APLICACIONES DE LA DERIVADA

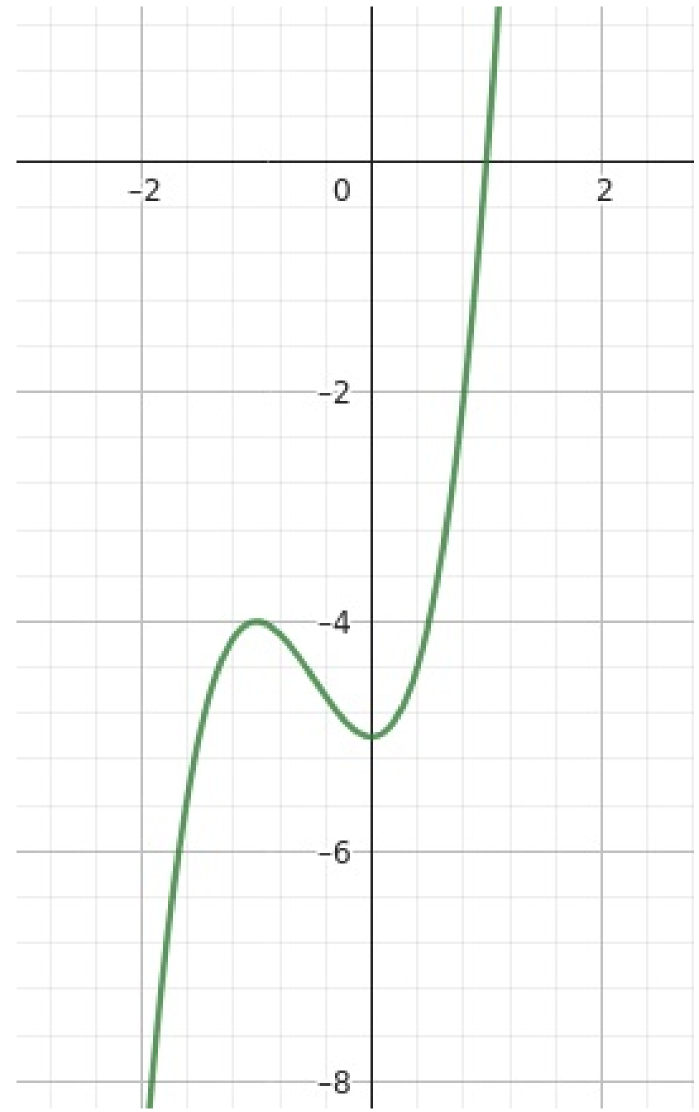
1. Calcula los intervalos de crecimiento y decrecimiento de las siguientes funciones:

i) $f(x) = 2x^3 + 3x^2 - 5$

$$f'(x) = 6x^2 + 6x = 6x(x+1) \Rightarrow f'(x) = 0 \text{ si } \begin{matrix} x = 0 \\ x = -1 \end{matrix}$$

$f'(x)$

	$(-\infty, -1)$	$(-1, 0)$	$(0, +\infty)$
$6x$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
	$+$	$-$	$+$
	Creciente	Decreciente	Creciente

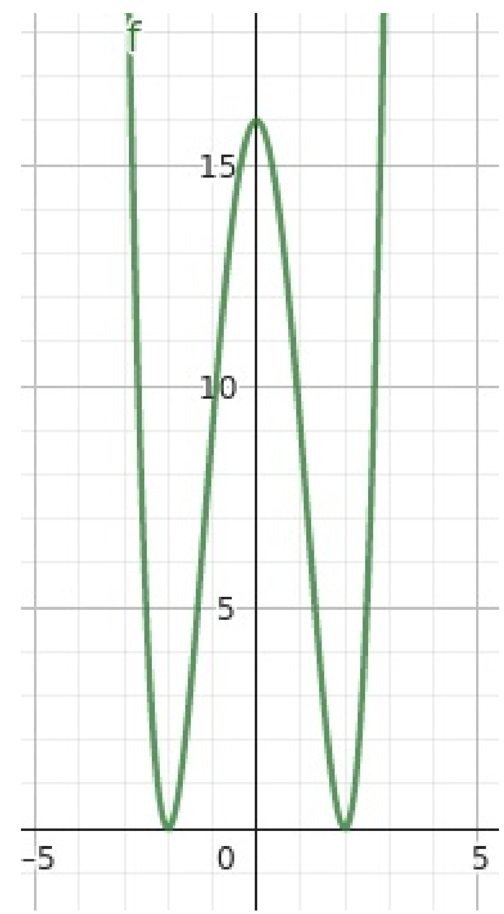


ii) $f(x) = (x^2 - 4)^2$

$$f'(x) = 2(x^2-4)\cdot 2x = 4x(x+2)(x-2) \Rightarrow$$

$$f'(x) = 0 \qquad x=0,\ x=-2,\ x=2$$

	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
$4x$	$-$	$-$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$
	$-$	$+$	$-$	$+$
	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$



v) $f(x) = \dfrac{x}{x^2+1}$

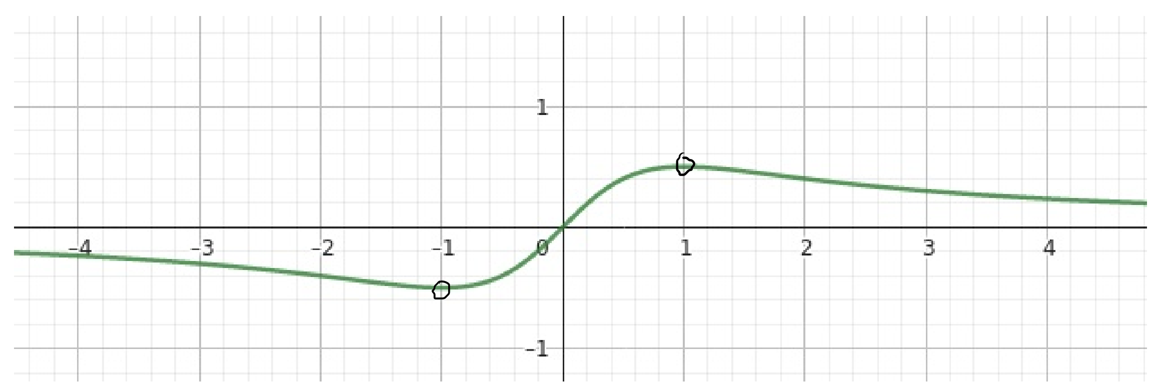
• f no es continua en $x = \pm 1 \Rightarrow$ No es derivable en $x = \pm 1$

• $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$ $\qquad f'(x) = 0 \Rightarrow 1 - x^2 = 0$

$$1 = x^2$$

$$x = \pm 1$$

	$(-\infty, -1)$	$(-1, 1)$	$(1, +\infty)$
$1-x^2$	$-$	$+$	$-$
$(x^2+1)^2$	$+$	$+$	$+$
	$-$	$+$	$-$
	$\searrow$	$\nearrow$	$\searrow$



vi) $f(x) = \frac{4x+5}{2x-3}$

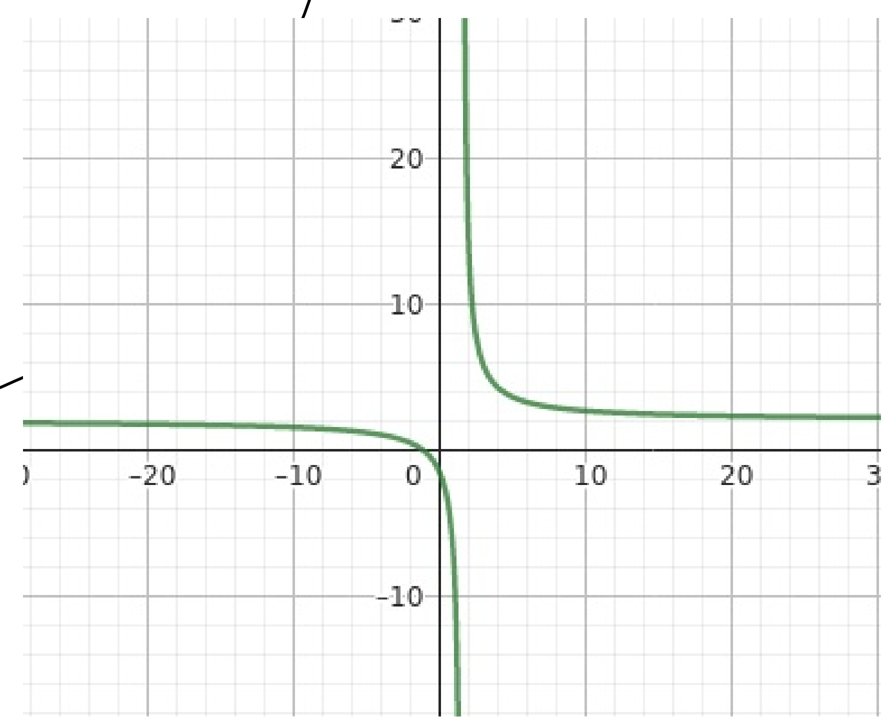
f es continua y derivable en $\mathbb{R} - \{\frac{3}{2}\}$ por ser de prop. inversa

$$f'(x) = \frac{4(2x-3) - 2(4x+5)}{(2x-3)^2} = \frac{-22}{\underbrace{(2x-3)^2}_{\geq 0}} < 0 \Rightarrow$$ f es decreciente en $(-\infty, \frac{3}{2})$ y $(\frac{3}{2}, +\infty)$

2ª FORMA

$f'(x) = 0 \Leftrightarrow \frac{-22}{(2x-3)^2} = 0 \Rightarrow -22 = 0$ No tiene solución

	$(-\infty, 3/2)$	$(3/2, \infty)$
-22	$-$	$-$
$(2x-3)^2$	$+$	$+$
	$-$	$-$
	↘	↘



viii) $f(x) = x^2.e^x$ $\Rightarrow$ Continua en $\mathbb{R}$ (y derivable) por serlo en su dominio

$$f'(x) = 2x\,e^x + x^2 \cdot e^x = (2x + x^2)\,e^x$$

$$f'(x) = 0 \Rightarrow (2x + x^2)\,e^x = 0 \underset{e^x > 0}{\Rightarrow} 2x + x^2 = 0$$

$$x(2+x) = 0$$

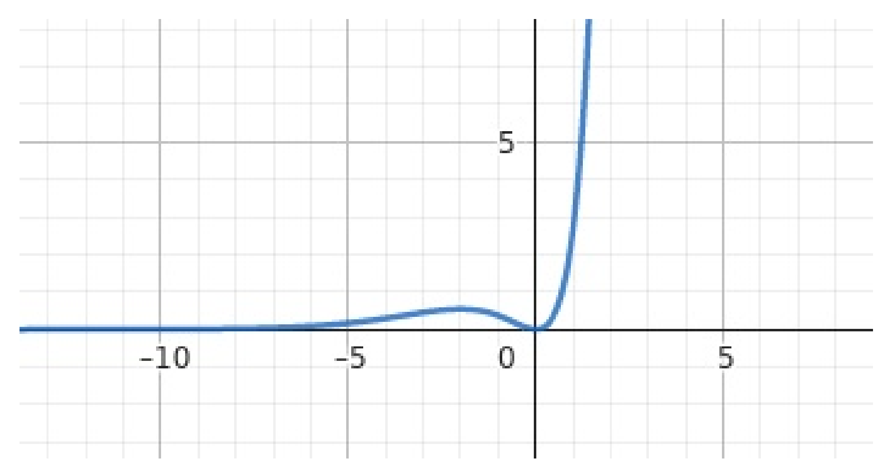
$$x = 0,\ x = -2$$

$$f'(x) = x(2+x)\,e^x$$

	$(-\infty,-2)$	$(-2,0)$	$(0,+\infty)$
x	$-$	$-$	$+$
$(2+x)$	$-$	$+$	$+$
e^x	$+$	$+$	$+$
	$+$	$-$	$+$
	$\nearrow$	$\searrow$	$\nearrow$

f es creciente en $(-\infty,-2)$ y $(0,+\infty)$

f es decreciente en $(-2,0)$



ix) $f(x) = \dfrac{e^{x}}{x^{3}}$ $\longrightarrow$ f es continua en $\mathbb{R}-\{0\}$ por serlo en su dominio

$$f'(x) = \frac{e^x \cdot x^3 - 3x^2 \cdot e^x}{(x^3)^2} = \frac{e^x\,(x^3 - 3x^2)}{x^6} = \frac{e^x \cdot x^2\,(x-3)}{x^6}$$ $\longrightarrow$ f es derivable en $\mathbb{R}-\{0\}$

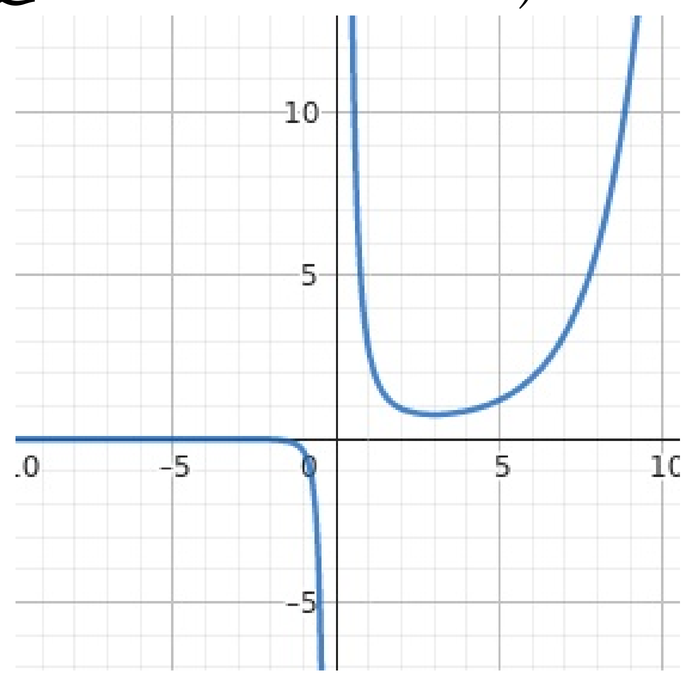
Estudio del crecimiento decrecimiento:

$f'(x) = 0 \Rightarrow e^x \cdot x^2(x-3) = 0 \Rightarrow x^2(x-3) = 0 \longrightarrow x=0,\ x=3$

	$(-\infty,0)$	$(0,3)$	$(3,+\infty)$
e^x	$+$	$+$	$+$
x^2	$+$	$+$	$+$
$x-3$	$-$	$-$	$+$
x^6	$+$	$+$	$+$
	$-$	$-$	$+$
	$\searrow$	$\searrow$	$\nearrow$

f decrece de $(-\infty,0)$ y $(0,3)$

f crece en $(3,+\infty)$



xvii) f(x) = 2.sen x – sen 2x

Continua y derivable en $\mathbb{R}$ por serlo en su dominio

$$f'(x) = 2\cos x - 2\cdot\cos 2x$$

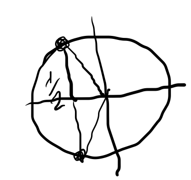
$$f'(x) = 0 \Rightarrow 2\cos x - 2\cos 2x = 0 \Rightarrow 2(\cos x - \cos 2x) = 0 \Rightarrow$$

$$\Rightarrow \cos x - \cos 2x = 0 \Rightarrow \cos x - \cos^2 x + \sin^2 x = 0 \Rightarrow$$

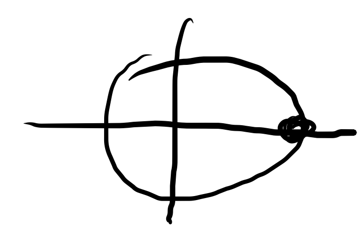
$$\Rightarrow \cos x - \cos^2 x + 1 - \cos^2 x = 0 \Rightarrow -2\cos^2 x + \cos x + 1 = 0$$

$$\underset{\cos x = t}{\Rightarrow} \quad t^2 - t - 1 = 0 \Rightarrow t = \left\langle \begin{matrix} -1/2 \\ 1 \end{matrix} \right.$$

$$\cos x = -\frac{1}{2} \Rightarrow \begin{cases} x = 120^\circ + 360^\circ k = \frac{2\pi}{3} \\ x = 240^\circ + 360^\circ k = \frac{4\pi}{3} \end{cases}$$



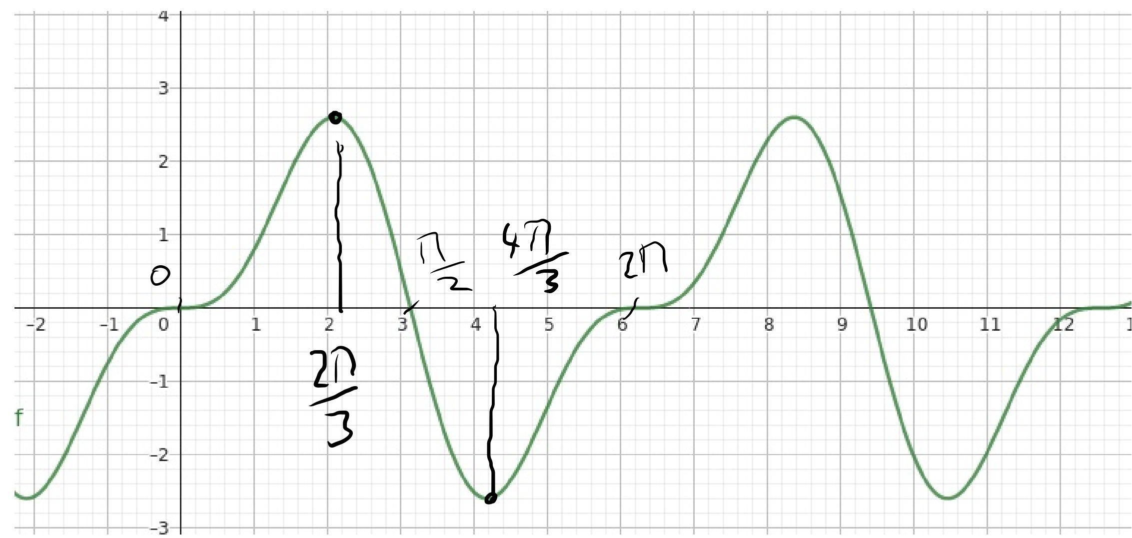
$$\cos x = 1 \Rightarrow x = 0 + 360k$$



En la 1ª Vuelta y en radianes: $f'(x)=0$ si $x=0 = 2\pi$, $x=\frac{2\pi}{3}$, $x=\frac{4\pi}{3}$

$$f'(x) = 2(\cos x - \cos 2x)$$

	$\left(0, \frac{2\pi}{3}\right)$	$\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$	$\left(\frac{4\pi}{3}, 2\pi\right)$
2	+	+	+
$\cos x - \cos 2x$	+	−	+
	90°	180°	270
	$0-(-1) = 1$	$-1-(+1) = -2$	$0-(-1) = +1$
	+	−	+
	↗	↘	↗



