# Diffusion Model

Computer Vision - Project

Clément, Grégoire, Nathan

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# First generation: Denoising Diffusion Probabilistic Models

# General Idea

Consider the set of hand-written digits $D$. Can you give a probability distribution $q$ such that $x \sim q(x)$ ?

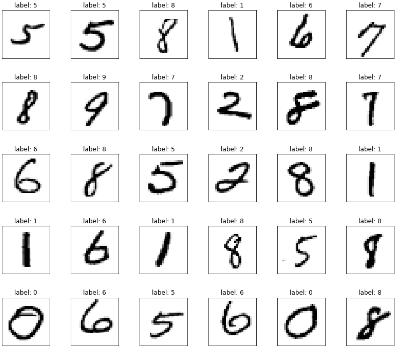


**Figure 1:** Source: ludwig.ai

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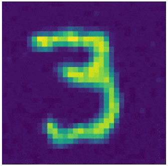

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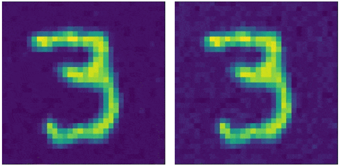

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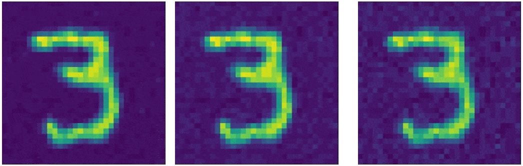

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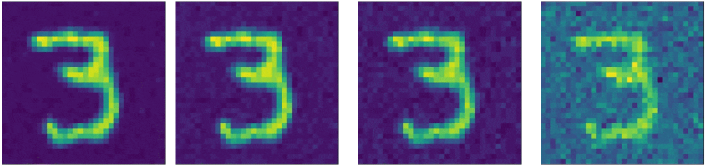

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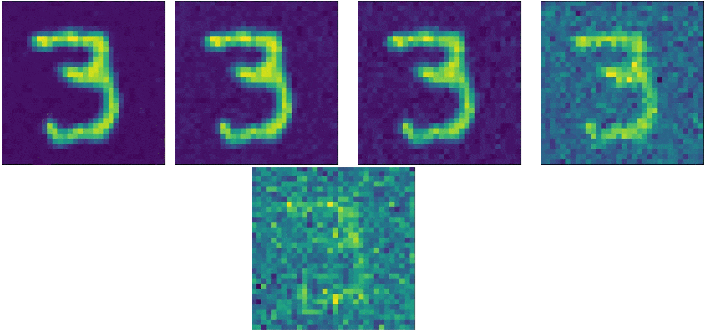

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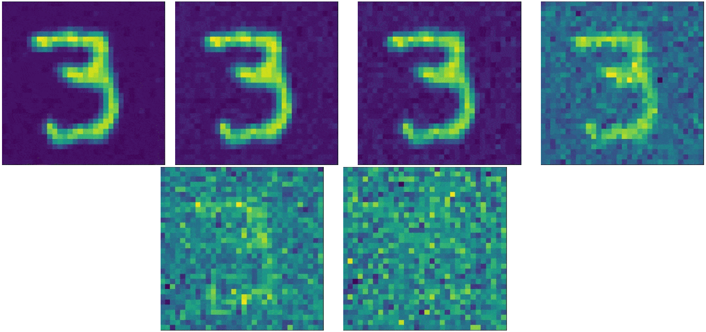

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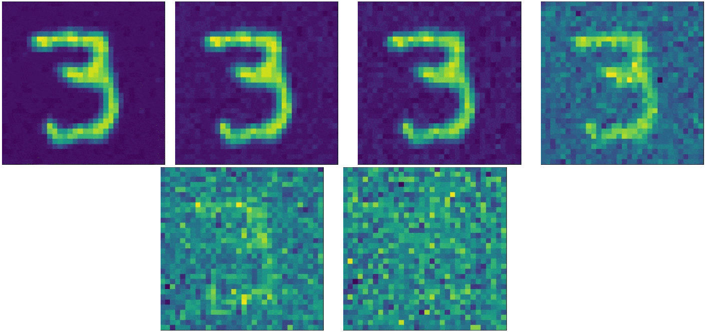
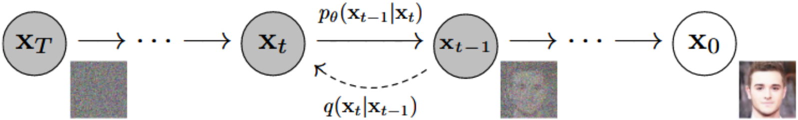


Formally: $q(x_{t+1} \mid x_t) := \mathcal{N}(x_{t+1}; \sqrt{1-\beta_t} x_t, \beta_t I)$ for some schedule $(\beta_t)_t$. Can we learn to reverse this process ?

# What we want to learn

Given a noisy image $x_t$, we train a model to predict $x_{t-1}$.



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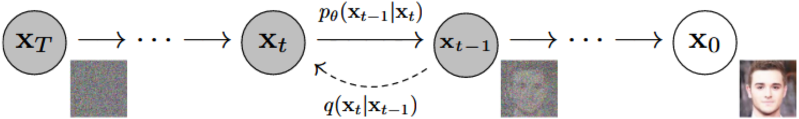


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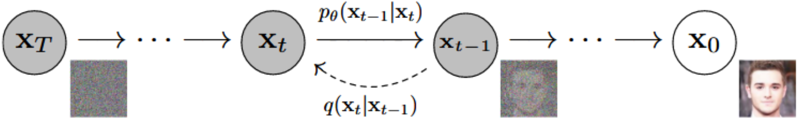


- Given a data image $x_0$, we sample $(x_t)_{1:T}$ according to $q(x_{1:T} \mid x_0) := \prod_{t=1}^{T} q(x_t \mid x_{t-1})$,
- Given a noisy image $x_t$ and $t$, we sample according to $p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t))$.

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We have $x_t = \sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon$.

# Training

For now, our model is learning $\mu$ and $\Sigma$, i.e. we sample according to

$$p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t))$$

They've found that fixing $\Sigma_\theta$ to a constant gives the same result. So,

$$p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \sigma_t I)$$

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$$E_q \left[ \frac{1}{2\sigma_t^2} \|\tilde{\mu}_t(x_t, x_0) - \mu_\theta(x_t, t)\|^2 \right]$$

where $\tilde{\mu}$ is the optimal mean that depends on $x_0$ which we don't know.

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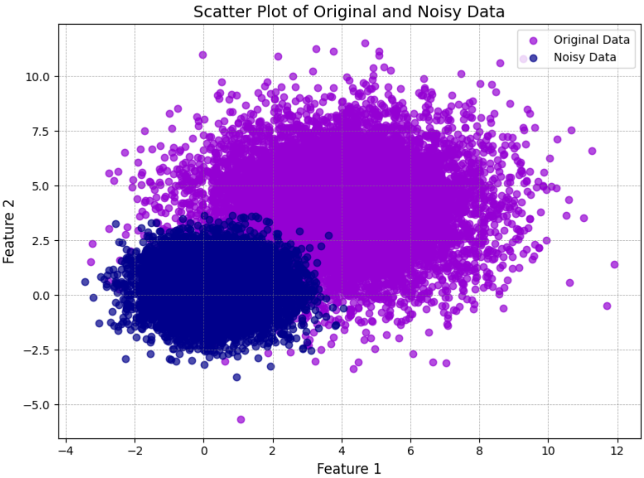
where $\tilde{\mu}$ is the optimal mean that depends on $x_0$. Using $x_t(x_0, \epsilon) = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$ we have a loss we can train on.

# Our results - Gaussian

We have started with Gaussian generation:



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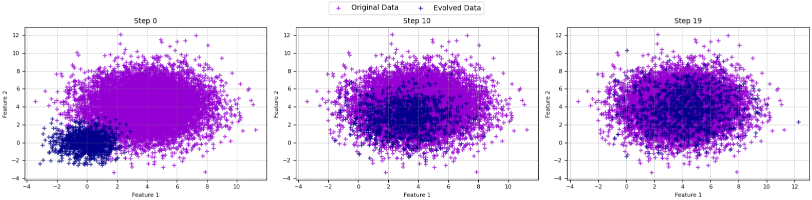
We have started with Gaussian generation and got satisfying results:



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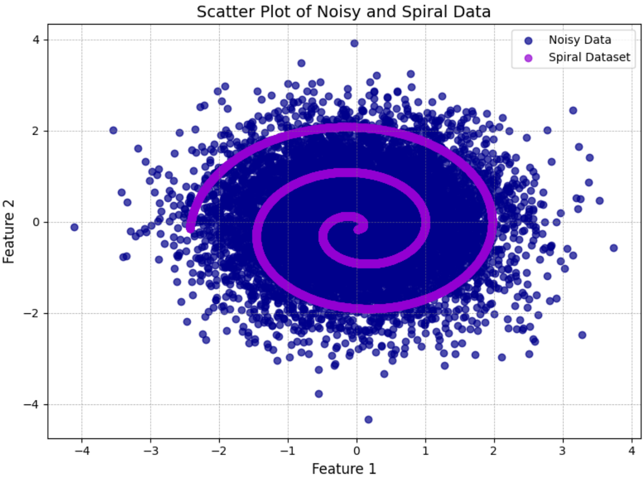

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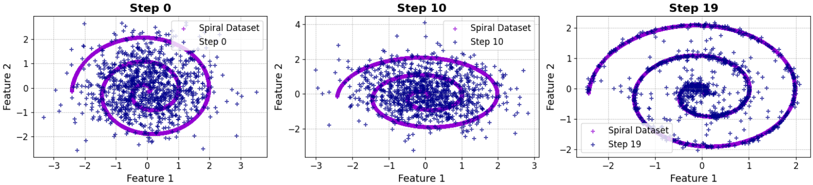

# Second generation

# OpenAI's incrementation

**Improved Denoising Diffusion Probabilistic Models**

**Alex Nichol** [*1] **Prafulla Dhariwal** [*1]

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Recall that:

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This paper tackle these problems.

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Ho et al. fixed $\Sigma_\theta(x_t, t) = \sigma_t^2 I$ with:

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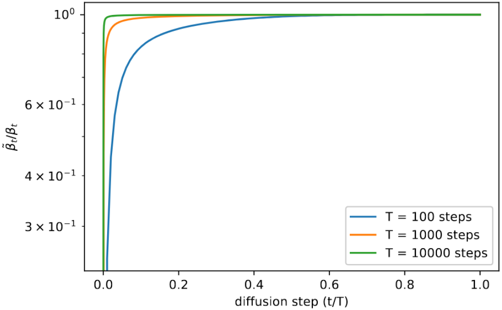


*Figure 1.* The ratio $\tilde{\beta}_t/\beta_t$ for every diffusion step for diffusion processes of different lengths.

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Ho et al. have found that the impact is negligeable. But it depends of other hyperparameters.

Hence, they interpolate between the two extreme values, and let the model learn $v(t)$:

$$\Sigma_\theta(x_t, t) := \exp(v_\theta(t)\log(\beta_t) + (1 - v_\theta(t))\log(\bar{\beta}_t))$$

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This loss is prone to gradient exploding and we need importance sampling to implement it.

# Our results

TODO

# Classifier Guidance

# Importance of labels

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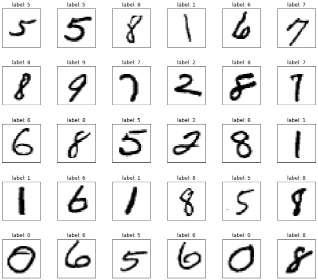


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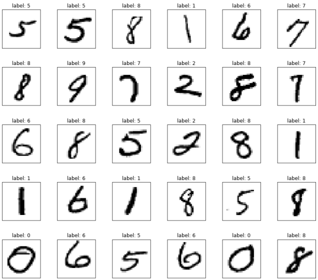


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A DDPM can generate new images that look like digits, but the model can't distinguish a mix of two digits and a real digit.

# Importance of labels

Let's get back to hand-written digits generation:

If we have a classifer that gives $p_\phi(y \mid x_t)$, we can sample using

$$p_{\theta,\phi}(x_t \mid x_{t+1}, y) = Z p_\theta(x_t \mid x_{t+1}) p_\phi(y \mid x_t)$$

This way if we set $y = 3$, we can trick our model to generate somthing that looks like a 3.

# Langevin Dynamics

Given $x_0 \sim \pi(x)$ an unknown distribution, if we iterate through $x_{i+1} \leftarrow x_i + \epsilon \nabla_x \log p(x) + \sqrt{2\epsilon} z_i$ with $\epsilon \to 0$ and $z_i \sim \mathcal{N}(0, I)$, we can sample from $p(x)$.

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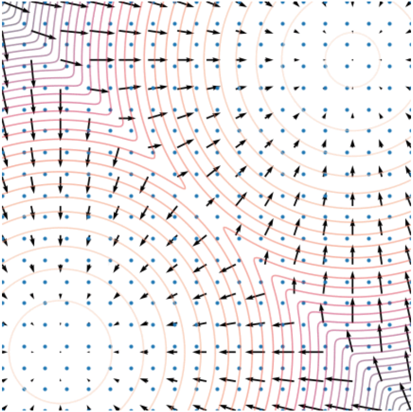

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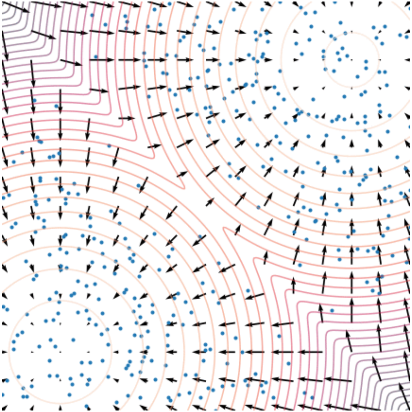

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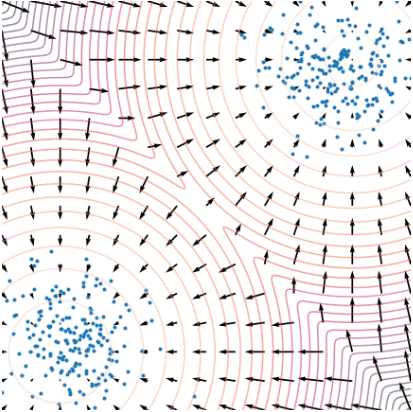

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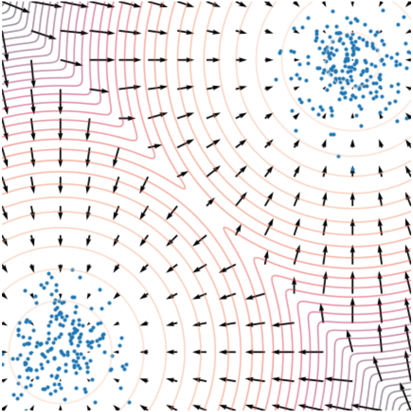

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So we need to know $\nabla_x \log p(x)$, but don't need to know $p(x)$.

# Langevin Dynamics

Given $x_0 \sim \pi(x)$ an unknown distribution, if we iterate through $x_{i+1} \leftarrow x_i + \epsilon \nabla_x \log p(x) + \sqrt{2\epsilon} z_i$ with $\epsilon \to 0$ and $z_i \sim \mathcal{N}(0, I)$, we can sample from $p(x)$.

So we need to know $\nabla_x \log p(x)$, but don't need to know $p(x)$.

From the classifier $p_\phi$, one can get an approximation of $\nabla_x \log p(x)$.

# Low vs High temperature

**Figure 1:** Classifier-free Diffusion Guidance

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# Low vs High temperature

**Figure 1:** Classifier-free Diffusion Guidance

- Low-temperature optimizes FID score
- High-temperature optimizes Inception score

We can do a trade-of by following more $p_\phi$ or $p_\theta$ between exploration and distance to the original distribution.

# GLIDE: draw what you prompt

# Classifier-Free Guidance

Previous guidance need a trained classifier. They define:

- An unconditional DDPM, that predicts $p_\theta(z)$.
- A conditional DPPM, that predicts $p_\theta(z \mid c)$.

# Classifier-Free Guidance

Previous guidance need a trained classifier. They define:

- An unconditional DDPM, that predicts $p_\theta(z)$.
- A conditional DPPM, that predicts $p_\theta(z \mid c)$.

