

1. **Teoría:** Reducción al primer cuadrante

(a) Ángulos complementarios (suman 90°)

$$\begin{aligned}\operatorname{sen}(90^\circ - \alpha) &= \cos \alpha \\ \cos(90^\circ - \alpha) &= \operatorname{sen} \alpha \\ \operatorname{tg}(90^\circ - \alpha) &= \operatorname{cotg} \alpha\end{aligned}$$

(c) Ángulos suplementarios (suman 180°)

$$\begin{aligned}\operatorname{sen}(180^\circ - \alpha) &= \operatorname{sen} \alpha \\ \cos(180^\circ - \alpha) &= -\cos \alpha \\ \operatorname{tg}(180^\circ - \alpha) &= -\operatorname{tg} \alpha\end{aligned}$$

(e) Ángulos cuya suma es 360°

$$\begin{aligned}\operatorname{sen}(360^\circ - \alpha) &= -\operatorname{sen} \alpha \\ \cos(360^\circ - \alpha) &= \cos \alpha \\ \operatorname{tg}(360^\circ - \alpha) &= -\operatorname{tg} \alpha\end{aligned}$$

(b) Ángulos que difieren 90°

$$\begin{aligned}\operatorname{sen}(90^\circ + \alpha) &= \cos \alpha \\ \cos(90^\circ + \alpha) &= -\operatorname{sen} \alpha \\ \operatorname{tg}(90^\circ + \alpha) &= -\operatorname{cotg} \alpha\end{aligned}$$

(d) Ángulos que difieren 90°

$$\begin{aligned}\operatorname{sen}(180^\circ + \alpha) &= -\operatorname{sen} \alpha \\ \cos(180^\circ + \alpha) &= -\cos \alpha \\ \operatorname{tg}(180^\circ + \alpha) &= \operatorname{tg} \alpha\end{aligned}$$

2. Demostrar que para cualquier ángulo α , se verifica: $\sec^2 \alpha + \operatorname{cosec}^2 \alpha = \sec^2 \alpha \cdot \operatorname{cosec}^2 \alpha$

Sol: $\frac{1}{\cos^2 \alpha} + \frac{1}{\operatorname{sen}^2 \alpha} = \frac{1}{\cos^2 \alpha} \cdot \frac{1}{\operatorname{sen}^2 \alpha} \rightarrow \frac{\operatorname{sen}^2 \alpha + \cos^2 \alpha}{\operatorname{sen}^2 \alpha \cdot \cos^2 \alpha} = \frac{1}{\operatorname{sen}^2 \alpha \cdot \cos^2 \alpha} \rightarrow$
 $\frac{1}{\operatorname{sen}^2 \alpha \cdot \cos^2 \alpha} = \frac{1}{\operatorname{sen}^2 \alpha \cdot \cos^2 \alpha} \quad (c.q.d.)$

3. Demostrar si son verdaderas o falsas las siguientes ecuaciones:

(a) $\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{cotg} \alpha + \operatorname{cotg} \beta} = \operatorname{tg} \alpha \cdot \operatorname{tg} \beta$

Sol: V: $\tan(x) \tan(y)$

(e) $\cotg^2 \alpha - \cos^2 \alpha = \cotg^2 \alpha \cdot \cos^2 \alpha$

Sol:

(b) $\frac{\operatorname{sen} \alpha \cdot \cos \alpha}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \frac{\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$

Sol:

(f) $\operatorname{sen} \alpha \cdot \cos \alpha \cdot \operatorname{tg} \alpha \cdot \operatorname{cotg} \alpha \cdot \sec \alpha \cdot \operatorname{cosec} \alpha = 1$

Sol:

(c) $\cotg \alpha - \frac{\cotg^2 \alpha - 1}{\cotg \alpha} = \operatorname{tg} \alpha$

Sol:

(g) $\frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha} = \frac{\cos \alpha + \operatorname{sen} \alpha}{\cos \alpha - \operatorname{sen} \alpha}$

Sol:

(d) $\frac{\operatorname{sen} \alpha + \cotg \alpha}{\operatorname{tg} \alpha + \operatorname{cosec} \alpha} = \cos \alpha$

Sol:

(h) $\frac{1 + \operatorname{tg}^2 \alpha}{\cotg \alpha} = \frac{\operatorname{tg} \alpha}{\cos^2 \alpha}$

Sol:

4. Simplificar las siguientes expresiones:

(a) $\operatorname{sen} \alpha \cdot \frac{1}{\operatorname{tg} \alpha}$

Sol: $\cos \alpha$

(b) $\operatorname{sen}^3 \alpha + \operatorname{sen} \alpha \cdot \cos^2 \alpha$

Sol: $\sin(\alpha)$

(c) $\sqrt{1 - \operatorname{sen} \alpha} \cdot \sqrt{1 + \operatorname{sen} \alpha}$

Sol: $\cos \alpha$

(d) $\operatorname{sen}^4 \alpha - \cos^4 \alpha$

Sol: $\operatorname{sen}^2 \alpha - \cos^2 \alpha$

(e) $\cos^3 \alpha + \cos^2 \alpha \cdot \operatorname{sen} \alpha + \cos \alpha \cdot \operatorname{sen}^2 \alpha + \operatorname{sen}^3 \alpha$

Sol:

(f) $\operatorname{sen} \alpha \cdot \cos \alpha \left(\operatorname{tg} \alpha + \frac{1}{\operatorname{tg} \alpha} \right)$

Sol:

(g) $\frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{\cos^4 \alpha - \operatorname{sen}^4 \alpha}$

Sol:

(h) $\frac{\sec^2 \alpha + \cos^2 \alpha}{\sec^2 \alpha - \cos^2 \alpha}$

Sol:

(i) $\frac{\cos^2 \alpha}{1 - \operatorname{sen} \alpha}$

Sol:

(j) $\frac{\operatorname{cosec} \alpha}{1 + \cot^2 \alpha}$

Sol:5. Calcular las restantes razones trigonométricas de α , conocida:

(a) $\cos \alpha = \frac{4}{5}$ y $\alpha \in \text{I}$

Sol:

(d) $\sec \alpha = 2$ y $\alpha \in \text{IV}$

Sol:

(b) $\operatorname{sen} \alpha = \frac{3}{5}$ y $\alpha \in \text{II}$

Sol:

(e) $\operatorname{cosec} \alpha = -2$ y $\alpha \in \text{III}$

Sol:

(c) $\operatorname{tg} \alpha = -\frac{3}{4}$ y $\alpha \in \text{II}$

Sol:

(f) $\cot g \alpha = -2$ y $\alpha \in \text{IV}$

Sol:6. Si $\operatorname{tg} \alpha = \frac{3}{4}$, halla el valor de las siguientes razones trigonométricas:

(a) $\operatorname{tg} \left(\frac{\pi}{2} - \alpha \right) =$

Sol:**Sol:****Sol:**

(b) $\operatorname{sen}(\pi - \alpha) =$

(c) $\cos(\pi + \alpha) =$

(d) $\operatorname{tg}(\pi - \alpha) =$

Sol:

(e) $\cotg\left(\frac{\pi}{2} + \alpha\right) =$

Sol:

(f) $\sec(\pi - \alpha) =$

Sol:

(g) $\operatorname{cosec}\left(\frac{3\pi}{2} - \alpha\right)$

Sol:

(h) $\cos\left(\frac{3\pi}{2} + \alpha\right) =$

Sol:

(i) $\sin(\pi + \alpha) =$

Sol:

(j) $\operatorname{tg}\left(\frac{\pi}{2} + \alpha\right)$

Sol:

(k) $\cotg(\pi + \alpha) =$

Sol:

(l) $\sec(\pi - \alpha) =$

Sol:

7. Resolver las siguientes ecuaciones:

(a) $\sin 2x = \frac{1}{2}$

Sol: $x = \frac{\pi}{12} + \pi \cdot n$
 $x = \frac{5\pi}{12} + \pi \cdot n$

Sol: $x = \frac{\pi}{3} + 2\pi \cdot n$

(c) $\sin\left(3x - \frac{\pi}{2}\right) = -\frac{1}{2}$

(b) $\operatorname{tg} \frac{x}{2} = \frac{\sqrt{3}}{3}$

Sol:

8. Resolver las siguientes ecuaciones:

(a) $2 \sin x + \operatorname{cosec} x = 2\sqrt{2}$

Sol: $x = \frac{\pi}{4} + 2\pi \cdot n$
 $x = \frac{3\pi}{4} + 2\pi \cdot n$

(d) $\sin x \cdot \cos x = 2 \cdot \sin x$

Sol:

(b) $\sin x = \cos^2 x + 1$

Sol:

(e) $2 \cos x - 3 \operatorname{tg} x = 0$

Sol:

(c) $\operatorname{tg} x - \sin x = 0$

Sol:

(f) $\operatorname{tg} x + 3 \cdot \cotg x = 4$

Sol: