

## Departamento de Matemáticas 2º Bachillerato

Integral indefinida



1. p38e01 - Calcula las siguientes integrales inmediatas:

(a) 
$$\int 4x^6 \cdot dx$$

**Sol:** 
$$\frac{4x^7}{7} + K$$

(b) 
$$\int 6x^3 + 8x^2 + 3 \cdot dx$$

**Sol:** 
$$\frac{3x^4}{2} + \frac{8x^3}{3} + 3x + K$$

(c) 
$$\int \sqrt{2}\sqrt{x} \cdot dx$$

**Sol:** 
$$\frac{2\sqrt{2}x^{\frac{3}{2}}}{3} + K$$

(d) 
$$\int \frac{1}{\sqrt[5]{x}} \cdot dx$$

**Sol:** 
$$\frac{5x^{\frac{4}{5}}}{4} + K$$

(e) 
$$\int \frac{\sqrt[3]{5}\sqrt[3]{x^2} + \sqrt{3}\sqrt{x^3}}{\sqrt{2}\sqrt{x}} \cdot dx$$

**Sol:** 
$$\frac{\sqrt{2}\left(\frac{6\sqrt[3]{5}\sqrt{x}\sqrt[3]{x^2}}{7} + \frac{\sqrt{3}\sqrt{x}\sqrt{x^3}}{2}\right)}{2} + K$$

(f) 
$$\int \frac{1}{x-1} \cdot dx$$

**Sol:** 
$$\log(x-1) + K$$

(g) 
$$\int \frac{5}{4x^2+1} \cdot dx$$

**Sol:** 
$$\frac{5 \tan{(2x)}}{2} + K$$

(h) 
$$\int \frac{2}{9x^2+1} \cdot dx$$

**Sol:** 
$$\frac{2 \tan{(3x)}}{3} + K$$

(i) 
$$\int \frac{1}{1-4x^2} \cdot dx$$

**Sol:** 
$$-\frac{\log(x-\frac{1}{2})}{4} + \frac{\log(x+\frac{1}{2})}{4} + K$$

(j) 
$$\int \frac{1}{x^2+4} \cdot dx$$

Sol: 
$$\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2} + K$$

(k) 
$$\int \frac{2x-3}{x^2-3x+9} \cdot dx$$

**Sol:** 
$$\log(x^2 - 3x + 9) + K$$

(1) 
$$\int 2e^{x^2}x \cdot dx$$

**Sol:** 
$$e^{x^2} + K$$

(m) 
$$\int \frac{1}{x-4} \cdot dx$$

**Sol:** 
$$\log(x-4) + K$$

(n) 
$$\int \frac{1}{(x-4)^3} \cdot dx$$

Sol: 
$$-\frac{1}{2x^2-16x+32}+K$$

(
$$\tilde{\mathbf{n}}$$
)  $\int e^{5x} \cdot dx$ 

**Sol:** 
$$\frac{e^{5x}}{5} + K$$

(o) 
$$\int \frac{\sqrt{x}+x}{x^2} \cdot dx$$

**Sol:** 
$$\log(x) - \frac{2}{\sqrt{x}} + K$$

(p) 
$$\int \frac{\log(x)^3}{x} \cdot dx$$

**Sol:** 
$$\frac{\log(x)^4}{4} + K$$

(q) 
$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}} \cdot dx$$

**Sol:** 
$$-2\cos(\sqrt{x}) + K$$

(r) 
$$\int e^{9+(-2)x} \cdot dx$$

**Sol:** 
$$-\frac{e^{9-2x}}{2} + K$$

2. p38e01cont - Calcula las siguientes integrales inmediatas:

(a) 
$$\int 3^x - x^3 \cdot dx$$

**Sol:** 
$$\frac{3^x}{\log(3)} - \frac{x^4}{4} + K$$

(b) 
$$\int \frac{3x^2 + 2e^{2x} + \cos(x)}{x^3 + e^{2x} + \sin(x)} \cdot dx$$

**Sol:** 
$$\log (x^3 + e^{2x} + \sin (x)) + K$$

(c) 
$$\int 6\cos(2x-1)\cdot dx$$

**Sol:** 
$$3\sin(2x-1) + K$$

(d) 
$$\int \frac{x+1}{x^2+2x+3} \cdot dx$$

**Sol:** 
$$\frac{\log(x^2+2x+3)}{2} + K$$

(e) 
$$\int \sin(x) \cos(x) \cdot dx$$

**Sol:** 
$$\frac{\sin^2(x)}{2} + K$$

(f) 
$$\int e^{4x-3} \cdot dx$$

**Sol:** 
$$\frac{e^{4x-3}}{4} + K$$

(g) 
$$\int \frac{4}{x^2+3} \cdot dx$$

Sol: 
$$\frac{4\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3} + K$$

(h) 
$$\int \sqrt{(x+3)^3} \cdot dx$$

**Sol:** 
$$\int \sqrt{(x+3)^3} \, dx + K$$

(i) 
$$\int \frac{\sin(x)}{\cos(x)} \cdot dx$$

**Sol:** 
$$-\log(\cos(x)) + K$$

## 3. p38e02 - Calcula las siguientes integrales, por cambio de variable:

(a) 
$$\int x \sin(x^2) \cdot dx$$

**Sol:** 
$$-\frac{\cos(x^2)}{2} + K$$

(b) 
$$\int \frac{x}{\sqrt{x^2+5}} \cdot dx$$

**Sol:** 
$$\sqrt{x^2 + 5} + K$$

(c) 
$$\int \frac{\sin(x)}{\cos^5(x)} \cdot dx$$

**Sol:** 
$$\frac{1}{4\cos^4(x)} + K$$

(d) 
$$\int \frac{x}{(x^2+3)^5} \cdot dx$$

Sol: 
$$-\frac{1}{8x^8+96x^6+432x^4+864x^2+648}+K$$

(e) 
$$\int \frac{\tan^2(x)+1}{\sqrt{\tan(x)}} \cdot dx$$

Sol: 
$$2\sqrt{\tan(x)} + K$$

(f) 
$$\int (x-1)\sqrt{x^2-2x}\cdot dx$$

**Sol:** 
$$\frac{x^2\sqrt{x^2-2x}}{3} - \frac{2x\sqrt{x^2-2x}}{3} + K$$

(g) 
$$\int \sin(x) \sin(\cos(x)) \cdot dx$$

Sol: 
$$\cos(\cos(x)) + K$$

(h) 
$$\int \frac{\log(x)^2 + 1}{x} \cdot dx$$

**Sol:** 
$$\frac{\log(x)^3}{3} + \log(x) + K$$

(i) 
$$\int \sqrt{(\cos(x)+1)^3} \sin(x) \cdot dx$$

**Sol:** 
$$\int \sqrt{(\cos(x) + 1)^3} \sin(x) \, dx + K$$

4. p38e03 - Calcula las siguientes integrales, por partes:

(a) 
$$\int e^x x \cdot dx$$

**Sol:** 
$$(x-1)e^x + K$$

(b) 
$$\int \log(x) \cdot dx$$

**Sol:** 
$$x \log(x) - x + K$$

(c) 
$$\int x^3 \log(x) \cdot dx$$

**Sol:** 
$$\frac{x^4 \log(x)}{4} - \frac{x^4}{16} + K$$

(d) 
$$\int x \sin(x) \cdot dx$$

**Sol:** 
$$-x\cos(x) + \sin(x) + K$$

(e) 
$$\int \operatorname{atan}(x) \cdot dx$$

**Sol:** 
$$x \arctan(x) - \frac{\log(x^2+1)}{2} + K$$

(f) 
$$\int a\sin(x) \cdot dx$$

**Sol:** 
$$x \sin(x) + \sqrt{1 - x^2} + K$$

(g) 
$$\int x \cos(3x) \cdot dx$$

**Sol:** 
$$\frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + K$$

(h) 
$$\int e^{2x}x^2 \cdot dx$$

**Sol:** 
$$\frac{(2x^2-2x+1)e^{2x}}{4} + K$$

(i) 
$$\int \frac{x}{e^x} \cdot dx$$

**Sol:** 
$$(-x-1)e^{-x} + K$$

(j) 
$$\int \frac{\log(x)}{x^2} \cdot dx$$

**Sol:** 
$$-\frac{\log(x)}{x} - \frac{1}{x} + K$$

5. p38e04 - Resuelve las siguientes integrales, que luego aparecerán al integrar funciones racionales:

(a) 
$$\int \frac{3}{2x-1} \cdot dx$$

**Sol:** 
$$\frac{3\log(2x-1)}{2} + K$$

(b) 
$$\int \frac{2}{(x+2)^3} \cdot dx$$

**Sol:** 
$$-\frac{1}{x^2+4x+4}+K$$

(c)  $\int \frac{7x-5}{x^2+4} \cdot dx$ 

**Sol:** 
$$\frac{7\log(x^2+4)}{2} - \frac{5\tan(\frac{x}{2})}{2} + K$$

(d)  $\int \frac{x+2}{x^2+x+1} \cdot dx$ 

Sol: 
$$\frac{\log(x^2+x+1)}{2} + \sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}(2x+1)}{3}\right) + K$$

(e)  $\int \frac{1}{3x-4} \cdot dx$ 

**Sol:** 
$$\frac{\log(3x-4)}{3} + K$$

(f) 
$$\int \frac{7}{(2x-3)^2} \cdot dx$$

**Sol:** 
$$-\frac{7}{4x-6} + K$$

(g)  $\int \frac{3x+4}{x^2+2} \cdot dx$ 

Sol: 
$$\frac{3\log(x^2+2)}{2} + 2\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) + K$$

(h)  $\int \frac{x-1}{x^2+2x+3} \cdot dx$ 

Sol: 
$$\frac{\log(x^2+2x+3)}{2} - \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}(x+1)}{2}\right) + K$$

6. p38e05 - Calcula las siguientes integrales racionales:

(a) 
$$\int \frac{x-2}{x^2+x} \cdot dx$$

**Sol:** 
$$-2\log(x) + 3\log(x+1) + K$$

(b) 
$$\int \frac{x^3 + 4x^2 - 10x + 7}{x^3 - 7x - 6} \cdot dx$$

**Sol:** 
$$x + 2 \log(x - 3) - 5 \log(x + 1) + 7 \log(x + 2) + K$$

(c) 
$$\int \frac{2x^2+5x-1}{x^3+x^2-2x} \cdot dx$$

**Sol:** 
$$\frac{\log(x)}{2} + 2\log(x-1) - \frac{\log(x+2)}{2} + K$$

(d) 
$$\int \frac{2x+3}{x^2+3x-10} \cdot dx$$

**Sol:** 
$$\log(x^2 + 3x - 10) + K$$

(e) 
$$\int \frac{1}{x^3 + x^2 - 6x} \cdot dx$$

**Sol:** 
$$-\frac{\log(x)}{6} + \frac{\log(x-2)}{10} + \frac{\log(x+3)}{15} + K$$

(f) 
$$\int \frac{x^2 + 2x}{x^2 - 1} \cdot dx$$

**Sol:** 
$$x + \frac{3\log(x-1)}{2} + \frac{\log(x+1)}{2} + K$$

(g) 
$$\int \frac{2x^2 + 7x - 1}{x^3 + x^2 - x - 1} \cdot dx$$

**Sol:** 
$$\frac{2(x+1)\log(x-1)-3}{x+1} + K$$

(h) 
$$\int \frac{2x-4}{(x-1)^2(x+3)} \cdot dx$$

Sol: 
$$\frac{5(x-1)(\log(x-1)-\log(x+3))+4}{8(x-1)}+K$$

(i) 
$$\int \frac{x^2+x}{x^4+2x^3-3x^2-4x+4} \cdot dx$$

Sol: 
$$\frac{-12x+5(\log{(x-1)}-\log{(x+2)})(x^2+x-2)-6}{27(x^2+x-2)} +$$

$$(j) \quad \int \frac{x+2}{x^3 - 2x^2 + x} \cdot dx$$

**Sol:** 
$$\frac{2(x-1)(\log(x)-\log(x-1))-3}{x-1}+K$$

(k) 
$$\int \frac{8x^2 - 2x - 1}{x^3 - x^2 + 4x - 4} \cdot dx$$

Sol: 
$$\log(x-1) + \frac{7\log(x^2+4)}{2} + \frac{5\tan(\frac{x}{2})}{2} + K$$

(l) 
$$\int \frac{6x+8}{x^2+2x+5} \cdot dx$$

**Sol:** 
$$3 \log (x^2 + 2x + 5) + \operatorname{atan}(\frac{x}{2} + \frac{1}{2}) + K$$

(m) 
$$\int \frac{3x-2}{x^3-3x^2+12x-10} \cdot dx$$

Sol: 
$$\frac{\log(x-1)}{9} - \frac{\log(x^2-2x+10)}{18} + \tan(\frac{x}{3} - \frac{1}{3}) + K$$

7. p38e06 - Calcula las siguientes integrales trigonométricas:

(a) 
$$\int \cos^3(x) dx$$

**Sol:** 
$$-\frac{\sin^3(x)}{3} + \sin(x) + K$$

(b) 
$$\int \sin^3(x) dx$$

**Sol:** 
$$\frac{\cos^3(x)}{3} - \cos(x) + K$$

(c) 
$$\int \sin^3(x) \cos(x) dx$$

**Sol:** 
$$\frac{\sin^4(x)}{4} + K$$

(d) 
$$\int \sin(x) \cos^4(x) dx$$

**Sol:** 
$$-\frac{\cos^5(x)}{5} + K$$

(e) 
$$\int \cos^2(x) dx$$

**Sol:** 
$$\frac{x}{2} + \frac{\sin(2x)}{4} + K$$

(f)  $\int \sin^2(x) dx$ 

**Sol:** 
$$\frac{x}{2} - \frac{\sin(2x)}{4} + K$$

(g)  $\int \sin^2(x) \cos^2(x) dx$ 

**Sol:** 
$$\frac{x}{8} - \frac{\sin(4x)}{32} + K$$

(h)  $\int \cos^4(x) dx$ 

**Sol:** 
$$\frac{3x}{8} + \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + K$$

(i)  $\int \frac{\sin(x) + \tan(x)}{\cos(x)} dx$ 

**Sol:** 
$$-\log(\cos(x)) + \frac{1}{\cos(x)} + K$$

 $(j) \int \frac{\sin(x)}{(\cos^2(x)+1)\cos(x)} dx$ 

**Sol:** 
$$\frac{\log (\cos^2(x)+1)}{2} - \log (\cos (x)) + K$$

(k)  $\int \frac{\sin(x)\cos(x)}{1-\cos(x)} dx$ 

**Sol:** 
$$\log(\cos(x) - 1) + \cos(x) + K$$

(1)  $\int \frac{\sin^3(x)}{1-\cos(x)} dx$ 

**Sol:** 
$$-\frac{\cos^2(x)}{2} - \cos(x) + \frac{3}{2} + K$$

(m)  $\int \frac{1}{\cos(x)+2} dx$ 

Sol: 
$$\frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{x}{2\pi}-\frac{1}{2}\right\rfloor\right)}{3}+K$$

(n)  $\int \frac{1}{\cos(x)+2} dx$ 

Sol: 
$$\frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right) + \pi\left\lfloor\frac{x}{2\pi} - \frac{1}{2}\right\rfloor\right)}{3} + K$$

( $\tilde{n}$ )  $\int \frac{1}{\sin(x) + \cos(x) + 1} dx$ 

**Sol:** 
$$\log (\tan (\frac{x}{2}) + 1) + K$$

(o)  $\int \frac{\cos^2(x)}{\sin^2(x)} dx$ 

**Sol:** 
$$-x - \frac{1}{\tan{(x)}} + K$$

8. p38e07 - Calcula las siguientes integrales irracionales:

(a)  $\int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt{x}} dx$ 

Sol: 
$$-\frac{4x^{\frac{3}{4}}}{3} - 4\sqrt[4]{x} + 2\sqrt{x} + x + 4\log(\sqrt[4]{x} + 1) + K$$

(b)  $\int \frac{\sqrt[4]{x} + \sqrt{x}}{-\sqrt[4]{x} + \sqrt{x}} dx$ 

**Sol:** 
$$\frac{8x^{\frac{3}{4}}}{3} + 8\sqrt[4]{x} + 4\sqrt{x} + x + 8\log(\sqrt[4]{x} - 1) + K$$

(c)  $\int \frac{\sqrt{x}}{x+1} dx$ 

Sol: 
$$2\sqrt{x} - 2 \operatorname{atan}(\sqrt{x}) + K$$

(d)  $\int \frac{x}{\sqrt{x}+x} dx$ 

**Sol:** 
$$-2\sqrt{x} + x + 2\log(\sqrt{x} + 1) + K$$

(e)  $\int \frac{1}{-\sqrt[4]{x} + \sqrt{x}} dx$ 

**Sol:** 
$$4\sqrt[4]{x} + 2\sqrt{x} + 4\log(\sqrt[4]{x} - 1) + K$$

(f)  $\int \frac{1}{x\sqrt{1-x^2}} dx$ 

Sol: 
$$\begin{cases} -\cosh\left(\frac{1}{x}\right) & \text{for } \left|\frac{1}{x^2}\right| > 1\\ i \sin\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} + K$$

(g) 
$$\int \sqrt{x^2 + 1} \, dx$$

**Sol:** 
$$\frac{x\sqrt{x^2+1}}{2} + \frac{\sinh(x)}{2} + K$$