

Departamento de Matemáticas $1^{\underline{0}}$ Bachillerato



23 - Trigonometría

1	2020201	Erranoge on	nodionos!	100	giguiontog	án milos	dadaa an	ano do a
Ι.	bosseor -	Expresa en	radianes.	108	signiemes	anguios,	dados en	grados.

(a) 45° Sol: $\frac{\pi}{4}$

Sol: $\frac{5\pi}{12}$

Sol: $\frac{7\pi}{12}$

(b) 75°

(b)

(d) 230°

Sol: $\frac{23\pi}{18}$

$2.\ p039e02$ - Expresa en grados los siguientes ángulos dados en radianes:

(a) $\frac{3\pi}{4}$ **Sol:** 135

Sol: 300

Sol: 270

Sol: 810

(c) $\frac{3\pi}{2}$

 $(c) \frac{}{2}$

(c) 105°

(e) $\frac{4\pi}{3}$

(d) $\frac{97}{2}$

Sol: 240

3. p039e05y6 - Demostrar si son verdaderas o falsas las siguientes ecuaciones:

(a)
$$\sec^2 \alpha + \csc^2 \alpha = \sec^2 \alpha \cdot \csc^2 \alpha$$

Sol:
$$\left[\frac{8}{-\cos(4\alpha)+1}, \frac{8}{-\cos(4\alpha)+1}\right] \to \text{True}$$

(b)
$$\frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \tan \alpha \cdot \tan \beta$$

Sol:
$$[\tan(\alpha)\tan(\beta), \tan(\alpha)\tan(\beta)] \rightarrow \text{True}$$

(c)
$$\frac{\sin \alpha \cdot \cos \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \frac{\tan \alpha}{1 - \tan^2 \alpha}$$

Sol:
$$\left[\frac{\tan{(2\alpha)}}{2}, \frac{\tan{(2\alpha)}}{2}\right] \to \text{True}$$

(d)
$$\cot \alpha - \frac{\cot^2 \alpha - 1}{\cot \alpha} = \tan \alpha$$

Sol:
$$[\tan{(\alpha)}, \tan{(\alpha)}] \to \text{True}$$

(e)
$$\frac{\sin \alpha + \cot \alpha}{\tan \alpha + \csc \alpha} = \cos \alpha$$

Sol:
$$[\cos(\alpha), \cos(\alpha)] \to \text{True}$$

(f)
$$\cot^2 \alpha - \cos^2 \alpha = \cot^2 \alpha \cdot \cos^2 \alpha$$

Sol:
$$\left[-\cos^{2}\left(\alpha\right)+\cot^{2}\left(\alpha\right), \cos^{2}\left(\alpha\right)\cot^{2}\left(\alpha\right)\right] \rightarrow True$$

(g) $\sin \alpha \cos \alpha \tan \alpha \cot \alpha \sec \alpha \csc \alpha = 1$

Sol:
$$[1, 1] \rightarrow True$$

(h) $\frac{1+\tan\alpha}{1-\tan\alpha} = \frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}$

Sol:
$$\left[\frac{\tan{(\alpha)}+1}{-\tan{(\alpha)}+1}, \tan{(\alpha+\frac{\pi}{4})}\right] \to \text{True}$$

(i) $\frac{1+\tan^2\alpha}{\cot\alpha} = \frac{\tan\alpha}{\cos^2\alpha}$

Sol:
$$\left[\frac{\tan{(\alpha)}}{\cos^2{(\alpha)}}, \frac{\tan{(\alpha)}}{\cos^2{(\alpha)}}\right] \to \text{True}$$

4. p039e07 - Simplificar las siguientes expresiones:

(a) $\sin \alpha \cdot \frac{1}{\tan \alpha}$

Sol:
$$\cos(\alpha)$$

(b) $\sin^3 \alpha + \sin \alpha \cdot \cos^2 \alpha$

Sol:
$$\sin(\alpha)$$

(c) $\sqrt{(1-\sin\alpha)\cdot(1+\sin\alpha)}$

Sol:
$$\sqrt{\cos^2{(\alpha)}}$$

(d) $\sin^4 \alpha - \cos^4 \alpha$

Sol:
$$-\cos(2\alpha)$$

(e) $\cos^3 \alpha + \cos^2 \alpha \cdot \sin \alpha + \cos \alpha \cdot \sin^2 \alpha + \sin^3 \alpha$

Sol:
$$\sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right)$$

(f) $\sin \alpha \cdot \cos \alpha \cdot (\tan \alpha + \frac{1}{\tan \alpha})$

 $(g) \quad \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^4 \alpha - \sin^4 \alpha}$

Sol: 1

(h) $\frac{\sec^2\alpha + \cos^2\alpha}{\sec^2\alpha - \cos^2\alpha}$

Sol:
$$\frac{\left(-\cos^2\left(\alpha\right)+1\right)^2+2\cos^2\left(\alpha\right)}{-\cos^4\left(\alpha\right)+1}$$

(i) $\frac{\cos^2 \alpha}{1-\sin \alpha}$

Sol:
$$\sin{(\alpha)} + 1$$

 $(j) \quad \frac{\csc\alpha}{1+\cot^2\alpha}$

Sol:
$$\sin(\alpha)$$

5. p
039e08 - Calcular las restantes razones trigonométricas de α , conocida:

(a) $\cos \alpha = \frac{4}{5} \land \alpha \in I$

Sol:
$$\begin{bmatrix} 36,86989764584401, & \frac{3}{5}, & \frac{4}{5}, & \frac{3}{4} \end{bmatrix}$$

(b) $\sin \alpha = \frac{3}{5} \land \alpha \in II$

Sol:
$$\begin{bmatrix} 36,86989764584402, & \frac{3}{5}, & -\frac{4}{5}, & -\frac{3}{4} \end{bmatrix}$$

(c) $\tan \alpha = -\frac{3}{4} \wedge \alpha \in II$

Sol:
$$\begin{bmatrix} 36,86989764584402, & \frac{3}{5}, & -\frac{4}{5}, & -\frac{3}{4} \end{bmatrix}$$

(d) $\sec \alpha = 2 \land \alpha \in IV$

Sol:
$$\begin{bmatrix} 60,0, & -\frac{\sqrt{3}}{2}, & \frac{1}{2}, & -\sqrt{3} \end{bmatrix}$$

(e) $\csc \alpha = -2 \wedge \alpha \in III$

Sol:
$$\begin{bmatrix} 30,0, & -\frac{1}{2}, & -\frac{\sqrt{3}}{2}, & \frac{\sqrt{3}}{3} \end{bmatrix}$$

(f) $\cot \alpha = -2 \wedge \alpha \in IV$

Sol:
$$\left[26,56505117707799, -\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}, -\frac{1}{2}\right]$$

6.~p039e09 - Expresa las siguientes razones trigonométricas en función de ángulos del primer cuadrante:

(a) $\sin(-120)$

Sol:
$$\left[60, -\frac{\sqrt{3}}{2} \right]$$

(g) $\cot(-150)$

Sol:
$$[30, \sqrt{3}]$$

(b) $\sin(2700)$

(h) $\cot(4500)$

Sol:
$$[0, \quad \tilde{\infty}]$$

(c) $\cos(-30)$

Sol:
$$[30, \frac{\sqrt{3}}{2}]$$

(i) $\sec(-25)$

Sol:
$$[25, \sec(\frac{5\pi}{36})]$$

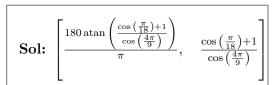
(d) $\cos(3000)$

Sol:
$$[60, -\frac{1}{2}]$$

(j) $\sec(745)$

Sol:
$$[25, \sec(\frac{149\pi}{36})]$$

(e) $\tan(-275)$



(k) $\csc(-155)$

Sol:
$$[25, -\csc(\frac{5\pi}{36})]$$

(f) $\tan(10330)$

Sol:
$$[70, \tan(\frac{7\pi}{18})]$$

(1) $\csc(4420)$

Sol:
$$[80, \csc(\frac{4\pi}{9})]$$

7. p
039e10 - Si sen $37^{0}=0,6$. Calcula, sin usar la calculadora, las razones trigonométricas de los siguientes ángulos dados en grados:

(a) 53

Sol: [0.8, -0.6, -1.33]

(c) 143

Sol:
$$[0,6, -0,8, -0,75]$$

- (b) 127
- 8. p041e27 Resolver las siguientes ecuaciones para ángulos en el primer cuadrante:
 - (a) $\sin 2x = \frac{1}{2}$