En $x=\frac{2\pi}{3} \Rightarrow$ Máximo relativo

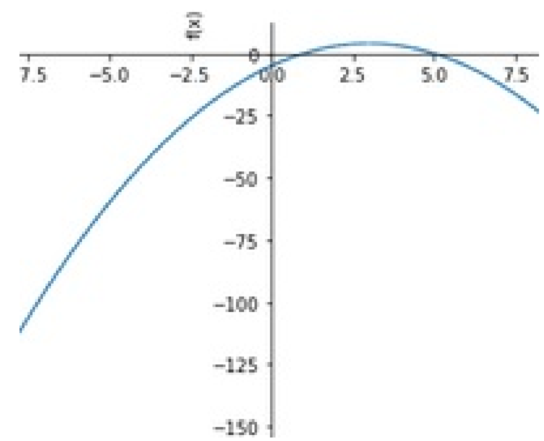
En $x=\frac{4\pi}{3} \Rightarrow$ Mínimo relativo

2. Halla los extremos relativos de las siguientes funciones:

i) $f(x) = -x^2 + 6x - 5$

$$f(x) = -x^2 + 6x - 5$$
$$f'(x) = 6 - 2x$$
$$f''(x) = -2$$
$$f'(3) = 0 \wedge f''(3) = -2$$

Hay un máximo relativo en $(3, 4)$



ii) $f(x) = x.(x-1)^2$

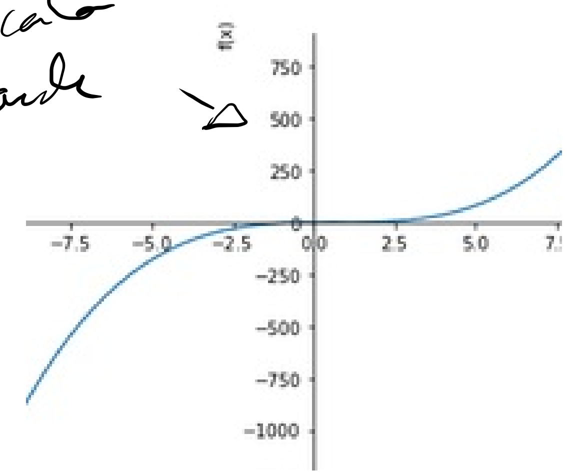
$$f(x) = x(x-1)^2$$
$$f'(x) = (x-1)(3x-1)$$
$$f''(x) = 6x - 4$$
$$f'(\tfrac{1}{3}) = 0 \wedge f''(\tfrac{1}{3}) = -2$$

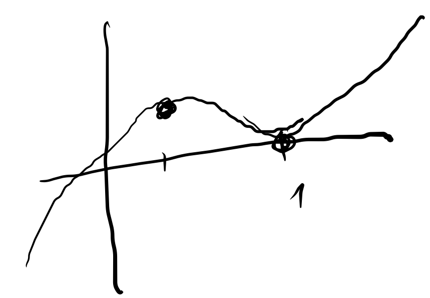
Hay un máximo relativo en $(\frac{1}{3}, \frac{4}{27})$

$$f'(1) = 0 \wedge f''(1) = 2$$

Hay un mínimo relativo en $(1, 0)$

Escala grande





iii) $f(x) = 2x^3 - 15x^2 + 36x - 12$

$$f(x) = 2x^3 - 15x^2 + 36x - 12$$

$$f'(x) = 6x^2 - 30x + 36$$

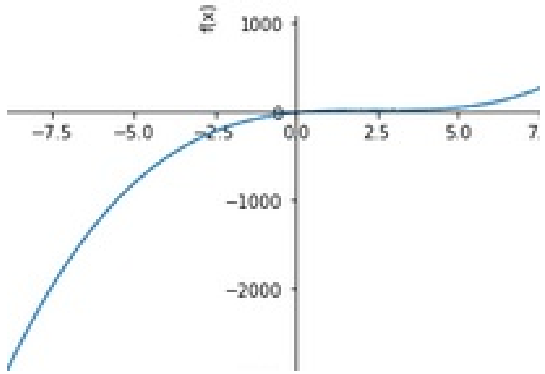
$$f''(x) = 12x - 30$$

$$f'(2) = 0 \wedge f''(2) = -6$$

Hay un máximo relativo en $(2, 16)$

$$f'(3) = 0 \wedge f''(3) = 6$$

Hay un mínimo relativo en $(3, 15)$



iv) $f(x) = \frac{2}{1+x^2}$

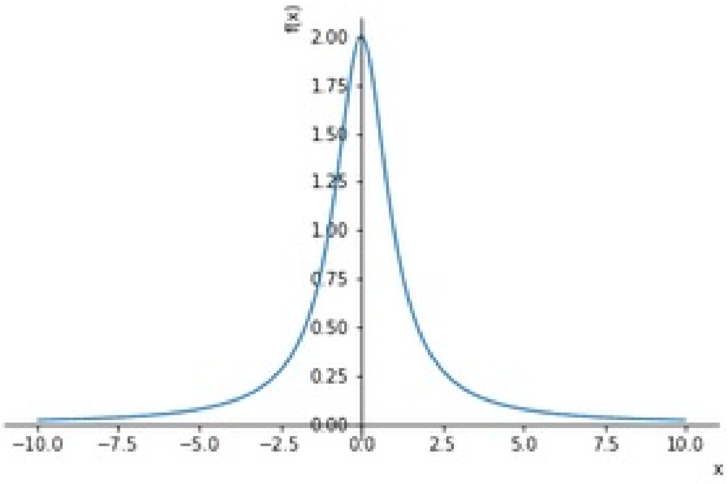
$$f(x) = \frac{2}{x^2+1}$$

$$f'(x) = -\frac{4x}{\left(x^2+1\right)^2}$$

$$f''(x) = \frac{4(3x^2-1)}{\left(x^2+1\right)^3}$$

$$f'(0) = 0 \wedge f''(0) = -4$$

Hay un máximo relativo en $(0, 2)$



v) $f(x) = \dfrac{8x}{x^2 + 2}$

$$f(x) = \frac{8x}{x^2+2}$$

$$f'(x) = \frac{8(2-x^2)}{x^4+4x^2+4}$$

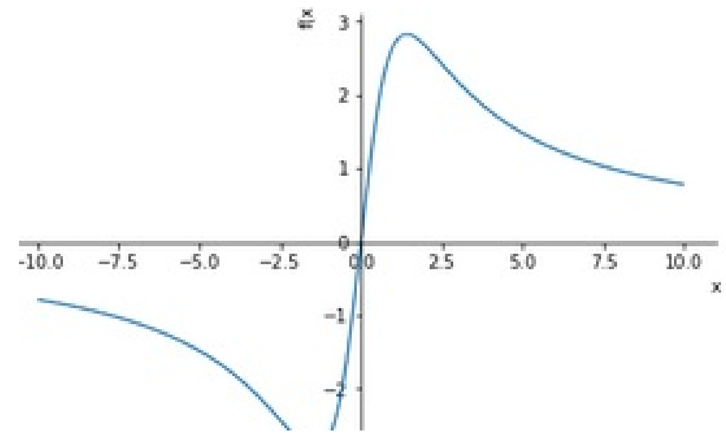
$$f''(x) = \frac{16x(x^2-6)}{(x^2+2)^3}$$

$$f'(-\sqrt{2}) = 0 \wedge f''(-\sqrt{2}) = \sqrt{2}$$

Hay un mínimo relativo en $(-\sqrt{2}, -2\sqrt{2})$

$$f'(\sqrt{2}) = 0 \wedge f''(\sqrt{2}) = -\sqrt{2}$$

Hay un máximo relativo en $(\sqrt{2}, 2\sqrt{2})$



vi) $f(x) = x.\ln x$

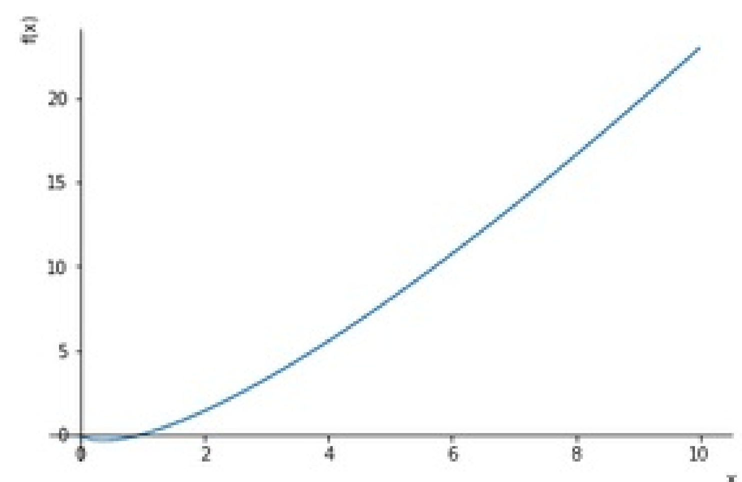
$$f(x) = x\log(x)$$

$$f'(x) = \log(x) + 1$$

$$f''(x) = \frac{1}{x}$$

$$f'(e^{-1}) = 0 \wedge f''(e^{-1}) = e$$

Hay un mínimo relativo en $(e^{-1}, -\frac{1}{e})$



vii) $f(x) = x^2.e^x$

$$f(x) = x^2 e^x$$

$$f'(x) = x(x+2)e^x$$

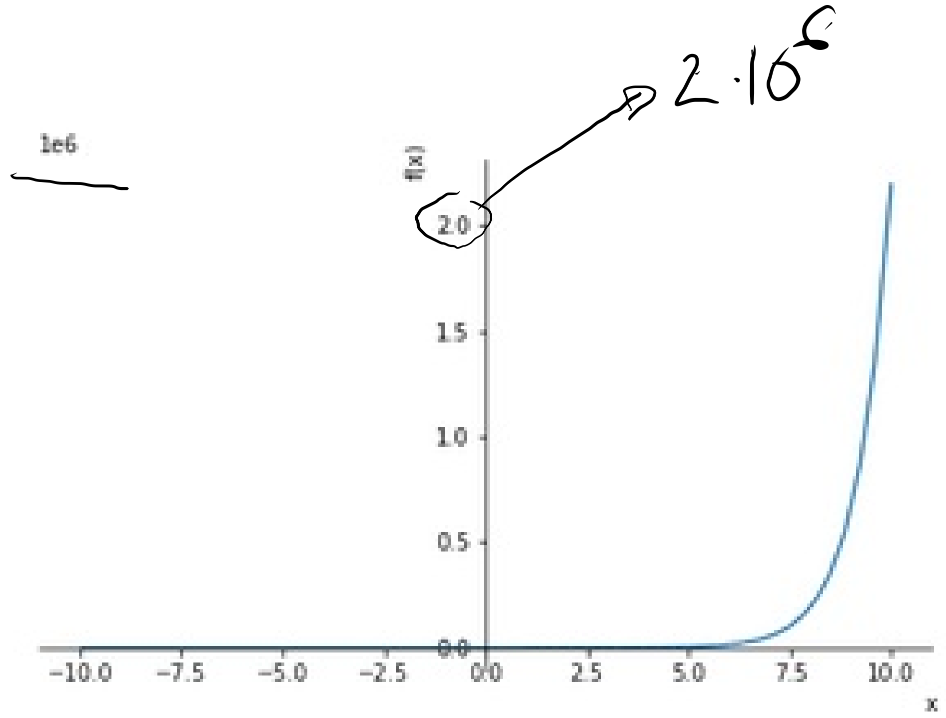
$$f''(x) = \left(x^2 + 4x + 2\right)e^x$$

$$f'(-2) = 0 \wedge f''(-2) = -\frac{2}{e^2}$$

Hay un máximo relativo en $\left(-2, \frac{4}{e^2}\right)$

$$f'(0) = 0 \wedge f''(0) = 2$$

Hay un mínimo relativo en $(0, 0)$



viii) $f(x) = \dfrac{x}{\ln x}$

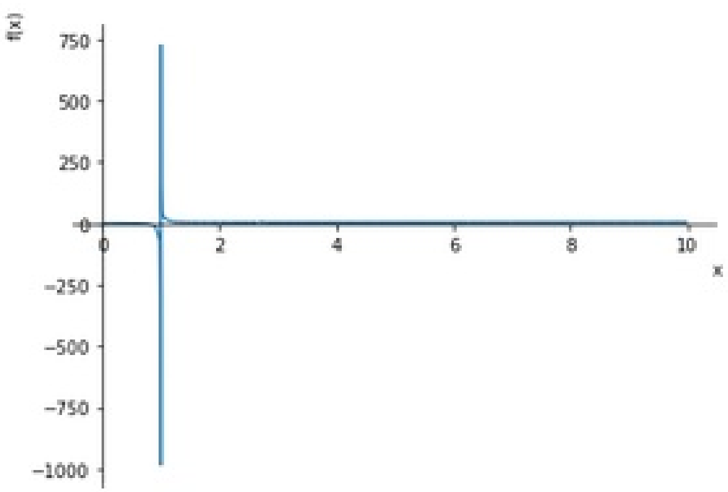
$$f(x) = \frac{x}{\log(x)}$$

$$f'(x) = \frac{\log(x) - 1}{\log(x)^2}$$

$$f''(x) = \frac{2 - \log(x)}{x \log(x)^3}$$

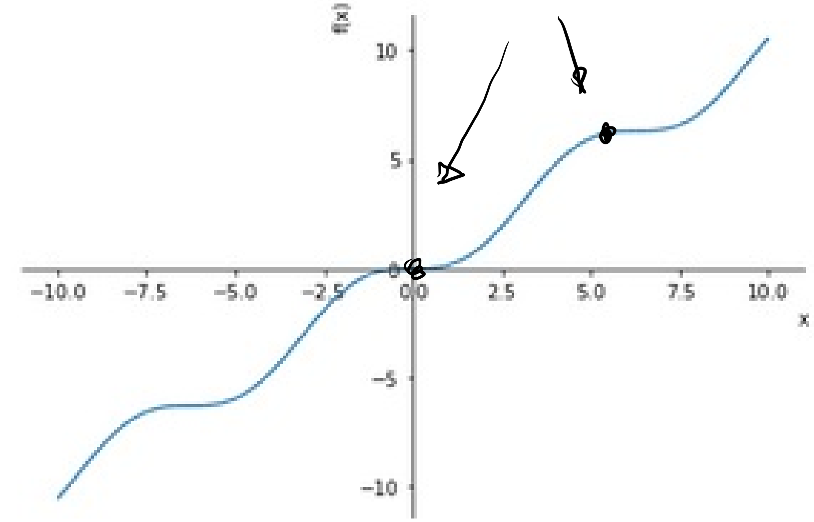
$$f'(e) = 0 \wedge f''(e) = e^{-1}$$

Hay un mínimo relativo en (e, e)



ix) f(x) = x – sen x

Pto de inflexión



$$f(x) = x - \sin(x)$$

$$f'(x) = 1 - \cos(x)$$

$$f''(x) = \sin(x)$$

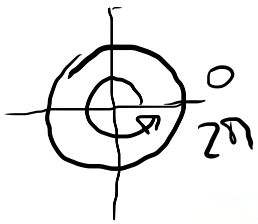
Candidatos en la 1ª Vuelta

$$f'(0) = 0 \wedge f''(0) = 0$$

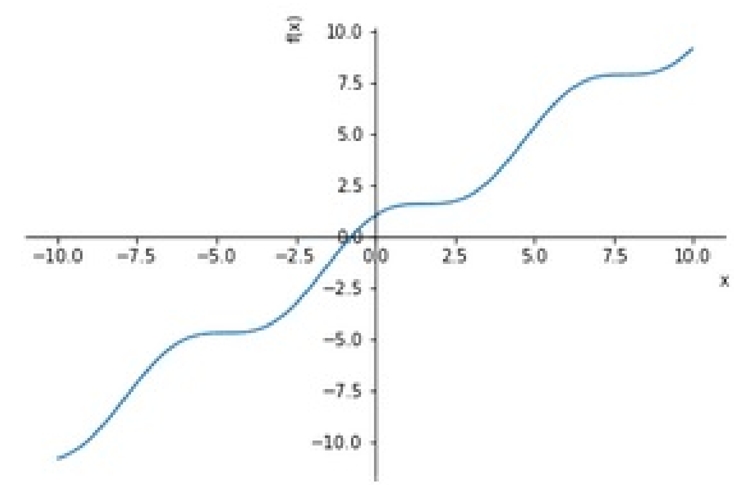
No hay mínimo ni máximo en $(0, 0)$

$$f'(2\pi) = 0 \wedge f''(2\pi) = 0$$

No hay mínimo ni máximo en $(2\pi, 2\pi)$



x) f(x) = x + cos x



$$f(x) = x + \cos(x)$$

$$f'(x) = 1 - \sin(x)$$

$$f''(x) = -\cos(x)$$

$$f'(\tfrac{\pi}{2}) = 0 \wedge f''(\tfrac{\pi}{2}) = 0$$

No hay mínimo ni máximo en $(\frac{\pi}{2}, \frac{\pi}{2})$

xi) f(x) = 2.sen x – sen 2x

$$f(x) = 2\sin(x) - \sin(2x)$$

$$f'(x) = 2\cos(x) - 2\cos(2x)$$

$$f''(x) = 2(4\cos(x) - 1)\sin(x)$$

$$f'(0) = 0 \wedge f''(0) = 0$$

No hay minimo ni máximo en $(0, 0)$

$$f'(-\tfrac{4\pi}{3}) = 0 \wedge f''(-\tfrac{4\pi}{3}) = -3\sqrt{3}$$

Hay un máximo relativo en $(-\frac{4\pi}{3}, \frac{3\sqrt{3}}{2})$

$$f'(-\tfrac{2\pi}{3}) = 0 \wedge f''(-\tfrac{2\pi}{3}) = 3\sqrt{3}$$

Hay un minimo relativo en $(-\frac{2\pi}{3}, -\frac{3\sqrt{3}}{2})$

$$f'(\tfrac{2\pi}{3}) = 0 \wedge f''(\tfrac{2\pi}{3}) = -3\sqrt{3}$$

Hay un máximo relativo en $(\frac{2\pi}{3}, \frac{3\sqrt{3}}{2})$

$$f'(\tfrac{4\pi}{3}) = 0 \wedge f''(\tfrac{4\pi}{3}) = 3\sqrt{3}$$

Hay un minimo relativo en $(\frac{4\pi}{3}, -\frac{3\sqrt{3}}{2})$

$$f'(2\pi) = 0 \wedge f''(2\pi) = 0$$

No hay minimo ni máximo en $(2\pi, 0)$