INTEGRAL INDEFINIDA

1.- Calcula las siguientes integrales inmediatas:

i) $\int 4x^6\,dx = 4\int x^6\,dx = \frac{4x^7}{7} + K$

ii) $\int (6x^3 + 8x^2 + 3)\,dx = \int 6x^3\,dx + \int 8x^2\,dx + \int 3\,dx = \frac{6x^4}{4} + \frac{8x^3}{3} + 3x + K = \frac{3}{2}x^4 + \frac{8}{3}x^3 + 3x + K$

iii) $\int \sqrt{2x}\,dx = \sqrt{2}\int \sqrt{x}\,dx = \sqrt{2}\int x^{1/2}\,dx = \sqrt{2}\,\frac{x^{3/2}}{3/2} + K = \frac{2\sqrt{2}}{3}\sqrt{x^3} + K$

iv) $\int \frac{dx}{\sqrt[5]{x}} = \int x^{-1/5}\,dx = \frac{x^{4/5}}{\frac{4}{5}} + K = \frac{5}{4}\sqrt[5]{x^4} + K$

v) $\int 3a^2x^5\,dx = 3a^2\int x^5\,dx = 3a^2\,\frac{x^6}{6} + K = \frac{1}{2}a^2x^6 + K$

vi) $\int \frac{\sqrt{3x^3} + \sqrt[3]{\sqrt{5x^2}}}{\sqrt{2x}}\,dx = \int \sqrt{\frac{3}{2}x}\,dx + \int \sqrt[6]{\frac{25x^4}{8x^3}}\,dx = \int \sqrt{\frac{3}{2}}\,x\,dx + \int \sqrt[6]{\frac{25x}{8}}\,dx = \frac{\sqrt{3}}{\sqrt{2}}\,\frac{x^2}{2} + \frac{\sqrt[6]{25}\cdot 6}{\sqrt[6]{8}\;7}\,x^{7/6} + K$

$= \frac{\sqrt{3}}{2\sqrt{2}}\,x^2 + \frac{6\sqrt[3]{5}}{7\sqrt{2}}\,x^{7/6} + K$

xiv) $\int \frac{dx}{x-4} = \ln|x-4| + K$

vii) $\int \frac{dx}{x-1} = \ln|x-1| + K$

viii) $\int \frac{5\,dx}{1+4x^2} = \int \frac{5\,dx}{1+(2x)^2} = \frac{5}{2}\int \frac{2\,dx}{1+(2x)^2} = \frac{5}{2}\,\operatorname{arctg}(2x) + K$

x) $\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(2x)^2}} = \frac{1}{2}\int \frac{2\,dx}{\sqrt{1-(2x)^2}} = \frac{1}{2}\,\arcsin(2x) + K$

xi) $\int \frac{dx}{x^2+4} = \int \frac{dx}{\frac{4x^2}{4}+4} = \int \frac{dx}{4\left(\frac{x^2}{4}+1\right)} = \frac{1}{4}\int \frac{dx}{\left(\frac{x}{2}\right)^2+1} = \frac{1}{2}\int \frac{\frac{1}{2}\,dx}{\left(\frac{x}{2}\right)^2+1} =$

$= \frac{1}{2}\,\operatorname{arctg}\frac{x}{2} + K$

xii) $\int \frac{2x-3}{x^2-3x+9} = \ln|x^2-3x+9| + K$

xiii) $\int 2x.e^{x^2}dx = e^{x^2} + K$

xiv) $\int \frac{dx}{x-4} = \ln|x-4| + K$

xv) $\int \frac{dx}{(x-4)^3} = \int (x-4)^{-3}dx = \frac{(x-4)^{-2}}{-2} + K = -\frac{1}{2(x-4)^2} + K$

xvi) $\int e^{5x}dx = \frac{1}{5}\int 5e^{5x}dx = \frac{1}{5}e^{5x} + K$

xvii) $\int \frac{x+\sqrt{x}}{x^2}\,dx = \int \frac{1}{x}\,dx + \int \frac{\sqrt{x}}{x^2}\,dx = \int \frac{1}{x}\,dx + \int x^{\frac{1}{2}-2}\,dx =$

$= \int \frac{1}{x}\,dx + \int x^{-3/2}\,dx = \ln|x| + \frac{x^{-1/2}}{-\frac{1}{2}} + K =$

$= \ln|x| - \frac{2}{\sqrt{x}} + K$

xviii) $\int \frac{1}{x}\ln^3 x\,dx = \frac{\ln^4 x}{4} + K$

xix) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx = 2\int \operatorname{sen}\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,dx = -2\cos\sqrt{x} + K$

xx) $\int e^{-2x+9}dx = \frac{1}{-2}\int -2 \cdot e^{-2x+9}\,dx = -\frac{1}{2}e^{-2x+9} + K$

xxi) $\int \left(3^x - x^3\right)dx = \int 3^x dx - \int x^3 dx = \frac{1}{\ln 3}\int 3^x \ln 3\, dx - \int x^3 dx =$

$= \frac{1}{\ln 3}3^x - \frac{x^4}{4} + K$

xxii) $\int \frac{3x^2 + \cos x + 2e^{2x}}{x^3 + \operatorname{sen} x + e^{2x}}dx = \ln\left|x^3 + \operatorname{sen} x + e^{2x}\right| + K$

xxiii) $\int 6.\cos(2x-1)dx = 3\int 2\cos(2x-1)\,dx = 3 \cdot \operatorname{sen}(2x-1) + K$

xxiv) $\int \frac{x+1}{x^2+2x+3}\,dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+3}\,dx = \frac{1}{2}\ln\left|x^2+2x+3\right| + K$

xxv) $\int \operatorname{sen} x \cos x\,dx = \frac{\operatorname{sen}^2 x}{2} + K$

xxvi) $\int e^{4x-3}\,dx = \frac{1}{4}\int 4e^{4x-3}\,dx = \frac{1}{4}e^{4x-3} + K$

xxvii) $\int \frac{4\,dx}{3+x^2} = \frac{4}{3}\int \frac{dx}{1+\left(\frac{x}{\sqrt{3}}\right)^2} = \frac{4}{3}\operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right) + K$

xxvii i) $\int \sqrt{(x+3)^3}\, dx = \int (x+3)^{3/2}\, dx = \dfrac{(x+3)^{5/2}}{5/2} + K =$

$$= \frac{2\sqrt{(x+3)^5}}{5} + K$$

xxix) $\int \frac{\text{sen } x}{\cos x} dx = -\int \frac{-\text{sen } x}{\cos x} dx = -\ln|\cos x| + K$

2.- Calcula las siguientes integrales, por cambio de variable:

i) $\int x.\operatorname{sen} x^2\, dx$ $= \frac{1}{2}\int 2x \cdot \operatorname{sen} x^2\, dx = \frac{1}{2}\int \operatorname{sen} u \cdot du = -\frac{1}{2}\cos u + K =$

$u = x^2 \Rightarrow du = 2x\, dx$

$= -\frac{1}{2}\cos x^2 + K$

ii) $\int \frac{x.dx}{\sqrt{x^2+5}}$ $= \frac{1}{2}\int \frac{2x\, dx}{\sqrt{x^2+5}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \int \frac{du}{2\sqrt{u}} = \sqrt{u} + K =$

$u = x^2 + 5$

$= \sqrt{x^2+5} + K$

$du = 2x\, dx$

iii) $\int \frac{\operatorname{sen} x\, dx}{\cos^5 x}$ $= -\int \frac{-\operatorname{sen} x\, dx}{\cos^5 x} = -\int \frac{du}{u^5} = -\int u^{-5}\, du = -\frac{u^{-4}}{-4} + K =$

$u = \cos x \quad du = -\operatorname{sen} x\, dx$

$= \frac{1}{4u^4} + K = \frac{1}{4\cos^4 x} + K$

iv) $\int \frac{x.dx}{\left(x^2+3\right)^5} = \frac{1}{2}\int \frac{2x\,dx}{(x^2+3)^5} = \frac{1}{2}\int \frac{du}{u^5} = \frac{1}{2}\int u^{-5}\,du = \frac{1}{2}\,\frac{u^{-4}}{-4} + K =$

