1. ae01-0 - Halla analíticamente el dominio de las siguientes funciones:

   (a) $f(x) = \frac{x+1}{\sqrt{x^2+1}}$

   Sol: $Dom\,(f) = (-\infty, \infty)$

   (b) $f(x) = \sqrt{\frac{x-1}{x}}$

   Sol: $Dom\,(f) = (-\infty, 0) \cup [1, \infty)$

   (c) $f(x) = \frac{1}{4x^2-1}$

   Sol: $Dom\,(f) = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

   (d) $f(x) = \ln x^2 - 3$

   Sol: $Dom\,(f) = (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

2. ae02-0 - Dadas las funciones $f(x) = x^3 + 2$, $g(x) = \frac{x+1}{x-3}$ y $h(x) = \sqrt{x-1}$. Calcula:

   (a) $g \circ f$

   Sol: $g(f(x)) = \frac{x^3+3}{x^3-1}$

   (b) $f \circ g$

   Sol: $f(g(x)) = 2 + \frac{(x+1)^3}{(x-3)^3}$

   (c) $h \circ g \circ f$

   Sol: $h(g(f(x))) = 2\sqrt{\frac{1}{x^3-1}}$

3. ae03 - Halla la función inversa de $f(x)$, siendo:

   (a) $f(x) = -\frac{1}{x+4}$

   Sol: $f^{-1}(x) = -4 - \frac{1}{x}$
   $f^{-1} \circ f(x) = x = x$

   (b) $f(x) = \frac{2x-1}{3x+4}$

   Sol: $f^{-1}(x) = -\frac{4x+1}{3x-2}$

   $f^{-1} \circ f(x) = \frac{-\frac{4(2x-1)}{3x+4} - 1}{\frac{3(2x-1)}{3x+4} - 2} = x$

   (c) $f(x) = E^{2x} + 5$

   Sol: $f^{-1}(x) = \log\left(-\sqrt{x-5}\right)$
   $f^{-1} \circ f(x) = \log\left(-e^{x}\right) = x + i\pi$

   (d) $f(x) = \log(3x+1)$

   Sol: $f^{-1}(x) = \frac{e^x}{3} - \frac{1}{3}$
   $f^{-1} \circ f(x) = x = x$

   (e) $f(x) = \sqrt{x^2-3}$

   Sol: $f^{-1}(x) = -\sqrt{x^2+3}$
   $f^{-1} \circ f(x) = -|x| = -|x|$

   (f) $f(x) = \sqrt{x^2-3}$

   Sol: $f^{-1}(x) = -\sqrt{x^2+3}$
   $f^{-1} \circ f(x) = -|x| = -|x|$

4. ae04 - Calcula los siguientes límites:

(a)

$$\lim_{x\to\infty}\left(\frac{2x^2-14x+12}{x^2-10x+4}\right)$$

Sol: 2

(b)

$$\lim_{x\to\infty}\left(\frac{(5x-4)\left(2x^2-3\right)}{2x^3-4x+1}\right)$$

Sol: 5

(c)

$$\lim_{x\to-1}\left(\frac{x^2-1}{x^2+3x+2}\right)$$

Sol: -2

(d)

$$\lim_{x\to 0}\left(\frac{2x^3+6x^2-3x}{2x^2+5x}\right)$$

Sol: $-\frac{3}{5}$

(e)

$$\lim_{x\to\infty}\left(\frac{2x^3+6x^2-3x}{2x^2+5x}\right)$$

Sol: ∞

(f)

$$\lim_{x\to-\infty}\left(\frac{2x^3+6x^2-3x}{2x^2+5x}\right)$$

Sol: $-\infty$

(g)

$$\lim_{x\to-\infty}\left(\frac{4x^2-x+3}{3x^2+x-3}\right)^{\frac{x}{1-x}}$$

Sol: $\frac{3}{4}$

(h)

$$\lim_{x\to\infty}\left(-x+\sqrt{x^3+x+1}\right)$$

Sol: ∞

(i)

$$\lim_{x\to\infty}\left(\frac{x^2+3}{3x^2-5}\right)^{\frac{x^2}{2-x}}$$

Sol: ∞

(j)

$$\lim_{x\to 3}\,(x-2)^{\frac{1}{x-3}}$$

Sol: e

5. ae05: - Halla a y b de modo que las siguientes funciones sean continuas:

(a)

$$f(x)=\begin{cases}1-2x & \text{for } x<-2\\ ax+2 & \text{for } x<2\\ b+x^2 & \text{otherwise}\end{cases}$$

Sol: $\left\{a:-\frac{3}{2},\ b:-5\right\}$

(b)

$$f(x)=\begin{cases}\log(x) & \text{for } x<1\\ ax^2+b & \text{otherwise}\end{cases}$$

Sol: $\{-b\}$

6. ae06: - Calcula el valor de k para que las siguientes funciones sean continuas:

(a)

$$f(x)=\begin{cases}\frac{e^{kx}}{x^2+2} & \text{for } x<0\\ 2kx+k+x^2 & \text{otherwise}\end{cases}$$

Sol: $\left\{\frac{1}{2}\right\}$

(b)

$$f(x) = \begin{cases} \log(x) & \text{for } x \leq 1 \\ kx^2 + 2 & \text{otherwise} \end{cases}$$

Sol: $\{-2\}$

(c)

$$f(x) = \begin{cases} kx + x^2 & \text{for } x \leq -2 \\ k - x^2 & \text{otherwise} \end{cases}$$

Sol: $\left\{\frac{8}{3}\right\}$

7. ae07 - Calcula las asíntotas de las funciones:

(a) $\frac{2x-4}{x+2}$

Sol: Asíntotas:
$x = -2$
$y = 2$
$y = 2$
$y = 2$
$y = 2$

(b) $\frac{x^2-3}{x-2}$

Sol: Asíntotas:
$x = 2$
$y = x + 2$
$y = x + 2$

8. ae08 - Calcula las siguientes derivadas:

(a) $y = 2x^5 - x^2 + 5x + 2$

Sol: $y' = 10x^4 - 2x + 5$

(b) $y = x(x+2)(x+3)$

Sol: $y' = x(x+2) + x(x+3) + (x+2)(x+3)$

(c) $y = \frac{x^3}{\sqrt{x^2-x}}$

Sol: $y' = \frac{x^3\left(\frac{1}{2}-x\right)}{(x^2-x)^{\frac{3}{2}}} + \frac{3x^2}{\sqrt{x^2-x}}$

(d) $y = x^4 x^{\frac{1}{4}}$

Sol: $y' = \frac{17x^{\frac{13}{4}}}{4}$

(e) $y = \frac{\sqrt{x^3}}{x}$

Sol: $y' = \frac{\sqrt{x^3}}{2x^2}$

(f) $y = \left(1 - x^2\right)^4$

Sol: $y' = -8x\left(1 - x^2\right)^3$

(g) $y = \sqrt[3]{2x^2 + 5x + 7}$

Sol: $y' = \frac{\frac{4x}{3} + \frac{5}{3}}{(2x^2+5x+7)^{\frac{2}{3}}}$

(h) $y = \sqrt{\frac{2-x}{x+2}}$

Sol: $y' = \frac{\sqrt{\frac{2-x}{x+2}}(x+2)\left(-\frac{2-x}{2(x+2)^2} - \frac{1}{2(x+2)}\right)}{2-x}$

(i) $y = e^{\sin(x)}$

Sol: $y' = e^{\sin(x)}\cos(x)$

(j) $y = 2^{5\cos(x)}$

Sol: $y' = -5 \cdot 2^{5\cos(x)}\log(2)\sin(x)$

(k) $y = 8^{3\tan^2(x)-1}$

Sol: $y' = 3 \cdot 8^{3\tan^2(x)-1}\left(2\tan^2(x)+2\right)\log(8)\tan(x)$

(l) $y = \log\left(\frac{2x-1}{2x+1}\right)$

Sol: $y' = \frac{(2x+1)\left(-\frac{2(2x-1)}{(2x+1)^2}+\frac{2}{2x+1}\right)}{2x-1}$

(m) $y = \cos^3\left(x^3+1\right)$

Sol: $y' = -9x^2\sin\left(x^3+1\right)\cos^2\left(x^3+1\right)$

(n) $y = \tan^3(5x)$

Sol: $y' = \left(15\tan^2(5x)+15\right)\tan^2(5x)$

(ñ) $y = \log(-\sin(x-1))$

Sol: $y' = \frac{\cos(x-1)}{\sin(x-1)}$

(o) $y = \sqrt[3]{\sin(x)}$

Sol: $y' = \frac{\cos(x)}{3\sin^{\frac{2}{3}}(x)}$

(p) $y = \sin^4(x)\cos(x)$

Sol: $y' = -\sin^5(x)+4\sin^3(x)\cos^2(x)$

(q) $y = 2^{\log(\cos(x))}$

Sol: $y' = -\frac{2^{\log(\cos(x))}\log(2)\sin(x)}{\cos(x)}$

(r) $y = \left(x^2\right)^{\log(\cos(x))}$

Sol: $y' = \left(-\frac{\log\left(x^2\right)\sin(x)}{\cos(x)}+\frac{2\log(\cos(x))}{x}\right)\left(x^2\right)^{\log(\cos(x))}$

(s) $y = \cos^{e^x}(x)$

Sol: $y' = \left(e^x\log(\cos(x))-\frac{e^x\sin(x)}{\cos(x)}\right)\cos^{e^x}(x)$

(t) $y = x^{\tan(x)}$

Sol: $y' = x^{\tan(x)}\left(\left(\tan^2(x)+1\right)\log(x)+\frac{\tan(x)}{x}\right)$

(u) $y = \cos^{\frac{1}{x}}(x)$

Sol: $y' = \left(-\frac{\sin(x)}{x\cos(x)}-\frac{\log(\cos(x))}{x^2}\right)\cos^{\frac{1}{x}}(x)$