NFL Time Series

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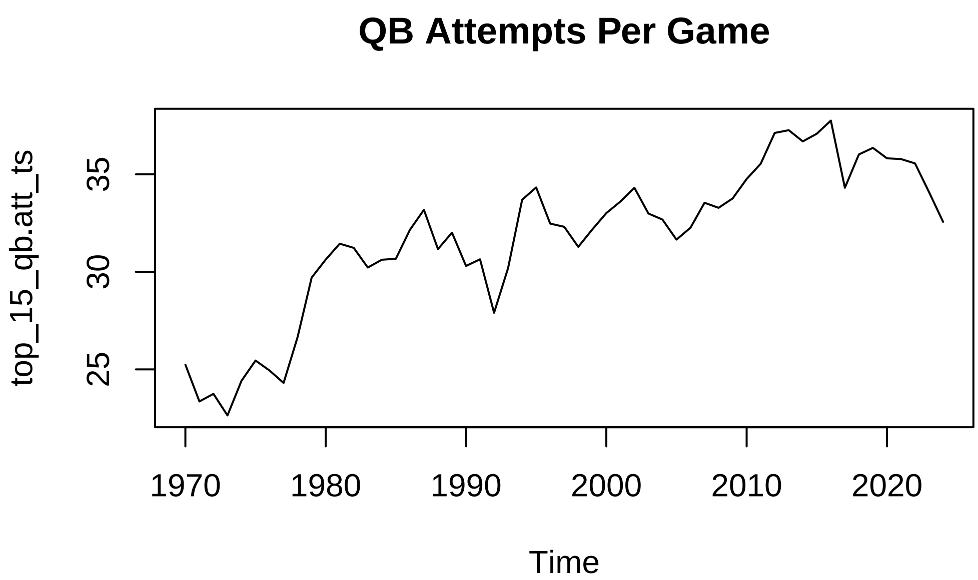
1. Data Collection and Description: QB Attempts Per Game

Data is obtained from Pro Football Reference. In particular, the "Fantasy Ranks" page from 1970-2024 was scraped (see separate Jupyter notebook), e.g. 1970 Fantasy Ranks. Included on these pages is a wide variety of regular season statistics, including the number of passing attempts made by a player throughout the season and the number of games they started or participated in. Of particular interest here is the **number of passing attempts per game among the top 15 quarterbacks sorted by total pass attempts**. A few clarifying statements.

- For a given season, quarterbacks are first ranked by the raw number of passing attempts they had. Even though we ultimately need to scale by number of games played, we want to exclude players not representative of consistent starting quarterbacks (e.g. someone who started one game but threw many passes).
- The top 15 quarterbacks by total pass attempts are selected in order to capture roughly half the league.
- The raw number of pass attempts is scaled by games played to account for the shift in regular season schedule length.

```
fantasy_data <- read.csv("fantasy_data.csv")

years <- unique(fantasy_data$Year)

avg_top15_by_year <- numeric(length(years))
names(avg_top15_by_year) <- years

for (y in years) {
    category_year <- subset(fantasy_data, Year == y & FantPos == "QB")
    category_year <- category_year[order(category_year$Passing_Att, decreasing = TRUE), ]

    top15 <- head(category_year, 15)
```

```
    avg_among_top_15 <- mean(top15$Passing_Att / top15$Games_G)

    avg_top15_by_year[as.character(y)] <- avg_among_top_15
}

top_15_qb.att_ts <- ts(data = avg_top15_by_year, start = 1970, frequency = 1)

ts.plot(top_15_qb.att_ts, main = "QB Attempts Per Game")
```

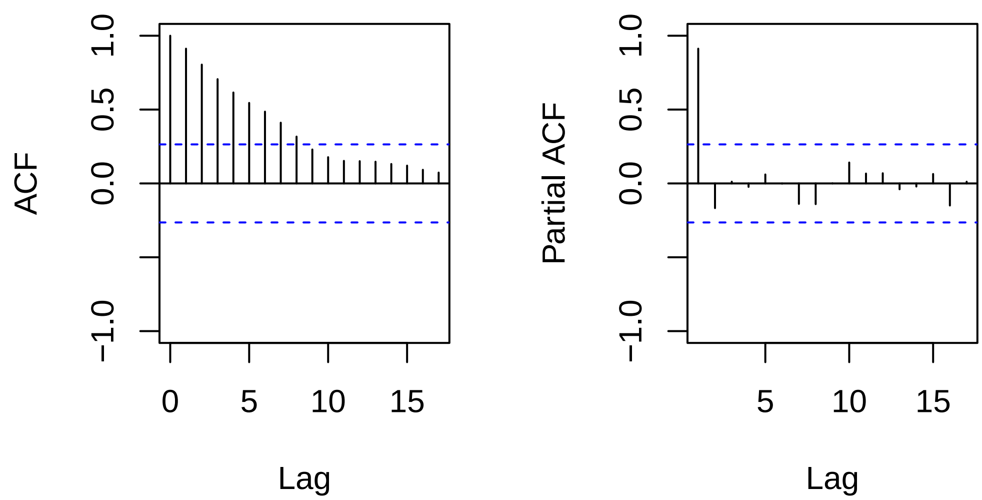


First observations about the data: values range between a minimum of 22.6381 attempts per game in 1973 and a maximum of 37.7581 in 2016; the time series is evidently nonstationary at least in the mean; there may be some periodic or seasonal dips, but it is difficult to say with certainty that they are not just noise.

2. Exploratory Data Analysis

Having visualized the series above, we want further confirmation that it is nonstationary. Our first effort in this direction is to plot the sample autocorrelation (ACF) function and partial autocorrelation (PACF) functions.

```r
par(mfrow = c(1, 2))
acf(top_15_qb.att_ts, main = "", ylim = c(-1, 1))
pacf(top_15_qb.att_ts, main = "", ylim = c(-1, 1))
```




```r
pacf(top_15_qb.att_ts, plot = FALSE)[1]
```

```
Partial autocorrelations of series 'top_15_qb.att_ts', by lag

    1
0.912
```

The ACF does not decay quickly, with significant correlation values (according to the critical value envelope) as far as lag 8. Moreover, the PACF at lag 1 is 0.912, providing some further support of nonstationarity.

For now we will ignore the possibility of (conditional) heteroscedasticity. Instead, we proceed by first estimating the trend of the data and then fitting a model with seasonality modeled by trigonometric functions. Only later will we move toward a (S)ARIMA model.

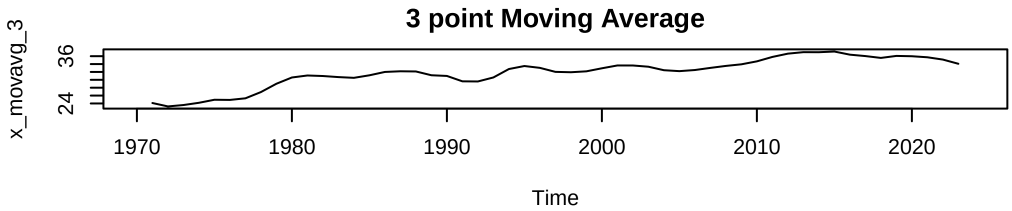
3. Trend Regression and Harmonic Seasonality Model

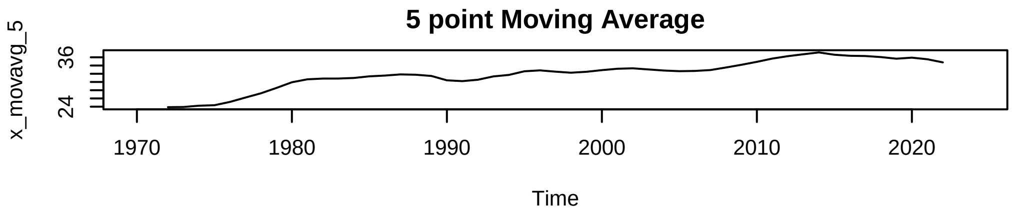
We start with a simple model in which we fit the trend of the data using linear regression and estimate the seasonal component with trigonometric functions.

