Diffusion Model

Computer Vision - Project

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Denoising Diffusion Probabilistic Models

General Idea

Consider the set of hand-written digits D. Can you give a probability distribution q such that $x \sim q(x)$?

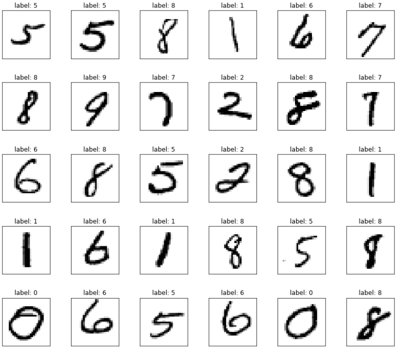

Figure 1: Source: ludwig.ai

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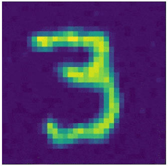
Consider the set of hand-written digits D. It is hard to find q such that $x \sim q(x)$, we need a clever way to sample hand-written digits. Consider the following process:



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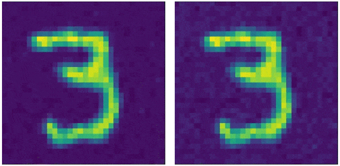

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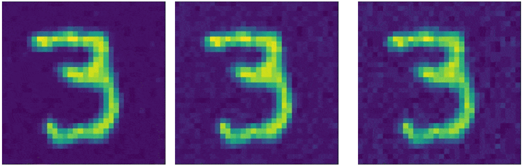

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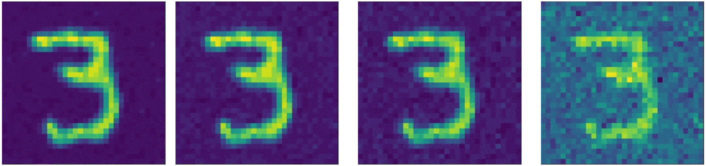

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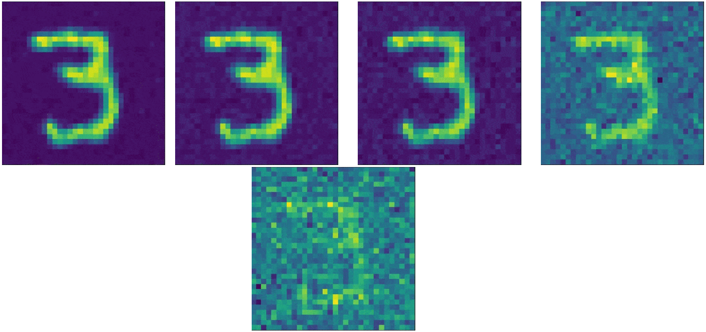

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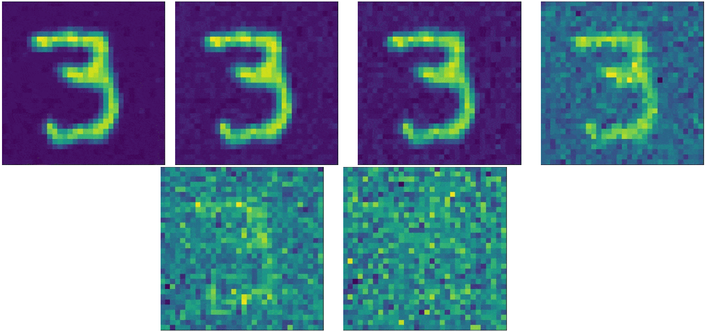

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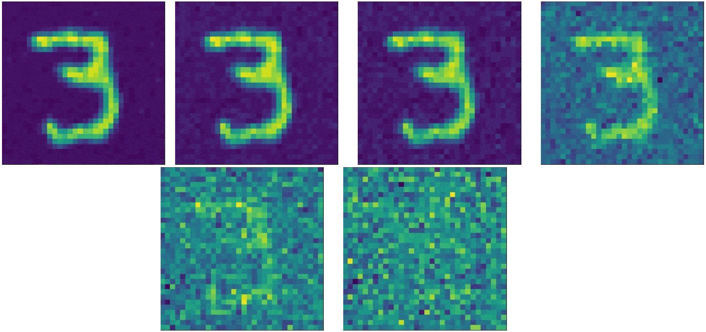
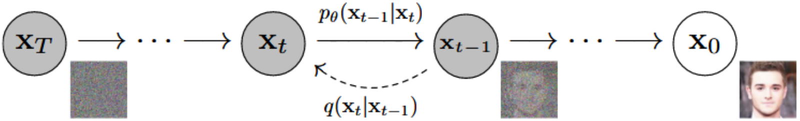


Formally: $q(x_{t+1} \mid x_t) := \mathcal{N}(x_{t+1}; \sqrt{1-\beta_t}x_t, \beta_t I)$ for some schedule $(\beta_t)_t$. Can we learn to reverse this process ?

What we want to learn

Given a noisy image x_t, we train a model to predict x_{t-1}.



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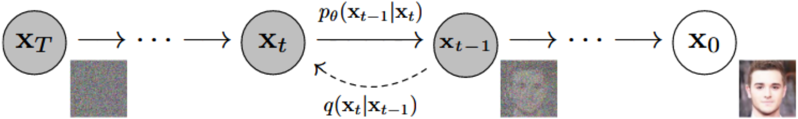

- Given a data image x_0, we sample $(x_t)_{1:T}$ according to $q(x_{1:T} \mid x_0) := \prod_{t=1}^{T} q(x_t \mid x_{t-1})$,

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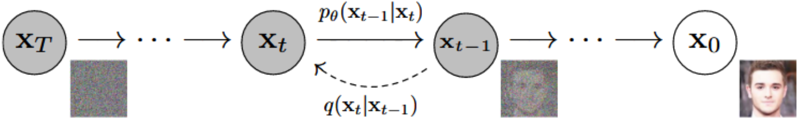

- Given a data image x_0, we sample $(x_t)_{1:T}$ according to $q(x_{1:T} \mid x_0) := \prod_{t=1}^{T} q(x_t \mid x_{t-1})$,
- Given a noisy image x_t and t, we sample according to $p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t))$.

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Given a data image x_0, compute x_t takes t sampling on q. But a simple trick, allows to do only one.

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Remember that $q(x_{t+1} \mid x_t) := \mathcal{N}(x_{t+1}; \sqrt{1-\beta_t} x_t, \beta_t I)$. Let $\alpha_t = 1 - \beta_t$ and $\bar{\alpha}_t = \prod_{i=1}^{t} \alpha_i$.

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$$x_t = \sqrt{\alpha_t} x_{t-1} + \sqrt{1-\alpha_t} \epsilon_{t-1}$$

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Let $G_1 \sim \mathcal{N}(0, \sigma_1^2 I)$, $G_2 \sim \mathcal{N}(0, \sigma_2^2 I)$, the sum of them gives $g_2 \sim \mathcal{N}(0, (\sigma_1^2 + \sigma_2^2) I)$.

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We have $x_t = \sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon$.

Training

For now, our model is learning μ and Σ, i.e. we sample according to

$$p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t))$$

They've found that fixing Σ_θ to a constant gives the same result. So,

$$p_\theta(x_{t-1} \mid x_t) := \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \sigma_t I)$$

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The probability for our model to generate x_0 is $p_\theta(x_0) := \int p_\theta(x_{0:T}) dx_{1:T}$.

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The probability for our model to generate x_0 is $p_\theta(x_0) := \int p_\theta(x_{0:T}) dx_{1:T}$. Using negative log likelihood and computatios, we want to minimize:

$$E_q \left[\frac{1}{2\sigma_t^2} \|\tilde{\mu}_t(x_t, x_0) - \mu_\theta(x_t, t)\|^2 \right]$$

where $\tilde{\mu}$ is the optimal mean that depends on x_0 which we don't know.

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where $\tilde{\mu}$ is the optimal mean that depends on x_0. Using $x_t(x_0, \epsilon) = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$ we have a loss we can train on.