Departamento de Matemáticas 1º Bachillerato



7 - Sistemas de ecuaciones



1. p019e01 - Resuelve los sistemas:

(a)
$$\begin{cases} 3x - 2y = 1 \\ x + 6y = 7 \end{cases}$$

Sol:
$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & \frac{20}{3} & \frac{20}{3} \end{bmatrix} \rightarrow \{x:1, y:1\}$$

(b)
$$\begin{cases} 6x - 2y = 14\\ 3x - y = 7 \end{cases}$$

Sol:
$$\begin{bmatrix} 6 & -2 & 14 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \left\{ x : \frac{y}{3} + \frac{7}{3} \right\}$$

(c)
$$\begin{cases} 6x - 2y = 9 \\ 3x - y = 10 \end{cases}$$

Sol:
$$\begin{bmatrix} 6 & -2 & 9 \\ 0 & 0 & \frac{11}{2} \end{bmatrix} \rightarrow []$$

(d)
$$\begin{cases} 4x + 7y = -3 \\ 7x + 4y = 36 \end{cases}$$

Sol:
$$\begin{bmatrix} 4 & 7 & -3 \\ 0 & -\frac{33}{4} & \frac{165}{4} \end{bmatrix} \rightarrow \{x:8, y:-5\}$$
 (1) $\begin{cases} 10(x-2)+y=1 \\ x+3(x-y)=5 \end{cases}$

(e)
$$\begin{cases} 4x + 16 = 5y \\ 5y - 19 = 3x \end{cases}$$

Sol:
$$\begin{bmatrix} 4 & -5 & -16 \\ 0 & \frac{5}{4} & 7 \end{bmatrix} \rightarrow \left\{ x : 3, \quad y : \frac{28}{5} \right\}$$
 (m) $\overline{\left\{ \frac{x-y}{2} + \frac{x-y}{3} = 5 \\ \frac{x+7}{4} + y = 3 \right\}}$

(f)
$$\begin{cases} x - 5 = y + 2 \\ 1 + 3x + 2y = x - 4 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -1 & 7 \\ 0 & 4 & -19 \end{bmatrix} \rightarrow \left\{ x : \frac{9}{4}, \quad y : -\frac{19}{4} \right\}$$
 (n) $\begin{cases} \frac{3(y+2x+2)}{4} = \frac{4x+y-1}{3} \\ \frac{1}{3}(x+y) - \frac{1}{6}(x-y) = \frac{y-1}{6} \end{cases}$

(g)
$$\begin{cases} x - 5 = y + 2 \\ 3x - 2y = x - 5 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -1 & 7 \\ 0 & 0 & -19 \end{bmatrix} \rightarrow \begin{bmatrix} \end{bmatrix}$$

(h)
$$\begin{cases} x + 3y = 6 \\ 6y - 5 = 7 - 2x \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \{x : -3y + 6\}$$

(i)
$$\begin{cases} x - y = 8 \\ x + y = 24 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -1 & 8 \\ 0 & 2 & 16 \end{bmatrix} \rightarrow \{x : 16, y : 8\}$$

$$(j) \begin{cases} x + 2y = 11 \\ 2x - y = 2 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 2 & 11 \\ 0 & -5 & -20 \end{bmatrix} \rightarrow \{x:3, y:4\}$$

$$(k) \quad \begin{cases} 3x - 4y = -9\\ 2x + y = 5 \end{cases}$$

Sol:
$$\begin{bmatrix} 3 & -4 & -9 \\ 0 & \frac{11}{3} & 11 \end{bmatrix} \rightarrow \{x:1, y:3\}$$

(1)
$$\begin{cases} 10(x-2) + y = 1\\ x + 3(x-y) = 5 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \{x : 2, y : 1\}$$

m)
$$\begin{cases} \frac{x-y}{2} + \frac{x-y}{3} = 5\\ \frac{x+7}{4} + y = 3 \end{cases}$$

Sol:
$$\begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 5 \end{bmatrix} \rightarrow \left\{ x : \frac{29}{5}, \quad y : -\frac{1}{5} \right\}$$

$$\begin{cases} \frac{3(y+2x+2)}{4} = \frac{4x+y-1}{3} \\ \frac{1}{3}(x+y) - \frac{1}{6}(x-y) = \frac{y-1}{6} \end{cases}$$

Sol:
$$\begin{bmatrix} -\frac{4}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{24} & -\frac{1}{8} \end{bmatrix} \rightarrow \{x: 39, y: -20\}$$

(n)
$$\begin{cases} x - 2(x+y) = 3y - 2\\ \frac{x}{3} + \frac{y}{2} = 3 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & \frac{3}{2} & \frac{11}{3} \end{bmatrix} \rightarrow \{x : 12, y : -2\}$$

(o)
$$\begin{cases} \frac{3-2y}{4} - \frac{1}{4} = \frac{1-2x}{6} \\ \frac{25}{8} - 1 = \frac{x+3}{2} - \frac{3(1+y)}{8} \end{cases}$$
Sol:
$$\begin{bmatrix} \frac{3}{8} & \frac{x}{2} - \frac{25}{8} + \frac{17}{8} \\ 0 & -\frac{x}{2} + \frac{5}{8} \end{bmatrix} \to \{x:5, y:4\}$$

$$\begin{cases}
\frac{3-2y}{4} - \frac{1}{4} = \frac{1-2x}{6} \\
\frac{25}{8} - 1 = \frac{x+3}{2} - \frac{3(1+y)}{8}
\end{cases}$$
(p)
$$\begin{cases}
\frac{4y-5x}{6} + \frac{3x-2y}{2} = 1 - \frac{2}{9}(x+y) \\
\frac{4y+x-8}{8} - x = \frac{2(y-2x)}{3}
\end{cases}$$
Sol:
$$\begin{bmatrix}
\frac{3}{8} & \frac{x}{2} - \frac{25}{8} + \frac{17}{8} \\
0 & -\frac{x}{3} + \frac{5}{12}
\end{bmatrix}
\rightarrow \{x: 5, y: 4\}$$
(p)
$$\begin{cases}
\frac{4y-5x}{6} + \frac{3x-2y}{2} = 1 - \frac{2}{9}(x+y) \\
\frac{4y+x-8}{8} - x = \frac{2(y-2x)}{3}
\end{cases}$$
Sol:
$$\begin{bmatrix}
\frac{3}{8} & \frac{x}{2} - \frac{25}{8} + \frac{17}{8} \\
0 & -1 - \frac{3}{2}
\end{bmatrix}
\rightarrow \{x: \frac{4}{7}, y: -\frac{31}{7}\}$$

2. p021e23 - Resuelve los sistemas:

(a)
$$\begin{cases} x - 2y + 5z = 13 \\ 2x - 5y + z = 19 \\ x + 3y - 2z = -4 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 5 & -2 & 13 \\ 0 & -9 & -1 & -7 \\ 0 & 0 & \frac{52}{9} & -\frac{104}{9} \end{bmatrix}$$
$$\{x:4, y:-2, z:1\}$$

(b)
$$\begin{cases} x - y + z = 7 \\ x + y - 3z = 1 \\ 2x + y - 4z = 5 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 1 & -1 & 7 \\ 0 & -4 & 2 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\{x: z+4, \quad y: 2z-3\}$$

(c)
$$\begin{cases} x - 2y + z = 13 \\ 3x - 4y + 2z = 1 \\ 2x - 2y + z = 0 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 1 & -2 & 13 \\ 0 & -1 & 2 & -38 \\ 0 & 0 & 2 & -38 \end{bmatrix} \rightarrow \begin{bmatrix} \end{bmatrix}$$

(d)
$$\begin{cases} x - y + z = 1 \\ x + z = 4 \\ y - 3z = -15 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -3 & -18 \end{bmatrix} \rightarrow \{x: -2, y: 3, z: 6\}$$

(e)
$$\begin{cases} 2x - y + z = 6 \\ x + y - 2z = 1 \\ x - 2y + 3z = 0 \end{cases}$$

Sol:
$$\begin{bmatrix} 2 & 1 & -1 & 6 \\ 0 & -\frac{5}{2} & \frac{3}{2} & -2 \\ 0 & 0 & \frac{9}{5} & -\frac{12}{5} \end{bmatrix} \rightarrow []$$

(f)
$$\begin{cases} x + 2y - 3z = 9 \\ 2x - y = 6 \\ 4x + 3y - 6z = 24 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -3 & 2 & 9 \\ 0 & 6 & -5 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\left\{ x : \frac{3z}{5} + \frac{21}{5}, \quad y : \frac{6z}{5} + \frac{12}{5} \right\}$$

