

Primary insights into the superconducting twin qubit

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Short decoherence times stand as one of the main obstacles hindering industrial scale implementation of quantum processors based on superconducting qubits. The simplicity and flexibility of fabricating superconducting qubits compared with other qubit architectures, most notably trapped ions, has to be set against their diminished ability of handling sequential state operations - the most successful processor prototype has a decoherence time of $\sim 60\ \mu\text{s}$ and a gate operation time of $\sim 20\ \mu\text{s}$ [1], so within 3 gate operations it's quantum state becomes substantially deteriorated.

Right now, in the embryonic stages of quantum infrastructure development, probability theory of the multi-armed bandit dictates [2]: to maximize expected gains in combating this decoherence problem, resources should be allocated to exploring new alternatives, as opposed to exploiting the possibly sub-optimal (requiring a supporting qubit overhead that is double the size of the original system) error correction mechanism already in place [3].

In this work we look at boosting the qubit's decoherence time using a new geometry - a fusion of two flux qubits joined by a common Josephson Junction. At the degeneracy flux-bias point, $\frac{1}{2}\Phi_0$, the twin qubit is measured to have energy spectrum plateaus with curvature several orders of magnitude lower than that in previous flux qubits. This flatness makes the qubit more robust to flux noise. We experimentally show the transmission spectrum, Rabi oscillations and simulate dipole transitions on this new type of qubit, laying the out the initial characteristics for future development.

I. INTRODUCTION

Superconducting qubits are one of the promising trends for implementing quantum computing technology. Being nothing more than strips of aluminum on a chip, their geometry can be designed to select an operating energy, state transition rates and sensitivity that would fit the use-case of very specific environments. Over the past decade they have been carried out the functionality of a transistor [4][5] (a control field was used to pass or block a second field at a different frequency) multiplexer [6] (2 input signals can be mixed on an artificial atom to controllably generates a single output signal) and serial bus [7]. Superconducting qubits can be produced using industry standard fabrication techniques and integrated at scale into large coherent circuits [8]. All

speaks to a strong case of servicing future quantum electronic platforms with this technology.

One of the inherent limitation superconducting qubits face is a comparatively short coherence time, τ_{dec} . It the time over which the ρ_{01} and ρ_{10} components of the system's density matrix, $\rho = (\begin{smallmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{smallmatrix})$, decay to zero and computational information becomes lost [9]. For example, the transmon qubits in the revolutionary 5-qubit quantum experience from IBM (research.ibm.com/ibm-q) have a coherence time of $\sim 60\ \mu\text{s}$ [1]. For reference, qubits on trapped ${}^+{}^{\text{Be}}{}^9$ ions had a coherence time of $\sim 1\ \text{ms}$ over 20 years ago [10], and to house a sufficient number of quantum logic operations for multi-stage computations, $\sim 10^4$, coherence times need to surpass the $100\ \mu\text{s}$ barrier [12].

Strong decoherence in superconducting qubits is

a consequence of the large capacitances inherent to their geometry, the loop of the flux qubits are typically $1\text{ }\mu\text{m}$ or greater in size [4][5][8], which couples them to charge variations in the external environment. The charge fluctuations in the environment result in random changes of the qubit's energy levels, leading to an erratic evolution of the quantum state that 'averages' out information on the off-diagonal elements of the density matrix [11].

Flux qubit architectures have been developed to address this decoherence problem by making the energy of their Josephson junctions (JJ), dominate over the charging energy, $E_J/E_C \gg 1$, which lowered the device's charge sensitivity [12] [13] [14]. A whole family of flux qubit designs have lead to improved coherence times: quantronium $\sim 500\text{ ns}$ [15] [16], shunted phase qubit $\sim 10\text{ }\mu\text{s}$ [17], shunted flux qubit $\sim 80\text{ }\mu\text{s}$ [18], 4-JJ [19], fluxonium $\sim 1\text{ ms}$ [20].

We extended this list with a 'twin' qubit, consisting of two symmetrical flux qubits, linked by a common α -Josephson Junction (Fig. 1). A chain of 15 such qubits was recently placed into a coplanar waveguide to demonstrate flux-tunable transmission of microwaves [22]. Of particular interest to us was the weak flux dependence of the systems transition energy when it was biased to the degeneracy point $\frac{1}{2}\Phi_0$, making it benefit from both low flux and charge sensitivities.

In this work we isolate one of these twin qubits and provide experimental evidence for: strong anharmonicity with respect to the $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ transitions; weak flux dependence of the transition energies at the degeneracy bias $\sim \frac{1}{2}\Phi_0 = \frac{\hbar}{4e}$; compliance with simulations of the energy structure and controllability of the $|1\rangle \leftrightarrow |2\rangle$ transition rate.

II. FABRICATION

The qubits and transmission line are fabricated on an undoped 100 silicon substrate, which is pre-patterned with 10 nm NiCr - 90 nm Au ground planes and markers. We begin by cleaning the wafer for ~ 10 minutes at 60 C in acetone rinsing in de-ionized water. Two layers of electron resist are sequentially spun and post baked (3 minutes at 60 C) onto the wafer: Copolymer 13%, 700 nm; ZEP520a:Anisole 2:1, 60 nm. We use an electron beam lithographer to expose the resist using a 30 kV, 10 pA beam delivering a dose of $70\text{ }\mu\text{C}/\text{cm}^2$. We develop the pattern in Pylxylen for 35 seconds followed by a 5 minute submersion in Isopropanol:H₂O 93:7 and rinse in Isopropanol. Shadow evaporation of aluminum in a Plassys [23] simultaneously deposits the JJ and

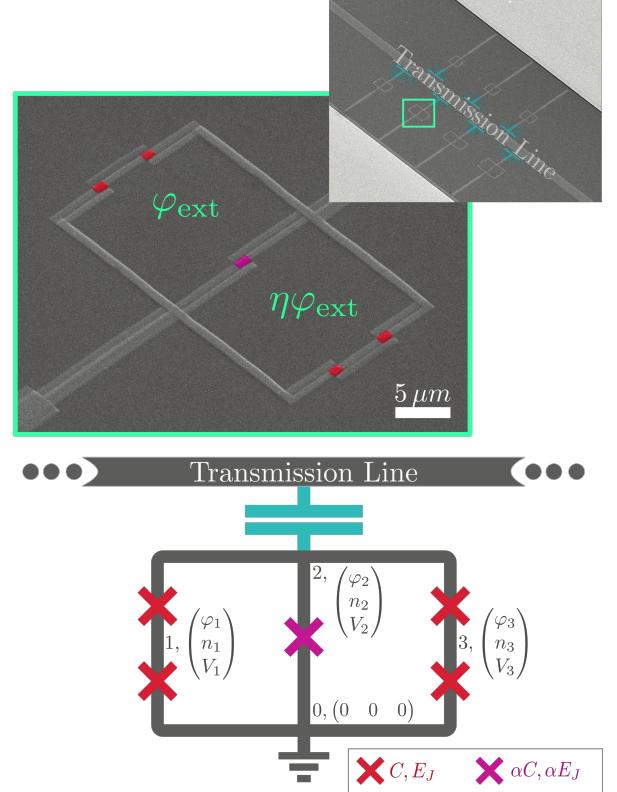


FIG. 1: **Geometry of a twin qubit.** a) Scanning electron microscope image of the qubit flux with phase biases φ , $\eta\varphi$ applied to its superconducting loops. The repeated line structures are the byproduct of the double-angle evaporation of aluminum that creates the Al-AlO_x-Al JJs highlighted in red and pink. Each of the qubits is coupled to the transmission line with a T-shaped capacitor (inset); b) The twin qubit is a symmetrical arrangement of two individual flux qubits (as described in [12]) sharing the central JJ. Islands are labeled with a Cooper pair occupation n_i , phase φ_i and voltage V_i , with the ground setting a reference of 0 for all three variables. JJs (marked with a cross) mediate capacitive and JJ interactions between the islands. The central junction has a capacitance of αC and an energy αE_J , compared with the C_J , E_J values of the outside ones.

transmission line structures. We first use **argon etching** to remove oxide layers for good galvanic contact, and, maintaining high vacuum, we deposit 20 nm of Al and perform static oxidation for 10 minutes at 0.3 mBar to generate the intermediate AlO_x insulating barrier for the JJ. A second 30 nm layer of Al completes the process.

These steps give us the 5-JJ structure of the twin qubits and T-shaped capacitors coupling them to the transmission line, see Fig. 1. Each JJ has an area of

$400 \times 200 \text{ nm}^2$. The coplanar transmission line with impedance $Z_0 \sim 50 \Omega$ runs to the opening between the ground planes in the center of the chip.

We bond the sample chip to a printed circuit board and mount it on a holder with a niobium-coil magnet on the 13 mK stage of a dilution refrigerator. A metal cover is used to shield the holder from stray magnetic fields, while attenuators, -50 dBm on the 50K stage, -30 dBm on the 4 K stage, thermalize the input lines to the respective temperatures. We attach a 4 K stage circulator on the output line for isolation, and amplify the signal by +35 dBm on the 4K stage and +35 dBm at room temperature. This system of attenuators and amplifiers facilitate power conversion between the laboratory equipment and qubit microwaves. We surround each temperature stage of the system with a case, wrapping it like a Russian doll, that thermally shield from higher stages. Prior to performing characterization measurement, we took the microwave transmission spectrum with the qubit detuned, and corrected all further measurements by this background transmission profile.

Our pilot experiment did not go to the depths of using chemical and physical treatment of the substrate surface to remove two-level system defects in the silicon oxide layer [24] or employing infra-red filters to eliminate stray light during measurement [25]. We plan on doing these improvements in further development cycles of the twin qubit.

III. OPERATIONS WITH QUBITS

We record the energy spectrum of the twin qubit while sweeping the biasing magnetic flux. Because of a small asymmetry, η , the fluxes linked through the left and right loops are $\Phi = \frac{\varphi}{2\pi}\Phi_0$ and $\eta\Phi$.

The $|1\rangle \leftrightarrow |2\rangle$ transition, ω_{21} , is mapped with a network analyzer, which measures the transmission of signal ω_{NA} through the system. For the most part, the signal passes through without any interaction with the qubit and after correcting for line losses, transmission is $\sim 100\%$. Only near resonance ($\omega_{NA} = \omega_{21}$), does the qubit exchange photons with the driving field as it evolves between the ground and excited states [31]. This evolution emits a wave that is exactly in anti-phase with the driving field [26], and the destructive superposition in the output line results in a transmission dip, see Fig. 2. The bottom of this dip is plotted with blue circles for different points in the magnetic flux to get the transition spectrum of ω_{21} , see inset of Fig. 2.

The $|2\rangle \leftrightarrow |3\rangle$ transition, ω_{32} , is mapped using two-

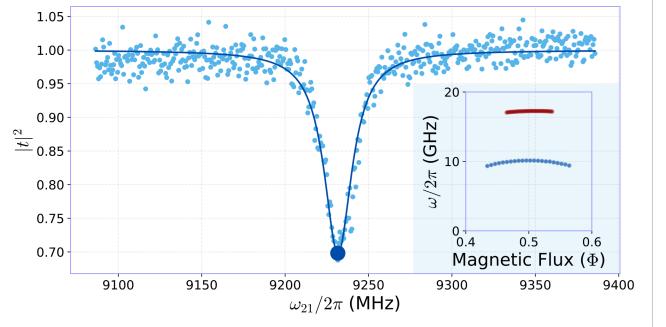


FIG. 2: **Mapping the qubit transition spectrum.**

For the lower transition ω_{21} (blue, inset) a network analyzer measures the power transmission coefficient, $|t|^2$, at flux bias Φ and microwave frequency $\omega_{21}/2\pi$. A Lorentzian fit [4] to the transmission profile establishes the resonant frequency, which is marked with blue points on the flux-frequency spectrum. For transition ω_{32} (red, inset) a two-tone measurement is run by monitoring changes to a weak ω_{21} probe while sweeping a second frequency in search of the higher transition. Any changes to the probe's transmission are indicative of hitting the higher transition, which is marked with a red point on the spectrum. Readings are taken about the degeneracy point $\Phi \sim \Phi_0/2$, where the low curvature of transition energies, allows for stable measurements with respect to fluctuations in the field.

tone spectroscopy. The network analyzer is tuned to the transition frequency ω_{21} , found with the first measurement and called the probe signal, while an additional generator sweeps a second frequency, ω_{GEN} . Whenever the generator strikes the $|2\rangle \rightarrow |3\rangle$ transition ($\omega_{GEN} = \omega_{32}$), the qubit will undergo a ladder of excitations, $|1\rangle \xrightarrow{\omega_{21}} |2\rangle \xrightarrow{\omega_{32}} |3\rangle$, depopulating states $|1\rangle$ and $|2\rangle$. Because of this depopulation, the probe signal becomes less involved with driving and its transmission moves out of the dip in Fig. 2. This identifies ω_{32} which is mapped with red circles.

We match the experimental data points to simulations made on the system's Hamiltonian, $\mathcal{H} = T + U$, developed using the standard approach for quantum electrodynamics [12]. Islands, isolated by the JJ in Fig. 1, are labeled with Cooper pair (CP) occupation $\vec{n} = |n_1, n_2, n_3\rangle$, phase $\vec{\varphi} = |\varphi_1, \varphi_2, \varphi_3\rangle$ and voltage $\vec{V} = |V_1, V_2, V_3\rangle$ states. An inspection of the system's capacitor system links the charges and voltages

$$2e\vec{n} = \hat{C}\vec{V}$$

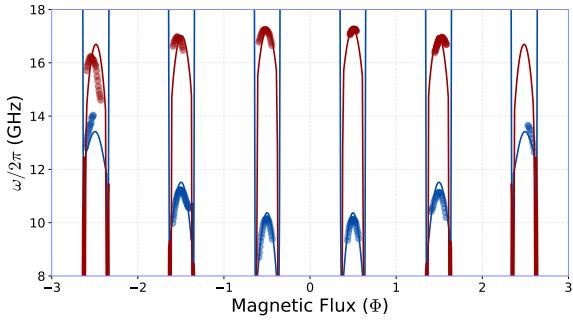


FIG. 3: Simulating the transition spectrum:

Transition frequencies between levels $|1\rangle \leftrightarrow |2\rangle$, ω_{21} (blue), and $|2\rangle \leftrightarrow |3\rangle$, ω_{32} (red). Readings for ω_{32} are in a narrow flux range because away from $\Phi = (n + \frac{1}{2})\Phi_0, n \in \mathbb{Z}$, it gets harder to tune the VNA to ω_{21} (as part of the two-tone spectroscopy procedure) which prevents the accurate mapping of ω_{32} with the second tone. Asymmetry in the flux penetrating the left and right loops results in the gradual change of transition frequencies with every Φ_0 period - ω_{21} creeps up, while ω_{32} creeps down, breaking the usual spectrum periodicity of flux qubits.

through the capacitance matrix

$$\hat{C} = |C| \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 + \alpha & -1 \\ 0 & -1 & 2 \end{pmatrix},$$

where $|C|$ is the capacitance of the outer JJs. The interaction of the CPs, carrying a charge $\vec{Q} = 2e\vec{n}$, and voltages on their respective islands gives rise to the kinetic term of the Hamiltonian:

$$\begin{aligned} T &= \frac{1}{2} \sum_{i=1}^3 Q_i V_i = \frac{(2e)^2}{2} \vec{n} \hat{C}^{-1} \vec{n}^T \\ &= E_C |C| \langle \hat{C}^{-1} \rangle_{|n_1, n_2, n_3\rangle}, \end{aligned}$$

where we define $E_C = (2e)^2 / 2 |C|$.

Each JJ with a phase difference of $\Delta\varphi_i$ across it, contributes an energy of $E_{Ji}(1 - \cos(\Delta\varphi_i))$. The flux quantization condition for the left and right loops, $\sum_i^{\text{loop}} \varphi_i = 2\pi n, n \in \mathbb{Z}$, enters as a dependence on φ_{ext} and $\eta\varphi_{\text{ext}}$ on two of the junctions in the potential term:

$$U = E_J [4 + \alpha - \alpha \cos(\varphi_2) - \cos(\varphi_1) - \cos(\varphi_3) - \cos(\varphi_2 - \varphi_1 - \varphi_{\text{ext}}) - \cos(\varphi_2 - \varphi_3 + \eta\varphi_{\text{ext}})].$$

The Hamiltonian is solved in the CP-basis, \vec{n} , where it's matrix representation takes the form show

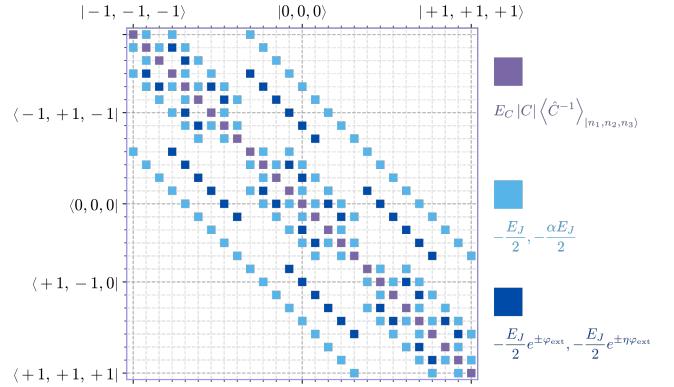


FIG. 4: Hamiltonian in the CP-basis

representation: We label some system states on the axes. In this scenario each island is allowed to be in charge states $n_i = -1, 0, 1$. Kinetic terms (T , purple) naturally fall on the diagonal axis of the matrix. Potential terms (U , shades of blue) are expanded to complex exponentials, which become off diagonal elements - see derivation in [30]. For example $\cos(\varphi_2)$ becomes $\mathbb{I}_1 \otimes \frac{1}{2} [\sum_{n_2} |n_2 + 1\rangle \langle n_2| + |n_2 - 1\rangle \langle n_2|] \otimes \mathbb{I}_3$, where identity operators $\mathbb{I}_{1,3}$ carry the states of the non-involved islands. Dark blue squares denote terms that depend on the external flux $\varphi_{\text{ext}}, \eta\varphi_{\text{ext}}$, the light blue ones are for all the others.

in Fig. 4. The eigenenergies of the resulting Hamiltonian are compared with the experimental data in Fig. 3 using $E_J = 91.0$ GHz, $E_C = 13.50$ GHz, $\alpha = 1.023$, $\eta = 1.011$. The asymmetry value, η , is close the visual loop area difference of 3% seen from the SEM image in Fig. 1. The resonance is periodic in flux, with a tendency of higher ω_{21} at higher magnetic flux numbers.

An important qubit parameter is the curvature at the turning points in the energy spectrum, where qubit operations are carried out. A low curvature is desirable, to make the qubit less sensitive to external flux changes, which would improve decoherence time. At the twin qubits' degeneracy point $\Phi = (n + \frac{1}{2})\Phi_0, n \in \mathbb{Z}$, the curvature is -550 ± 10 GHz/ Φ_0^2 . It is substantially smaller than 13×10^4 GHz/ Φ_0^2 on the 4-JJ flux qubit [17], 8.4×10^4 [21] and 37×10^4 GHz/ Φ_0^2 [32] on the 3-JJ flux qubits demonstrated recently. The decoherence time in our qubits was however relatively small, only $\tau_{\text{dec}} = 42$ ns. We get τ_{dc} from measurement of Rabi oscillations, see Fig. 5 [31]. We explain this by poisoning of the sample with infrared radiation, and simplified technology used for fabrication as we mentioned above.

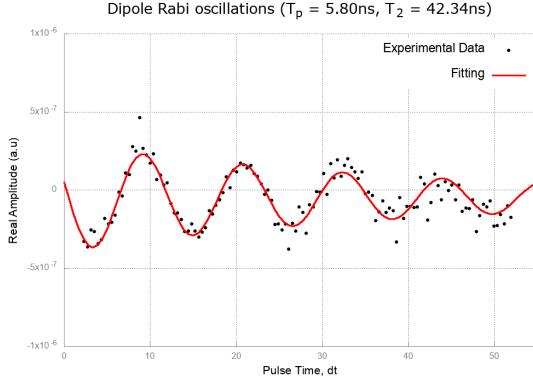


FIG. 5: Rabi oscillation taken at the degeneracy point, $\Phi_0/2$, by driving the qubit with resonant microwaves, $\omega_{\text{VNA}} = \omega_{21}$ for different time periods, dt , and monitoring the signal in the output line [31]. The a decoherence time of $\tau_{\text{dec}} = 42$ ns is extracted from the decay envelope, e^{-dt/τ_φ} , of the oscillations.

IV. DIPOLE TRANSITION

We analyse the tuneability of the $|2\rangle \rightarrow |1\rangle$ transition rate with reference to Fermi's golden rule

$$\frac{1}{T_{21}} = \frac{1}{Z_0} \frac{\hbar \omega_{21} |\langle 2 | T | 1 \rangle|^2}{\hbar^2}.$$

The kinetic term T plays the role of the dipole operator for this transition and is evaluated with the eigenstates $|1\rangle, |2\rangle$ of the Hamiltonian. Highest transition rates are achieved around the degeneracy biases, $\varphi_{\text{ext}} = (2n+1)\pi, n \in \mathbb{Z}$, see Fig. 6. This region is thus favourable for quick state operations with external driving fields.

V. CONCLUSION

We have fabricated and characterised the first isolated twin qubit, the distinguishing feature of which is the weak flux sensitivity in the working region of $\Phi = (n + \frac{1}{2})\Phi_0, n \in \mathbb{Z}$. Improved fabrication techniques can further advance the qubit's decoherence time to place it in competition for the building block in quantum electronic circuits.

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- $$\Psi = \alpha |0\rangle + e^{i\varphi} (1 - \alpha) |1\rangle.$$
- When written as a density matrix, the phase information is mapped onto the off-diagonal elements:
- $$\rho = |\Psi\rangle \langle \Psi| = \begin{pmatrix} |\alpha|^2 & \alpha(1 - \alpha)e^{-i\varphi} \\ \alpha(1 - \alpha)e^{+i\varphi} & |(1 - \alpha)|^2 \end{pmatrix}.$$
- The off-diagonal elements decay to $1/e$ of the initial value, over a time period τ_{dec} , and phase information becomes lost.
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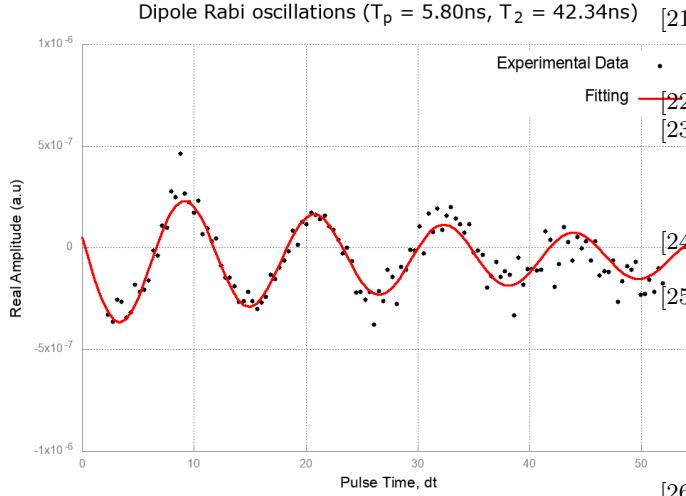


FIG. 6: The dipole transition rate Γ is proportional to the dipole transition element $\langle 2 | T | 1 \rangle$, where T is the charge operator responsible for the transition, and $|1\rangle$ and $|2\rangle$ are the eigenstates of the system.

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$$[\hat{n}, e^{\pm i\hat{\phi}}] = \left[\hat{n}, \sum_{\alpha=0}^{\infty} \frac{(\pm i\hat{\phi})^\alpha}{\alpha!} \right] = \sum_{\alpha=0}^{\infty} (\pm i)^\alpha \frac{[\hat{n}, \hat{\phi}^\alpha]}{\alpha!} = \sum_{\alpha=0}^{\infty} (\pm i)^\alpha \frac{-\alpha i \hat{\phi}^{\alpha-1}}{\alpha!} = \pm \sum_{\alpha=1}^{\infty} i^{\alpha-1} \frac{(\pm \hat{\phi})^{\alpha-1}}{(\alpha-1)!} = \pm e^{\pm i\hat{\phi}}.$$

Operating with the number operator yields

$$\hat{n} [e^{\pm i\hat{\phi}} |n\rangle] = [\pm e^{\pm i\hat{\phi}} + e^{\pm i\hat{\phi}} \hat{n}] |n\rangle = (n \pm 1) [e^{\pm i\hat{\phi}} |n\rangle],$$

meaning that the exponential phase operator is a ladder operator for the $|n\rangle$ state:

$$e^{\pm i\hat{\phi}} |n\rangle = |n \pm 1\rangle \Rightarrow e^{\pm i\varphi} = \sum_i |n_i \pm 1\rangle \langle n_i|.$$

Now phase a φ_2 operator can be expressed in the number basis as follows:

$$\cos(\varphi_2) = \frac{1}{2} (e^{i\varphi_2} + e^{-i\varphi_2}) = \frac{1}{2} \left(\sum_i |n_i^{(2)} \pm 1\rangle \langle n_i^{(2)}| \right) \otimes \mathbb{I}^{(1)} \otimes \mathbb{I}^{(3)}$$

with $\mathbb{I}^{(1)} = \sum_i |n_i^{(1)}\rangle \langle n_i^{(1)}|$ and $\mathbb{I}^{(3)} = \sum_i |n_i^{(3)}\rangle \langle n_i^{(3)}|$ being the identity operators in the $n^{(1)}$ and $n^{(3)}$ subspaces.

- [31] The Hamiltonian during driving is a combination of the qubit's two-level system

$$\mathcal{H}_q = -\frac{\hbar\omega_{21}}{2} \sigma_z,$$

where $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, and the resonant drive of strength $\hbar\Omega$

$$\mathcal{H}_d = \hbar\Omega \cos(\omega_{21}t) \sigma_x$$

that couples the two levels through the transition operator $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Applying the unitary transformation $U(t) = \exp(-i\frac{\omega_{21}t}{2}\sigma_z)$ to $\mathcal{H} = \mathcal{H}_q + \mathcal{H}_d$ evaluates to

$$\begin{aligned} \mathcal{H}' &= U\mathcal{H}U^\dagger - i\hbar U\dot{U}^\dagger \\ &\approx \frac{\hbar\Omega}{2} \sigma_x \end{aligned}$$

under the approximation that non-conserving energy terms $e^{\pm 2i\omega_{21}t}$ are neglected. The evolution of a ground state in this rotated frame is

$$\begin{aligned} U'(t)|0\rangle &= e^{-i\frac{\mathcal{H}'}{\hbar}t}|0\rangle \\ &= \cos(\Omega t/2)|0\rangle + i \sin(\Omega t/2)|1\rangle. \end{aligned}$$

Rabi oscillations are proportional to $\langle\sigma_-\rangle$ [4], $\sigma_- = |0\rangle\langle 1|$:

$$\begin{aligned} \langle\sigma_-\rangle &= \langle 0|U'^\dagger(t)|0\rangle\langle 1|U'(t)|0\rangle \\ &= \sin(\Omega t) \end{aligned}$$

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[33] <http://microchem.com/pdf/PMGI-Resists-data-sheetV-rhcedit-102206.pdf>

[34] <http://www.nanophys.kth.se/nanophys/facilities/nfl/resists/S181>

[35] [datasheet](#)