$u = x^2+3$

$du = 2x\,dx$

$= -\frac{1}{8(x^2+3)^4} + K$

v) $\int \frac{1+\mathrm{tg}^2 x}{\sqrt{\mathrm{tg}\,x}}\,dx = \int \frac{du}{\sqrt{u}} = 2\int \frac{1}{2\sqrt{u}}\,du = 2\sqrt{u} + K = 2\sqrt{\mathrm{tg}\,x} + K$

$\mathrm{tg}\,x = u$

$(1+\mathrm{tg}^2 x)\,dx = du$

vi) $\int \sqrt{x^2-2x}\,(x-1)\,dx = \frac{1}{2}\int \sqrt{x^2-2x}\;\;2(x-1)\,dx = \frac{1}{2}\int \sqrt{u}\cdot du =$

$x^2-2x = u$

$(2x-2)\,dx = du$

$2(x-1)\,dx = du$

$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{\cancel{2}}\,\frac{u^{3/2}}{3/\cancel{2}} + K = \frac{\sqrt{u^3}}{3} + K =$

$= \frac{\sqrt{(x^2-2x)^3}}{3} + K$

vii) $\int \text{sen}(\cos x).\ \text{sen}\ x\, dx = -\int \text{sen}(\cos x)\cdot(-\text{sen}\ x \cdot dx) = -\int \text{sen}\ u \cdot du =$

$\cos x = u$

$-\text{sen}\ x\, dx = du$

$= -(-\cos u) + K = \cos(\cos x) + K$

viii) $\int \frac{(1+\ln x)^2}{x} dx = \int u^2 \cdot du = \frac{u^3}{3} + K = \frac{(1+\ln x)^3}{3} + K$

$1+\ln x = u \quad \frac{1}{x} dx = du$

ix) $\int \sqrt{(1+\cos x)^3}\ \text{sen}\ x\, dx = -\int \sqrt{u^3} \cdot du = -\int u^{3/2} du = -\frac{u^{5/2}}{5/2} + K =$

$1+\cos x = u \quad -\text{sen}\ x \cdot dx = du$

$= -\frac{2}{5}\sqrt{u^5} + K = -\frac{2}{5}\sqrt{(1+\cos x)^5} + K$

3.- Calcula las siguientes integrales, por partes:

$(uv)' = u'v + uv' \Rightarrow \int (uv)' = \int u'v + \int uv' \Rightarrow \int uv' = uv - \int u'v$

$\int u\,dv = uv - \int v\,du$

i) $\int x.e^x dx = x e^x - \int e^x dx = x e^x - e^x + K = e^x(x-1) + K$

$u = x$, $du = dx$

$e^x dx = dv$, $v = e^x$

ii) $\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + K = x(\ln x - 1) + K$

$u = \ln x$ $\quad du = \frac{1}{x} dx$

$1 \cdot dx = dv$ $\quad v = x$

iii) $\int x^3 . \ln x \, dx = \ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4x} dx = \ln x \frac{x^4}{4} - \frac{x^4}{16} + K = \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + K$

$\ln x = u$ $\quad \frac{1}{x} dx = du$

$x^3 dx = dv$ $\quad v = \frac{x^4}{4}$

iv) $\int x.\text{sen } x\, dx = -x\cos x + \int \cos x\, dx = -x\cos x + \text{sen } x + K$

$u = x \qquad du = dx$

$\text{sen } x\, dx = dv \qquad v = -\cos x$

v) $\int \text{arc tg } x\, dx = x \,\text{arctg } x - \int \frac{x}{1+x^2} dx =$

$u = \text{arctg } x \qquad du = \frac{1}{1+x^2} dx$

$dx = dv \qquad v = x$

$= x\,\text{arctg } x - \frac{1}{2} \ln|1+x^2| + K$

vi) $\int \text{arc sen } x\, dx = x \,\text{arcsen} x - \int \frac{x\, dx}{\sqrt{1-x^2}} = x\,\text{arcsen} x - \frac{1}{2}\int \frac{2x\, dx}{\sqrt{1-x^2}} =$

$u = \text{arcsen} x \qquad du = \frac{dx}{\sqrt{1-x^2}}$

$dx = dv \qquad v = x$

$= x\,\text{arcsen} x + \sqrt{1-x^2} + K$

vii) $\int x.\cos 3x\, dx$ $= \frac{1}{3} x\, \text{sen} 3x - \frac{1}{3}\int \text{sen}\, 3x\, dx = \frac{1}{3} x\, \text{sen} 3x + \frac{1}{9}\cos 3x + K$

$u = x \qquad du = dx$

$\cos 3x\, dx = dv \qquad v = \frac{1}{3}\text{sen}\, 3x$

viii) $\int x^2.e^{2x}\, dx$ $= \frac{x^2 e^{2x}}{2} - \int \frac{\cancel{2}x\, e^{2x}}{\cancel{2}} dx = \frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{1}{2}\int e^{2x} dx =$

$= \frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{e^{2x}}{4} + K$

$u = x^2 \qquad du = 2x\, dx$

$e^{2x} dx = dv \qquad v = \frac{e^{2x}}{2}$

$x = u \qquad dx = du$

$e^{2x} dx = dv \qquad v = \frac{e^{2x}}{2}$

ix) $\int \frac{x}{e^x} dx$ $= -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + K = -e^{-x}(1+x) + K$

$u = x \qquad du = dx$

$e^{-x} dx = dv \qquad v = -e^{-x}$

x) $\int \frac{\ln x}{x^2} dx$ $= -\frac{\ln|x|}{x} + \int \frac{1}{x}\frac{1}{x} dx = -\frac{\ln|x|}{x} - \frac{1}{x} + K = -\frac{1}{x}(\ln|x| + 1) + K$

$u = \ln|x| \qquad du = \frac{1}{x} dx$

$x^{-2} dx = dv \qquad v = -\frac{1}{x}$

4.- Resuelve las siguientes integrales, que luego aparecerán al integrar funciones racionales:

i) $\int \frac{3}{2x-1}dx = \frac{3}{2}\int \frac{2}{2x-1}dx = \frac{3}{2}\ln|2x-1|+K$

ii) $\int \frac{2}{(x+2)^3}dx = 2\int \frac{dx}{(x+2)^3} = \cancel{2}\,\frac{(x+2)^{-2}}{-\cancel{2}} = -\frac{1}{(x+2)^2}+K$

iii) $\int \frac{7x-5}{x^2+4}dx = \int \frac{7x\,dx}{x^2+4} - \int \frac{5\,dx}{x^2+4} = \frac{7}{2}\ln|x^2+4| - \frac{5}{\cancel{4}_2}\int \frac{2\cdot\frac{1}{2}dx}{\frac{x^2}{4}+1} =$

$= \frac{7}{2}\ln|x^2+4| - \frac{5}{2}\,\mathrm{arctg}\left(\frac{x}{2}\right)+K$

$x^2+x+1 = \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

iv) $\int \frac{x+2}{x^2+x+1}dx = \frac{1}{2}\int \frac{2x+4}{x^2+x+1}dx = \frac{1}{2}\left(\int \frac{2x+1}{x^2+x+1}dx + \int \frac{3\,dx}{x^2+x+1}\right) =$

$\frac{1}{2}\ln|x^2+x+1| + \frac{\cancel{3}}{\cancel{2}}\int \frac{\cancel{4}/\cancel{3}\;2}{\frac{4}{3}\left[\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right]}dx = \frac{1}{2}\ln|x^2+x+1| + 2\int \frac{dx}{\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}\right)^2+1} = \frac{1}{2}\ln|x^2+x+1| +$

$+\sqrt{3}\,\mathrm{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)+K$

v) $\int \frac{5dx}{3x-4} = \frac{5}{3} \ln|3x-4| + K$

vi) $\int \frac{7dx}{(2x-3)^2} = \frac{7}{2} \int \frac{2dx}{(2x-3)^2} = -\frac{7}{2(2x-3)} + K$

vii) $\int \frac{3x+4}{x^2+2} dx = \int \frac{3x}{x^2+2} dx + \int \frac{4}{x^2+2} dx = \frac{3}{2} \int \frac{2x}{x^2+2} dx + \frac{4}{2} \int \frac{dx}{\frac{x^2}{2}+1} =$

