Departamento de Matemáticas 1º Bachillerato Derivadas



1. p80e16 - Calcula las siguientes derivadas:

(a)
$$y = 2x$$

Sol:
$$y' = 2$$

(b)
$$y = 3x - 5$$

Sol:
$$y' = 3$$

(c)
$$y = 7x^5 - 3x^2 + x + 2345$$

Sol:
$$y' = 35x^4 - 6x + 1$$

(d)
$$y = x(x+2)$$

Sol:
$$y' = 2x + 2$$

(e)
$$y = (x-1)(x+1)$$

Sol:
$$y' = 2x$$

(f)
$$y = \frac{5x^4}{7} - \frac{x^3}{55} - \frac{3x^2}{4} + x - 1255$$

Sol:
$$y' = \frac{20x^3}{7} - \frac{3x^2}{55} - \frac{3x}{2} + 1$$

(g)
$$y = (x+1)^3$$

Sol:
$$y' = 3(x+1)^2$$

(h)
$$y = (x^3 + x + 1)^4$$

Sol:
$$y' = (12x^2 + 4)(x^3 + x + 1)^3$$

(i)
$$y = -(3x-1)^2 + (3x+1)^2$$

Sol:
$$y' = 12$$

$$(j) \quad y = \frac{1}{x^2}$$

Sol:
$$y' = -\frac{2}{x^3}$$

$$(k) \quad y = \frac{1}{x+1}$$

Sol:
$$y' = -\frac{1}{(x+1)^2}$$

(1)
$$y = \frac{x^2 - 3}{x^3 + x}$$

Sol:
$$y' = \frac{-x^4 + 10x^2 + 3}{x^2(x^4 + 2x^2 + 1)}$$

(m)
$$y = \frac{x+1}{x}$$

Sol:
$$y' = -\frac{1}{x^2}$$

(n)
$$y = \frac{xx^2(x^2-1)}{3} - 3$$

Sol:
$$y' = \frac{5x^4}{3} - x^2$$

$$(\tilde{\mathbf{n}}) \quad y = \frac{1}{x^3}$$

Sol:
$$y' = -\frac{3}{x^4}$$

(o)
$$y = x^{\frac{1}{2}}$$

Sol:
$$y' = \frac{1}{2\sqrt{x}}$$

(p)
$$y = x^{\frac{2}{3}}$$

Sol:
$$y' = \frac{2}{3\sqrt[3]{x}}$$

(q)
$$y = x^{\frac{-2}{3}}$$

Sol:
$$y' = -\frac{2}{3x^{\frac{5}{3}}}$$

(r)
$$y = x^{\frac{1}{2}} + x^{\frac{1}{5}} + x^{\frac{1}{6}}$$

Sol:
$$y' = \frac{\frac{x\frac{49}{30}}{2} + \frac{x\frac{13}{10}}{6} + \frac{x^{\frac{4}{3}}}{5}}{\frac{32}{x^{\frac{15}{15}}}}$$

(s)
$$y = \sqrt{3}\sqrt{x}$$

Sol:
$$y' = \frac{\sqrt{3}}{2\sqrt{x}}$$

2. p80e16-cont - Calcula las siguientes derivadas:

(a)
$$y = \frac{x^3}{\sqrt{x}}$$

Sol:
$$y' = \frac{5x^{\frac{3}{2}}}{2}$$

(b)
$$y = x^3 x^{\frac{1}{3}}$$

Sol:
$$y' = \frac{10x^{\frac{7}{3}}}{3}$$

(c)
$$y = \frac{\sqrt{x}}{x}$$

Sol:
$$y' = -\frac{1}{2x^{\frac{3}{2}}}$$

(d)
$$y = (1 - x^2)^3$$

Sol:
$$y' = -6x(x^2 - 1)^2$$

(e)
$$y = \sqrt{2x - 4}$$

Sol:
$$y' = \frac{\sqrt{2}}{2\sqrt{x-2}}$$

(f)
$$y = \sqrt{2 - x}$$

Sol:
$$y' = -\frac{1}{2\sqrt{2-x}}$$

(g)
$$y = \sqrt[3]{2}\sqrt[3]{x^2}$$

Sol:
$$y' = \frac{2\sqrt[3]{2} \operatorname{sign}(x)}{3\sqrt[3]{|x|}}$$

(h)
$$y = \sqrt{3x^2 - 1}$$

Sol:
$$y' = \frac{3x}{\sqrt{3x^2-1}}$$

(i)
$$y = \frac{2x}{\sqrt{x-1}}$$

Sol:
$$y' = \frac{x-2}{(x-1)^{\frac{3}{2}}}$$

$$(j) \quad y = \sqrt{\frac{1-x}{x+1}}$$

Sol:
$$y' = \frac{\sqrt{\frac{1-x}{x+1}}}{x^2-1}$$

$$(k) \quad y = e^{2x}$$

Sol:
$$y' = 2e^{2x}$$

(l)
$$y = 2^{5x}$$

Sol:
$$y' = 5 \cdot 32^x \log(2)$$

(m)
$$y = 8^{3x^2 - 1}$$

Sol:
$$y' = 9 \cdot 2^{9x^2 - 2} x \log(2)$$

(n)
$$y = a^x x^a$$

Sol:
$$y' = a^x x^{a-1} (a + x \log(a))$$

(
$$\tilde{\mathbf{n}}$$
) $y = e^{\sqrt{x}}$

Sol:
$$y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

(o)
$$y = \frac{\log(2x-1)}{\log(10)}$$

Sol:
$$y' = \frac{2}{(2x-1)\log(10)}$$

$$(p) \quad y = \log(x+3)$$

Sol:
$$y' = \frac{1}{x+3}$$

(q)
$$y = \cos(5x)$$

Sol:
$$y' = -5\sin(5x)$$

(r)
$$y = 3\tan(2x)$$

Sol:
$$y' = \frac{6}{\cos^2{(2x)}}$$

$$(s) \quad y = \sin^2(x)$$

Sol: $y' = \sin(2x)$

(t)
$$y = \sin(x^2)$$

Sol: $y' = 2x \cos(x^2)$

3. p80e17 - Calcula las siguientes derivadas:

(a) $y = \log(3x^2 - 7)$

Sol: $y' = \frac{6x}{3x^2 - 7}$

(b) $y = \log\left((x-2)^2\right)$

Sol: $y' = \frac{2}{x-2}$

(c) $y = \frac{\log(x^2 - 2x)}{\log(10)}$

Sol: $y' = \frac{2(x-1)}{x(x-2)\log(10)}$

(d) $y = \frac{\log(2x^3 + 3x^2)}{\log(2)}$

Sol: $y' = \frac{6(x+1)}{x(2x+3)\log(2)}$

(e) $y = \sqrt{\log(x)}$

Sol: $y' = \frac{1}{2x\sqrt{\log(x)}}$

 $(f) \quad y = \frac{\log(x)}{x}$

Sol: $y' = \frac{1 - \log(x)}{x^2}$

(g) $y = \log\left(\frac{1-x}{x+1}\right)$

Sol: $y' = \frac{2}{x^2 - 1}$

 $(h) \quad y = \log\left(x^{\frac{3}{4}}\right)$

Sol: $y' = \frac{3}{4x}$

(i) $y = \frac{\log(2x+1)}{\log(4)}$

Sol: $y' = \frac{1}{(2x+1)\log(2)}$

 $(j) \quad y = \log\left(\frac{e^x}{e^x - 1}\right)$

Sol: $y' = \frac{1}{1 - e^x}$

 $(k) \quad y = \frac{1 - \log(x)}{\log(x) + 1}$

Sol: $y' = -\frac{2}{x(\log(x)+1)^2}$

(l) $y = \frac{e^x}{x-1}$

Sol: $y' = \frac{(x-2)e^x}{x^2 - 2x + 1}$

(m) $y = e^{-x} + \frac{e^x - e^{-x}}{e^x}$

Sol: $y' = (2 - e^x) e^{-2x}$

(n) $y = e^{\sqrt{x^2+1}}$

Sol: $y' = \frac{xe^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

 $(\tilde{\mathbf{n}})$ $y = \sin(2x)$

Sol: $y' = 2\cos(2x)$

(o) $y = \sin(7x - 3)$

Sol: $y' = 7\cos(7x - 3)$

(p) $y = \cos(5x)$

Sol: $y' = -5\sin(5x)$

(q) $y = 3\tan(2x)$

Sol: $y' = \frac{6}{\cos^2{(2x)}}$

 $(\mathbf{r}) \quad y = \sin^2\left(x\right)$

Sol:
$$y' = \sin(2x)$$

(s) $y = \sin(x^2)$

4. p80e17-cont - Calcula las siguientes derivadas:

(a)
$$y = \cos^2(x^2 + 1)$$

Sol: $y' = -4x \sin(x^2 + 1) \cos(x^2 + 1)$

(b) $y = \tan^3(5x)$

Sol:
$$y' = (15 \tan^2 (5x) + 15) \tan^2 (5x)$$

(c) $y = \sin^3(4x)$

Sol:
$$y' = 12\sin^2(4x)\cos(4x)$$

(d) $y = \sqrt{\sin(2x)}$

Sol:
$$y' = \frac{\cos(2x)}{\sqrt{\sin(2x)}}$$

(e) $y = \log(-\tan(x-1))$

Sol:
$$y' = -\frac{-\tan^2(x-1)-1}{\tan(x-1)}$$

(f) $y = \sqrt[3]{\sin(x)}$

Sol:
$$y' = \frac{\cos(x)}{3\sin^{\frac{2}{3}}(x)}$$

(g) $y = \sin^3(x)\cos(x)$

Sol:
$$y' = -\sin^4(x) + 3\sin^2(x)\cos^2(x)$$

(h) $y = \sec(5x + 2)$

Sol:
$$y' = 5 \tan(5x + 2) \sec(5x + 2)$$

(i) $y = a\sin(2x)$

Sol:
$$y' = \frac{2}{\sqrt{1-4x^2}}$$

Sol:
$$y' = 2x \cos(x^2)$$

(j) $y = a\cos(x^2)$

Sol:
$$y' = -\frac{2x}{\sqrt{1-x^4}}$$

(k) $y = \operatorname{atan}\left(\frac{x-1}{1-x}\right)$

Sol:
$$y' = \frac{\frac{1}{1-x} + \frac{x-1}{(1-x)^2}}{1 + \frac{(x-1)^2}{(1-x)^2}}$$

(1) $y = \operatorname{asin}\left(\frac{x+1}{x-1}\right)$

Sol:
$$y' = \frac{\frac{1}{x-1} - \frac{x+1}{(x-1)^2}}{\sqrt{1 - \frac{(x+1)^2}{(x-1)^2}}}$$

(m) $y = \tan^2(\sin(x))$

Sol:
$$y' = 2 (\tan^2 (\sin (x)) + 1) \cos (x) \tan (\sin (x))$$

 $(n) \quad y = \sin^{\frac{1}{x}}(x)$

Sol:
$$y' = \left(\frac{\cos(x)}{x\sin(x)} - \frac{\log(\sin(x))}{x^2}\right)\sin^{\frac{1}{x}}(x)$$

 $(\tilde{\mathbf{n}}) \quad y = x^{\tan{(x)}}$

Sol:
$$y' = x^{\tan(x)} \left(\left(\tan^2(x) + 1 \right) \log(x) \right) + \frac{\tan(x)}{x} \right)$$

(o) $y = 2^{\log(\cos(x))}$

Sol:
$$y' = -\frac{2^{\log(\cos(x))}\log(2)\sin(x)}{\cos(x)}$$

 $(p) \quad y = \sin^{\operatorname{atan}(x)}(x)$

Sol:
$$y' = \left(\frac{\cos(x) \tan(x)}{\sin(x)} + \frac{\log(\sin(x))}{x^2 + 1}\right) \sin^{\tan(x)}(x)$$

 $(\mathbf{q}) \quad y = \operatorname{atan}^{x}(x)$

Sol:
$$y' = \left(\frac{x}{(x^2+1)\operatorname{atan}(x)} + \log(\operatorname{atan}(x))\right) \operatorname{atan}^x \left(\text{Sol: } y' = x^{\sec(x)} \left(\log(x)\operatorname{tan}(x)\sec(x) + \frac{\sec(x)}{x}\right)\right)$$

- $(\mathbf{r}) \quad y = x^{\sec(x)}$
- 5. p81e18 Calcula las siguientes derivadas:

(a)
$$y = x^{\log(x)}$$

Sol:
$$y' = \frac{2x^{\log(x)}\log(x)}{x}$$

(b)
$$y = x^{\frac{1}{x}}$$

Sol:
$$y' = x^{\frac{1}{x}} \left(-\frac{\log(x)}{x^2} + \frac{1}{x^2} \right)$$

(c)
$$y = \cos^{\sin(x)}(x)$$

Sol:
$$y' = \left(\log\left(\cos\left(x\right)\right)\cos\left(x\right) - \frac{\sin^2\left(x\right)}{\cos\left(x\right)}\right)\cos^{\sin\left(x\right)}\left(x\right)$$