Departamento de Matemáticas
1º Bachillerato
23 - Trigonometría

1. p039e01 - Expresa en radianes los siguientes ángulos, dados en grados:

   (a) 45º

   Sol: $\frac{\pi}{4}$

   (b) 75º

   Sol: $\frac{5\pi}{12}$

   (c) 105º

   Sol: $\frac{7\pi}{12}$

   (d) 230º

   Sol: $\frac{23\pi}{18}$

2. p039e02 - Expresa en grados los siguientes ángulos dados en radianes:

   (a) $\frac{3\pi}{4}$

   Sol: 135

   (b) $\frac{5\pi}{3}$

   Sol: 300

   (c) $\frac{3\pi}{2}$

   Sol: 270

   (d) $\frac{9\pi}{2}$

   Sol: 810

   (e) $\frac{4\pi}{3}$

   Sol: 240

3. p039e05y6 - Demostrar si son verdaderas o falsas las siguientes ecuaciones:

   (a) $\sec^2\alpha + \csc^2\alpha = \sec^2\alpha \cdot \csc^2\alpha$

   Sol: $\left[\frac{8}{-\cos(4\alpha)+1}, \quad \frac{8}{-\cos(4\alpha)+1}\right] \to \text{True}$

   (b) $\frac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} = \tan\alpha \cdot \tan\beta$

   Sol: $[\tan(\alpha)\tan(\beta), \quad \tan(\alpha)\tan(\beta)] \to \text{True}$

   (c) $\frac{\sin\alpha \cdot \cos\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{\tan\alpha}{1-\tan^2\alpha}$

   Sol: $\left[\frac{\tan(2\alpha)}{2}, \quad \frac{\tan(2\alpha)}{2}\right] \to \text{True}$

   (d) $\cot\alpha - \frac{\cot^2\alpha - 1}{\cot\alpha} = \tan\alpha$

   Sol: $[\tan(\alpha), \quad \tan(\alpha)] \to \text{True}$

   (e) $\frac{\sin\alpha + \cot\alpha}{\tan\alpha + \csc\alpha} = \cos\alpha$

   Sol: $[\cos(\alpha), \quad \cos(\alpha)] \to \text{True}$

   (f) $\cot^2\alpha - \cos^2\alpha = \cot^2\alpha \cdot \cos^2\alpha$

Sol: $\left[-\cos^2(\alpha)+\cot^2(\alpha), \quad \cos^2(\alpha)\cot^2(\alpha)\right] \to \text{True}$

(g) $\sin\alpha\cos\alpha\tan\alpha\cot\alpha\sec\alpha\csc\alpha = 1$

Sol: $[1, \quad 1] \to \text{True}$

(h) $\frac{1+\tan\alpha}{1-\tan\alpha} = \frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}$

Sol: $\left[\frac{\tan(\alpha)+1}{-\tan(\alpha)+1}, \quad \tan\left(\alpha+\frac{\pi}{4}\right)\right] \to \text{True}$

(i) $\frac{1+\tan^2\alpha}{\cot\alpha} = \frac{\tan\alpha}{\cos^2\alpha}$

Sol: $\left[\frac{\tan(\alpha)}{\cos^2(\alpha)}, \quad \frac{\tan(\alpha)}{\cos^2(\alpha)}\right] \to \text{True}$

4. p039e07 - Simplificar las siguientes expresiones:

(a) $\sin\alpha \cdot \frac{1}{\tan\alpha}$

Sol: $\cos(\alpha)$

(b) $\sin^3\alpha + \sin\alpha\cdot\cos^2\alpha$

Sol: $\sin(\alpha)$

(c) $\sqrt{(1-\sin\alpha)\cdot(1+\sin\alpha)}$

Sol: $\sqrt{\cos^2(\alpha)}$

(d) $\sin^4\alpha - \cos^4\alpha$

Sol: $-\cos(2\alpha)$

(e) $\cos^3\alpha + \cos^2\alpha\cdot\sin\alpha + \cos\alpha\cdot\sin^2\alpha + \sin^3\alpha$

Sol: $\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)$

(f) $\sin\alpha\cdot\cos\alpha\cdot(\tan\alpha + \frac{1}{\tan\alpha})$

Sol: 1

(g) $\frac{\cos^2\alpha-\sin^2\alpha}{\cos^4\alpha-\sin^4\alpha}$

Sol: 1

(h) $\frac{\sec^2\alpha+\cos^2\alpha}{\sec^2\alpha-\cos^2\alpha}$

Sol: $\frac{\left(-\cos^2(\alpha)+1\right)^2+2\cos^2(\alpha)}{-\cos^4(\alpha)+1}$

(i) $\frac{\cos^2\alpha}{1-\sin\alpha}$

Sol: $\sin(\alpha)+1$

(j) $\frac{\csc\alpha}{1+\cot^2\alpha}$

Sol: $\sin(\alpha)$

5. p039e08 - Calcular las restantes razones trigonométricas de α, conocida:

(a) $\cos\alpha=\frac{4}{5}\wedge\alpha\in I$

Sol: $\left[36{,}86989764584401,\quad \frac{3}{5},\quad \frac{4}{5},\quad \frac{3}{4}\right]$

(b) $\sin\alpha=\frac{3}{5}\wedge\alpha\in II$

Sol: $\left[36{,}86989764584402,\quad \frac{3}{5},\quad -\frac{4}{5},\quad -\frac{3}{4}\right]$

(c) $\tan\alpha=-\frac{3}{4}\wedge\alpha\in II$

Sol: $\left[36{,}86989764584402,\quad \frac{3}{5},\quad -\frac{4}{5},\quad -\frac{3}{4}\right]$

(d) $\sec\alpha=2\wedge\alpha\in IV$

Sol: $\left[60{,}0,\quad -\frac{\sqrt{3}}{2},\quad \frac{1}{2},\quad -\sqrt{3}\right]$

(e) $\csc\alpha=-2\wedge\alpha\in III$

Sol: $\left[30{,}0,\quad -\frac{1}{2},\quad -\frac{\sqrt{3}}{2},\quad \frac{\sqrt{3}}{3}\right]$

(f) $\cot\alpha=-2\wedge\alpha\in IV$

Sol: $\left[26{,}56505117707799,\quad -\frac{\sqrt{5}}{5},\quad \frac{2\sqrt{5}}{5},\quad -\frac{1}{2}\right]$

6. p039e09 - Expresa las siguientes razones trigonométricas en función de ángulos del primer cuadrante:

(a) $\sin(-120)$

Sol: $\left[-60, \quad -\frac{\sqrt{3}}{2}\right]$

(b) $\sin(2700)$

Sol: $[0, \quad 0]$

(c) $\cos(-30)$

Sol: $\left[30, \quad \frac{\sqrt{3}}{2}\right]$

(d) $\cos(3000)$

Sol: $\left[120, \quad -\frac{1}{2}\right]$

(e) $\tan(-275)$

Sol: $\left[\frac{180\,\operatorname{atan}\left(\frac{\cos\left(\frac{\pi}{18}\right)+1}{\cos\left(\frac{4\pi}{9}\right)}\right)}{\pi}, \quad \frac{\cos\left(\frac{\pi}{18}\right)+1}{\cos\left(\frac{4\pi}{9}\right)}\right]$

(f) $\tan(10330)$

Sol: $\left[70, \quad \tan\left(\frac{7\pi}{18}\right)\right]$

(g) $\cot(-150)$

Sol: $\left[30, \quad \sqrt{3}\right]$

(h) $\cot(4500)$

Sol: $[0, \quad \tilde{\infty}]$

(i) $\sec(-25)$

Sol: $\left[25, \quad \sec\left(\frac{5\pi}{36}\right)\right]$

(j) $\sec(745)$

Sol: $\left[25, \quad \sec\left(\frac{149\pi}{36}\right)\right]$

(k) $\csc(-155)$

Sol: $\left[\frac{180 \operatorname{acsc}\left(-\csc\left(\frac{5\pi}{36}\right)\right)}{\pi}, \quad -\csc\left(\frac{5\pi}{36}\right)\right]$

(l) $\csc(4420)$

Sol: $\left[80, \quad \csc\left(\frac{4\pi}{9}\right)\right]$