Super Phasors

Monday, February 22, 2016 3:31 PM

WHAT IS THE VOLTAGE ACROSS RESISTOR

$v(t) = L\frac{di}{dt} = 0.01\frac{d}{dt}\,5\cos(2000t)$

$= 0.01(-10000\sin(2000t))$

$= -100\sin(2000t)$

$= 100\sin\left(2000t + \frac{180}{180}\pi\right)$

$= 100\cos\left(2000t + \frac{180\pi}{180} - \frac{90}{180}\pi\right)$

$= 100\cos\left(2000t + \frac{90}{180}\pi\right)$

$i(t) = 5\cos 2000t$ → 0.01H, + $v(t)$ −

to write voltage with positive amplitude: $-\sin(\alpha) = \sin\left(\alpha + \frac{180\pi}{180}\right)$

to write voltage WRT cosine as given: $\sin(\alpha) = \cos\left(\alpha - \frac{90\pi}{180}\right)$

SAME EXERCISE, NOW USING PHASORS

$Z_L = j\omega L \rightarrow Z_L = j\left(2000\frac{rad}{s}\right)0.01H = j20\Omega$

$\bar{I} = \left(\frac{5}{\sqrt{2}}\angle 0°\right)$

$\bar{V} = \bar{I}Z_L = \left(\frac{100}{\sqrt{2}}\angle 90°\right)$

$\hookrightarrow v(t) = 100\cos\left(2000t + \frac{90\pi}{180}\right)$

Ex) TWO PURE ELEMENTS ARE CONNECTED IN SERIES WITH THE FOLLOWING V & I, WHAT ARE THEY?

$v(t) = 255\sin\left(300t + \frac{45}{180}\pi\right)$ → $\bar{V} = \frac{255}{\sqrt{2}}\angle 45°$

$i(t) = 8.5\sin\left(300t + \frac{15}{180}\pi\right)$ → $\bar{I} = \frac{8.5}{\sqrt{2}}\angle 15°$

$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{\frac{255}{\sqrt{2}}\angle 45}{\frac{8.5}{\sqrt{2}}\angle 15} = 30\angle 30\Omega = 25.98 + j15$

$R = 25.98\Omega$

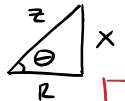
$\omega L = 15$

$L = \frac{15}{30} = 50mH$

ANGLES

$\bar{Z} = R + jX = Z\angle\theta$

$\theta = \text{atan}\frac{X}{R}$



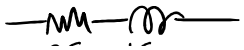
the angle between the voltage & current is the same as the angle b/w the resistance and the reactance

$\bar{V} = V\angle\alpha \quad \bar{I} = I\angle\beta$

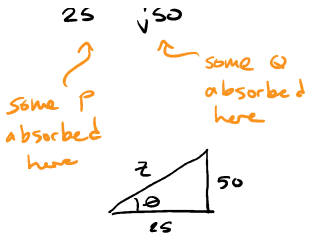
$Z\angle\theta = \frac{\bar{V}}{\bar{I}} = \frac{V}{I}\angle(\alpha - \beta)$

$\theta = \alpha - \beta$ (phase V − phase I)

RL BRANCH



$\theta = \text{atan}\frac{50}{25} = 63.435°$

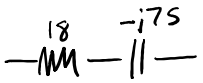


$\rightarrow \theta = \angle V - \angle I$

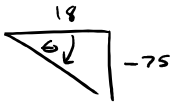
$\rightarrow \angle V = \angle I + 63.435°$

↖ the current lags by 63.435°

RC BRANCH



$\theta = \text{atan}\frac{-75}{18} = -76.504$

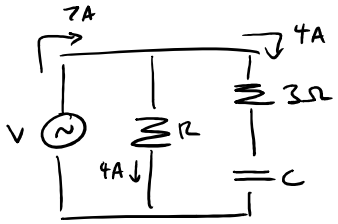


$\rightarrow \theta = \angle V - \angle I = -76.504°$

$\rightarrow \angle I = \angle V + 76.504°$

current leads by 76.5°

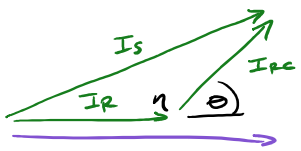
Ex) IN THE CIRCUIT BELOW, THE CURRENT HAVE BEEN MEASURED WITH AN AC AMMETER?



DETERMINE R, C, & V_{SOURCE}

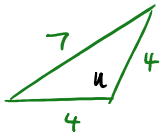
$f = 60Hz$

1. Choose any voltage or current as base reference,
2. we'll choose V to be 0°
3. the current through the R branch, I_R, will be in-phase with voltage
4. the current through RC must be slightly leading



cosine law

$c^2 = a^2 + b^2 - 2ab\cos(\eta)$



$\eta = 122.0955°$

$\hookrightarrow \theta = 180 - \eta = 57.9045°$

$\theta = \text{atan}\frac{X}{R}$ (angle actually (-)ive b/c capacitive)

$\hookrightarrow X_C = R\tan\theta = 3\tan 57.904 = 4.784\Omega$

$X = \frac{1}{\omega C} \rightarrow C = \frac{1}{2\pi 60 (4.784)} = \boxed{554\mu F}$

$V = IZ \rightarrow V_{RMS} = I_{RMS} Z_{RC} = 4A\left(\sqrt{3^2 + 4.784^2}\right) = \boxed{22.59V}$

$Z = \sqrt{R^2 + X^2}$

$R = \frac{V_{RMS}}{I_{RMS}} = \frac{22.59V}{4A} = \boxed{5.647\Omega}$

POWER

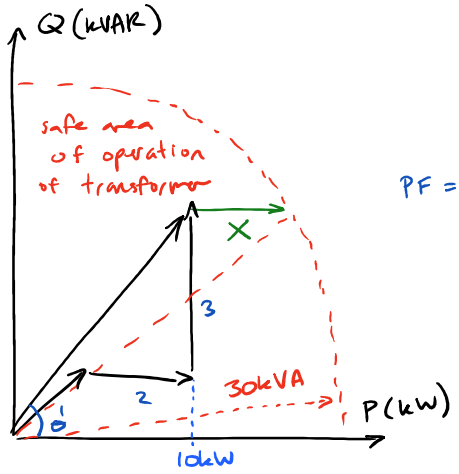
POWER

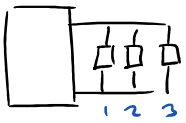
↗ inductive load

→ resistive load

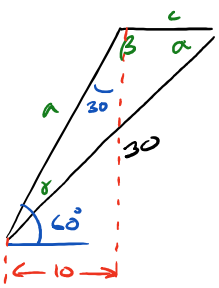
↘ capacitive load

Ex) transformer rated for 30kVA, it feeds a load of 10kW @ PF = 0.5 lagging, how much resistive load can you add to bring to full rated power?



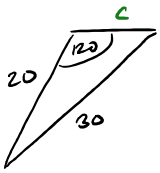


PF = 0.5 → θ = 60



$a = \frac{10}{\cos(60)} = 20$ $\beta = (180 - 90 - \theta) + 90 = 120$

from cosine law



c = 14.5kW