EJERCICIO 7

e) $2x^2(x^2-1)-3x^2=-3-x^3+x$

$2x^4-2x^2-3x^2+3+x^3-x=0$

$\boxed{2x^4+x^3-5x^2-x+3=0}$

* Factorizamos $P(x)=2x^4+x^3-5x^2-x+3$

Posibles raíces enteras $=\text{Div}(+3)=\{\pm1,\pm3\}$

$$\begin{array}{c|ccccc}
 & 2 & +1 & -5 & -1 & +3 \\
1 & & +2 & +3 & -2 & -3 \\
\hline
 & 2 & +3 & -2 & -3 & \underline{|0} \\
1 & & +2 & +5 & +3 & \\
\hline
 & 2 & +5 & +3 & \underline{|0} &
\end{array}$$

Primera división: $\Rightarrow$ 1 es raíz y $(x-1)$ factor $\Rightarrow P(x)=(x-1)(2x^3+3x^2-2x-3)$

Segunda división: $\Rightarrow$ 1 es raíz y $(x-1)$ factor $\Rightarrow P(x)=(x-1)^2\underbrace{(2x^2+5x+3)}_{(*)}$

$2x^2+5x+3=0$

$x=\dfrac{-5\pm\sqrt{25-24}}{4}=\dfrac{-5\pm1}{4}$ ↗ $x=-1$ es raíz y $(x+1)$ factor ↘ $x=-3/2$ es raíz y $(x+\frac{3}{2})$ factor $\Rightarrow$

$\Rightarrow 2x^2+5x+3=\underbrace{2(x+1)(x+3/2)}_{(*)}$

Luego $\boxed{\begin{array}{l} P(x)=2(x-1)^2(x+1)(x+3/2) \\ \text{Raíces}=\{1\,(\text{doble}),-1,-3/2\}\end{array}}$

* La ecuación inicial es equivalente a $2(x-1)^2(x+1)(x+\frac{3}{2})=0 \Rightarrow$

$$\Rightarrow \begin{cases} (x-1)^2=0 \Rightarrow \boxed{x=1} \text{ (doble)} \\ x+1=0 \Rightarrow \boxed{x=-1} \\ x+\frac{3}{2}=0 \Rightarrow \boxed{x=-3/2} \end{cases}$$

SOLUCIONES

g) $6x^5 - 15x^2 = -23x^4 + 38x^3$

$6x^5 + 23x^4 - 38x^3 - 15x^2 = 0$

* Factorizamos $P(x) = 6x^5 + 23x^4 - 38x^3 - 15x^2$

Extraemos factor común $P(x) = x^2 \cdot \underline{(6x^3 + 23x^2 - 38x - 15)}$

Posibles raíces enteras $= Div(-15) = \{\pm 1, \pm 3, \pm 5, \pm 15\}$

$$\begin{array}{r|rrrr} & 6 & +23 & -38 & -15 \\ -5 & & -30 & +35 & +15 \\ \hline & 6 & -7 & -3 & \boxed{0} \end{array}$$

$\Rightarrow$ -5 es raíz y $(x+5)$ factor $\Rightarrow$ $P(x) = x^2 \cdot (x+5) \cdot \underbrace{(6x^2 - 7x - 3)}_{(*)}$

$6x^2 - 7x - 3 = 0$

$x = \dfrac{7 \pm \sqrt{49 + 72}}{12} = \dfrac{7 \pm 11}{12}$ ↗ $3/2$ es raíz y $(x - \frac{3}{2})$ factor ↘ $-1/3$ es raíz y $(x + \frac{1}{3})$ factor $\Rightarrow$

$\Rightarrow$ $6x^2 - 7x + 3 = \underbrace{6\left(x - \frac{3}{2}\right)\left(x + 1/3\right)}_{(*)}$

Por tanto

$$\boxed{\begin{array}{l} P(x) = 6x^2 (x+5)\left(x - \frac{3}{2}\right)\left(x + \frac{1}{3}\right) \\ \text{Raíces} = \{0 \text{(doble)}, -5, 3/2, -1/3\} \end{array}}$$

* La ecuación inicial es equivalente a $6x^2 (x+5)\left(x - \frac{3}{2}\right)\left(x + \frac{1}{3}\right) = 0 \Rightarrow$

$$\Rightarrow \left\{ \begin{array}{l} 6x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0 \text{ (doble)} \\ x + 5 = 0 \Rightarrow x = -5 \\ x - \frac{3}{2} = 0 \Rightarrow x = 3/2 \\ x + \frac{1}{3} = 0 \Rightarrow x = -1/3 \end{array} \right.$$

SOLUCIONES

EJERCICIO 8

d) $\left(x+1+\frac{6}{x}\right)\cdot\left(x-1+\frac{6}{x}\right)=24$

$$x^2 \cancel{-x}+6\cancel{+x}-1\cancel{+\frac{6}{x}}+6\cancel{-\frac{6}{x}}+\frac{36}{x^2}=24$$

$$x^2+11+\frac{36}{x^2}=24$$

$$\frac{x^4+11x^2+36}{x^2}=\frac{24x^2}{x^2}$$

$$\boxed{x^4-13x^2+36=0 \qquad (\text{con } x\neq 0)}$$ BICUADRADA

* $x^2=t \Rightarrow t^2-13t+36=0$
* $t=\frac{13\pm\sqrt{169-144}}{2}=\frac{13\pm 5}{2}$ → $t=9$, $t=4$
* $t=9 \Rightarrow x^2=9 \Rightarrow x=\pm\sqrt{9} \Rightarrow \boxed{x=\pm 3}$

  $t=4 \Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{4} \Rightarrow \boxed{x=\pm 2}$ SOLUCIONES

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h) $\frac{7x+7}{3x^2+17x+10}-\frac{1}{3x+2}=\frac{1}{x^2+5x}$

* $3x^2+17x+10=\boxed{3(x+2/3)(x+5)}$

  $3x^2+17x+10=0 \Rightarrow x=\frac{-17\pm\sqrt{289-120}}{6}=\frac{-17\pm 13}{6}$ → $x=-2/3$, $x=-5$

* $3x+2=\boxed{3\left(x+\frac{2}{3}\right)}$
* $x^2+5x=\boxed{x\cdot(x+5)}$

$$\frac{7x+7}{3\left(x+\frac{2}{3}\right)(x+5)} - \frac{1}{3\left(x+\frac{2}{3}\right)} = \frac{1}{x(x+5)}$$

$$\frac{x\cdot(7x+7) - x(x+5)}{3x\cancel{\left(x+\frac{2}{3}\right)(x+5)}} = \frac{3\left(x+\frac{2}{3}\right)}{3x\cancel{\left(x+\frac{2}{3}\right)(x+5)}}$$

$$7x^2+7x - x^2 - 5x = 3x+2$$

$$\boxed{6x^2 - x - 2 = 0} \quad (\text{con } x \neq 0,\ x \neq -5,\ x \neq -2/3)$$

$$x = \frac{1 \pm \sqrt{1+48}}{12} = \frac{1 \pm 7}{12} \begin{cases} x = 2/3 \ \checkmark \\ x = -1/2 \ \checkmark \end{cases}$$

Soluciones $\boxed{x = \frac{2}{3} \text{ y } x = -\frac{1}{2}}$

EJERCICIO 9

i) $\sqrt{x^2+x+4} = 2 + \sqrt{x^2-2x+1}$

$$\left(\sqrt{x^2+x+4}\right)^{\cancel{2}} = \underbrace{\left(2+\sqrt{x^2-2x+1}\right)^2}_{\text{IN}}$$

$$x^2+x+4 = (2)^2 + 2\cdot 2\cdot \sqrt{x^2-2x+1} + \left(\sqrt{x^2-2x+1}\right)^{\cancel{2}}$$

$$x^2+x+4 = 4 + 4\sqrt{x^2-2x+1} + x^2-2x+1$$

$$\cancel{x^2}+x+\cancel{4}-\cancel{4}-\cancel{x^2}+2x-1 = 4\sqrt{x^2-2x+1}$$

$$(3x-1)^2 = \left(4\sqrt{x^2-2x+1}\right)^2$$

$$9x^2-6x+1 = 16(x^2-2x+1)$$

$$9x^2-6x+1 = 16x^2-32x+16$$

$$\boxed{-7x^2+26x-15=0}$$

$$x = \frac{-26 \pm \sqrt{676-420}}{-14} = \frac{-26\pm 16}{-14} \begin{cases} x = 5/7 \\ x = 3 \end{cases}$$

COMPROBACIÓN

$x=3$ $\quad \sqrt{3^2+3+4} = 2+\sqrt{3^2-2\cdot 3+1}$

$$\sqrt{16} = 2+\sqrt{4}$$

$$4 = 2+2$$

$$2=2 \checkmark$$

SOLUCIONES

$$\boxed{x=3 \text{ y } x=\frac{5}{7}}$$

$x=\frac{5}{7}$ $\quad \sqrt{\left(\frac{5}{7}\right)^2+\frac{5}{7}+4} = 2+\sqrt{\left(\frac{5}{7}\right)^2-2\cdot\left(\frac{5}{7}\right)+1}$

$$\sqrt{\frac{256}{49}} = 2+\sqrt{\frac{4}{49}}$$

$$\frac{16}{7} = 2+\frac{2}{7} \Rightarrow \frac{16}{7} = \frac{16}{7} \checkmark$$

n) $\sqrt{2x+7} - \sqrt{x+3} = 1$

$$\left(\sqrt{2x+7}\right)^2 = \left(1 + \sqrt{x+3}\right)^2$$

$$2x+7 = (1)^2 + 2 \cdot 1 \cdot \sqrt{x+3} + \left(\sqrt{x+3}\right)^2$$

$$2x+7 = 1 + 2\sqrt{x+3} + x+3$$

$$2x+7-1-x-3 = 2\sqrt{x+3}$$

$$(x+3)^2 = \left(2\sqrt{x+3}\right)^2$$

$$x^2+6x+9 = 4(x+3)$$

$$x^2+6x+9 = 4x+12$$

$$\boxed{x^2+2x-3=0}$$

$$x = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm 4}{2} \quad x=1 \quad x=-3$$

COMPROBACIÓN

$x=1$ $\quad \sqrt{2 \cdot 1+7} - \sqrt{1+3} = 1$

$$3 - 2 = 1$$

$$1 = 1 \ ✓$$

$x=-3$ $\quad \sqrt{2 \cdot (-3)+7} - \sqrt{-3+3} = 1$

$$1 - 0 = 1$$

$$1 = 1$$

SOLUCIONES $\boxed{x=1 \text{ y } x=-3}$

$\boxed{\text{EJERCICIO 10}}$

15) $2^{x-2} + 28 = 2^{x+2} - 2$

$$\frac{2^x}{2^2} + 28 = 2^x \cdot 2^2 - 2$$

$2^x = t \Rightarrow \frac{t}{4} + 28 = 4t - 2 \Rightarrow \frac{t + 112}{\cancel{4}} = \frac{16t - 8}{\cancel{4}} \Rightarrow$

$\Rightarrow -15t = -120 \Rightarrow t = \frac{-120}{-15} \Rightarrow t = 8$

SOLUCIÓN

$* \; t = 8 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow \boxed{x = 3}$

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16) $5^{2x-3} - 5^{2x+1} = 5^{2x-2} - 3145$

$$\frac{5^{2x}}{5^3} - 5^{2x} \cdot 5^1 = \frac{5^{2x}}{5^2} - 3145$$

$5^{2x} = t \Rightarrow \frac{t}{125} - 5t = \frac{t}{25} - 3145 \Rightarrow \frac{t - 625t}{\cancel{125}} = \frac{5t - 393125}{\cancel{125}}$

$\Rightarrow -629t = -393125 \Rightarrow t = \frac{-393125}{-629} \Rightarrow t = 625$

SOLUCIÓN

$* \; t = 625 \Rightarrow 5^{2x} = 625 \Rightarrow 5^{2x} = 5^4 \Rightarrow 2x = 4 \Rightarrow \boxed{x = 2}$

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20) $4^{x+1} + 2^{x+3} - 320 = 0$

$$4^x \cdot 4^1 + 2^x \cdot 2^3 - 320 = 0$$

$4 \cdot 2^{2x} + 8 \cdot 2^x - 320 = 0 \Rightarrow 4 \cdot (2^x)^2 + 8 \cdot 2^x - 320 = 0$

$* \; 2^x = t \Rightarrow 4t^2 + 8t - 320 = 0 \xRightarrow[(:4)]{} t^2 + 2t - 80 = 0 \Rightarrow$

$\Rightarrow t = \frac{-2 \pm \sqrt{4 + 320}}{2} = \frac{-2 \pm 18}{2}$ $\begin{cases} t = 8 \\ t = -10 \end{cases}$

SOLUCIÓN

* $t=8 \Rightarrow 2^x=8 \Rightarrow 2^x=2^3 \Rightarrow \boxed{x=3}$

$t=-10 \Rightarrow 2^x=-10 \Rightarrow \nexists$ solución

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24) $25^x - 6\cdot 5^{x+1} + 125 = 0$

$5^{2x} - 6\cdot 5^x \cdot 5^1 + 125 = 0$

$(5^x)^2 - 30\cdot 5^x + 125 = 0$

* $5^x = t \Rightarrow t^2 - 30t + 125 = 0 \Rightarrow t = \frac{30 \pm \sqrt{900-500}}{2} = \frac{30\pm 20}{2}$ $\nearrow t=25$ $\searrow t=5$

* $t=25 \Rightarrow 5^x = 25 \Rightarrow 5^x = 5^2 \Rightarrow \boxed{x=2}$

$t=5 \Rightarrow 5^x = 5^1 \Rightarrow \boxed{x=1}$ SOLUCIONES

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25) $13^{2x} - 6\cdot 13^x + 5 = 0$

$(13^x)^2 - 6\cdot 13^x + 5 = 0$

* $13^x = t \Rightarrow t^2 - 6t + 5 = 0 \Rightarrow t = \frac{6\pm\sqrt{36-20}}{2} = \frac{6\pm 4}{2}$ $\nearrow t=5$ $\searrow t=1$

* $t=1 \Rightarrow 13^x = 1 \Rightarrow \boxed{x=0}$ SOLUCIONES

$t=5 \Rightarrow 13^x = 5 \Rightarrow \log 13^x = \log 5 \Rightarrow x\cdot \log 13 = \log 5 \Rightarrow$

$\Rightarrow \boxed{x = \frac{\log 13}{\log 5}} \Rightarrow \boxed{x = \log_5 13}$

$\boxed{\text{EJERCICIO 11}}$

17) $\log 2 - \log(x+1) + \log(x+3) = \log 6$

$$\log 2 + \log(x+3) = \log 6 + \log(x+1)$$

$$\log\left(2\cdot(x+3)\right) = \log\left(6\cdot(x+1)\right)$$

$$2\cdot(x+3) = 6\cdot(x+1)$$

$:2$

$$x+3 = 3(x+1) \Rightarrow x+3 = 3x+3 \Rightarrow -2x = 0 \Rightarrow x=0$$

COMPROBACIÓN

$$\log 2 - \log(0+1) + \log(0+3) = \log 6 \Rightarrow \log 2 - \log 1 + \log 3 = \log 6 \checkmark$$

SOLUCIÓN: $\boxed{x=0}$

19) $\log(x-2) = 1 + \log 2 - \log(x-3)$

$$\log(x-2) = \log 10 + \log 2 - \log(x-3)$$

$$\log(x-2) = \log\left(\frac{10\cdot 2}{x-3}\right)$$

$$x-2 = \frac{20}{x-3} \Rightarrow (x-2)\cdot(x-3) = 20 \Rightarrow$$

$$\Rightarrow x^2 - 3x - 2x + 6 - 20 = 0 \Rightarrow \boxed{x^2 - 5x - 14 = 0}$$

$$x = \frac{5 \pm \sqrt{25+56}}{2} = \frac{5\pm 9}{2} \quad x=7 \quad x=-2$$

SOLUCIÓN: $\boxed{x=7}$

COMPROBACIÓN

$x=7$: $\log(7-2) = 1 + \log 2 - \log(7-3) \Rightarrow \log 5 = 1 + \log 2 - \log 4 \checkmark$

$x=-2$: $\log(-2-2) = 1 + \log 2 - \log(-2-3) \Rightarrow \log(-4) = 1 + \log 2 - \log(-5) \Rightarrow$

$\Rightarrow x=-2$ <u>no</u> es solución $\log(-4)$ y $\log(-5)$ <u>no</u> existen

21) $\log(x-1) + \log(x+1) = 3\log 2 + \log(x-2)$

$$\log[(x-1)(x+1)] = \log 2^3 + \log(x-2)$$

$$\log(x^2-1) = \log(8(x-2))$$

$$x^2 - 1 = 8x - 16 \Rightarrow \boxed{x^2 - 8x + 15 = 0}$$

$$x = \frac{8 \pm \sqrt{64-60}}{2} = \frac{8 \pm 2}{2} \begin{cases} x=5 \\ x=3 \end{cases}$$

Comprobación

$x=5$ $\quad \log(5-1) + \log(5+1) = 3\log 2 + \log(5-2) \Rightarrow \log 4 + \log 6 = 3\log 2 + \log 3$ ✓

$x=3$ $\quad \log(3-1) + \log(3+1) = 3\log 2 + \log(3-2) \Rightarrow \log 2 + \log 4 = 3\log 2 + \log 1$ ✓

Soluciones $\boxed{x=5 \text{ y } x=3}$

23) $(2+x)\log 2^{2-x} + \log 1250 = 4$

$$\log\left(2^{2-x}\right)^{2+x} = \log 10^4 - \log 1250$$

$$\log 2^{(2-x)\cdot(2+x)} = \log\left(\frac{10\,000}{1250}\right)$$

$$\log 2^{4-x^2} = \log 8$$

$$2^{4-x^2} = 8 \Rightarrow 2^{4-x^2} = 2^3 \Rightarrow 4 - x^2 = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Comprobación

$x=1$ $\quad (2+1)\cdot\log 2^{2-1} + \log 1250 = 4 \Rightarrow 3\cdot\log 2 + \log 1250 = 4$ ✓

$x=-1$ $\quad (2-1)\cdot\log 2^{2+1} + \log 1250 = 4 \Rightarrow \log 2^3 + \log 1250 = 4$ ✓

Soluciones $\boxed{x=1 \text{ y } x=-1}$