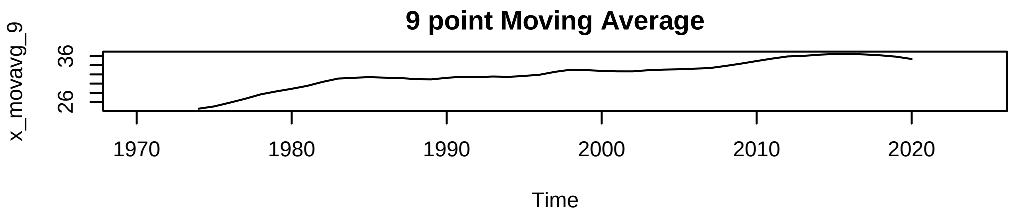
3.1 Model Selection: Estimate of Trend

Two methods for visualizing and/or identifying the trend of the data which we will employ here are moving average and linear regression. First, we utilize 3, 5, and 9-point moving averages to provide varying levels of smooth approximation to the data trend.

```
par(mfrow = c(3,1),mar = c(4, 4, 2.5, 2))
# Moving Average of QB Att/G
#ts.plot(top_15_qb.att_ts, ylab = "Att/G", main = "QB Historical Attempts Per Game")

# 3 point MA
x_movavg_3 <- stats::filter(top_15_qb.att_ts, sides = 2, rep(1,3)/3)
ts.plot(x_movavg_3, main = "3 point Moving Average")

#5 point MA
x_movavg_5 <- stats::filter(top_15_qb.att_ts, sides = 2, rep(1,5)/5)
ts.plot(x_movavg_5, main = "5 point Moving Average")


#7 point MA
#x_movavg_7 <- filter(top_15_qb.att_ts, sides = 2, rep(1,7)/7)
#ts.plot(x_movavg_7, main = "7 point Moving Average")

#One more for fun: 9 point MA
x_movavg_9 <- stats::filter(top_15_qb.att_ts, sides = 2, rep(1,9)/9)
ts.plot(x_movavg_9, main = "9 point Moving Average")
```







The three plots reveal (to varying degrees) the generally positive trend or upward movement of attempts per game as the role of passing in the NFL has grown. It is interesting to note that even in the case of 9-point moving average, there are still discernible periods in which pass attempts has increased rapidly but then subsequently plateaued.

Next, for a more tangible representation of trend we will use standard regression techniques to fit four different models. To evaluate the predictive capabilities of each, we split the data into a training portion from 1970-2013 and a forecasting portion from 2014-2024.

```
#Create training and forecasting sets
att.g_train <- window(top_15_qb.att_ts, end=2013)
num_fit <- length(att.g_train)

att.g_fore <- window(top_15_qb.att_ts, start=2014, frequency = 1)
num_fore <- length(att.g_fore)

#Create training df with t, t^2/2!, sqrt(t)

train_trend_df <- tibble(
t_fit = 1:num_fit,
t_sqfit = t_fit^2/factorial(2),
t_cubefit = t_fit^3/factorial(3),
t_sqrt = sqrt(t_fit)
)
```

```r
#Fit desired models:
#(1) Linear - \beta_0 + \beta_1 t
#(2) Quadratic - \beta_0 + \beta_1 t + \beta_2 t^2/2
#(3) Cubic - \beta_0 + \beta_1 t + \beta_2 t^2/2 + \beta_3 t^3/3!
#(4) Square root model - \beta_0 + \beta_1 \sqrt(t)

mlr.line <- lm(att.g_train ~ t_fit, data = train_trend_df)
mlr.quadr <- lm(att.g_train ~ t_fit + t_sqfit, data = train_trend_df)
mlr.cubic <- lm(att.g_train ~ t_fit + t_sqfit + t_cubefit, data = train_trend_df)
mlr.sqrt <- lm(att.g_train ~ t_sqrt, data = train_trend_df)
```

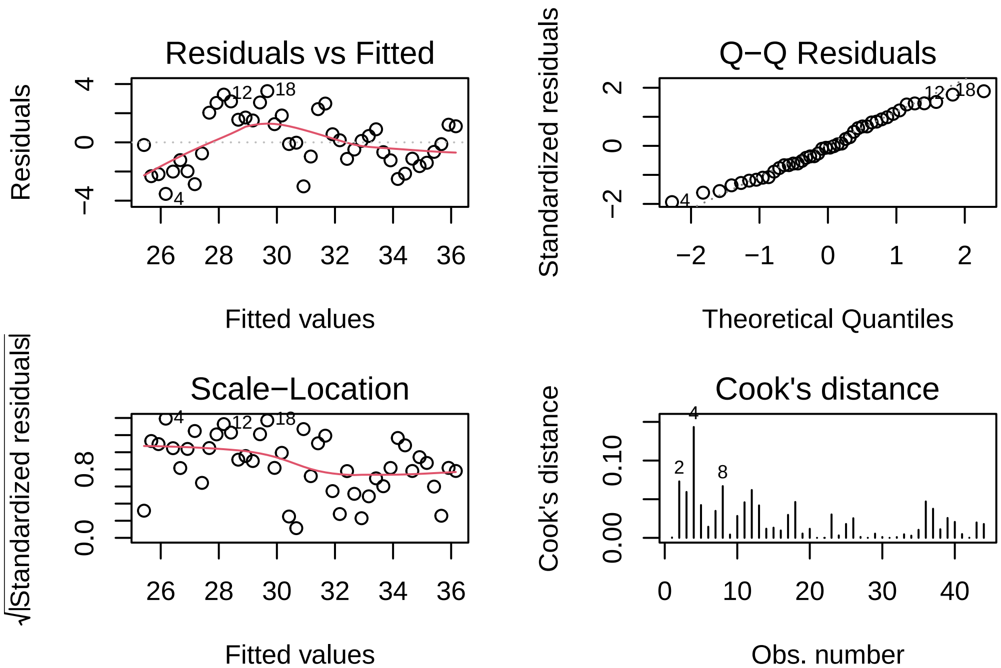
We examine the performance of each model and its residuals on the training set before considering forecast evaluation criteria.

```r
summary(mlr.line)
```

```
Call:
lm(formula = att.g_train ~ t_fit, data = train_trend_df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.5347 -1.2746 -0.1186  1.5142  3.5134 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 25.17395    0.58045   43.37  < 2e-16 ***
t_fit        0.24973    0.02247   11.12 4.35e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.892 on 42 degrees of freedom
Multiple R-squared:  0.7463,    Adjusted R-squared:  0.7403 
F-statistic: 123.6 on 1 and 42 DF,  p-value: 4.35e-14
```

The linear model takes the form $25.1740 + 0.2497t$, with the coefficient on the t-term being significant according to p-value. With an adjusted R-squared value of approximately 0.74, the model is able to explain 74% of the variance in attempts per game over time. Viewing diagnostic plots, we see some nonlinear aspect(s) of the trend have not been captured:

```r
# Residual Diagnostic Plots for mlr.lin
par(mfrow = c(2, 2), mar = c(4, 4, 2.5, 2)) # plot 4 figures, 2 in each of 2 rows
plot(mlr.line, main = "", which = 1:4)
```



Next, we want to see if the quadratic model captures the nonlinearity of the trend any better than the linear model.

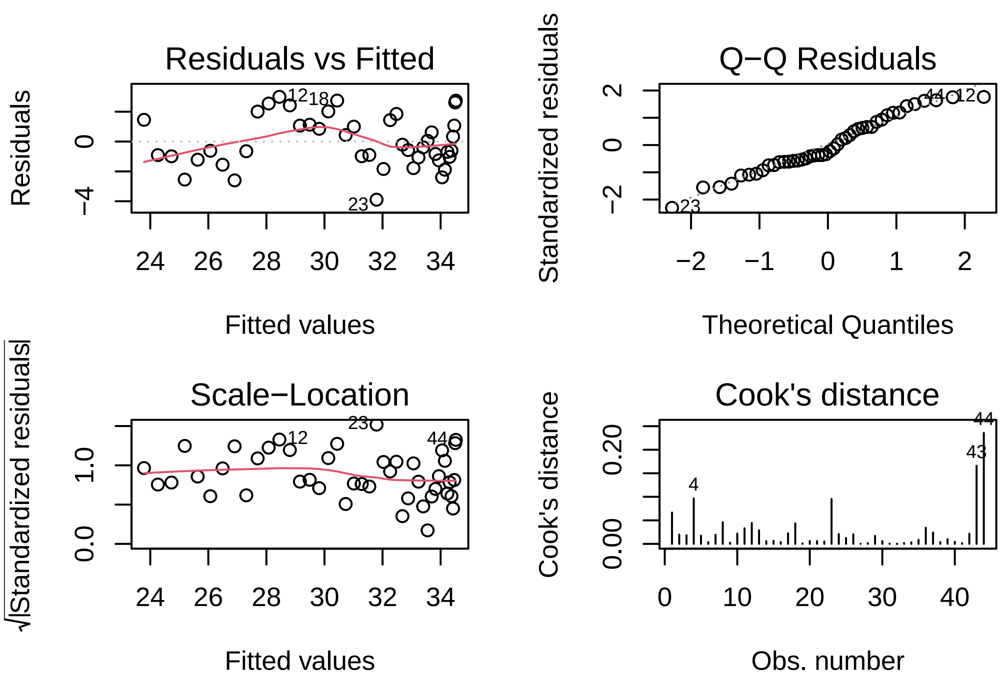
```r
summary(mlr.quadr)
```

```

Call:
lm(formula = att.g_train ~ t_fit + t_sqfit, data = train_trend_df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8942 -1.0353 -0.4776  1.2058  2.9964 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 23.295867   0.821450  28.359  < 2e-16 ***
t_fit        0.494693   0.084199   5.875 6.51e-07 ***
t_sqfit     -0.010887   0.003629  -3.000  0.00457 ** 
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.734 on 41 degrees of freedom
Multiple R-squared:  0.792, Adjusted R-squared:  0.7818
F-statistic: 78.05 on 2 and 41 DF,  p-value: 1.05e-14
```

The model takes the form $23.2959 + 0.4947t - 0.0109\left(\frac{t^2}{2}\right)$, with both coefficients being significant. The negative t^2 term is expected, since the number of pass attempts per game in recent years has leveled off or even decreased slightly. The adjusted R-squared of 0.78 here shows a 4% improvement in variance explained. The same diagnostic plots from before still show patterns in the residuals.

```
# Residual Diagnostic Plots for mlr.quadr
par(mfrow = c(2, 2), mar = c(4, 4, 2.5, 2)) # plot 4 figures, 2 in each of 2 rows
plot(mlr.quadr, main = "", which = 1:4)
```



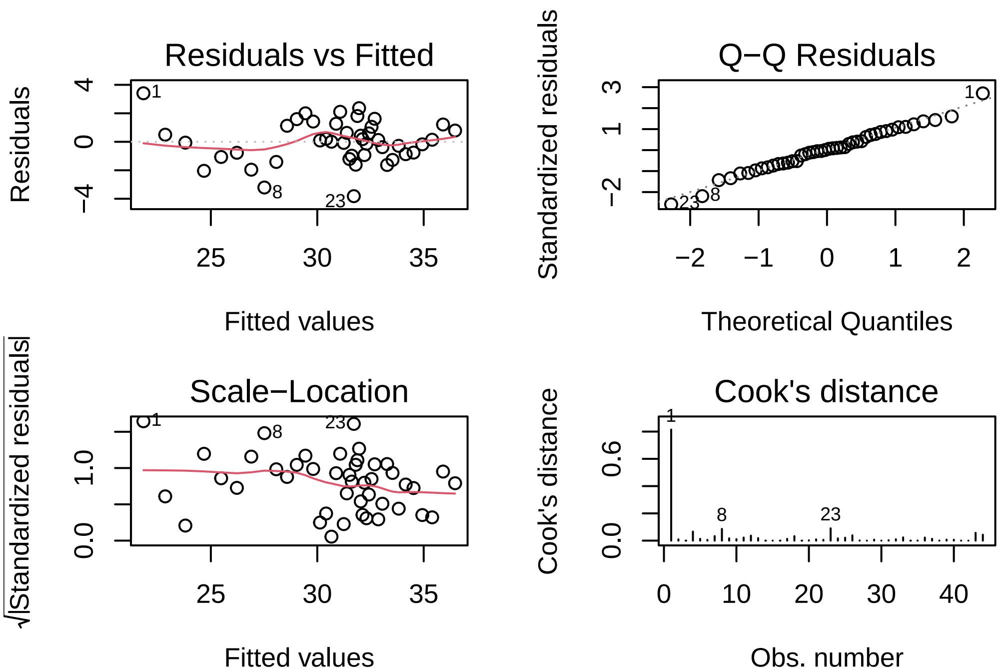
Next, the most complicated of the polynomial models: the cubic.

```
summary(mlr.cubic)
```

```
Call:
lm(formula = att.g_train ~ t_fit + t_sqfit + t_cubefit, data = train_trend_df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8178 -0.9433  0.0475  1.0700  3.4094 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 20.7301570  0.9974718  20.783  < 2e-16 ***
t_fit        1.1427528  0.1898141   6.020 4.42e-07 ***
t_sqfit     -0.0820912  0.0194851  -4.213 0.000139 ***
t_cubefit    0.0031646  0.0008545   3.704 0.000642 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.515 on 40 degrees of freedom
Multiple R-squared:  0.8451,    Adjusted R-squared:  0.8335 
F-statistic: 72.74 on 3 and 40 DF,  p-value: 3.001e-16
```

The model appears as $20.7302 + 1.1428t - 0.0821(t^2/2) + 0.0032(t^3/3!)$, where all coefficients are deemed significant. The adjusted R-squared of 0.83 indicates another 5% improvement in explained variance.

```r
# Residual Diagnostic Plots for mlr.quadr
par(mfrow = c(2, 2), mar = c(4, 4, 2.5, 2)) # plot 4 figures, 2 in each of 2 rows
plot(mlr.cubic, main = "", which = 1:4)
```



The diagnostic plots show a bit less structure, further supporting the strength of the model fit to the training set. Finally, we take a closer look at the model which includes a square root term and intercept.

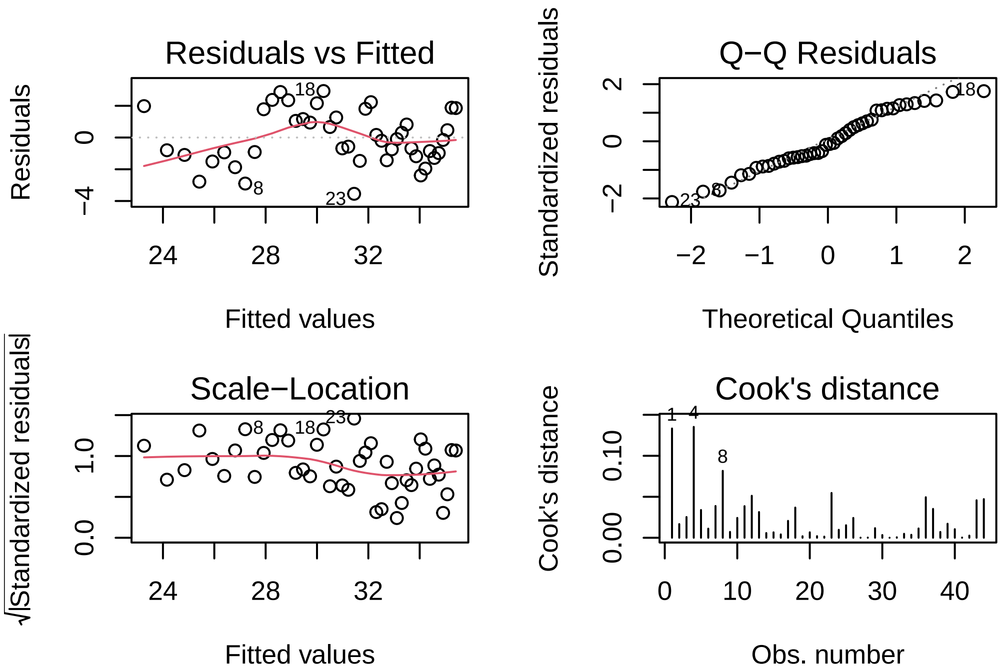
```
summary(mlr.sqrt)
```

```
Call:
lm(formula = att.g_train ~ t_sqrt, data = train_trend_df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.5459 -1.1177 -0.1777  1.3928  2.9293 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  21.1060     0.7939   26.59  < 2e-16 ***
t_sqrt        2.1560     0.1674   12.88 3.52e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.689 on 42 degrees of freedom
Multiple R-squared:  0.798, Adjusted R-squared:  0.7932
```

```
F-statistic: 165.9 on 1 and 42 DF,  p-value: 3.517e-16
```

This model has the functional form $21.1060 + 2.1560\sqrt{t}$ with significant coefficients ($p << 0.05$). The adjusted R-squared of 0.79 is better than the linear or quadratic models, but not as strong as the cubic model.

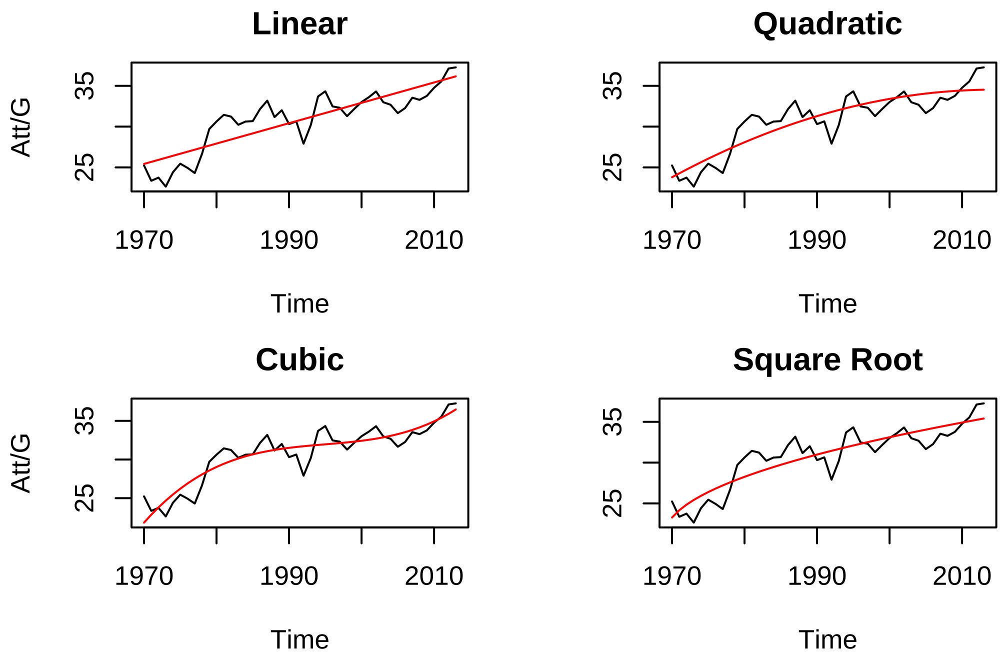
```
# Residual Diagnostic Plots for mlr.sqrt
par(mfrow = c(2, 2), mar = c(4, 4, 2.5, 2)) # plot 4 figures, 2 in each of 2 rows
plot(mlr.sqrt, main = "", which = 1:4)
```



The residuals still show a fairly distinct pattern. We can plot the prediction curves for each model, first with only the training data.

```
#Make plots showing fitted curves (4 plots)
par(mfrow = c(2, 2), mar = c(4, 4, 2.5, 2))

#Create a table with xfit and line fit models; access the fitted values using model$fitted
plin <- cbind(att.g_train, mlr.line$fitted)

#Use ts.plot on plin; specify the xfit plot is in black while model is in red
ts.plot(plin, main="Linear", col=c("black", 'red'), ylab="Att/G")

#Same idea for quadratic and cubic fits
```

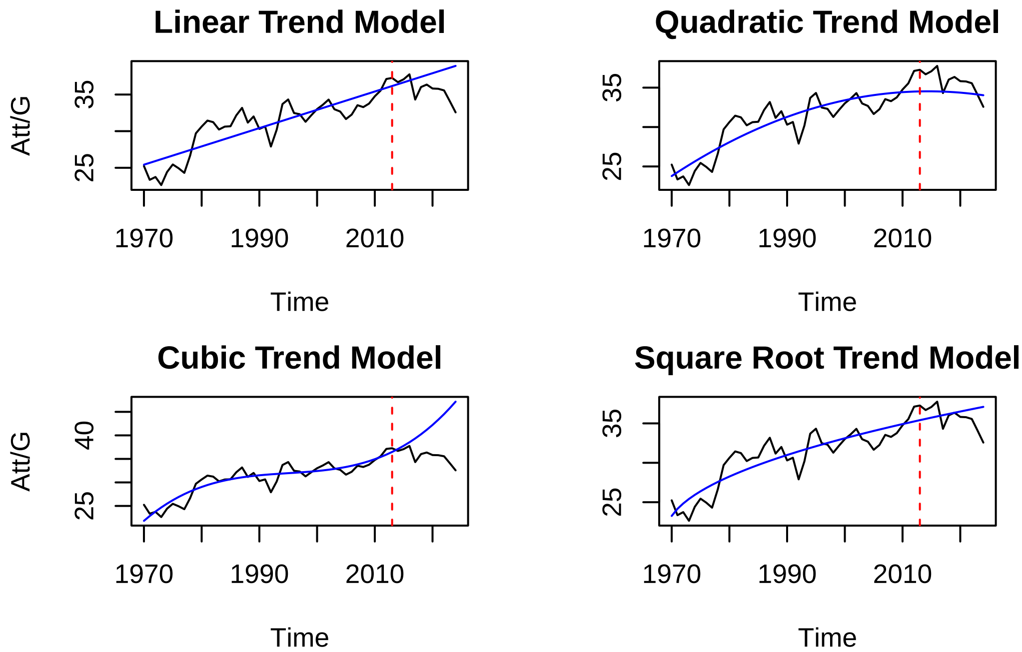
```r
pquad <- cbind(att.g_train, mlr.quadr$fitted)
ts.plot(pquad, main="Quadratic", col=c("black",'red'))

pcub <- cbind(att.g_train, mlr.cubic$fitted)
ts.plot(pcub, main="Cubic", col=c("black", 'red'), ylab = "Att/G")

psqrt <- cbind(att.g_train, mlr.sqrt$fitted)
ts.plot(psqrt, main="Square Root", col=c("black", 'red'))
```



Visually the cubic trend model does capture the uptick in attempts from 2009-2013 better than the other models; however, we know that the seasons 2014-2024 do not continue to follow this pattern, so incorporating the forecast set will tell a different story.

```r
trend_fore_df <- tibble(t_fit = (num_fit + 1):(length(top_15_qb.att_ts)),
t_sqfit = t_fit^2/factorial(2),
t_cubefit = t_fit^3/factorial(3),
t_sqrt = sqrt(t_fit))

#Make dataframe of forecast times -- t only


# use predict function
pfore.lin  <- predict(mlr.line,  newdata = trend_fore_df,       se.fit = TRUE)
```

```r
pfore.quad <- predict(mlr.quadr, newdata = trend_fore_df,     se.fit = TRUE)
pfore.cub  <- predict(mlr.cubic, newdata = trend_fore_df,   se.fit = TRUE)
pfore.sqrt <- predict(mlr.sqrt,  newdata = trend_fore_df, se.fit = TRUE)


#Plot of time series with fits and out-of-sample forecasts
# Linear Trend Model fits and forecasts
linff <- c(mlr.line$fitted, pfore.lin$fit)
# Quadratic Trend Model fits and forecasts
quadff <- c(mlr.quadr$fitted, pfore.quad$fit)
# Cubic Trend Model fits and forecasts
cubff <- c(mlr.cubic$fitted, pfore.cub$fit)
#Sqrt trend model fits and forecasts
sqrtff <- c(mlr.sqrt$fitted, pfore.sqrt$fit)

# observed data: top_15_qb.att_ts from 1970-2024
# Bind observed data and fits+forecasts

obslin <- cbind(top_15_qb.att_ts, linff)
obsquad <- cbind(top_15_qb.att_ts, quadff)
obscub <- cbind(top_15_qb.att_ts, cubff)
obssqrt <- cbind(top_15_qb.att_ts, sqrtff)

par(mfrow = c(2,2), mar = c(4, 4, 2.5, 2))
ts.plot(obslin, main = "Linear Trend Model", col = c("black", "blue"),
        ylab = "Att/G")
abline(v = time(top_15_qb.att_ts)[num_fit], col = "red", lty = "dashed")

ts.plot(obsquad, main = "Quadratic Trend Model", col = c("black", "blue"))
abline(v = time(top_15_qb.att_ts)[num_fit], col = "red", lty = "dashed")

ts.plot(obscub, main = "Cubic Trend Model", col = c("black", "blue"),
        ylab="Att/G")
abline(v = time(top_15_qb.att_ts)[num_fit], col = "red", lty = "dashed")

ts.plot(obssqrt, main = "Square Root Trend Model", col = c("black", "blue"))
abline(v = time(top_15_qb.att_ts)[num_fit], col = "red", lty = "dashed")
```



The red line in the plots above indicates the end of the training portion. As feared, the cubic model outgrows the data in the forecast portion, as does the linear model. The square root model does a somewhat better job because of its slower growth, and the quadratic model recognizes the downturn in the data. We can evaluate model forecast performance with standard metrics.

```
nfore <- length(att.g_fore)

# Forecast/Prediction errors: Observed - Predicted
efore.lin <- att.g_fore - pfore.lin$fit
efore.quad <- att.g_fore - pfore.quad$fit
efore.cub <- att.g_fore - pfore.cub$fit
efore.sqrt <- att.g_fore - pfore.sqrt$fit

# Forecast evaluation criteria
me.lin <- mean(efore.lin)                              # Mean Error
mpe.lin <- 100*(mean(efore.lin/att.g_fore))                 # Mean Percent Error
mse.lin <- sum(efore.lin**2)/nfore                     # Mean Squared Error
mae.lin <- mean(abs(efore.lin))                        # Mean Absolute Error
mape.lin <- 100*(mean(abs((efore.lin)/att.g_fore)))    # Mean Absolute Percent Error
fec.lin <- data.frame(linear = rbind(me.lin, mpe.lin,
                                      mse.lin, mae.lin, mape.lin),
                      row.names = c("me", "mpe", "mse", "mae", "mape"))
```

```
# Forecast evaluation criteria
me.quad <- mean(efore.quad)                              # Mean Error
mpe.quad <- 100*(mean(efore.quad/att.g_fore))           # Mean Percent Error
mse.quad <- sum(efore.quad**2)/nfore                     # Mean Squared Error
mae.quad <- mean(abs(efore.quad))                        # Mean Absolute Error
mape.quad <- 100*(mean(abs((efore.quad)/att.g_fore)))  # Mean Absolute Percent Error
fec.quad <- data.frame(quadratic = rbind(me.quad, mpe.quad,
                                      mse.quad, mae.quad, mape.quad),
                      row.names = c("me", "mpe", "mse", "mae", "mape"))

# Forecast evaluation criteria
me.cube <- mean(efore.cub)                              # Mean Error
mpe.cube <- 100*(mean(efore.cub/att.g_fore))           # Mean Percent Error
mse.cube <- sum(efore.cub**2)/nfore                     # Mean Squared Error
mae.cube <- mean(abs(efore.cub))                        # Mean Absolute Error
mape.cube <- 100*(mean(abs((efore.cub)/att.g_fore)))  # Mean Absolute Percent Error
fec.cube <- data.frame(cubic = rbind(me.cube, mpe.cube,
                                      mse.cube, mae.cube, mape.cube),
                      row.names = c("me", "mpe", "mse", "mae", "mape"))

# Forecast evaluation criteria
me.sqrt <- mean(efore.sqrt)                              # Mean Error
mpe.sqrt <- 100*(mean(efore.sqrt/att.g_fore))           # Mean Percent Error
mse.sqrt <- sum(efore.sqrt**2)/nfore                     # Mean Squared Error
mae.sqrt <- mean(abs(efore.sqrt))                        # Mean Absolute Error
mape.sqrt <- 100*(mean(abs((efore.sqrt)/att.g_fore)))  # Mean Absolute Percent Error
fec.sqrt <- data.frame(sqroot = rbind(me.sqrt, mpe.sqrt,
                                      mse.sqrt, mae.sqrt, mape.sqrt),
                      row.names = c("me", "mpe", "mse", "mae", "mape"))

round(fec.lin, digits = 4)
```

```
      linear
me   -2.0208
mpe  -5.9087
mse   8.3657
mae   2.3018
mape  6.6606
```

```r
round(fec.quad, digits = 4)
```

```
     quadratic
me      1.2727
mpe     3.4291
mse     3.2828
mae     1.5838
mape    4.3779
```

```r
round(fec.cube, digits = 4)
```

```
        cubic
me    -5.9244
mpe  -17.0935
mse   54.2081
mae    5.9244
mape  17.0935
```

```r
round(fec.sqrt, digits = 4)
```

```
      sqroot
me   -0.7040
mpe  -2.1818
mse   3.7380
mae   1.4953
mape  4.3051
```

According to these criterion, the square root and quadratic models have roughly the same forecast performance. We can also use information based criterion for model selection, e.g. the Akaike information criterion (AIC) or Bayesian information criterion (BIC).

```r
# Linear Trend Model, Sqrt Model: k=1
k_1 <- 1
k_2 <- 2
k_3 <- 3
# Akaike Information Criterion,AIC
AIC.lin <- AIC(mlr.line, k=k_1)
AIC.quad <- AIC(mlr.quadr, k=k_2)
AIC.cubic <- AIC(mlr.cubic, k=k_3)
```

```
AIC.sqrt <- AIC(mlr.sqrt, k=k_1)

AIC_vals <- data.frame(AIC = rbind(AIC.lin, AIC.quad,
                                   AIC.cubic, AIC.sqrt),
                      row.names = c("Linear", "Quadratic", "Cubic", "Square Root"))
AIC_vals
```

```
                 AIC
Linear      181.9499
Quadratic   178.2156
Cubic       172.2424
Square Root 171.9216
```

Of these, the AIC model prefers the square root model. We will select the square root estimate of the trend, and next add in a seasonal component if it proves to be applicable.

3.2 Model Selection: Seasonal Component

We will use trigonometric functions to estimate any seasonal patterns in the data. For instance, it could be argued that the data goes through a repeating pattern every number of years, though it is not clear what that number of years is. With that in mind:

```
#Maximum period willing to consider
per <- 20

for (n in 1:per) {
  c1fit_0 <- cos(2 * pi * train_trend_df$t_fit / n)
  s1fit_0 <- sin(2 * pi * train_trend_df$t_fit / n)
  att.g_trig1 <- lm(att.g_train ~ train_trend_df$t_sqrt + c1fit_0 + s1fit_0)
  print(paste("n = ", n, "R^2 = ", summary(att.g_trig1)$adj.r.squared))
}
```

```
[1] "n =  1 R^2 =  0.788954521382315"
[1] "n =  2 R^2 =  0.783409733413258"
[1] "n =  3 R^2 =  0.783799163496362"
[1] "n =  4 R^2 =  0.78968234894235"
[1] "n =  5 R^2 =  0.789046501269144"
[1] "n =  6 R^2 =  0.793649135930864"
[1] "n =  7 R^2 =  0.800501241662951"
[1] "n =  8 R^2 =  0.818555806557192"
[1] "n =  9 R^2 =  0.790986636198813"
```

```
[1] "n =  10 R^2 =  0.788835125543901"
[1] "n =  11 R^2 =  0.793587581040882"
[1] "n =  12 R^2 =  0.783053108928284"
[1] "n =  13 R^2 =  0.799126899606411"
[1] "n =  14 R^2 =  0.832482018144459"
[1] "n =  15 R^2 =  0.857132925357403"
[1] "n =  16 R^2 =  0.859764440553288"
[1] "n =  17 R^2 =  0.842537731740622"
[1] "n =  18 R^2 =  0.820110090926656"
[1] "n =  19 R^2 =  0.803889752022249"
[1] "n =  20 R^2 =  0.796697595820688"
```

The largest adjusted R-squared occurs for $n = 16$, indicating a period of 16 years. To see the full summary for this model, we rerun the same phrase but with fixed $n = 16$ and find that all coefficients are significant according to their p-values.

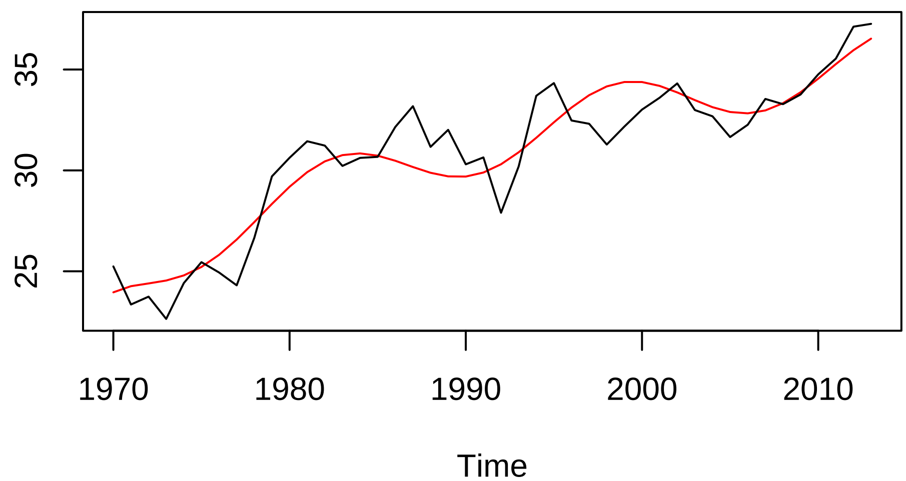
```
#Model 1a
n_0 <- 16
c1fit_0 <- cos(2 * pi * train_trend_df$t_fit / n_0)
s1fit_0 <- sin(2 * pi * train_trend_df$t_fit / n_0)
t_sqrt <- train_trend_df$t_sqrt
att.g_trig1 <- lm(att.g_train ~ t_sqrt + c1fit_0 + s1fit_0)
summary(att.g_trig1)
```

```
Call:
lm(formula = att.g_train ~ t_sqrt + c1fit_0 + s1fit_0)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.88103 -0.72136 -0.08461  0.87861  3.01469 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  21.5220     0.6730  31.977  < 2e-16 ***
t_sqrt        2.0882     0.1421  14.692  < 2e-16 ***
c1fit_0       0.8545     0.3040   2.811 0.007607 ** 
s1fit_0      -1.1569     0.3010  -3.843 0.000426 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.391 on 40 degrees of freedom
```

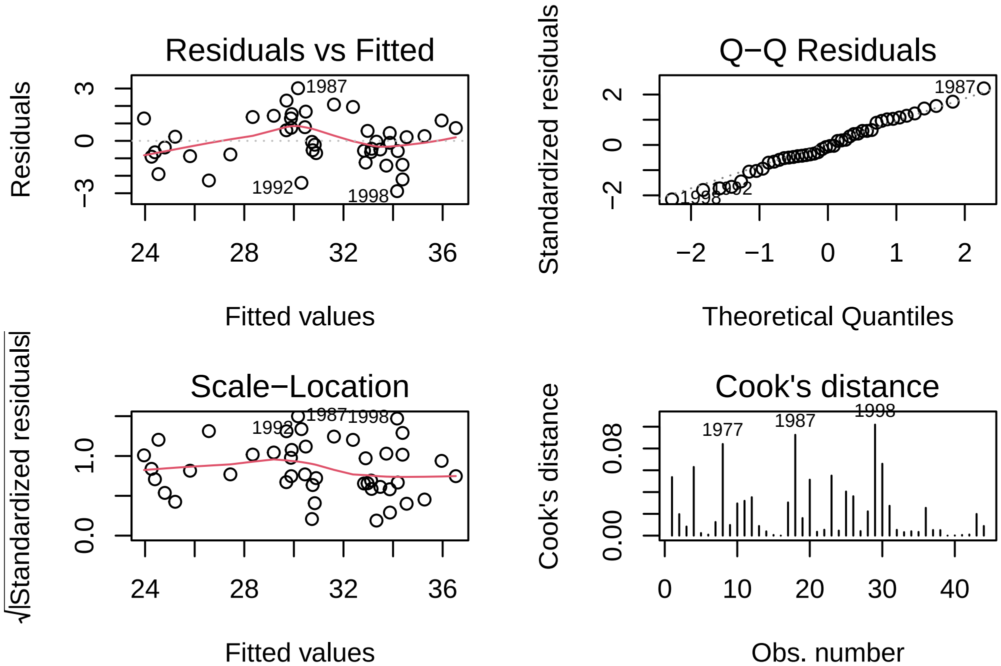
```
Multiple R-squared:  0.8695,	Adjusted R-squared:  0.8598 
F-statistic: 88.88 on 3 and 40 DF,  p-value: < 2.2e-16
```

We plot the predicted values for this model on the training set, and it does seem to follow the recurrent peaks and valleys of the series.

```r
ts.plot(cbind(att.g_trig1$fitted,att.g_train), col= c("red", "black"))
```




```r
# Residual Diagnostic Plots for co2_trig1
par(mfrow = c(2, 2), mar = c(4, 4, 2.5, 2)) # plot 4 figures, 2 in each of 2 rows
plot(att.g_trig1, main = "", which = 1:4)
```



Several of the residuals (1998, 1987 in particular) of `att.g_trig1` are highly influential, lending themselves to the consistent (though still expected) presence of structure in the residual plot. Before evaluating the performance of `att.g_trig1` on the forecast set, it is reasonable to try including more harmonics to see if the trend, low-frequency harmonic (16 year period), and high-frequency harmonic(s) are more capable of describing the data.

It is possible to add many harmonic pairs of trigonometric functions, but of course a greater number of parameters reduces parsimony, increases the possibility of overfitting, and may not lead to a significant increase in explained variance. For these reasons we will fit a model denoted 1b with two harmonics and model 1c with three harmonics. Then, using F-tests plus information and forecast criterion, we will select the best model.

```
#Models 1b and 1c: 2 and 3 harmonics

seasonal_train_df <- tibble(

#Harmonic 1
t_sqrt = t_sqrt,
t_fit = train_trend_df$t_fit,
c1fit = cos(2 * pi * t_fit / n_0),
s1fit = sin(2 * pi * t_fit / n_0),

#Harmonic 2
c2fit = cos(2 * pi * 2 * t_fit / n_0),
```

```r
s2fit = sin(2 * pi * 2 * t_fit / n_0),

#Harmonic 3
c3fit = cos(2 * pi * 3 * t_fit / n_0),
s3fit = sin(2 * pi * 3 * t_fit / n_0)
)

#Model 1a
att.g_trig1 <- lm(att.g_train ~ t_sqrt + c1fit + s1fit, data = seasonal_train_df)

#Model 1b
att.g_trig2 <- lm(att.g_train ~ t_sqrt + c1fit + s1fit + c2fit + s2fit, data = seasonal_train

#Model 1c
att.g_trig3 <- lm(att.g_train ~ t_sqrt + c1fit + s1fit + c2fit + s2fit + c3fit + s3fit,
                  data = seasonal_train_df)

summary(att.g_trig2)
```

```
Call:
lm(formula = att.g_train ~ t_sqrt + c1fit + s1fit + c2fit + s2fit,
    data = seasonal_train_df)

Residuals:
     Min       1Q   Median       3Q      Max
-2.32014 -1.18458  0.07177  0.99439  2.30610

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  21.3891     0.6298  33.960  < 2e-16 ***
t_sqrt        2.1057     0.1328  15.857  < 2e-16 ***
c1fit         0.7785     0.2852   2.729 0.009562 **
s1fit        -1.0868     0.2821  -3.852 0.000437 ***
c2fit        -0.1663     0.2774  -0.600 0.552392
s2fit         0.7717     0.2798   2.758 0.008883 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.297 on 38 degrees of freedom
Multiple R-squared:  0.8921,    Adjusted R-squared:  0.8779
F-statistic: 62.84 on 5 and 38 DF,  p-value: < 2.2e-16
```

```r
summary(att.g_trig3)
```

```

Call:
lm(formula = att.g_train ~ t_sqrt + c1fit + s1fit + c2fit + s2fit + 
    c3fit + s3fit, data = seasonal_train_df)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.84346 -0.75039 -0.00447  0.64137  2.50943 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  21.4209     0.6078  35.246  < 2e-16 ***
t_sqrt        2.0958     0.1281  16.363  < 2e-16 ***
c1fit         0.7732     0.2751   2.810 0.007958 ** 
s1fit        -1.0511     0.2721  -3.863 0.000448 ***
c2fit        -0.1255     0.2684  -0.468 0.642814    
s2fit         0.8136     0.2701   3.012 0.004732 ** 
c3fit         0.5684     0.2717   2.092 0.043538 *  
s3fit         0.2054     0.2650   0.775 0.443523    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.249 on 36 degrees of freedom
Multiple R-squared:  0.9053,	Adjusted R-squared:  0.8869 
F-statistic: 49.17 on 7 and 36 DF,  p-value: < 2.2e-16
```

Per R-squared values, both Model 1b and Model 1c have improved performance over their subset counterparts. To determine whether this increase in explained variance is significant based on the added parameters, we can use the Extra Sum of Squares F-test. First, to compare Model 1a (one harmonic) with Model 1b (two harmonics), we have:

```r
anova(att.g_trig1)
```

```
Analysis of Variance Table

Response: att.g_train
          Df Sum Sq Mean Sq  F value    Pr(>F)    
t_sqrt     1 473.12  473.12 244.6917 < 2.2e-16 ***
c1fit      1  13.86   13.86   7.1678 0.0107117 *  
```

```
s1fit      1  28.55   28.55  14.7674 0.0004255 ***
Residuals 40  77.34    1.93
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(att.g_trig2)
```

```
Analysis of Variance Table

Response: att.g_train
          Df Sum Sq Mean Sq  F value    Pr(>F)
t_sqrt     1 473.12  473.12 281.0693 < 2.2e-16 ***
c1fit      1  13.86   13.86   8.2334 0.0066815 **
s1fit      1  28.55   28.55  16.9629 0.0001983 ***
c2fit      1   0.57    0.57   0.3379 0.5644632
s2fit      1  12.81   12.81   7.6088 0.0088827 **
Residuals 38  63.96    1.68
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From these tables, we calculate

$$\text{Extra SS} = 77.34 - 63.96 = 13.38 \text{Extra DF} = 40 - 38 = 2 \text{MS Residuals Full} = 1.68$$

Then the Extra SS F-statistic is

$$\frac{\text{Extra SS / Extra DF}}{\text{MS Residuals Full}} = \frac{(13.38/2)}{1.68)} = 3.9821$$

This is compared with the F-critical value, which may be computed using qf(0.95, Extra DF, DF Residuals Full):

```r
qf(0.95, 2, 38)
```

```
[1] 3.244818
```

Since $3.9821 > 3.2448$, we reject the null hypothesis (i.e. we reject that the addition of a second harmonic was insignificant), and thus prefer Model 1b to Model 1a. Repeating the same calculations with Model 1c as the full model:

```r
anova(att.g_trig3)
```

```
Analysis of Variance Table

Response: att.g_train
          Df Sum Sq Mean Sq  F value    Pr(>F)    
t_sqrt     1 473.12  473.12 303.3868 < 2.2e-16 ***
c1fit      1  13.86   13.86   8.8872 0.0051236 ** 
s1fit      1  28.55   28.55  18.3098 0.0001325 ***
c2fit      1   0.57    0.57   0.3648 0.5496647    
s2fit      1  12.81   12.81   8.2129 0.0069045 ** 
c3fit      1   6.89    6.89   4.4170 0.0426449 *  
s3fit      1   0.94    0.94   0.6003 0.4435226    
Residuals 36  56.14    1.56                       
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$\text{Extra SS} = 63.96 - 56.14 = 7.82 \text{Extra DF} = 38 - 36 = 2 \text{MS Residuals Full} = 1.56$$

$$\implies \frac{\text{Extra SS / Extra DF}}{\text{MS Residuals Full}} = \frac{(7.82/2)}{1.56)} = 2.50641$$

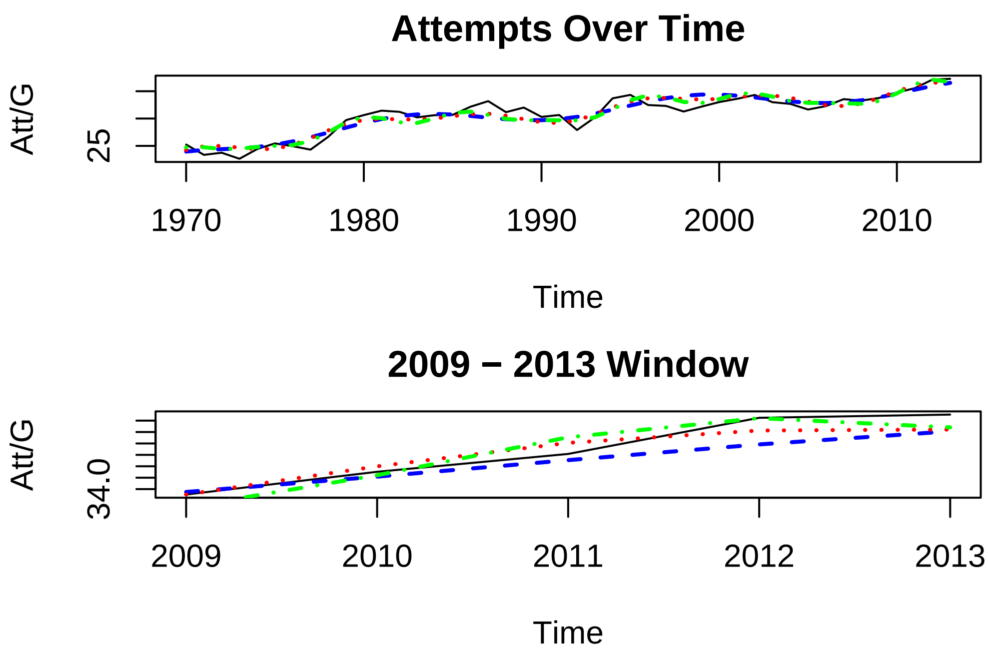
```r
qf(0.95, 2, 36)
```

```
[1] 3.259446
```

Since $2.5064 < 3.2594$, the Extra SS F-test does *not* prefer Model 1c over 1b. This points us toward Model 1b. For completeness, we include plots depicting the predicted values from all three models as well as a version zoomed in on the end of the training set.

```r
par(mfrow = c(2, 1), mar = c(4, 4, 2.5, 2))
plot(att.g_train, xlab = "Time", ylab = "Att/G",
     main = "Attempts Over Time")
lines(ts(att.g_trig1$fit,start=1970,freq=1),
      col="blue", lty = "dashed", lwd = 2)
lines(ts(att.g_trig2$fit,start=1970,freq=1),
      col="red", lty = "dotted", lwd = 2)
lines(ts(att.g_trig3$fit,start=1970,freq=1),
      col="green", lty = "dotdash", lwd = 2)
```

```r
### Zoom in on the last five years of training data
plot(window(att.g_train, start=2009), xlab = "Time", ylab = "Att/G",
     main = "2009 - 2013 Window")
lines(window(ts(att.g_trig1$fit, start=1970, freq=1),
             start=2009), col = "blue", lty = "dashed", lwd = 2)
lines(window(ts(att.g_trig2$fit, start=1970, freq=1),
             start=2009), col = "red", lty = "dotted", lwd = 2)
lines(window(ts(att.g_trig3$fit, start=1970, freq=1),
             start=2009), col = "green", lty = "dotdash", lwd = 2)
```



Next, we compare the models using AIC, AICc, and BIC.

```r
num_fit <- length(att.g_train)
k1 <- length(coef(att.g_trig1))
k2 <- length(coef(att.g_trig2))
k3 <- length(coef(att.g_trig3))

# Information criteria
# Verify with each anova table
sigsq.1a <- summary(att.g_trig1)$sigma^2
sigsq.1b <- summary(att.g_trig2)$sigma^2
sigsq.1c <- summary(att.g_trig3)$sigma^2
```

```r
# AIC
AIC.1a <- AIC(att.g_trig1)
AIC.1b <- AIC(att.g_trig2)
AIC.1c <- AIC(att.g_trig3)
# Or use AIC(logLik(co2.model1a))
# Or use AIC(logLik(co2_trig1))
# Or use AIC(logLik(co2_trig2))

# Corrected AIC
AICc.1a <- AIC.1a + (2*k1^2 + 2*k1)/(num_fit - k1 - 1)
AICc.1b  <- AIC.1b  + (2*k2^2 + 2*k2)/(num_fit - k2 - 1)
AICc.1c  <- AIC.1c  + (2*k3^2 + 2*k3)/(num_fit - k3 - 1)


# BIC
BIC.1a <- BIC(att.g_trig1)
BIC.1b <- BIC(att.g_trig2)
BIC.1c <- BIC(att.g_trig3)

IC.1a <- data.frame(model1a = rbind(AIC.1a, AICc.1a, BIC.1a),
                    row.names = c("AIC", "AICc", "BIC"))
IC.1b <- data.frame(model1b = rbind(AIC.1b, AICc.1b, BIC.1b),
                    row.names = c("AIC", "AICc", "BIC"))
IC.1c <- data.frame(model1c = rbind(AIC.1c, AICc.1c, BIC.1c),
                    row.names = c("AIC", "AICc", "BIC"))
round(cbind(IC.1a, IC.1b, IC.1c), digits = 4)
```

```
      model1a  model1b  model1c
AIC  159.6844 155.3290 153.5881
AICc 160.7100 157.5992 157.7024
BIC  168.6053 167.8183 169.6458
```

Model 1b is preferred by two of the three information criterion. Finally, we look to forecast criterion for one more piece of evidence.

```r
# Set up for prediction
t_fore <- seq_len(11) + 44   # future times

seasonal_fore_df_1b <- tibble(
  t_sqrt = sqrt(t_fore),
```

```
  c1fit  = cos(2 * pi * t_fore / n_0),
  s1fit  = sin(2 * pi * t_fore / n_0),
  c2fit  = cos(2 * pi * 2 * t_fore / n_0),
  s2fit  = sin(2 * pi * 2 * t_fore / n_0)
)


seasonal_fore_df <-tibble(
  t_fit = t_fore,
  t_sqrt = sqrt(t_fit),
  c1fit  = cos(2 * pi * t_fore/n_0),
  s1fit  = sin(2 * pi * t_fore/n_0),
  c2fit  = cos(2 * pi * 2 * t_fore/n_0),
  s2fit  = sin(2 * pi * 2 * t_fore/n_0),
  c3fit  = cos(2 * pi * 3 * t_fore/n_0),
  s3fit  = sin(2 * pi * 3 * t_fore/n_0)
)


# Model 1a
pfore.1a <- predict(att.g_trig1, newdata = seasonal_fore_df, se.fit = TRUE)
efore.1a <- as.numeric(att.g_fore) - as.numeric(pfore.1a$fit)

me.1a <- mean(efore.1a)
mpe.1a <- 100 * (mean(as.numeric(efore.1a) / as.numeric(att.g_fore)))
mse.1a <- sum(efore.1a**2) / nfore
mae.1a <- mean(abs(efore.1a))
mape.1a <- 100 * (mean(abs(efore.1a) / as.numeric(att.g_fore)))
fec.1a <- data.frame(model1a = rbind(me.1a, mpe.1a, mse.1a, mae.1a, mape.1a),
                     row.names = c("me", "mpe", "mse", "mae", "mape"))

# Model 1b
pfore.1b <- predict(att.g_trig2, newdata = seasonal_fore_df, se.fit = TRUE)
efore.1b <- as.numeric(att.g_fore) - as.numeric(pfore.1b$fit)

me.1b <- mean(efore.1b)
mpe.1b <- 100 * (mean(efore.1b / att.g_fore))
mse.1b <- sum(efore.1b**2) / nfore
mae.1b <- mean(abs(efore.1b))
mape.1b <- 100 * (mean(abs(efore.1b) / att.g_fore))
fec.1b <- data.frame(model1b = rbind(me.1b, mpe.1b, mse.1b, mae.1b, mape.1b),
                     row.names = c("me", "mpe", "mse", "mae", "mape"))
```

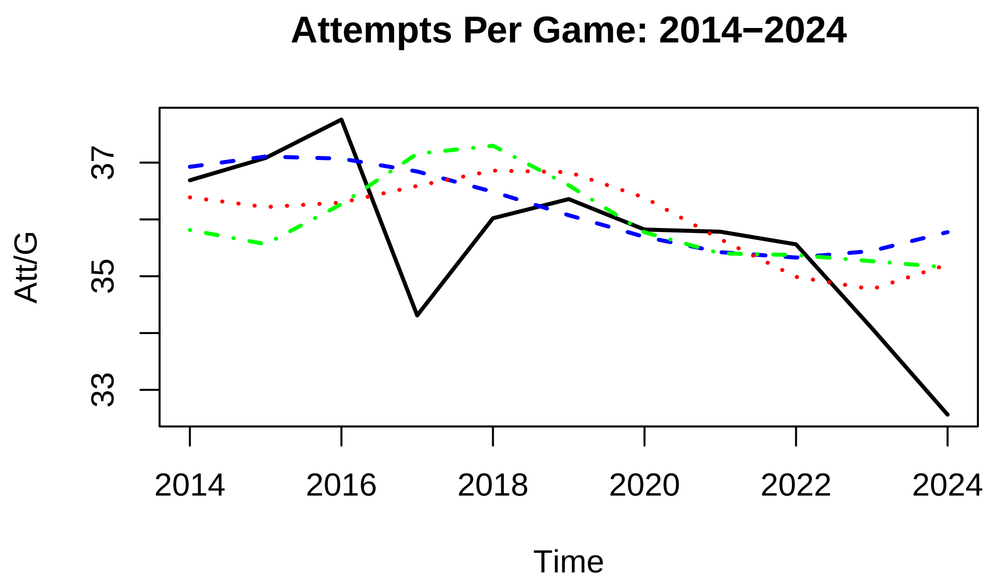
```r
# Model 1c
pfore.1c <- predict(att.g_trig3, seasonal_fore_df, se.fit = TRUE)
efore.1c <- att.g_fore - pfore.1c$fit

me.1c <- mean(efore.1c)
mpe.1c <- 100 * (mean(efore.1c / att.g_fore))
mse.1c <- sum(efore.1c**2) / nfore
mae.1c <- mean(abs(efore.1c))
mape.1c <- 100 * (mean(abs(efore.1c) / att.g_fore))
fec.1c <- data.frame(model1c = rbind(me.1c, mpe.1c, mse.1c, mae.1c, mape.1c),
                     row.names = c("me", "mpe", "mse", "mae", "mape"))
round(cbind(fec.1a, fec.1b, fec.1c), digits = 4)
```

```
     model1a model1b model1c
me   -0.5582 -0.3776 -0.3308
mpe  -1.6929 -1.1797 -1.0708
mse   1.7834  1.5653  2.1247
mae   0.8659  0.9832  1.1489
mape  2.5330  2.8160  3.2803
```

Interestingly, Model 1a is decently strong, with the best MAPE and MAE. Model 1b claims the superior MSE, while Model 1c produces the best ME and MPE. To see the corresponding plots:

```r
# Plot of time series with fits and out-of-sample forecasts
# Model 1a fits and forecasts
ff.1a <- ts(pfore.1a$fit,
             start = start(att.g_fore),
             frequency = frequency(att.g_fore))

ff.1b <- ts(pfore.1b$fit,
             start = start(att.g_fore),
             frequency = frequency(att.g_fore))

ff.1c <- ts(pfore.1c$fit,
             start = start(att.g_fore),
             frequency = frequency(att.g_fore))

# Plot
ts.plot(cbind(att.g_fore, ff.1a, ff.1b, ff.1c), main = "Attempts Per Game: 2014-2024", ylab =
```

```
        col = c("black", "blue", "red", "green"), lwd = 2,
        lty = c("solid", "dashed", "dotted", "dotdash"))
```



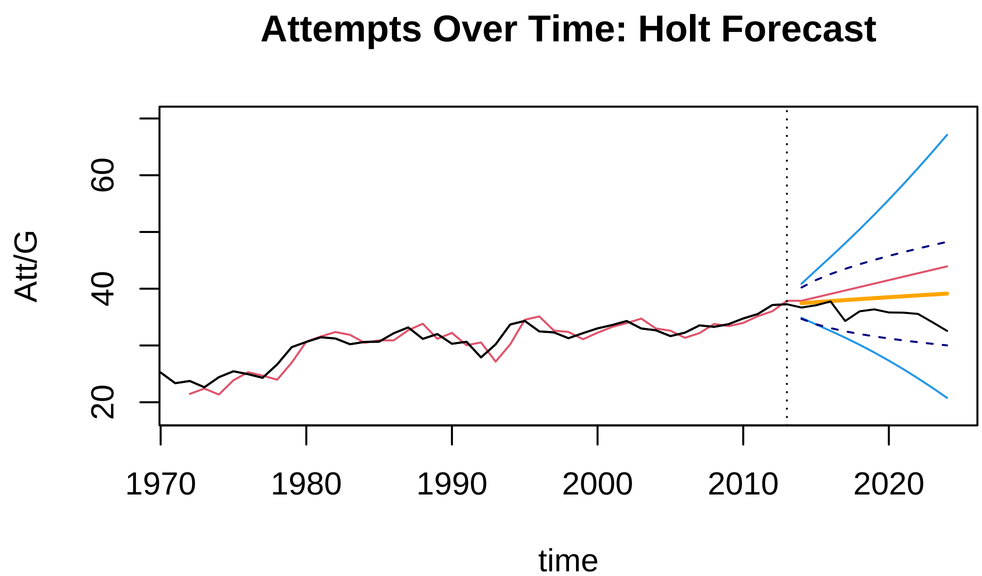
None of the models seems to follow what happens with the data beyond the training set, which is not surprising given the sizeable downturn after 2016. We will next explore a different model class.

4. Smoothing with Holt-Winters

We can use Holt-Winters exponential smoothing to estimate the level and trend of the data, though not with a seasonal parameter because our data is annual (frequency 1). Attempts with and without damping are made.

```
library(forecast)

# Attempt to damp
hw.damped <- holt(att.g_train, damped=TRUE, h=11)

#Undamped
dexp.smooth <- HoltWinters(att.g_train, gamma = F)

#Forecast values with interval about them
```

```r
dexp.fore <- predict(dexp.smooth, 11, prediction.interval = TRUE)
dexp.damped.fore <- predict(hw.damped, 11, prediction.interval = TRUE)
#Visualize result
plot(dexp.smooth, dexp.fore, xlab = "time", ylab = "Att/G",
     ylim = c(18, 70), main = "Attempts Over Time: Holt Forecast")
lines(2014:2024, hw.damped$mean, col = "orange", lwd = 2)
lines(2014:2024, hw.damped$lower[,2], col = "navy", lty = 2)
lines(2014:2024, hw.damped$upper[,2], col = "navy", lty = 2)
lines(x = 1970: 2024, y = top_15_qb.att_ts)
```



Unfortunately, the trend from ~ 2005-2013 is captured by both models and then extended in some fashion for 2014-2024, which does not match reality. The predicted values for the undamped model are depicted in red, with solid light blue confidence intervals. The forecast values for the damped model are displayed in orange, with dark blue dashed confidence intervals.

5. Conclusions

While there is a distinct increasing trend $\sim \sqrt{t}$ in the number of attempts per game among the top fifteen passers of the ball each year, periodicity of this phenomenon is more difficult to conclusively identify. In a future work, one might consider the use of an ARIMA-type model, in particular ARIMA(0,1,0), as this small data set resembles a random walk in some respects. Indeed, we part with:

```
att.g_arima <- auto.arima(top_15_qb.att_ts, ic='aicc', max.d=2, seasonal = FALSE, allowdrift
```

```
 ARIMA(2,1,2) with drift         : Inf
 ARIMA(0,1,0) with drift         : 190.6718
 ARIMA(1,1,0) with drift         : 192.8998
 ARIMA(0,1,1) with drift         : 192.8965
 ARIMA(0,1,0)                    : 189.0477
 ARIMA(1,1,1) with drift         : 195.2094

 Best model: ARIMA(0,1,0)
```