

1. Calcular el valor del número x:

(a) $x^{-1} = 3$

Sol: $x^{-1} = 3 \rightarrow$
 $\frac{1}{x} = 3 \rightarrow 1 = 3x \rightarrow$
 $x = \frac{1}{3}$

Sol: $(2x)^{-2} = 4 \rightarrow$
 $\frac{1}{(2x)^2} = 4 \rightarrow \frac{1}{4x^2} =$
 $4 \rightarrow 16x^2 = 1 \rightarrow$
 $x^2 = \frac{1}{16} \rightarrow x =$
 $\pm \frac{1}{4} \rightarrow x = \frac{1}{4}$

Sol: $(0,2)^{-1/2} =$
 $x \rightarrow \left(\frac{2}{10}\right)^{-1/2} =$
 $x \rightarrow \left(\frac{1}{5}\right)^{-1/2} =$
 $x \rightarrow 5^{1/2} = x \rightarrow$
 $x = \sqrt{5}$

(b) $3^{1/4} = x$

Sol: $3^{1/4} = x \rightarrow$
 $x = \sqrt[4]{3}$

(d) $\left(\frac{1}{4}\right)^{-\frac{2}{3}} = x$

Sol: $\left(\frac{1}{4}\right)^{-\frac{2}{3}} = x \rightarrow$
 $4^{2/3} = x \rightarrow x =$
 $\sqrt[3]{2^4} \rightarrow x = 2\sqrt[3]{2}$

(f) $x^{-1} = 3$

Sol: $x^{-1} = 3 \rightarrow$
 $\frac{1}{x} = 3 \rightarrow 1 = 3x \rightarrow$
 $x = \frac{1}{3}$

(c) $(2x)^{-2} = 4$

(e) $(0,2)^{-1/2} = x$

2. Calcular los siguientes logaritmos aplicando la definición:

(a) $\log_{\sqrt{2}} \frac{1}{64}$

Sol: $\log_{\sqrt{2}} \frac{1}{64} =$
 $x \rightarrow (\sqrt{2})^x =$
 $2^{-6} \rightarrow 2^{x/2} =$
 $2^{-6} \rightarrow \frac{x}{2} = -6 \rightarrow$
 $x = -12$

(c) $(0,125)^x = 16$

Sol: $(0,125)^x =$
 $16 \rightarrow \left(\frac{125}{1000}\right)^x =$
 $2^4 \rightarrow \left(\frac{5^3}{2^3 \cdot 5^3}\right)^x =$
 $2^4 \rightarrow (2^{-3})^x =$
 $2^4 \rightarrow 2^{-3x} = 2^4 \rightarrow$
 $-3x = 4 \rightarrow x =$
 $-\frac{4}{3}$

Sol: $\left(\frac{1}{\sqrt{3}}\right)^x =$
 $3^3 \rightarrow \left(3^{-1/2}\right)^x =$
 $3^3 \rightarrow 3^{-x/2} = 3^3 \rightarrow$
 $-\frac{x}{2} = 3 \rightarrow x = -6$

(b) $\log_2 2\sqrt{2}$

Sol: $\log_2 2\sqrt{2} =$
 $x \rightarrow 2^x =$
 $2 \cdot 2^{1/2} \rightarrow 2^x =$
 $2^{3/2} \rightarrow x = \frac{3}{2}$

(d) $\left(\frac{1}{\sqrt{3}}\right)^x = 3^3$

(e) $(0,2)^x = 5^4$

Sol: $(0,2)^x =$
 $5^4 \rightarrow \left(\frac{2}{10}\right)^x = 5^4 \rightarrow$
 $\left(\frac{1}{5}\right)^x = 5^4 \rightarrow 5^{-x} =$
 $5^4 \rightarrow -x = 4 \rightarrow$
 $x = -4$

3. Calcular:

(a) $\log_2 2^3 \cdot 2^5$

Sol: $\log_2 2^3 \cdot 2^5 =$
 $\log_2 2^8 = 8 \cdot \log_2 2 =$
 8

(b) $\log_5 5^2 \cdot 5^3$

$$\begin{aligned}\text{Sol: } \log_5 5^2 \cdot 5^3 &= \\ \log_5 5^5 &= 5 \cdot \log_5 5 = \\ 5\end{aligned}$$

(c) $\log_{1/5} 5^4$

$$\begin{aligned}\text{Sol: } \log_{1/5} 5^4 &= \\ 4 \log_{1/5} 5 &= 4 \frac{\log_5 5}{\log_5 \frac{1}{5}} =\end{aligned}$$

$$\frac{4^{\frac{1}{\log_5 1 - \log_5 5}}}{\frac{4}{0-1}} = -4$$

4. Sabiendo que $\log 2 = 0,30103$ y que $\log 3 = 0,47712$ calcular los siguientes logaritmos:

(a) $\log_2 3$

$$\begin{aligned} \text{Sol: } \log_2 3 &= \\ \frac{\log 3}{\log 2} &= \\ \frac{0,47712}{0,30103} &= \\ 1,58 & \end{aligned}$$

(b) $\log 5$

$$\begin{aligned} \text{Sol: } \log 5 &= \\ \log \frac{10}{2} &= \\ \log 10 - \log 2 &= \\ 1 - 0,30103 &= \\ 0,69897 & \end{aligned}$$

$$\begin{aligned} \text{Sol: } \log 0,6 &= \\ \log \frac{6}{10} &= \\ \log 6 - \log 10 &= \\ \log (2 \cdot 3) - 1 &= \\ \log 2 + \log 3 - 1 &= \\ 0,30103 + 0,47712 - 1 &= \\ -0,22 & \end{aligned}$$

$$\text{Sol: } \log 0,0125 =$$

$$\begin{aligned} \log \frac{125}{10000} &= \\ \log 125 - \log 10000 &= \\ \log 5^3 - \log 10^4 &= \\ 3 \log 5 - 4 \log 10 &= \\ 3 \log \frac{10}{2} - 4 &= \\ 3(\log 10 - \log 2) - 4 &= \\ 3 - 3 \log 2 - 4 &= \\ -1 - \log 2 &= \\ -1 - 0,30103 &= \\ -1,30103 & \end{aligned}$$

(d) $\log 0,0125$

(c) $\log 0,6$

5. Sin utilizar la calculadora, resuelve los siguientes logaritmos:

(a) $\log_3 27$

$$\begin{aligned} \text{Sol: } \log_3 27 &= \\ \log_3 3^3 &= 3 \end{aligned}$$

$$\begin{aligned} \text{Sol: } \log_{1/3} 27 &= \\ \log_{1/3} 3^3 &= 3 \frac{\log_3 3}{\log_3 \frac{1}{3}} = \\ 3 \frac{1}{0-1} &= -3 \end{aligned}$$

(e) $\log_5 \sqrt{125}$

$$\begin{aligned} \text{Sol: } \log_5 \sqrt{125} &= \\ \log_5 5^{3/2} &= \frac{3}{2} \end{aligned}$$

(b) $\log_3 \frac{1}{81}$

$$\begin{aligned} \text{Sol: } \log_3 \frac{1}{81} &= \\ \log_3 3^{-4} &= -4 \end{aligned}$$

(d) $\log_{1/3} \frac{1}{81}$

$$\begin{aligned} \text{Sol: } \log_{1/3} \frac{1}{81} &= \\ \log_{1/3} \left(\frac{1}{3}\right)^4 &= 4 \end{aligned}$$

(f) $\log_{1/5} 625$

$$\begin{aligned} \text{Sol: } \log_{1/5} 625 &= \\ \log_{1/5} 5^4 &= 4 \frac{\log_5 5}{\log_5 \frac{1}{5}} = \\ 4 \frac{1}{0-1} &= -4 \end{aligned}$$

6. Escribe las siguientes expresiones como el logaritmo de una sola expresión, lo más simple posible:

(a) $\log a + 2 \log b - \frac{3}{2} \log c + \frac{5}{2} \log d$

Sol: $\log a + 2 \log b - \frac{3}{2} \log c + \frac{5}{2} \log d = \log a^3 + \log b^2 - \log c^{3/2} + \log d^{5/2} =$
 $\log \frac{a^3 b^2}{c^{3/2}} + \log d^{5/2} =$
 $= \log \left(\frac{a^3 b^2}{c^{3/2}} \cdot \sqrt[5]{d^5} \right) = \log \frac{a^3 b^2 d^2 \sqrt{d}}{c^{3/2}}$

(b) $\frac{1}{2} \log (x^2 + 4) + \frac{1}{2} \log (x + 3) + \frac{1}{2} \log (x - 3)$

Sol: $\frac{1}{2} \log (x^2 + 4) + \frac{1}{2} \log (x + 3) + \frac{1}{2} \log (x - 3) = \log \sqrt{x^2 + 4} + \log \sqrt{x + 3} +$
 $\log \sqrt{x - 3} =$
 $= \log \sqrt{(x^2 + 4)(x + 3)(x - 3)} = \log \sqrt{(x^2 + 4)(x^2 - 9)} = \log \sqrt{x^4 - 5x^2 - 36}$

7. Calcula los siguientes logaritmos:

(a) $\log_9 \frac{1}{\sqrt[5]{3}} + \log_{0,5} \sqrt{128} - \log_{32} \frac{1}{\sqrt{8}}$

Sol: $-\frac{33}{10}$

(b) $3 \cdot \log_4 \sqrt{512} + 2 \cdot \log_8 0,25 - 8 \cdot \log_9 \frac{1}{\sqrt[4]{3}} + 3 \cdot \log \sqrt{0,00001}$

Sol: $-\frac{13}{12}$

(c) $\frac{1}{2} \cdot \log_8 \sqrt[3]{0,25} + 2 \log_{25} \frac{1}{5} - \log_{81} 3 - \log_{49} \sqrt{7 \sqrt[3]{7}}$

Sol: $-\frac{61}{36}$

8. Resolver las siguientes ecuaciones logarítmicas:

(a) $\log x = \log 2 + \log (x - 3)$

Sol: $\log x = \log 2 + \log (x - 3) \rightarrow$
 $x = 2(x - 3) \rightarrow x = 6$

Sol: $\log (20x) + \log (2x) = 3 \rightarrow$
 $20x, 2x = 1000 \rightarrow 40x = 1000 \rightarrow$
 $x^2 = 25 \rightarrow x = 5$

(b) $\log (3x + 1) - \log (2x - 3) = 1 - \log 5$ (d) $\log (x + 2) + \log (10x + 20) = 3$

Sol: $\log (3x + 1) - \log (2x - 3) =$
 $1 - \log 5 \rightarrow \frac{3x+1}{2x-3} = \frac{10}{5} \rightarrow 3x + 1 =$
 $4x - 6 \rightarrow x = 7$

Sol: $\log (x + 2) + \log (10x + 20) =$
 $3 \rightarrow (x + 2)(10x + 20) = 1000 \rightarrow$
 $(x + 2)(x + 2) 10 = 1000$
 $x^2 + 4x + 4 = 100 \rightarrow x^2 + 4x - 96 =$
 $0 \rightarrow x = 8$

(c) $\log (20x) + \log (2x) = 3$