Departamento de Matemáticas

4º Académicas

Trigonometría 2

1. **Teoría:** Reducción al primer cuadrante

(a) Ángulos complementarios (suman 90º)

$$\begin{aligned} sen\ (90º - \alpha) &= \cos\alpha \\ \cos\ (90º - \alpha) &= \operatorname{sen}\alpha \\ tg\ (90º - \alpha) &= \operatorname{cotg}\alpha \end{aligned}$$

(b) Ángulos que difieren 90º

$$\begin{aligned} sen\ (90º + \alpha) &= \cos\alpha \\ \cos\ (90º + \alpha) &= -\operatorname{sen}\alpha \\ tg\ (90º + \alpha) &= -\operatorname{cotg}\alpha \end{aligned}$$

(c) Ángulos suplementarios (suman 180º)

$$\begin{aligned} sen\ (180º - \alpha) &= \operatorname{sen}\alpha \\ \cos\ (180º - \alpha) &= -\cos\alpha \\ tg\ (180º - \alpha) &= -\operatorname{tg}\alpha \end{aligned}$$

(d) Ángulos que difieren 90º

$$\begin{aligned} sen\ (180º + \alpha) &= -\operatorname{sen}\alpha \\ \cos\ (180º + \alpha) &= -\cos\alpha \\ tg\ (180º + \alpha) &= \operatorname{tg}\alpha \end{aligned}$$

(e) Ángulos cuya suma es 360º

$$\begin{aligned} sen\ (360º - \alpha) &= -\operatorname{sen}\alpha \\ \cos\ (360º - \alpha) &= \cos\alpha \\ tg\ (360º - \alpha) &= -\operatorname{tg}\alpha \end{aligned}$$

2. Demostrar que para cualquier ángulo α, se verifica: $\sec^2\alpha + \operatorname{cosec}^2\alpha = \sec^2\alpha \cdot \operatorname{cosec}^2\alpha$

Sol: $\dfrac{1}{\cos^2\alpha} + \dfrac{1}{\operatorname{sen}^2\alpha} = \dfrac{1}{\cos^2\alpha} \cdot \dfrac{1}{\operatorname{sen}^2\alpha} \rightarrow \dfrac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\operatorname{sen}^2\alpha \cdot \cos^2\alpha} = \dfrac{1}{\operatorname{sen}^2\alpha \cdot \cos^2\alpha} \rightarrow$
$\dfrac{1}{\operatorname{sen}^2\alpha \cdot \cos^2\alpha} = \dfrac{1}{\operatorname{sen}^2\alpha \cdot \cos^2\alpha}$ *(c.q.d.)*

3. Demostrar si son verdaderas o falsas las siguientes ecuaciones:

(a) $\dfrac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{\operatorname{cotg}\alpha + \operatorname{cotg}\beta} = \operatorname{tg}\alpha \,.\, \operatorname{tg}\beta$

Sol: V: $\tan(x)\tan(y)$

(b) $\dfrac{\operatorname{sen}\alpha \,.\, \cos\alpha}{\cos^2\alpha - \operatorname{sen}^2\alpha} = \dfrac{\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$

Sol:

(c) $\operatorname{cotg}\alpha - \dfrac{\operatorname{cotg}^2\alpha - 1}{\operatorname{cotg}\alpha} = \operatorname{tg}\alpha$

Sol:

(d) $\dfrac{\operatorname{sen}\alpha + \operatorname{cotg}\alpha}{\operatorname{tg}\alpha + \operatorname{cosec}\alpha} = \cos\alpha$

Sol:

(e) $\operatorname{cotg}^2\alpha - \cos^2\alpha = \operatorname{cotg}^2\alpha \,.\, \cos^2\alpha$

Sol:

(f) $\operatorname{sen}\alpha \,.\, \cos\alpha \,.\, \operatorname{tg}\alpha \,.\, \operatorname{cotg}\alpha \,.\, \sec\alpha \,.\, \operatorname{cosec}\alpha = 1$

Sol:

(g) $\dfrac{1 + \operatorname{tg}\alpha}{1 - \operatorname{tg}\alpha} = \dfrac{\cos\alpha + \operatorname{sen}\alpha}{\cos\alpha - \operatorname{sen}\alpha}$

Sol:

(h) $\dfrac{1 + \operatorname{tg}^2\alpha}{\cot g\ \alpha} = \dfrac{\operatorname{tg}\alpha}{\cos^2\alpha}$

Sol:

4. Simplificar las siguientes expresiones:

(a) $sen\ \alpha\ .\ \frac{1}{\text{tg}\ \alpha}$

Sol: $\cos\alpha$

(b) $sen^3\alpha +$ sen α . $\cos^2\alpha$

Sol: $\sin(\alpha)$

(c) $\sqrt{1 \text{ - sen } \alpha}\ .\ \sqrt{1\ +\ \text{sen}\ \alpha}$

Sol: $\cos\alpha$

(d) $sen^4\alpha$ - $\cos^4\alpha$

Sol: $\text{sen}^2\,\alpha - \cos^2\,\alpha$

(e) $\cos^3\alpha + \cos^2\alpha$. sen α + cos α . $\text{sen}^2\alpha$ + $\text{sen}^3\alpha$

Sol:

(f) $sen\ \alpha\ .\ \cos\ \alpha\ \left(\text{tg}\ \alpha\ +\ \dfrac{1}{\text{tg}\ \alpha}\right)$

Sol:

(g) $\dfrac{\cos^2\alpha \text{ - sen}^2\alpha}{\cos^4\alpha \text{ - sen}^4\alpha}$

Sol:

(h) $\dfrac{\sec^2\alpha\ +\ \cos^2\alpha}{\sec^2\alpha \text{ - } \cos^2\alpha}$

Sol:

(i) $\dfrac{\cos^2\alpha}{1 \text{ - sen }\alpha}$

Sol:

(j) $\dfrac{\cos ec\ \alpha}{1\ +\ \cot g^2\alpha}$

Sol:

5. Calcular las restantes razones trigonométricas de α, conocida:

(a) $\cos\ \alpha\ =\ \frac{4}{5}$ y $\alpha\ \in$ I

Sol:

(b) $sen\ \alpha\ =\ \frac{3}{5}$ y $\alpha\ \in$ II

Sol:

(c) $tg\ \alpha\ =$ - $\frac{3}{4}$ y $\alpha\ \in$ II

Sol:

(d) $\sec\ \alpha\ =\ 2$ y $\alpha\ \in$ IV

Sol:

(e) $\cos ec\ \alpha\ =$ - 2 y $\alpha\ \in$ III

Sol:

(f) $\cot g\ \alpha\ =$ - 2 y $\alpha\ \in$ IV

Sol:

6. Si $\text{tg}\,\alpha = \dfrac{3}{4}$, halla el valor de las siguientes razones trigonométricas:

(a) tg $\left(\frac{\pi}{2} \text{ - } \alpha\right)\ =$

Sol:

(b) $\text{sen}(\pi - \alpha) =$

Sol:

(c) $\cos(\pi + \alpha) =$

Sol:

(d) $\text{tg}(\pi - \alpha) =$

Sol:

Sol:

Sol:

(e) cotg $\left(\frac{\pi}{2}\ +\ \alpha\right) =$

Sol:

(f) $\sec(\pi - \alpha) =$

Sol:

(g) cosec $\left(\frac{3\pi}{2}\ \text{-}\ \alpha\right)$

(h) cos $\left(\frac{3\pi}{2}\ +\ \alpha\right) =$

Sol:

(i) $\text{sen}(\pi + \alpha) =$

Sol:

(j) tg $\left(\frac{\pi}{2}\ +\ \alpha\right)$

(k) $\text{cotg}(\pi + \alpha) =$

Sol:

(l) $\sec(\pi - \alpha) =$

Sol:

7. Resolver las siguientes ecuaciones:

   (a) sen 2x = $\frac{1}{2}$

   Sol: $x = \frac{\pi}{12} + \pi \cdot n$
   $x = \frac{5\pi}{12} + \pi \cdot n$

   (b) tg $\frac{x}{2}$ = $\frac{\sqrt{3}}{3}$

   Sol: $x = \frac{\pi}{3} + 2\pi \cdot n$

   (c) sen $\left(3\text{x - } \frac{\pi}{2}\right)$ = - $\frac{1}{2}$

   Sol:

8. Resolver las siguientes ecuaciones:

   (a) 2 sen x + cosec x = $2\sqrt{2}$

   Sol: $x = \frac{\pi}{4} + 2\pi \cdot n$
   $x = \frac{3\pi}{4} + 2\pi \cdot n$

   (b) $\text{sen}\, x = \cos^2 x + 1$

   Sol:

   (c) tg x – sen x = 0

   Sol:

   (d) sen x . cos x = 2.sen x

   Sol:

   (e) 2 cos x – 3tg x = 0

   Sol:

   (f) tg x + 3.cotg x = 4

   Sol: