Diffusion Model

Computer Vision - Project

Clément, Grégoire, Nathan

January 9, 2025

First generation: Denoising Diffusion Probabilistic Models

General Idea

Consider the set of hand-written digits D. Can you give a probability distribution q such that $x \sim q(x)$?

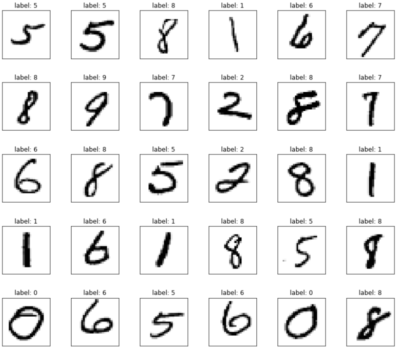

Figure 1: Source: ludwig.ai

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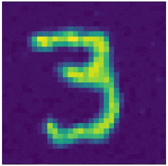

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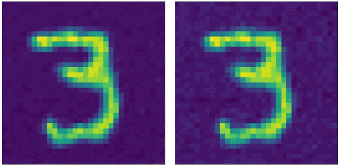

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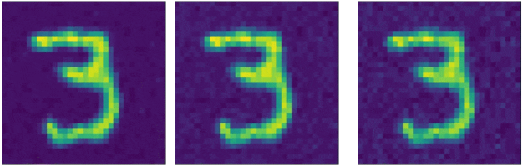

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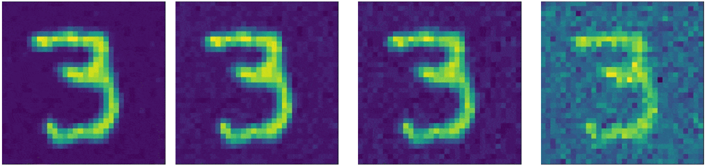

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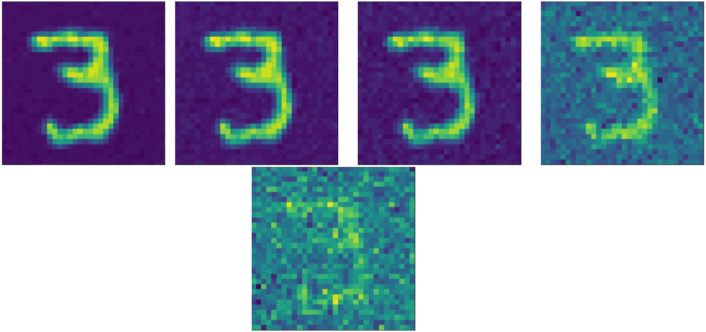

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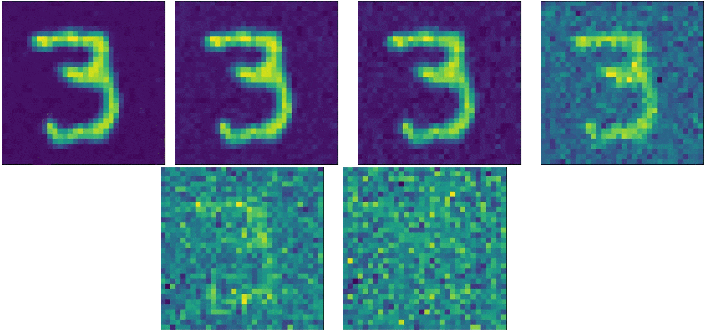

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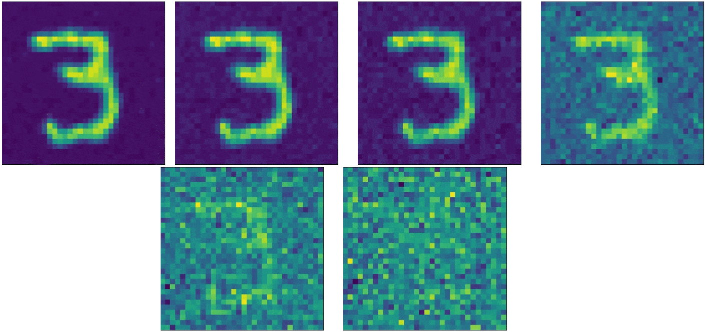
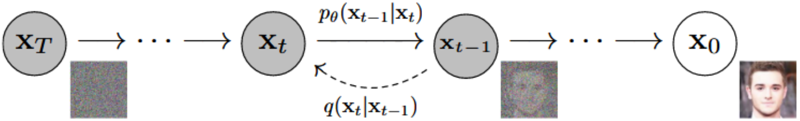


Formally: $q(x_{t+1} \mid x_t) := \mathcal{N}(x_{t+1}; \sqrt{1-\beta_t}x_t, \beta_t I)$ for some schedule $(\beta_t)_t$. Can we learn to reverse this process ?

What we want to learn

Given a noisy image x_t, we train a model to predict x_{t-1}.



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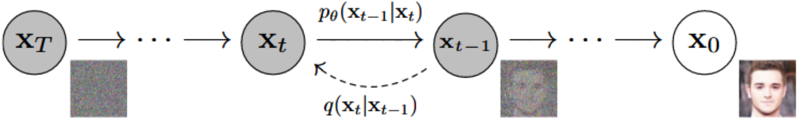

- Given a data image x_0, we sample $(x_t)_{1:T}$ according to $q(x_{1:T} \mid x_0) := \prod_{t=1}^{T} q(x_t \mid x_{t-1})$,

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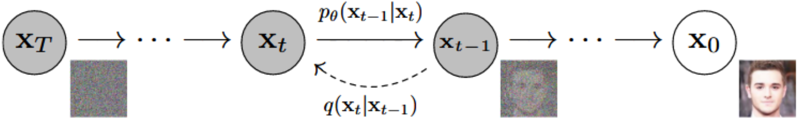

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- Given a noisy image x_t and t, we sample according to $p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t))$.

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Given a data image x_0, compute x_t takes t sampling on q. But a simple trick, allows to do only one.

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Remember that $q(x_{t+1} \mid x_t) := \mathcal{N}(x_{t+1}; \sqrt{1-\beta_t} x_t, \beta_t I)$. Let $\alpha_t = 1 - \beta_t$ and $\bar{\alpha}_t = \prod_{i=1}^{t} \alpha_i$.

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$$x_t = \sqrt{\alpha_t} x_{t-1} + \sqrt{1-\alpha_t}\epsilon_{t-1}$$

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We have $x_t = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon$.

Training

For now, our model is learning μ and Σ, i.e. we sample according to

$$p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t))$$

They've found that fixing Σ_θ to a constant gives the same result. So,

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$$E_q \left[\frac{1}{2\sigma_t^2} \|\tilde{\mu}_t(x_t, x_0) - \mu_\theta(x_t, t)\|^2 \right]$$

where $\tilde{\mu}$ is the optimal mean that depends on x_0 which we don't know.

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where $\tilde{\mu}$ is the optimal mean that depends on x_0. Using $x_t(x_0, \epsilon) = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t}\epsilon$ we have a loss we can train on.

Our results

TODO.

Second generation

OpenAI's incrementation

Improved Denoising Diffusion Probabilistic Models

Alex Nichol [*1] **Prafulla Dhariwal** [*1]

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- Σ_θ was set to a constant,
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This paper tackle these problems.

How to modelize Σ_θ

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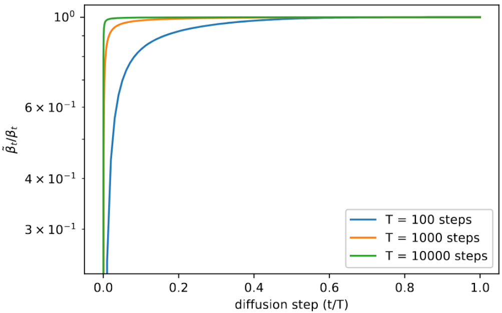

Figure 1. The ratio $\tilde{\beta}_t/\beta_t$ for every diffusion step for diffusion processes of different lengths.

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Ho et al. have found that the impact is negligeable. But it depends of other hyperparameters.

Hence, they interpolate between the two extreme values, and let the model learn $v(t)$:

$$\Sigma_\theta(x_t, t) := \exp(v_\theta(t) \log(\beta_t) + (1 - v_\theta(t)) \log(\bar{\beta}_t))$$

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This loss is prone to gradient exploding and we need importance sampling to implement it.

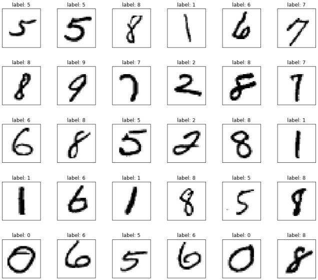
Our results

TODO

Classifier Guidance

Importance of labels

Let's get back to hand-written digits generation:



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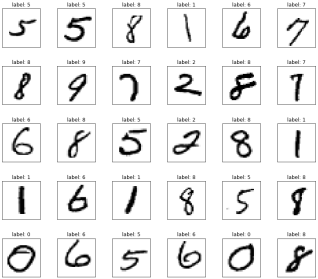

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A DDPM can generate new images that look like digits, but the model can't distinguish a mix of two digits and a real digit.

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Let's get back to hand-written digits generation:

If we have a classifer that gives $p_\phi(y \mid x_t)$, we can sample using

$$p_{\theta,\phi}(x_t \mid x_{t+1}, y) = Z p_\theta(x_t \mid x_{t+1}) p_\phi(y \mid x_t)$$

This way if we set $y = 3$, we can trick our model to generate somthing that looks like a 3.

Langevin Dynamics

Given $x_0 \sim \pi(x)$ an unknown distribution, if we iterate through $x_{i+1} \leftarrow x_i + \epsilon \nabla_x \log p(x) + \sqrt{2\epsilon} z_i$ with $\epsilon \to 0$ and $z_i \sim \mathcal{N}(0, I)$, we can sample from $p(x)$.

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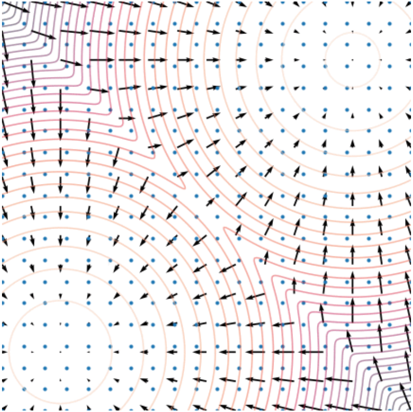

Figure 1: Visualizations from Yang Song's work.

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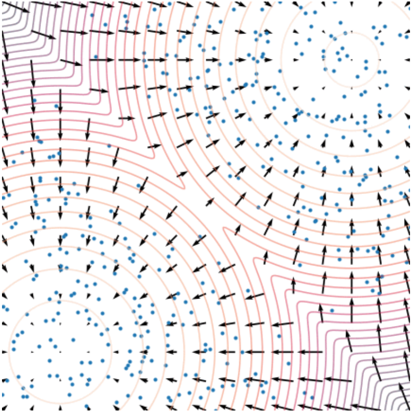

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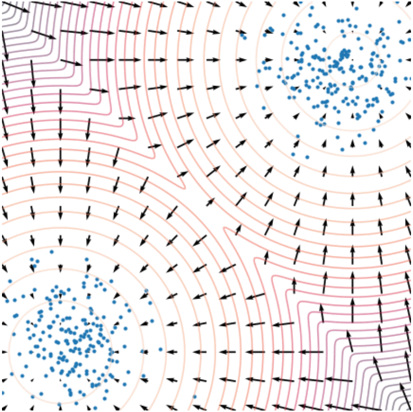

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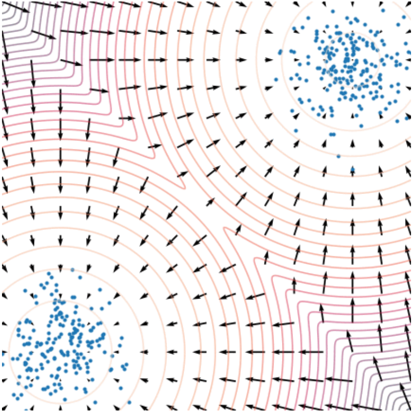

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So we need to know $\nabla_x \log p(x)$, but don't need to know $p(x)$.

From the classifier p_ϕ, one can get an approximation of $\nabla_x \log p(x)$.

Low vs High temperature

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We can do a trade-of by following more p_ϕ or p_θ between exploration and distance to the original distribution.

State-of-the-art: Video generator