(g)
$$\begin{cases} 4x - 2y = 2\\ 6y - 3z = 1\\ 3z - 4x = -1 \end{cases}$$

Sol:
$$\begin{bmatrix} 4 & -2 & 0 & 2 \\ 0 & 6 & -3 & 1 \\ 0 & 0 & 2 & \frac{4}{3} \end{bmatrix} \rightarrow \left\{ x : \frac{3}{4}, \quad y : \frac{1}{2}, \quad z : \frac{2}{3} \right\}$$

(h)
$$\begin{cases} x + 2y = 5 \\ 2x + y = -1 \\ -x + 3y = 6 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -3 & -11 \\ 0 & 0 & -\frac{22}{3} \end{bmatrix} \rightarrow []$$

(i)
$$\begin{cases} x - 3y = 1\\ 4y - z = 1\\ 2x - z = 1 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 4 & -1 & 1 \\ 0 & 0 & \frac{1}{2} & -\frac{5}{2} \end{bmatrix}$$
$$\{x: -2, \quad y: -1, \quad z: -5\}$$

(j)
$$\begin{cases} x+2 = -y \\ -y+3 = 2x \\ 4x - y = 6 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & -21 \end{bmatrix} \to []$$

(k)
$$\begin{cases} x - 2y + 3z = 2\\ 2x - 3y + z = 1\\ 3x - y + 2z = 9 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & -5 & 1 & -3 \\ 0 & 0 & \frac{18}{5} & \frac{36}{5} \end{bmatrix}$$
$$\{x:3, y:2, z:1\}$$

(1)
$$\begin{cases} x - 6 + y = 0 \\ -3y + x - 2 = 0 \\ 5x - 26 + y = 0 \end{cases}$$

Sol:
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \{x:5, y:1\}$$

(m)
$$\begin{cases} 2x + 2y = -2 \\ x + 6 = y \\ 3x + 5y = 1 \end{cases}$$

Sol:
$$\begin{bmatrix} 2 & 2 & -2 \\ 0 & -2 & -5 \\ 0 & 0 & -1 \end{bmatrix} \to []$$

(n)
$$\begin{cases} x+y+z=4\\ x-2y+3z=13\\ x+3y+4z=11 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 2 & -3 & 9 \\ 0 & 0 & \frac{13}{2} & -\frac{13}{2} \end{bmatrix} \rightarrow \{x:2, y: -1, z:3\}$$

(n)
$$\begin{cases} z - 2(x+y) = -9\\ 3x - y = 3\\ 3y - z = 9 \end{cases}$$

Sol:
$$\begin{bmatrix} 3 & -1 & 0 & 3 \\ 0 & 3 & -1 & 9 \\ 0 & 0 & 1 & -9 \end{bmatrix}$$
$$\{x: 3, y: 6, z: 9\}$$

(o)
$$\begin{cases} \frac{x}{2} + \frac{y}{3} + z = 7\\ x - \frac{y}{2} + \frac{z}{3} = 11\\ \frac{x}{3} - y - \frac{z}{2} = 5 \end{cases}$$

Sol:
$$\begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{3} & 7\\ 0 & -\frac{5}{3} & -\frac{7}{6} & -3\\ 0 & 0 & -\frac{73}{180} & \frac{73}{30} \end{bmatrix}$$
$$\{x:6, y:-6, z:6\}$$

(p)
$$\begin{cases} \frac{x}{2} + \frac{y}{3} + \frac{z}{3} = 9\\ \frac{x}{3} - \frac{y}{9} + \frac{z}{3} = 6\\ \frac{x}{6} + \frac{y}{2} + \frac{z}{2} = 13 \end{cases}$$

Sol:
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & 9\\ 0 & \frac{1}{9} & -\frac{1}{3} & 0\\ 0 & 0 & \frac{14}{9} & 10 \end{bmatrix} \rightarrow \left\{ x : \frac{6}{7}, \quad y : \frac{45}{7}, \quad z : \frac{135}{7} \right\}$$

(q)
$$\begin{cases} x - y + z = 5\\ \frac{x-1}{2} + \frac{y}{3} = 1\\ \frac{2x+y}{6} - \frac{3z+y}{8} = 4 \end{cases}$$

Sol:
$$\begin{bmatrix} 1 & 1 & -1 & 5 \\ 0 & -\frac{3}{8} & -\frac{1}{8} & 4 \\ 0 & 0 & \frac{1}{3} & 1 \end{bmatrix} \rightarrow \left\{ x : \frac{177}{29}, \quad y \right\} : -\frac{135}{29}, \quad z : -\frac{135}{29}$$