Sol:
$$\left[\frac{\pi}{12}, \frac{5\pi}{12}\right]$$

(b) $\tan \frac{x}{2} = \frac{\sqrt{3}}{3}$

Sol: $\left[\frac{\pi}{3}\right]$

(c) $\sin(3x - \frac{\pi}{2}) = -\frac{1}{2}$

Sol: $\left[\frac{\pi}{9}, \frac{5\pi}{9}\right]$

- 9. p041e28 Resolver las siguientes ecuaciones:
 - (a) $2\sin x + \csc x = 2\sqrt{2}$

Sol: [45, 135]

(b) $\sin x = \cos^2 x + 1$

Sol: [90]

(c) $\sin x \cos x = 0$

Sol: [0, 90, 180, 270]

(d) $\tan x - \sin x = 0$

Sol: [0, -180, 180, 360]

(e) $\sin x \cos x = 2 \sin x$

Sol: [0]

(f) $2\cos x - 3\tan x = 0$

Sol: $\left[150, 30, -\frac{180i\log\left(-i\left(-\sqrt{3}+2\right)\right)}{\pi}, -\frac{180i\log\left(-i\left(\sqrt{3}+2\right)\right)}{\pi}\right]$

(g) $\sin 2x = 2\cos x$

Sol: [-90, 90]

(h) $4\tan x = \frac{\sqrt{3}}{\cos^2 x}$

Sol: [-120, -150, 60, 30]

(i) $\sin x + \cos x = \sqrt{2}$

Sol: [45]

 $(j) \quad \sin 2x \cos x = 6 \sin^3 x$

Sol:
$$[0, 180, -150, 150, -30, 30]$$

 $(k) \quad 4\sin\frac{x}{2}\cos x = 3$

Sol: []

(1) $\tan x \tan 2x = 1$

 $(m) \quad 4\cos 2x + 3\cos x = 1$

Sol:
$$\left[180, -\frac{180i \log \left(\frac{5}{8} - \frac{\sqrt{39}i}{8} \right)}{\pi}, -\frac{180i \log \left(\frac{5}{8} + \frac{\sqrt{39}i}{8} \right)}{\pi} \right]$$

(n) $\tan x + 3 \cot x = 4$

Sol:
$$\left[45, \frac{180 \tan{(3)}}{\pi}\right]$$

(ñ) $4\sin(x-30)\cos(x-30) = \sqrt{3}$

Sol:
$$\left[\frac{180\left(-\frac{2\pi}{3}+30\right)}{\pi}, \frac{180\left(\frac{\pi}{6}+30\right)}{\pi}, \frac{180\left(\frac{\pi}{3}+30\right)}{\pi}, \frac{180\left(-2\operatorname{atan}\left(\sqrt{3}+2\right)+30\right)}{\pi}\right]$$

- 10. p042e01 Calcular los restantes elementos de un triángulo del que se conocen:
 - (a) El lado a = 6, y los ángulos $B=45^{\circ}$, $C=105^{\circ}$

Sol:
$$6\sqrt{2}$$
, $3\sqrt{2} + 3\sqrt{6}$, 30

(b) El lado a = 8, y los ángulos $B=30^{\circ}$, $C=60^{\circ}$

Sol:
$$4, 4\sqrt{3}, 90$$

- 11. p042e02 En el triángulo ABC se conocen:
 - (a) Los lados $a=10y b=7, y C=30^{\circ}$

Sol:
$$\sqrt{-70\sqrt{3}+149} = 5,268438428052338, 108,36878841450955, 41,63121158549045$$

- 12. p042e03 Determina si se puede construir un triángulo ABC sabiendo que
 - (a) El lado $a = 52 \text{ y b} = 32 \text{ y que B} = 40.5^{\circ}$.

Sol:
$$distancia_a = 33,77129851316955o$$

- 13. p
042e09 Tres puntos A, B y C están unidos por carreteras rectas y llanas. ¿Cuánto distan A y C?, si:
 - (a) La distancia AB es de 6km, la BC es=9km, y el ángulo que forman AB y BC es de 120°

Sol:
$$\sqrt{-108\cos(120) + 117} = 5{,}391516964930309$$