Ejercicio 8

i) $\dfrac{x}{x+1}-\dfrac{3x+1}{x^2-1}+\dfrac{2-x}{1-x}=0$ **(factorizamos denominadores)**

$$\frac{x}{(x+1)}-\frac{3x+1}{(x+1)(x-1)}+\frac{2-x}{-(x-1)}=0$$

$$\frac{x}{(x+1)}-\frac{3x+1}{(x+1)(x-1)}-\frac{2-x}{(x-1)}=0$$ **(reducimos a común denominador)**

$$\frac{x(x-1)-(3x+1)-(2-x)(x+1)}{(x+1)(x-1)}=\frac{0}{(x+1)(x-1)}$$

$x(x-1)-(3x+1)-(2-x)(x+1)=0 \quad (x\neq 1 \quad \text{y} \quad x\neq -1)$

$x^2-x-3x-1-2x-2+x^2+x=0$

$\boxed{2x^2-5x-3=0 \quad (x\neq 1 \quad \text{y} \quad x\neq -1)}$

$$x=\frac{5\pm\sqrt{(-5)^2-4\cdot 2\cdot(-3)}}{2\cdot 2}=\frac{5\pm\sqrt{25+24}}{4}=\frac{5\pm\sqrt{49}}{4}=\frac{5\pm 7}{4}\begin{cases}x=3\\ x=-\dfrac{1}{2}\end{cases}$$

Ejercicio 9

a) $\sqrt{2+\sqrt{x-5}}=\sqrt{13-x} \Rightarrow \left(\sqrt{2+\sqrt{x-5}}\right)^2=\left(\sqrt{13-x}\right)^2 \Rightarrow 2+\sqrt{x-5}=13-x \Rightarrow$

$\Rightarrow \sqrt{x-5}=11-x \Rightarrow (\sqrt{x-5})^2=(11-x)^2 \Rightarrow x-5=121-22x+x^2 \Rightarrow$

$$\Rightarrow x^2-23x+126=0 \Rightarrow x=\frac{23\pm\sqrt{529-504}}{2}=\frac{23\pm 5}{2}=\begin{cases}x=14\\ x=9\end{cases}$$

Comprobación

$\boxed{x=14}$

$\sqrt{2+\sqrt{14-5}}=\sqrt{13-14}$

$\sqrt{2+5}=\sqrt{-1}$

$x=14$ no es solución

$\boxed{x=9}$

$\sqrt{2+\sqrt{9-5}}=\sqrt{13-9}$

$\sqrt{2+2}=\sqrt{4}$

$2=2 \Rightarrow x=9$ es solución

Solución: $x=9$

Ejercicio 11

2) $2\log x - \log(3x-5) = \log 5x - 1$

$\log x^2 - \log(3x-5) = \log 5x - \log 10$

$\log\left(\frac{x^2}{3x-5}\right) = \log\left(\frac{5x}{10}\right) \Rightarrow \frac{x^2}{3x-5} = \frac{x}{2} \Rightarrow 2x^2 = 3x^2 - 5x \Rightarrow -x^2 + 5x = 0 \Rightarrow$

$\Rightarrow x\cdot(-x+5) = 0 \Rightarrow \begin{cases} x = 0 \text{ no es solución} \\ -x+5=0 \Rightarrow \boxed{x=5} \end{cases}$

3) $\log(x-1) + \log 2 = \log(x^2+3) - \log x$

$\log[(x-1)\cdot 2] = \log\left(\frac{x^2+3}{x}\right)$

$2(x-1) = \frac{x^2+3}{x} \Rightarrow 2x(x-1) = x^2+3 \Rightarrow 2x^2 - 2x = x^2 + 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow$

$\Rightarrow x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2\pm 4}{2} = \begin{cases} \boxed{x=3} \\ x=-1 \text{ no es solución} \end{cases}$

5) $(x^2 - 5x + 5)\log 5 + \log 20 = \log 4$

$\log 5^{(x^2-5x+5)} = \log 4 - \log 20$

$\log 5^{(x^2-5x+5)} = \log\left(\frac{4}{20}\right)$

$5^{(x^2-5x+5)} = \frac{1}{5} \Rightarrow 5^{(x^2-5x+5)} = 5^{-1} \Rightarrow x^2 - 5x + 5 = -1 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow$

$\Rightarrow x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5\pm 1}{2} = \begin{cases} \boxed{x=3} \\ \boxed{x=2} \end{cases}$