No tiene raíces reales

$= \frac{3}{2} \ln|x^2+2| + 2 \int \frac{dx \cdot \frac{1}{\sqrt{2}} \sqrt{2}}{\left(\frac{x}{\sqrt{2}}\right)^2+1} = \frac{3}{2} \ln|x^2+2| + 2\sqrt{2} \operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + K$

viii) $\int \frac{x-1}{x^2+2x+3} dx = \frac{1}{2} \int \frac{2x-2}{x^2+2x+3} = \frac{1}{2} \int \frac{2x+2-4}{x^2+2x+3} dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x+3} dx + \frac{1}{2} \int \frac{-4}{(x+1)^2+2} dx =$

$= \frac{1}{2} \ln|x^2+2x+3| - \cancel{2} \int \frac{dx}{\cancel{2}\left(\frac{(x+1)^2}{2}+1\right)} = \frac{1}{2} \ln|x^2+2x+3| - \sqrt{2} \int \frac{\frac{1}{\sqrt{2}}}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} dx = \frac{1}{2} \ln|x^2+2x+3| - \sqrt{2} \operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + K$

5.- Calcula las siguientes integrales racionales:

i) $\int \frac{x-2}{x^2+x}dx = \int \frac{x-2}{x(x+1)}dx = \int \frac{A}{x}dx + \int \frac{B}{x+1}dx = \int \frac{-2\,dx}{x} + \int \frac{3\,dx}{x+1} =$

$= -2\ln|x| + 3\ln|x+1| + K$

$\frac{x-2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} \Rightarrow x-2 = A(x+1) + Bx$

$x-2 = (A+B)x + A$

$\begin{cases} A = -2 \\ A+B = 1 \end{cases} \Rightarrow B = 3$

ii) $\int \frac{x^3+4x^2-10x+7}{x^3-7x-6}dx = \int \left(1 + \frac{4x^2-3x+13}{x^3-7x-6}\right)dx = x + \int \frac{2}{x-3}dx + \int \frac{-5}{x+1}dx + \int \frac{7}{x+2}dx =$

$= x + 2\ln|x-3| - 5\ln|x+1| + 7\ln|x+2| + K$

$$\begin{array}{l|l} x^3+4x^2-10x+7 & x^3-7x-6 \\ \hline -x^3 \qquad\quad +7x+6 & 1 \\ \hline 4x^2-3x+13 & \end{array}$$

$\frac{4x^2-3x+13}{(x-3)(x+1)(x+2)} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{x+2}$

$4x^2-3x+13 = A(x+1)(x+2) + B(x-3)(x+2) + C(x-3)(x+1)$

si $x=-1 \Rightarrow 20 = -4B \Rightarrow \underline{B=-5}$

si $x=-2 \Rightarrow 35 = 5C \Rightarrow \underline{C=7}$

$A+B+C = 4 \Rightarrow A-5+7 = 4$

$\underline{A=2}$

iii) $\int \frac{2x^2+5x-1}{x^3+x^2-2x}\,dx$ $= \int \frac{1}{2x}\,dx + \int \frac{2}{(x-1)}\,dx - \int \frac{1}{2(x+2)}\,dx = \frac{1}{2}\ln|x| + 2\ln|x-1| - \frac{1}{2}\ln|x+2| + K$

$$\frac{2x^2+5x-1}{x^3+x^2-2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

$$2x^2+5x-1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

si $x=1 \Rightarrow$ $2\cdot 1^2 + 5\cdot 1 - 1 = B\cdot 1\cdot 3$

$6 = 3B \Rightarrow \boxed{B=2}$

si $x=0$ $-1 = A\cdot(-1)\cdot(2)$

$-1 = -2A \Rightarrow \boxed{A=1/2}$

si $x=-2$ $-3 = 6C \Rightarrow \boxed{C=-1/2}$

iv) $\int \frac{2x+3}{x^2+3x-10}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{1}{x+5}\,dx = \ln|x-2| + \ln|x+5| + K$

$= \ln|(x-2)(x+5)| + K =$

$= \ln|x^2+3x-10| + K$

$$\frac{2x+3}{x^2+3x-10} = \frac{A}{(x-2)} + \frac{B}{(x+5)}$$

$$2x+3 = A(x+5) + B(x-2)$$

si $x=2$ $\quad 7 = A\cdot 7 \Rightarrow A=1$

si $x=-5$ $\quad -7 = -7B \Rightarrow \boxed{B=1}$

MÁS FÁCIL

INMEDIATA

iv) $\int \frac{\overbrace{2x+3}^{u'}}{\underbrace{x^2+3x-10}_{u}}\,dx = \ln|x^2+3x-10| + K$

v) $\int \frac{dx}{x^3+x^2-6x} = \int \left(-\frac{1}{6x} + \frac{1}{10(x-2)} + \frac{1}{15(x+3)} \right) dx = -\frac{1}{6} \ln|x| + \frac{1}{10} \ln|x-2| + \frac{1}{15} \ln|x+3| + K$

$$\frac{1}{x^3+x^2-6x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$1 = A(x-2)(x+3) + Bx(x+3) + Cx(x-2)$$

Si $x=0$ $\quad 1 = -6A \Rightarrow \boxed{A = -1/6}$

Si $x=2$ $\quad 1 = 10B \Rightarrow \boxed{B = 1/10}$

Si $x=-3$ $\quad 1 = 15B \Rightarrow \boxed{C = 1/15}$

vi) $\int \frac{x^2+2x}{x^2-1}dx = \int \left(1+\frac{2x+1}{x^2-1}\right)dx = x+\int \frac{2x+1}{x^2-1}\,dx = x+\int \frac{2x}{x^2-1}\,dx + \int \frac{dx}{x^2-1} =$

$$= \boxed{x+\ln|x^2-1| - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + K}$$

$$\frac{x^2+2x}{x^2-1} = 1+\frac{2x+1}{x^2-1}$$

$$\int \frac{dx}{x^2-1} = -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{dx}{x-1}$$

$$\frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+1)$$

Si $x=1$ $\quad 1 = 2B \Rightarrow B = \frac{1}{2}$

Si $x=-1$ $\quad 1 = -2A \Rightarrow A = -\frac{1}{2}$

vi) $\int \frac{x^2+2x}{x^2-1}\,dx = \int \left(1 + \frac{2x+1}{x^2-1}\right)dx = x + \int \frac{2x+1}{x^2-1}\,dx = \boxed{x + \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + K}$

$$\frac{x^2+2x}{x^2-1} = 1 + \frac{2x+1}{x^2-1}$$

$$\frac{2x+1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$2x+1 = A(x-1) + B(x+1)$$

si $x=1$ $\quad 3 = 2B \Rightarrow \boxed{B = 3/2}$

si $x=-1$ $\quad -1 = -2A \Rightarrow \boxed{A = 1/2}$

$$\text{vii)} \int \frac{2x^2+7x-1}{x^3+x^2-x-1}\,dx = \int \frac{2}{x-1}\,dx + 3\int \frac{dx}{(x+1)^2} = 2\ln|x-1| - \frac{3}{x+1} + k$$

$$\frac{2x^2+7x-1}{x^3+x^2-x-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$2x^2+7x-1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

si $x=1$ $\quad 8 = 4A \Rightarrow \boxed{A=2}$

si $x=-1$ $\quad -6 = -2C \Rightarrow \boxed{C=3}$

si $x=0$ $\quad -1 = A - B - C$

$$B = A - C + 1$$

$$B = 2-3+1 \Rightarrow \boxed{B=0}$$

viii) $\int \frac{2x-4}{(x-1)^2.(x+3)}\,dx = \frac{5}{8}\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{dx}{(x-1)^2} - \frac{5}{8}\int\frac{dx}{x+3} =$

$$\frac{2x-4}{(x-1)^2\cdot(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$$

$$2x-4 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

si $x=1$ $\quad -2 = 4B \Rightarrow \boxed{B = -1/2}$

si $x=-3$ $\quad -10 = 16C \Rightarrow \boxed{C = -\frac{5}{8}}$

si $x=0$ $\quad -4 = -3A + 3B + C$

$3A = 3B + C + 4$

$3A = -3/2 - 5/8 + 4$

$3A = \frac{-12-5+32}{8}$

$3A = \frac{15}{8}$

$\boxed{A = \frac{5}{8}}$

$$= \frac{5}{8}\ln|x-1| + \frac{1}{2(x-1)} - \frac{5}{8}\ln|x+3| + K$$

$$\text{ix)} \int \frac{x^2+x}{x^4+2x^3-3x^2-4x+4}\,dx = \frac{5}{27}\int \frac{1}{x-1}\,dx + \frac{2}{9}\int \frac{1}{(x-1)^2}\,dx - \frac{5}{27}\int \frac{dx}{x+2} + \frac{2}{9}\int \frac{dx}{(x+2)^2} =$$

$$(x-1)^2(x+2)^2$$

$$= \boxed{\frac{5}{27}\ln|x-1| - \frac{2}{9(x-1)} - \frac{5}{27}\ln|x+2| - \frac{2}{9(x+2)} + K}$$

$$\frac{x(x+1)}{(x-1)^2(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)} + \frac{D}{(x+2)^2}$$

$$x(x+1) = A(x-1)(x+2)^2 + B(x+2)^2 + C(x-1)^2(x+2) + D(x-1)^2$$

$x=1 \Rightarrow \quad 2 = 9B \Rightarrow \boxed{B = \frac{2}{9}}$

$x=-2 \Rightarrow (-2)\cdot(-1) = 9D \Rightarrow \boxed{D = \frac{2}{9}}$

$x=0 \Rightarrow \begin{cases} 0 = -4A + 4B + 2C + D \\ 0 = -2A + B + 4C + 4D \end{cases} \Rightarrow \begin{cases} -4A + 2C = -4\cdot\frac{2}{9} - \frac{2}{9} \\ -2A + 4C = -\frac{2}{9} - 4\cdot\frac{2}{9} \end{cases}$

$x=-1 \Rightarrow$

$\Rightarrow \begin{cases} -4A + 2C = -\frac{10}{9} \\ -2A + 4C = -\frac{10}{9} \end{cases} \to \begin{cases} -4A + 2C = -\frac{10}{9} \\ 4A - 8C = \frac{20}{9} \end{cases} \quad -6C = \frac{10}{9} \quad \boxed{C = -\frac{5}{27}}$

$2A = \frac{10}{9} - \frac{20}{27} \Rightarrow 2A = \frac{10}{27} \Rightarrow \boxed{A = \frac{5}{27}}$

$$x) \int \frac{x+2}{x^3-2x^2+x}\,dx = 2\int \frac{1}{x}\,dx - 2\int \frac{1}{x-1}\,dx + 3\int \frac{1}{(x-1)^2}\,dx =$$

$$x\cdot(x-1)^2$$

$$= \boxed{2\ln|x| - 2\ln|x-1| - \frac{3}{x-1} + K}$$

$$\frac{x+2}{x\cdot(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$x+2 = A(x-1)^2 + Bx(x-1) + Cx$$

$x=0$ $\quad \boxed{2 = A}$

$x=1$ $\quad \boxed{3 = C}$

$x=-1$ $\quad 1 = 4A + 2B - C$

$$1 = 8 + 2B - 3$$

$$-4 = 2B$$

$$\boxed{B = -2}$$

xi) $\int \frac{8x^2-2x-1}{x^3-x^2+4x-4}dx = \int \frac{dx}{x-1} + \int \frac{7x+5}{x^2+4}dx = \int \frac{dx}{x-1} + \frac{7}{2}\int \frac{2x}{x^2+4}dx + 5\int \frac{1}{x^2+4}dx =$

$\Leftrightarrow (x-1)\cdot(x^2+4)$

$= \ln|x-1| + \frac{7}{2}\ln|x^2+4| + \frac{5}{4}\int \frac{dx}{\left(\frac{x}{2}\right)^2+1} =$

$= \ln|x-1| + \frac{7}{2}\ln|x^2+4| + \frac{5}{2}\int \frac{1/2}{\left(\frac{x}{2}\right)^2+1}dx =$

$= \boxed{\ln|x-1| + \frac{7}{2}\ln|x^2+4| + \frac{5}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+K}$

$\frac{8x^2-2x-1}{x^3-x^2+4x-4} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$

$8x^2-2x-1 = A(x^2+4) + (Bx+C)(x-1)$

$x=1 \quad 5 = 5A \Rightarrow \boxed{A=1}$

$x=0 \quad -1 = 4A - C \Rightarrow -1 = 4 - C \Rightarrow \boxed{C=5}$

$x=-1 \quad 8+2-1 = 5A - 2\cdot(C-B)$

$9 = 5A + 2B - 2C$

$9 - 5A + 2C = 2B$

$14 = 2B \Rightarrow \boxed{B=7}$

xii) $\int \frac{6x+8}{x^2+2x+5}\,dx = 3\int \frac{2x+\frac{8}{3}}{x^2+2x+5}\,dx =$

$$= 3\int \frac{2x+2-2+\frac{8}{3}}{x^2+2x+5}\,dx = 3\int \frac{2x+2}{x^2+2x+5}\,dx + \int \frac{2}{x^2+2x+5}\,dx =$$

$$= 3\ln|x^2+2x+5| + 2\int \frac{dx}{x^2+2x+5} = 3\ln|x^2+2x+5| + 2\int \frac{dx}{(x+1)^2+4} =$$

$$= 3\ln|x^2+2x+5| + \frac{2\cdot 2}{4}\int \frac{\frac{1}{2}dx}{\left(\frac{x+1}{2}\right)^2+1} = \boxed{3\ln|x^2+2x+5 + \operatorname{arctg}\left(\frac{x+1}{2}\right)+K}$$

$$\text{xiii) } \int \frac{3x-2}{x^3-3x^2+12x-10}\,dx = \frac{1}{9}\int \frac{dx}{x-1} + \frac{1}{9}\int \frac{-x+28}{x^2-2x+10}\,dx = \frac{1}{9}\int \frac{dx}{x-1} - \frac{1}{18}\int \frac{2x-56}{x^2-2x+10}\,dx =$$

$$(x-1)(x^2-2x+10)$$

$$= \frac{1}{9}\ln|x-1| - \frac{1}{18}\int \frac{2x-2-54}{x^2-2x+10}\,dx =$$

$$= \frac{1}{9}\ln|x-1| - \frac{1}{18}\int \frac{2x-2}{x^2-2x+10}\,dx + \frac{54}{18}\int \frac{dx}{x^2-2x+10} =$$

$$= \frac{1}{9}\ln|x-1| - \frac{1}{18}\ln|x^2-2x+10| + 3\int \frac{dx}{(x-1)^2+9} =$$

$$= \frac{1}{9}\ln|x-1| - \frac{1}{18}\ln|x^2-2x+10| + \frac{3\cdot 3}{9}\int \frac{\frac{1}{3}\,dx}{\left(\frac{x-1}{3}\right)^2+1} =$$

$$= \boxed{\frac{1}{9}\ln|x-1| - \frac{1}{18}\ln|x^2-2x+10| + \operatorname{arctg}\left(\frac{x-1}{3}\right) + K}$$

$$\frac{3x-2}{(x-1)(x^2-2x+10)} = \frac{A}{x-1} + \frac{Bx+C}{x^2-2x+10}$$

$$3x-2 = A(x^2-2x+10) + (Bx+C)(x-1)$$

$x=1$ $\quad 1 = 9A \Rightarrow \boxed{A = 1/9}$

$x=0$ $\quad -2 = 10A - C \Rightarrow C = \frac{10}{9} + 2$

$$\boxed{C = \frac{28}{9}}$$

$x=-1$ $\quad -5 = 13A + 2B - 2C$

$$2B = -5 - 13A + 2C$$

$$2B = -5 - \frac{13}{9} + \frac{56}{9}$$

$$2B = \frac{-2}{9}$$

$$\boxed{B = -\frac{1}{9}}$$