Rather than using $p_\phi$, they train both models simultanously and use the gradient of $p_\theta(z \mid c)$ to estimate $\nabla \log p(z \mid c)$.

# CLIP

To force a label $c$, we train a model that estimates the probability for $x$ to be of class $c$, and uses its gradient.

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# CLIP

To force a sentence $c$, we train a model that estimates the distance between $x$ and the sentence $c$, and uses its gradient.

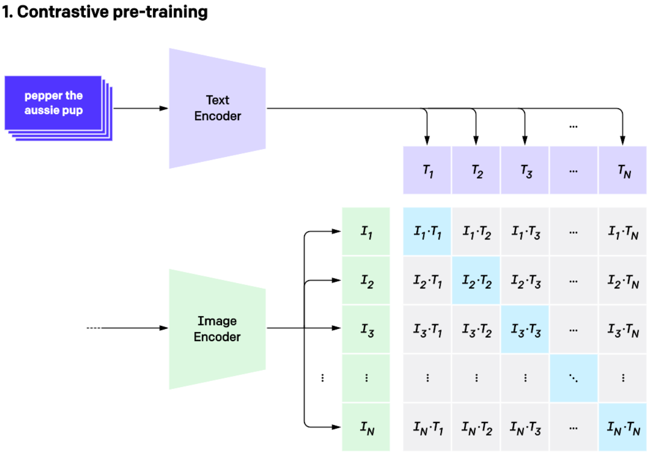
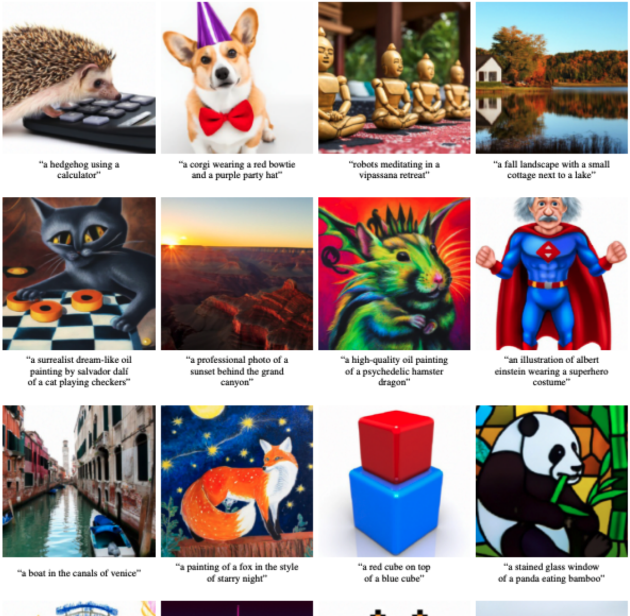



**Figure 2:** How to compare an image and a text

# Results (OpenAI GLIDE)



# State-of-the-art brief review

# ControlNet

GLIDE accepts one more input than text: a mask for inpainting.

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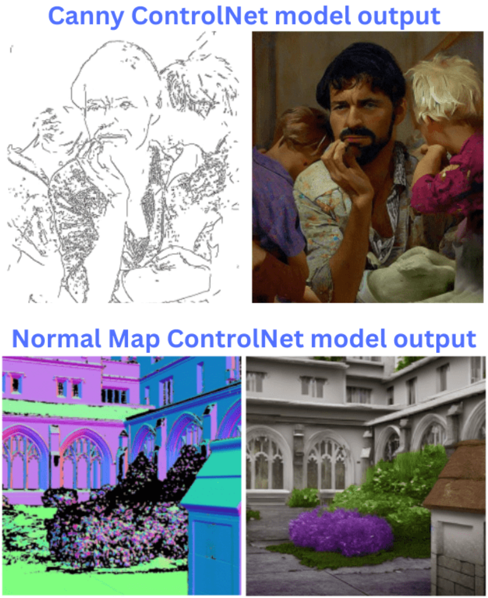
GLIDE accepts one more input than text: a mask for inpainting.

ControlNet generalises it by adding more optional inputs (e.g. Cany edges representation / Human Pose / Sketch).

# ControlNet


Canny ControlNet model output
Normal Map ControlNet model output