1. **Teoría:** Reducción al primer cuadrante

   (a) Ángulos complementarios (suman 90º)

   $$\begin{aligned} sen\ (90^o - \alpha) &= \cos\alpha \\ \cos\ (90^o - \alpha) &= \text{sen}\ \alpha \\ tg\ (90^o - \alpha) &= \text{cotg}\ \alpha \end{aligned}$$

   (b) Ángulos que difieren 90º

   $$\begin{aligned} sen\ (90^o + \alpha) &= \cos\alpha \\ \cos\ (90^o + \alpha) &= \text{-sen}\ \alpha \\ tg\ (90^o + \alpha) &= \text{-cotg}\ \alpha \end{aligned}$$

   (c) Ángulos suplementarios (suman 180º)

   $$\begin{aligned} sen\ (180^o - \alpha) &= \text{sen}\ \alpha \\ \cos\ (180^o - \alpha) &= \text{-cos}\ \alpha \\ tg\ (180^o - \alpha) &= \text{-tg}\ \alpha \end{aligned}$$

   (d) Ángulos que difieren 90º

   $$\begin{aligned} sen\ (180^o + \alpha) &= \text{-sen}\ \alpha \\ \cos\ (180^o + \alpha) &= \text{-cos}\ \alpha \\ tg\ (180^o + \alpha) &= \text{tg}\ \alpha \end{aligned}$$

   (e) Ángulos cuya suma es 360º

   $$\begin{aligned} sen\ (360^o - \alpha) &= \text{-sen}\ \alpha \\ \cos\ (360^o - \alpha) &= \cos\alpha \\ tg\ (360^o - \alpha) &= \text{-tg}\ \alpha \end{aligned}$$

2. Demostrar que para cualquier ángulo α, se verifica: $\sec^2\alpha + \text{cosec}^2\,\alpha = \sec^2\alpha \cdot \text{cosec}^2\,\alpha$

   Sol: $\dfrac{1}{\cos^2\alpha} + \dfrac{1}{\text{sen}^2\,\alpha} = \dfrac{1}{\cos^2\alpha} \cdot \dfrac{1}{\text{sen}^2\,\alpha} \rightarrow \dfrac{\text{sen}^2\,\alpha + \cos^2\alpha}{\text{sen}^2\,\alpha \cdot \cos^2\alpha} = \dfrac{1}{\text{sen}^2\,\alpha \cdot \cos^2\alpha} \rightarrow$
   $\dfrac{1}{\text{sen}^2\,\alpha \cdot \cos^2\alpha} = \dfrac{1}{\text{sen}^2\,\alpha \cdot \cos^2\alpha}$ *(c.q.d.)*

3. Demostrar si son verdaderas o falsas las siguientes ecuaciones:

   (a) $\dfrac{\text{tg}\ \alpha\ +\ \text{tg}\ \beta}{\text{cotg}\ \alpha\ +\ \text{cotg}\ \beta} = \text{tg}\ \alpha\ .\ \text{tg}\ \beta$

   Sol: V: $\tan(x)\tan(y)$

   (b) $\dfrac{\text{sen}\ \alpha\ .\ \cos\alpha}{\cos^2\alpha\ \text{-}\ \text{sen}^2\alpha} = \dfrac{\text{tg}\ \alpha}{1\ \text{-}\ \text{tg}^2\alpha}$

   Sol:

   (c) $\text{cotg}\,\alpha\ \text{-}\ \dfrac{\text{cotg}^2\alpha\ \text{-}\ 1}{\text{cotg}\ \alpha} = \text{tg}\ \alpha$

   Sol:

   (d) $\dfrac{\text{sen}\ \alpha\ +\ \text{cotg}\ \alpha}{\text{tg}\ \alpha\ +\ \text{cosec}\ \alpha} = \cos\alpha$

   Sol:

   (e) $\text{cotg}^2\,\alpha\ \text{-}\ \cos^2\alpha = \text{cotg}^2\,\alpha\ .\ \cos^2\alpha$

   Sol:

   (f) $\text{sen}\,\alpha\ .\ \cos\alpha\ .\ \text{tg}\ \alpha\ .\ \text{cotg}\alpha\ .\ \sec\alpha\ .\ \text{cosec}\ \alpha = 1$

   Sol:

   (g) $\dfrac{1 + \text{tg}\,\alpha}{1 - \text{tg}\,\alpha} = \dfrac{\cos\alpha + \text{sen}\,\alpha}{\cos\alpha - \text{sen}\,\alpha}$

   Sol:

   (h) $\dfrac{1\ +\ \text{tg}^2\alpha}{\cot g\ \alpha} = \dfrac{\text{tg}\ \alpha}{\cos^2\alpha}$

   Sol:

4. Simplificar las siguientes expresiones:

(a) $sen\ \alpha\ .\ \frac{1}{\text{tg}\ \alpha}$

Sol: $\cos\alpha$

(b) $sen^3\alpha +\ \text{sen}\ \alpha\ .\ \cos^2\alpha$

Sol: $\sin(\alpha)$

(c) $\sqrt{1\text{ - sen }\alpha}\ .\ \sqrt{1\ +\ \text{sen}\ \alpha}$

Sol: $\cos\alpha$

(d) $sen^4\alpha\text{ - }\cos^4\alpha$

Sol: $\text{sen}^2\,\alpha - \cos^2\,\alpha$

(e) $\cos^3\alpha + \cos^2\alpha\ .\ \text{sen}\ \alpha + \cos\ \alpha\ .\ \text{sen}^2\alpha + \text{sen}^3\alpha$

Sol:

(f) $sen\ \alpha\ .\ \cos\ \alpha\ \left(\text{tg}\ \alpha\ +\ \dfrac{1}{\text{tg}\ \alpha}\right)$

Sol:

(g) $\dfrac{\cos^2\alpha\text{ - sen}^2\alpha}{\cos^4\alpha\text{ - sen}^4\alpha}$

Sol:

(h) $\dfrac{\sec^2\alpha\ +\ \cos^2\alpha}{\sec^2\alpha\text{ - }\cos^2\alpha}$

Sol:

(i) $\dfrac{\cos^2\alpha}{1\text{ - sen }\alpha}$

Sol:

(j) $\dfrac{\cos ec\ \alpha}{1\ +\ \cot g^2\alpha}$

Sol:

5. Calcular las restantes razones trigonométricas de α, conocida:

(a) $\cos\ \alpha\ =\ \frac{4}{5}$ y $\alpha\ \in\ \text{I}$

Sol:

(b) $sen\ \alpha\ =\ \frac{3}{5}$ y $\alpha\ \in\ \text{II}$

Sol:

(c) $tg\ \alpha\ =\ \text{-}\ \frac{3}{4}$ y $\alpha\ \in\ \text{II}$

Sol:

(d) $\sec\ \alpha\ =\ 2$ y $\alpha\ \in\ \text{IV}$

Sol:

(e) $\cos ec\ \alpha\ =\ \text{-}\ 2$ y $\alpha\ \in\ \text{III}$

Sol:

(f) $\cot g\ \alpha\ =\ \text{-}\ 2$ y $\alpha\ \in\ \text{IV}$

Sol:

6. Si $\text{tg}\,\alpha = \dfrac{3}{4}$, halla el valor de las siguientes razones trigonométricas:

(a) $\text{tg}\ \left(\frac{\pi}{2}\text{ - }\alpha\right)\ =$

Sol:

(b) $\text{sen}(\pi - \alpha) =$

Sol:

(c) $\cos(\pi + \alpha) =$

Sol:

(d) $\text{tg}(\pi - \alpha) =$

Sol:

(e) $\text{cotg}\left(\frac{\pi}{2} + \alpha\right) =$

Sol:

(f) $\sec(\pi - \alpha) =$

Sol:

(g) $\text{cosec}\left(\frac{3\pi}{2} - \alpha\right)$

Sol:

(h) $\cos\left(\frac{3\pi}{2} + \alpha\right) =$

Sol:

(i) $\text{sen}(\pi + \alpha) =$

Sol:

(j) $\text{tg}\left(\frac{\pi}{2} + \alpha\right)$

Sol:

(k) $\text{cotg}(\pi + \alpha) =$

Sol:

(l) $\sec(\pi - \alpha) =$

Sol:

7. Resolver las siguientes ecuaciones:

(a) sen 2x = $\frac{1}{2}$

Sol: $x = \frac{\pi}{12} + \pi \cdot n$
$x = \frac{5\pi}{12} + \pi \cdot n$

(b) tg $\frac{x}{2}$ = $\frac{\sqrt{3}}{3}$

Sol: $x = \frac{\pi}{3} + 2\pi \cdot n$

(c) sen $\left(3x - \frac{\pi}{2}\right)$ = $-\frac{1}{2}$

Sol:

8. Resolver las siguientes ecuaciones:

(a) 2 sen x + cosec x = $2\sqrt{2}$

Sol: $x = \frac{\pi}{4} + 2\pi \cdot n$
$x = \frac{3\pi}{4} + 2\pi \cdot n$

(b) $\text{sen}\, x = \cos^2 x + 1$

Sol:

(c) tg x – sen x = 0

Sol:

(d) sen x . cos x = 2.sen x

Sol:

(e) 2 cos x – 3tg x = 0

Sol:

(f) tg x + 3.cotg x = 4

Sol: