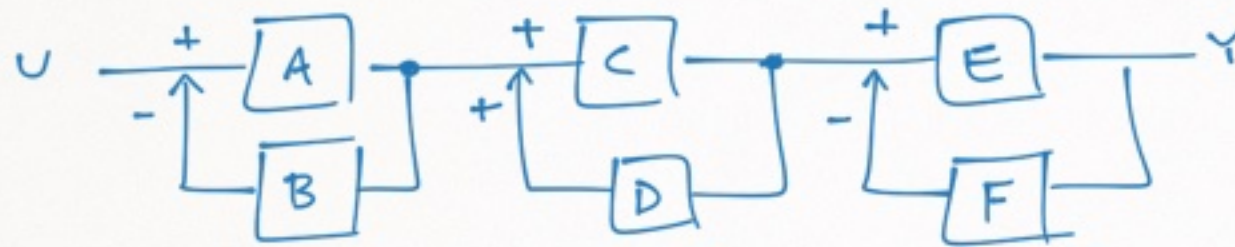
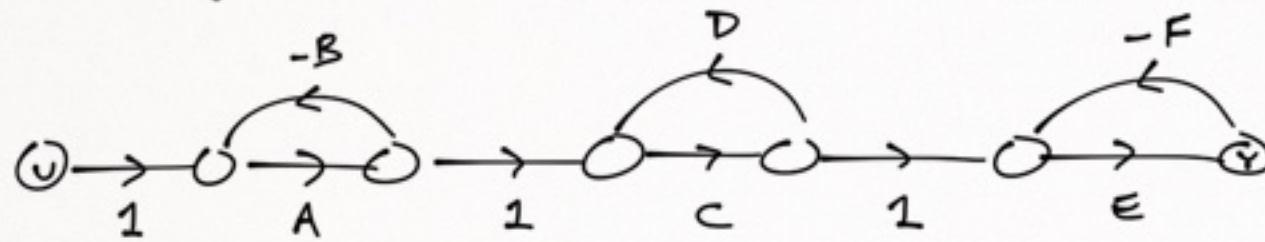


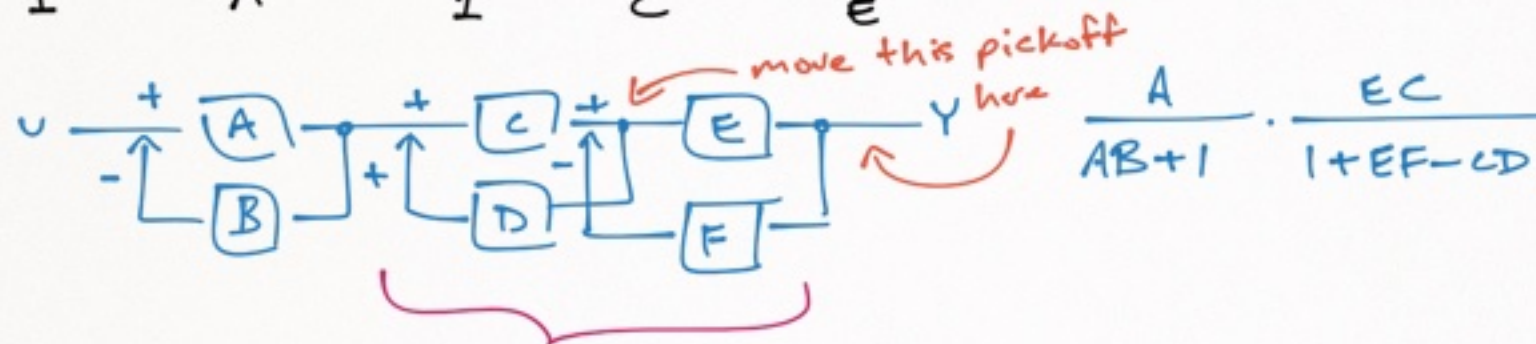
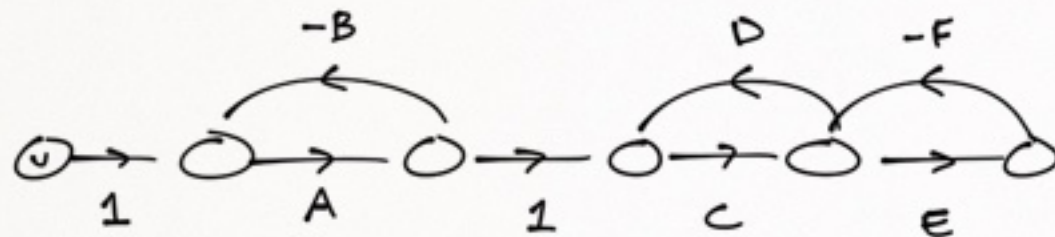
EECE 360 QUIZ #1

convert signal flow diagram to block diagram & find transfer function

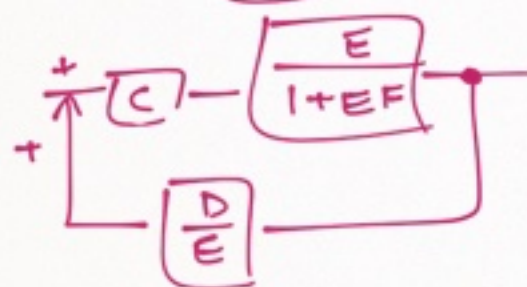
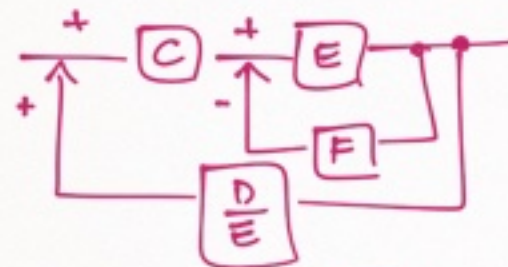


$$\frac{A}{AB+1} \cdot \frac{\cancel{C}}{\cancel{CD}+1} \cdot \frac{E}{EF+1} = \frac{Y}{U}$$

$$\frac{C}{1-CD}$$

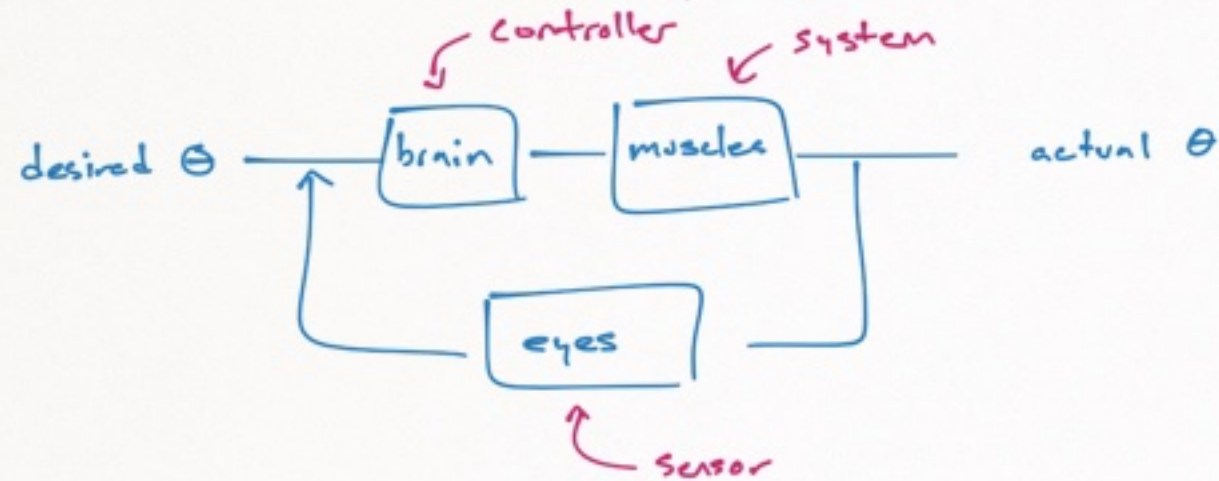


$$\frac{A}{AB+1} \cdot \frac{EC}{1+EF-CD}$$

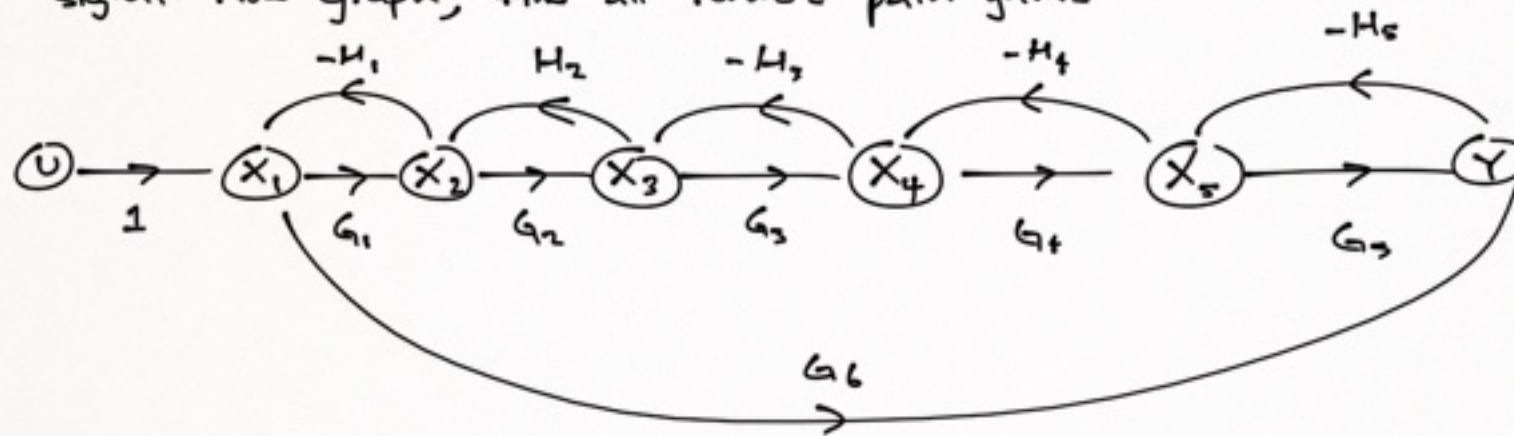


$$\frac{\frac{CE}{1+EF}}{1 + \frac{CE}{1+EF} \cdot \frac{D}{E}} = \frac{-EC}{DC-FE-1} = \frac{EC}{1+FE-DC}$$

gymnast uses her eyes, brain, stomach muscles to balance body on beam.
draw feedback control system, label all blocks/signals



for signal flow graph, find all forward path gains



$$P_1: G_1 G_2 G_3 G_4 G_5$$

$$P_2: G_6$$

find all loop gains

$$L_1: -G_1 H_1$$

$$L_2: G_2 H_2$$

$$L_3: -G_3 H_3$$

$$L_4: -G_4 H_4$$

$$L_5: -G_5 H_5$$

$$L_6: G_6 H_1 H_2 H_3 H_4 H_5$$

compute determinant Δ in terms of L & P

$$\sum \pi_1 = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 \quad (\text{all loops})$$

$$\sum \pi_2 = L_1 L_3 + L_1 L_4 + L_1 L_5 + L_2 L_4 + L_2 L_5 + L_3 L_5 \quad (\text{double loops that don't touch})$$

$$\sum \pi_3 = L_1 L_3 L_5$$

$$\Delta = 1 - \sum \pi_1 + \sum \pi_2 - \sum \pi_3$$

compute sub-determinant in terms of $L \nmid P$

go through forward paths identifying loops that don't touch it

$$P_1 = G_1 G_2 G_3 G_4 G_5 \leftarrow \text{touches all loops} \therefore \Delta_1 = 1$$

$$P_2 = G_6 \leftarrow \text{doesn't touch } L_2, L_3, L_4, \text{ doesn't touch} \therefore \Delta_2 = 1 - L_2 - L_3 - L_4 - L_2 L_4$$

combo of $L_2 L_4$
which also don't touch
each other

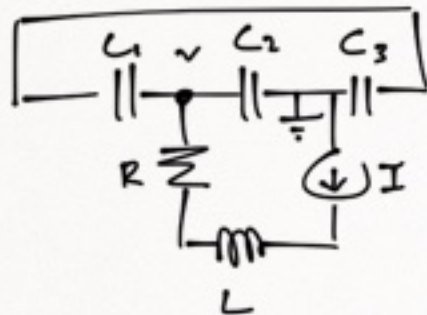
compute transfer function $\frac{Y}{U}$

$$TF = \frac{\sum P_i \Delta_i}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 G_5 + G_6 (1 - L_2 - L_3 - L_4 - L_2 L_4)}{\Delta}$$

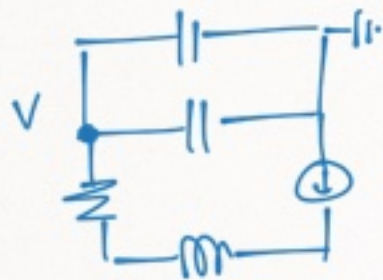
Δ
 \uparrow from previous page

QUIZ #2

convert following system into simplest possible mechanical equivalent & label w/ mech. symbols

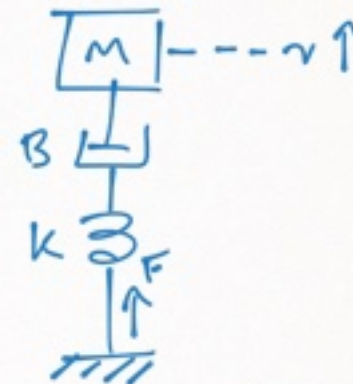


1. simplify circuit



$$(C_1^{-1} + C_3^{-1})^{-1} + C_2$$

2. draw mech equivalent



write formulas for mech. symbols in terms of electrical symbols

$$R = \frac{1}{B} \quad L = \frac{1}{K} \quad M = (C_1^{-1} + C_3^{-1})^{-1} + C_2 \quad v = V \quad F = I$$

convert following transfer function into time domain

$$\frac{Y(s)}{U(s)} = \frac{s^3 + s^2 + s + 1}{(s+1)(s+2)(s+3)}$$

partial fractions

inverse-laplace

$$1 + \frac{5}{s+2} - \frac{10}{s+3}$$

$$\delta(t) + 5e^{-2t} - 10e^{-3t}$$

↑
apparently it's more correct to multiply these by $u(t)$, the heaviside step function

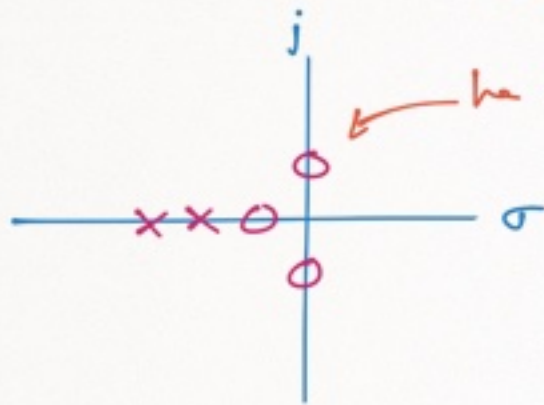
$$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$= \frac{A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)}{(s+1)(s+2)(s+3)}$$

$$\begin{array}{ll} s=-1 & A(1)(2) = -1 + 1 - 1 + 1 \\ s=-2 & B(-1)(1) = -8 + 4 - 2 + 1 = -5 \end{array}$$

Sketch the previous page's T.F. poles & zeros. What's the order of the system?
is it stable?

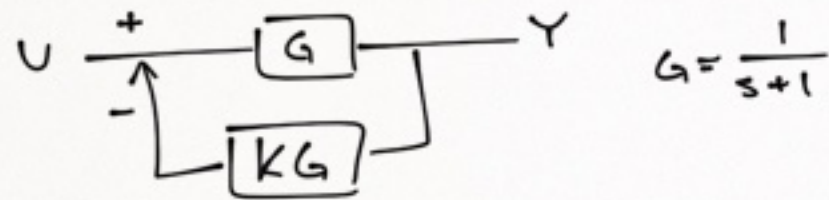
$$\frac{s^3 + s^2 + s + 1}{(s+1)(s+2)(s+3)} = \frac{\cancel{(s+1)}(s+j)(s-j)}{\cancel{(s+1)}(s+2)(s+3)} = \frac{(s+j)(s-j)}{(s+2)(s+3)} \quad \underline{\text{IT'S 2nd ORDER}}$$



he didn't label the complex poles for some reason

no poles in RHP \therefore stable

compute ζ & ω_n as a function of K of following system



$$\frac{Y}{U} = \frac{G}{1 + KG^2} = \frac{s+1}{k + s^2 + 2s + 1}$$

$$\zeta = \frac{a}{\sqrt{a^2 + \omega^2}}$$

$$\omega_n = \sqrt{a^2 + \omega^2}$$

$$CE: s^2 + 2s + 1 + k = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n^2 = 1 + k \rightarrow \underline{\omega_n = \sqrt{1+k}}$$

$$\cancel{\zeta} = \cancel{\zeta\omega_n} \rightarrow \underline{\zeta = \frac{1}{\omega_n} = \frac{1}{\sqrt{1+k}}}$$

what value of k results in critical damping?

$$\text{critically damped @ } \zeta = 1 \quad 1 = \frac{1}{\sqrt{1+k}} \quad \underline{k = 0}$$

note that when $K < 0$ the system turns into a positive feedback system.
 sketch the natural response when $K = -0.5$

$$@ K = -0.5 \quad \zeta = \frac{1}{\sqrt{1-0.5}} = 1.41$$

$\zeta > 1 \therefore$ overdamped

NATURAL RESPONSE?

use Routh-Hurwitz criteria to determine values of K for which system is stable

$$\frac{Y}{U} = \frac{s+1}{s^2+2s+(1+K)}$$

$$\begin{array}{ccc} s^2 & 1 & 1+K \\ s^1 & 2 & \emptyset \\ s^0 & m_1 & \end{array}$$

$$m_1 = -\frac{1}{b} \det \begin{vmatrix} a & c \\ b & d \end{vmatrix} = -\frac{1}{2} (\emptyset - 2(1+K)) = 1+K$$

$1+K > 0$ to remain stable

$$\therefore \underline{K > -1}$$

compute rise & settle time for the following system $\frac{Y}{U} = \frac{s+4.5}{s^2+3s+9}$

$$CE: s^2+3s+9$$

$$3s = 2\zeta\omega_n s \quad \omega_n^2 = 9 \rightarrow \omega_n = 3$$

$$\frac{3}{2} = \zeta\omega_n \rightarrow \zeta = \frac{3}{6} = \frac{1}{2}$$

$$0.3 < \zeta < 0.8 \therefore T_r \approx \frac{2.16\zeta + 0.6}{\omega_n} = 0.560$$

$$T_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{3/2} = \underline{2.67s}$$

$$T_r = \frac{1}{\omega_n \sqrt{1-\zeta^2}} \left(\pi - \tan^{-1} \left(\frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right) = \underline{0.806s}$$

compute the DC gain, final value of impulse response, & percentage overshoot

$$PO = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \times 100\% = \underline{16.3\%}$$

Final value theorem $\lim_{s \rightarrow 0} sF(s) = FV$

$$FV = \lim_{s \rightarrow 0} s \frac{s+45}{s^2+3s+9} = \underline{0}$$

$$TF(s) = K_{DC} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

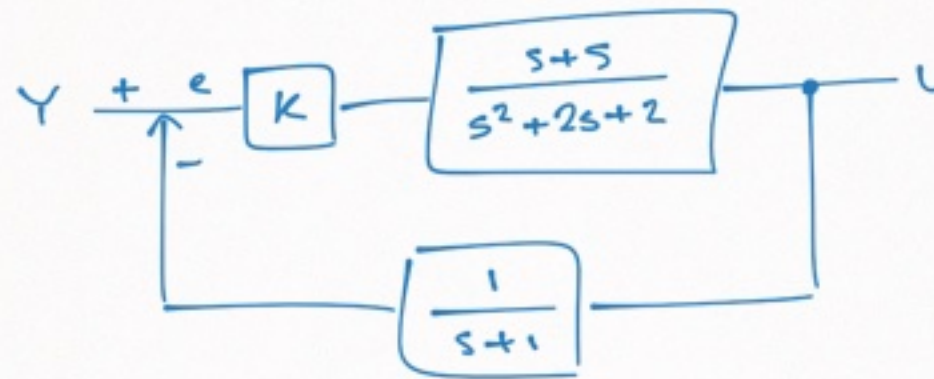
$$TF(0) = K_{DC} = \frac{0+45}{0+0+9} = \underline{5}$$

\uparrow $s = \text{frequency}$, so frequency = 0 therefore DC

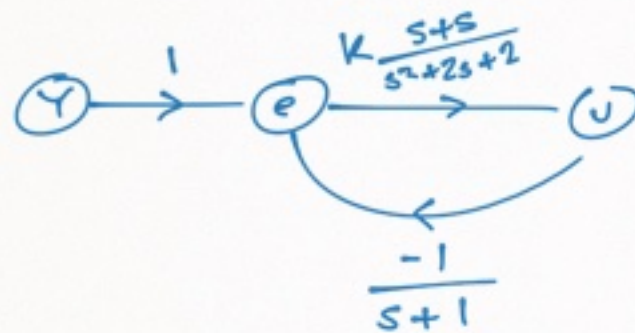
QUIZ #3

$$\frac{Y}{U} = \frac{\frac{K(s+5)}{s^2+2s+2}}{1 + \frac{K(s+5)}{(s+1)(s^2+2s+2)}}$$

for the following-closed loop transfer function create a block-diagram



redraw system as a signal flow graph



for above system, compute the open-loop poles & zeros

OPEN LOOP: KGM

zeros: -5

poles: -1-j, -1+j, -1

compute asymptotes of root locus

determine using angle criteria, $\angle S = \#P - \#Z = 3 - 1 = \underline{2}$
 \uparrow # asymptotes

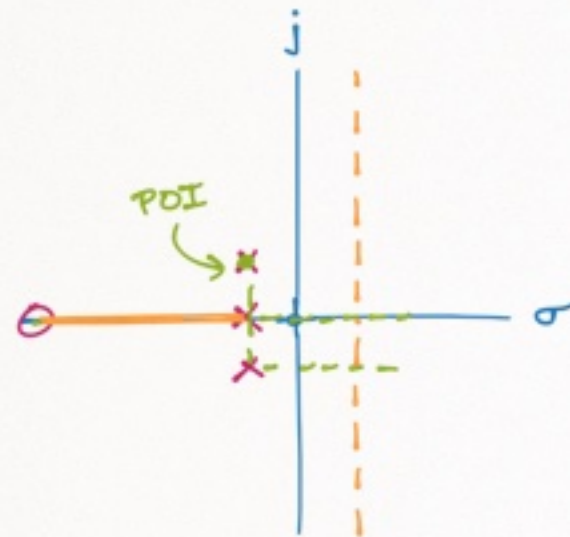
$$\sigma_A = \frac{\sum P - \sum Z}{\angle S} = \frac{-1 + (-1-j) + (-1+j) - (-5)}{2}$$

$\hookrightarrow 2$ ASYMPTOTES $\therefore \underline{\pm 90^\circ}$

$$\underline{\sigma_A = 1}$$



for all complex poles & zeros, compute departure / arrival angles



ANGLE CRITERION

$$\tan^{-1} \frac{1}{4} \quad \Theta = +\tan^{-1} \frac{1}{4}$$

DEPARTURE ANGLE OF COMPLEX POLES

$$\sum P - \sum Z = 180$$

$$90 + 90 + \Theta - \tan^{-1} \frac{1}{4} = 180$$

$$\Theta = \tan^{-1} \frac{1}{4} = 14.03^\circ$$

ARRIVAL ANGLE FOR COMPLEX POLES

there aren't any for some reason

compute breakpoints

$$\frac{d}{ds} \frac{1}{G(s)} = 0$$

$$\frac{d}{ds} \frac{s^2 + 2s + 2}{s + 5} = 0$$

$$\frac{d}{ds} (s + 5)^{-1} (s^2 + 2s + 2)$$

$$(s^2 + 2s + 2) \frac{d}{ds} (s + 5)^{-1} + (s + 5)^{-1} \frac{d}{ds} (s^2 + 2s + 2)$$

$$(s^2 + 2s + 2)(-1)(s + 5)^{-2} \cdot 1 + (s + 5)^{-1} (2s + 2)$$

$$\frac{-s^2 - 2s - 2}{(s + 5)^2} + \frac{2s + 2}{s + 5}$$

is system stable for all positive values of K ? If so, compute max K value.

$$\begin{array}{c} s^3 \\ s^2 \\ s^1 \\ s^0 \end{array} \quad \begin{array}{cc} 1 & 4 \\ 3 & 2+k \end{array}$$

~~$$\begin{aligned} -\frac{1}{3}(2+k-12) &> 0 \\ K &> 10 \end{aligned}$$~~

$$\begin{array}{c} s^2 \\ s^1 \\ s^0 \end{array} \quad \begin{array}{cc} 1 & 2+k \\ 2 & 0 \end{array}$$

~~$$\begin{aligned} -\frac{1}{2}(-2(2+k)) &> 0 \\ 2+k &> 0 \\ K &> 2 \end{aligned}$$~~

$$(s^2+2s+2)(1+s)$$

$$s^2+2s+2+s^3+2s^2+2s$$

$$s^3+3s^2+4s+2+K$$

GM

G

$$\frac{G}{1+GM} = \frac{\frac{K(s+5)}{s^2+2s+2}}{1 + \frac{K(s+5)}{(s^2+2s+2)(s+1)}} = \frac{\overbrace{K(s+5)}}{(s+1)(s^2+2s+2) + K(s+5)}$$

$$\cancel{s^2} + \cancel{2s} + 2 + \cancel{s^3} + \cancel{2s^2} + \cancel{2s} + \cancel{Ks} + Ks$$

$$s^3 + 3s^2 + (4+K)s + 2 + Ks$$

$$\begin{array}{cc} 1 & 4+K \\ 3 & 2+Ks \end{array}$$

$$-\frac{1}{3}(2+Ks-12-3K) > 0$$

$$-\frac{1}{3}(-10+2K) > 0$$

$$2K > 10$$

$$\underline{K > 5}$$

compute the frequency when the system is marginally stable

$$k=5$$

$$s^3 + 3s^2 + (4+k)s + 2+5k$$

$$s^3 + 3s^2 + 9s + 27$$

$$(s+3)(s^2+9)$$

$$\underbrace{\hspace{1.5cm}}$$

$$\underline{s \pm 3j}$$

↑ frequency is $3 \frac{\text{rad}}{\text{s}}$

using Ziegler nichols compute PID gains

$$K_p = 0.6 K_u = 0.6(5) = \underline{3}$$

$$K_I = 1.2 \frac{K_u}{T_u} = 1.2 \frac{5}{\frac{2\pi}{3}} = \underline{2.86}$$

$$K_D = 0.075 K_u T_u = 0.075 \cdot \frac{2\pi}{3} \cdot 5 = \underline{0.785}$$

$$T_u = \frac{2\pi}{f} = \frac{2\pi}{3}$$

QUIZ #4

$$KGH = \frac{10^8(s+100)}{(s+10)^2(s^2+1600s+10^6)}$$

write in correct form for drawing bode plot

$$K_{DC} = 10^8 10^2 \frac{1}{10^1 10^1 10^3 10^3}$$

$$= \frac{10^{10}}{10^8} = 10^2$$

$$10^2 \frac{1 + \frac{s}{100}}{\left(\frac{s}{10} + 1\right)^2 \left(\frac{s}{-800,600} + 1\right) \left(\frac{s}{-800,-600} + 1\right)}$$

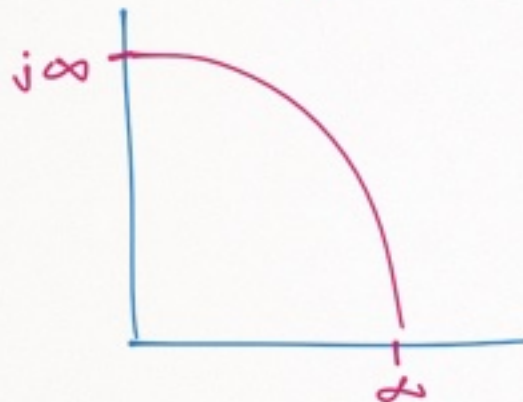
draw associated Nyquist contour in s-domain, transform any points that do not appear on the bode plot into the u+jv domain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1600}{2} \pm \sqrt{\frac{1600^2 - 4(10^6)}{2}}$$

$$= -800 \pm 600$$

no zeros or poles @ $\sigma = \emptyset$



draw the magnitude & frequency bode plots

$$\text{entry point} = 20 \log K_{DC} = 20 \log 100 = 20 \cdot 2 = 40 \text{ dB}$$

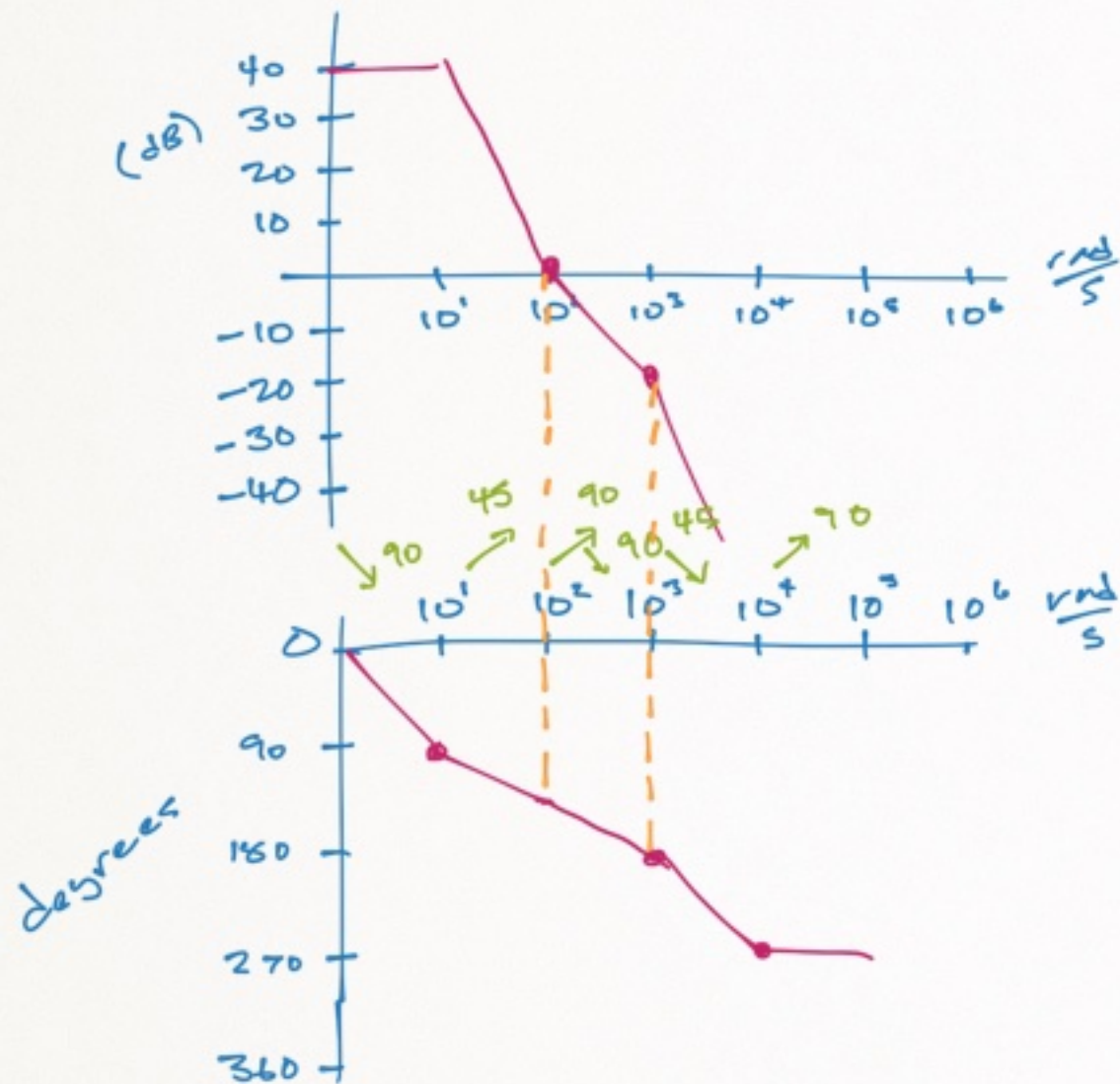
no pole or zero @ \emptyset so its slope coming in is @ $\frac{0 \text{ dB}}{\text{dec}}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

@ 10^1 2 poles so slope = $-40 \frac{\text{dB}}{\text{dec}}$

@ 10^2 1 zero so slope = $-20 \frac{\text{dB}}{\text{dec}}$

@ 10^3 2 pole so slope = $-60 \frac{\text{dB}}{\text{dec}}$



if magnitude entering at $0 \frac{\text{dB}}{\text{dec}}$
phase enters @ 0°

if mag enters @ $20 \frac{\text{dB}}{\text{dec}}$ phase enters @ 90°

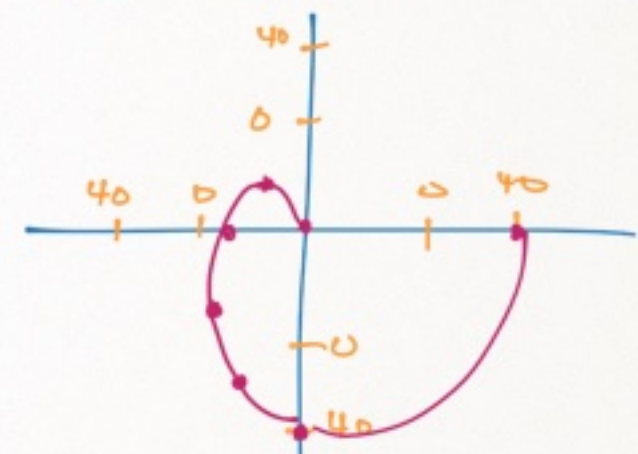
if mag enters @ $40 \frac{\text{dB}}{\text{dec}}$ phase enters @ 180°

determine gain margin, phase margin,
crossover frequency

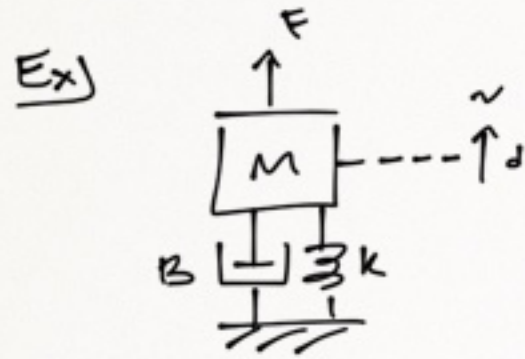
gain margin (gain @ $180^\circ + 180^\circ$)

phase margin (\emptyset - phase @ 0 dB)

draw nyquist



STATE SPACE REVIEW

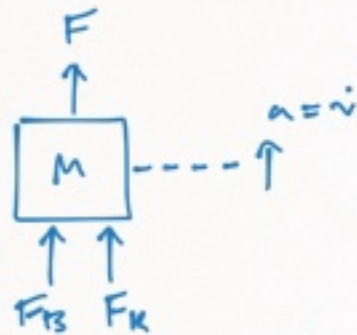


define states

independent values that define energy in the system

states: $\tilde{v} \neq d$

do free body diagram



$$F_B + F_K + F = m \dot{v}$$

$$B\tilde{v} + Kd + F = m \dot{v}$$

always choose forces in the same direction

these are (-)ive why?

put in the form $\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u}$

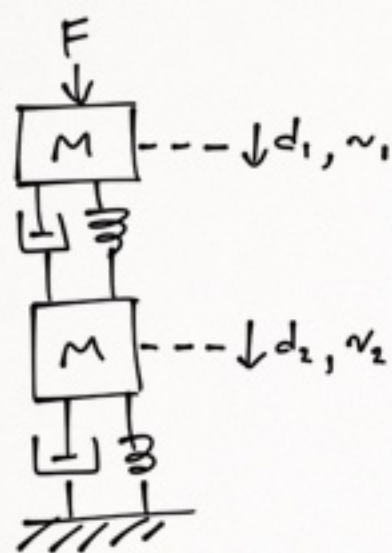
$$\dot{\bar{x}} = \begin{bmatrix} \dot{d} \\ \dot{v} \end{bmatrix} \quad \bar{x} = \begin{bmatrix} d \\ v \end{bmatrix} \quad \bar{u} = \begin{bmatrix} F \end{bmatrix}$$

$$\dot{d} = A_{11}d + A_{12}\tilde{v} + B_1F \longrightarrow \dot{d} = \tilde{v} \therefore A_{11} = B_1 = \emptyset, A_{12} = 1$$

$$\dot{v} = A_{21}d + A_{22}\tilde{v} + B_2F \longrightarrow \dot{v} = \frac{B\tilde{v}}{m} + \frac{Kd}{m} + \frac{F}{m}$$

$$\begin{bmatrix} \dot{d} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \emptyset & 1 \\ \frac{K}{m} & \frac{B}{m} \end{bmatrix} \begin{bmatrix} d \\ v \end{bmatrix} + \begin{bmatrix} \emptyset \\ \frac{1}{m} \end{bmatrix} F$$

Ex)



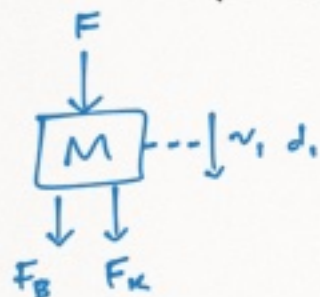
define states

$$v_1, v_2, d_1, d_2$$

define vectors

$$\bar{x} = \begin{bmatrix} d_1 \\ d_2 \\ v_1 \\ v_2 \end{bmatrix} \quad \dot{\bar{x}} = \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} \quad \bar{u} = \begin{bmatrix} F \end{bmatrix}$$

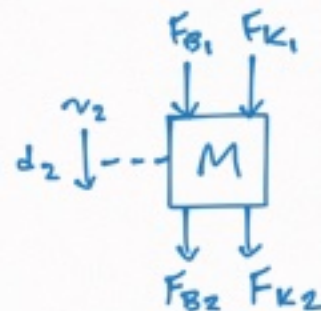
do free body diagrams



$$F_B + F_K + F = m\dot{v}_1$$

$$F_B = B(v_2 - v_1)$$

$$F_K = K(d_2 - d_1)$$



$$F_{B1} + F_{K1} + F_{B2} + F_{K2} = m\dot{v}_2$$

$$F_{B1} = B(v_1 - v_2)$$

$$F_{B2} = -Bv_2$$

$$F_{K1} = K(d_1 - d_2)$$

$$F_{K2} = -Kd_2$$

$v_2 > v_1$ WHY?

define matrices

$$\dot{d}_1 = v_1 \quad \dot{v}_1 = \frac{Bv_2}{m} + \frac{-Bv_1}{m} + \frac{Kd_2}{m} + \frac{-Kd_1}{m} + \frac{F}{m}$$

$$\dot{v}_2 = \frac{Bv_1}{m} + \frac{-Bv_2}{m} + \frac{-Bv_2}{m} + \frac{Kd_1}{m} + \frac{-Kd_2}{m} + \frac{-Kd_2}{m}$$

$$\begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{K}{m} & \frac{K}{m} & \frac{-B}{m} & \frac{B}{m} \\ \frac{K}{m} & \frac{-2K}{m} & \frac{B}{m} & \frac{-2B}{m} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/m \\ 0 \end{bmatrix} F$$

still don't get signs

define output equation

$$\bar{y} = \bar{C}\bar{x} + \bar{D}\bar{u}, \quad \text{outputs } v_1, v_2, d_1, d_2 : \Delta v, \Delta d$$

$$\bar{y} = \begin{bmatrix} \Delta d \\ \Delta v \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \bar{F}$$

transform into s-domain using laplace

$$\bar{\dot{x}} = \bar{A}\bar{x} + \bar{B}\bar{u} \xrightarrow{\mathcal{L}} s\bar{x} - \bar{x}_0 = \bar{A}\bar{x} + \bar{B}\bar{u}$$

$$s\bar{x} - \bar{A}\bar{x} = \bar{x}_0 + \bar{B}\bar{u}$$

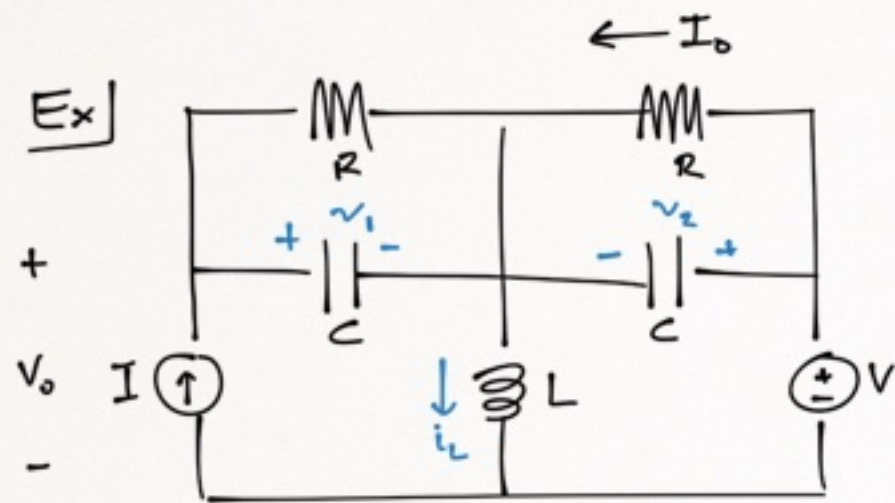
$$\bar{x}(s\bar{I} - \bar{A}) = \bar{x}_0 + \bar{B}\bar{u}$$

$$\underbrace{\bar{x}(s\bar{I} - \bar{A})}_{= \bar{\phi}}$$

$$\bar{x} = \underbrace{\bar{\phi}\bar{x}_0}_{\text{response to initial conditions}} + \underbrace{\bar{\phi}\bar{B}\bar{u}}_{\text{response to forcing function}}$$

response to initial conditions

response to forcing function



define state vectors

$$x = \begin{bmatrix} v_1 \\ v_2 \\ i_L \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{i}_L \end{bmatrix} \quad u = \begin{bmatrix} V \\ I \end{bmatrix}$$

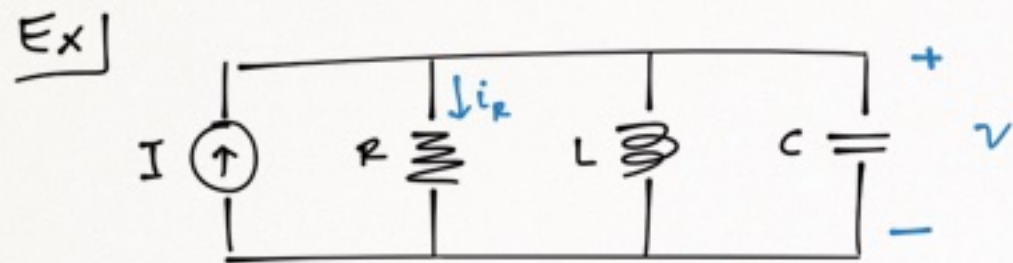
define matrices \mathbf{I}

$$i_L = C\dot{v}_1 + \frac{1}{R}v_1 + C\dot{v}_2 + \frac{1}{R}v_2 \longrightarrow \dot{v}_2 = \frac{i_L}{C} + \frac{-I}{C} + \frac{-1}{RC}v_2$$

$$I = C\dot{v}_1 + \frac{1}{R}v_1 \longrightarrow \dot{v}_1 = \frac{I}{C} + \frac{-1}{RC}v_1$$

$$V - v_2 - Li_L' = 0 \longrightarrow i_L' = \frac{V}{L} - \frac{v_2}{L}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{i}_L' \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 0 & 0 \\ 0 & -\frac{1}{RC} & \frac{1}{C} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{C} \\ 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}$$



state vectors

output: i_R

input: I

$$x = \begin{bmatrix} v_c \\ i_L \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} \quad \begin{aligned} I &= i_R + i_L + i_C \\ I &= \frac{v}{R} + i_L + C \dot{v} \end{aligned} \quad \begin{aligned} v_L &= i_L sL \\ \rightarrow s i_L &= \dot{i}_L = \frac{v}{L} \end{aligned} \quad u = |I|$$

state matrices

$$\rightarrow s v = \dot{v} = -\frac{v}{RC} + \frac{I}{C} + \frac{-i_L}{C}$$

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & \emptyset \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \emptyset \end{bmatrix} |I|$$

output equation

$$\bar{y} = \bar{C} \bar{x} + \bar{D} u$$

$$|i_R| = \left| \frac{1}{R} \ \emptyset \right| \begin{bmatrix} v_c \\ i_L \end{bmatrix} + |\emptyset| I$$

ϕ , **WHAT DOES THIS EVEN MEAN?**

$$\begin{aligned} \phi &= [sI - A]^{-1} \\ &= \left(\begin{bmatrix} s & \emptyset \\ \emptyset & s \end{bmatrix} - \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & \emptyset \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} s + \frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & s \end{bmatrix}^{-1} \end{aligned} \quad \phi = \frac{1}{s^2 + \frac{s}{RC} - \frac{1}{LC}} \begin{bmatrix} s & -\frac{1}{C} \\ \frac{1}{L} & s + \frac{1}{RC} \end{bmatrix}$$

get transfer function

$$T(s) = \frac{Y}{U} = \bar{C} \bar{\phi} \bar{B} + \bar{D}$$

$$\frac{Y}{U} = \begin{bmatrix} \frac{1}{R} & \emptyset \end{bmatrix} \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} s & -\frac{1}{C} \\ \frac{1}{L} & s + \frac{1}{RC} \end{bmatrix} \begin{bmatrix} \frac{1}{C} \\ \emptyset \end{bmatrix} + [\emptyset]$$

$$= \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} \frac{s}{R} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \frac{1}{C} \\ \emptyset \end{bmatrix}$$

$$= \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \quad \leftarrow \text{CHARACTERISTIC EQUATION}$$

get roots of CE

$$\det(\bar{A}) = \det(\lambda \bar{I} - \bar{A}) = \emptyset$$

$$\text{CE: } s^2 + \frac{1}{RC}s + \frac{1}{LC} = \emptyset$$

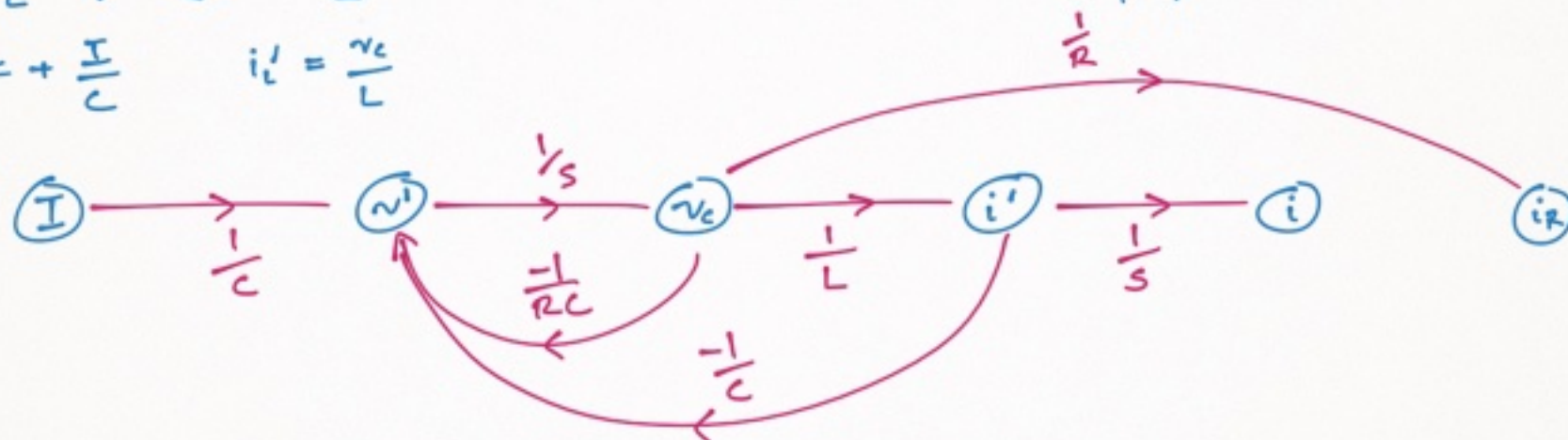
$$s = \frac{-\frac{1}{RC} \pm \sqrt{\frac{1}{R^2 C^2} - \frac{4}{LC}}}{2}$$

draw signal flow diagram

$$\begin{bmatrix} v_c' \\ i_L' \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & \emptyset \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \emptyset \end{bmatrix} I \quad \text{inputs: } I$$

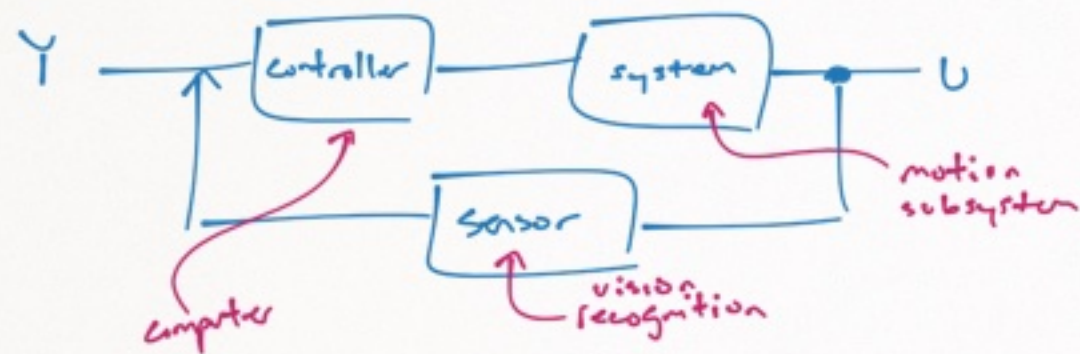
$$i_R = \left| \frac{1}{R} \emptyset \right| \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \emptyset I = \frac{v_c}{R}$$

$$v_c' = -\frac{v_c}{RC} - \frac{i_L}{C} + \frac{I}{C} \quad i_L' = \frac{v_c}{L}$$

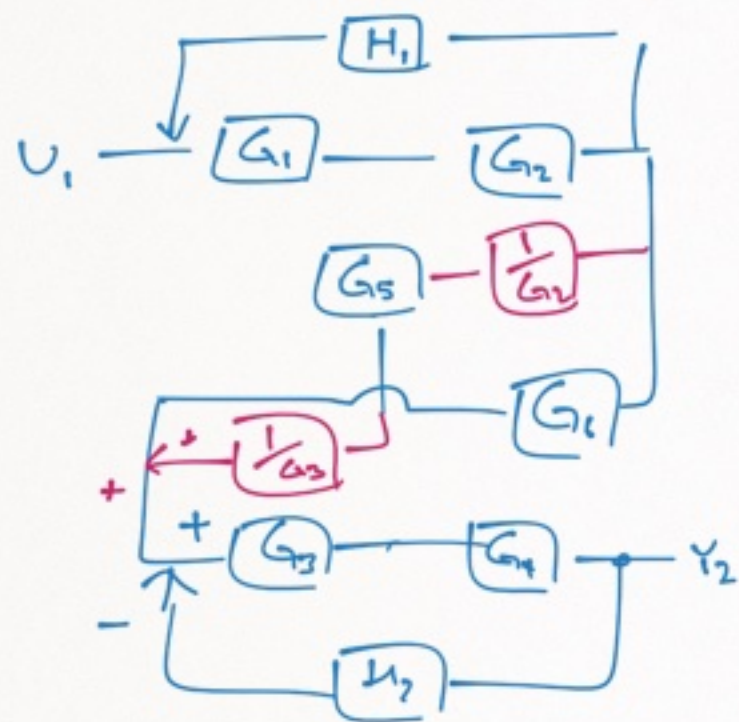


QUIZ #1

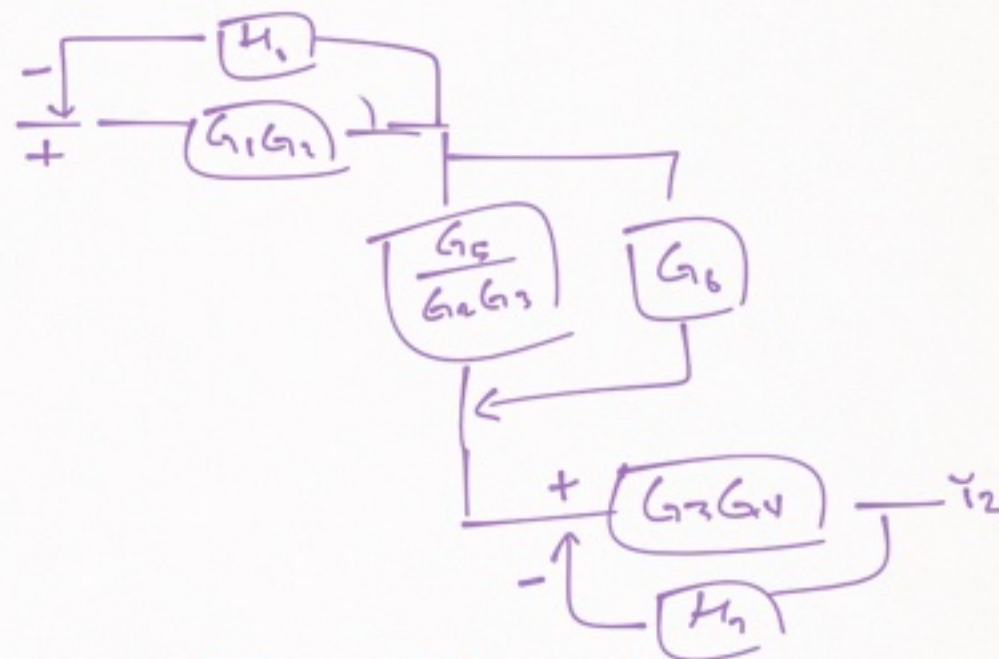
sketch block diagram of control whatever



determine $\frac{Y_2}{U_1}$



superposition $U_2 = \emptyset, Y_1 = \emptyset$



$$\frac{Y_2}{U_1} = \left(\frac{G_1 G_2}{1 + H_1 G_1 G_2} \right) \left(\frac{G_5}{G_2 G_3} + \frac{G_6}{G_2 G_3} \right) \left(\frac{G_3 G_4}{1 + H_2 G_3 G_4} \right)$$

$$\underline{G_5 + G_6 G_2 G_3} \quad G_5 = -G_6 G_2 G_3$$

Quiz #3

$$\frac{\pi}{\omega_n \beta} = \frac{4}{2 \omega_n} = \frac{\pi}{\sqrt{1-\zeta^2} \omega_n}$$

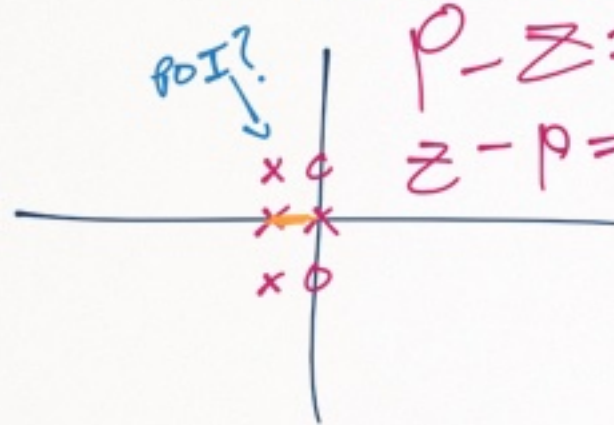
root locus of $\frac{k(s^2+1)}{s(s+1)(s^2+2s+2)} = KGH$

zeros @ $\pm j$

poles @ $-1, -1+j, -1-j, 0$

$$\frac{-2 \pm \sqrt{4-4 \cdot 2}}{2}$$

$$\frac{-1 \pm \sqrt{-4}}{2} = -1 \pm j$$



$$p-z=150$$

$$z-p=150$$

$$\#P - \#Z = \#S = 4 - 2 = 2$$

$$\frac{-1-1-1}{\#S} = \frac{-3}{2} = -\frac{3}{2}$$

$$\phi_A = \pm 90^\circ$$

$$\sigma_A = -\frac{3}{2}$$

$$(s^2+1)(s^2+2s+2) + k(s^2+1)$$

$$(s^4+2s^3+2s^2+s^2+2s^2+2s) + k(s^2+1)$$

$$(s^4+3s^2+4s^2+2s) + k(s^2+1)$$

$$\sum P_\theta - \sum Z_\theta = \pm 180$$

$$90 + 90 + 45 + \theta - 180 - 90 - 27 = 180$$

