Supplementary Notes: The superconducting twin qubit

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I. CAPACITANCE MATRIX

We are considering a capacitance network of the qubit, shown in Fig. (1). Each Josephson junction has capacitance C_{ij}, where i and j are island numbers (0, 1, 2, 3). The corresponding capacitance matrix is

$$\mathbf{C} = \begin{pmatrix} C_{01}+C_{12} & -C_{12} & 0 \\ -C_{12} & C_{12}+C_{02}+C_{23}+C_c & -C_{23} \\ 0 & -C_{23} & C_{03}+C_{23} \end{pmatrix}. \tag{1}$$

All Josephson junctions except the central one are identical and equal to C: $C_{01} = C_{12} = C_{03} = C_{23} = C$. The central junction is different, which together with small coupling capacitance C_c can be represented as $C_{02} + C_c = \alpha' C$. Note that $C_c \ll C$ and $\alpha' \approx \alpha$. The capacitance matrix is simplified to

$$\mathbf{C} = C \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2+\alpha & -1 \\ 0 & -1 & 2 \end{pmatrix}. \tag{2}$$

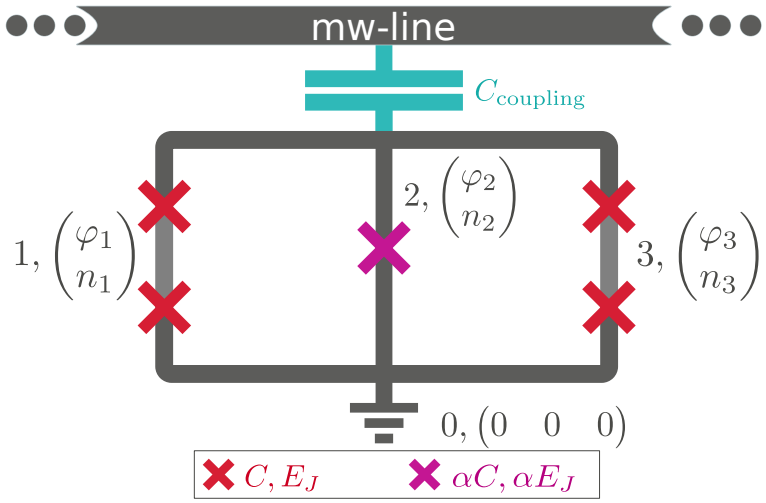

FIG. 1. **Topology of the twin flux qubit.**

The capacitance matrix couples charges on the islands $2e\vec{n} = 2e\{n_1, n_2, n_3\}$ with their potentials $\vec{V} = \{V_1, V_2, V_3\}$ according to $2e\vec{n} = \mathbf{C}\vec{V}$ and an electrostatic energy of the qubit

is simplified to kinetic (phase dynamics)

$$T = E_C C \vec{n}^T \mathbf{C}^{-1} \vec{n}, \tag{3}$$

where the charging energy $E_C = \frac{(2e)^2}{2C}$. The potential energy due to Josephson junction is

$$U = E_J[4+\alpha-\alpha\cos\varphi_2-\cos\varphi_1-\cos\varphi_3-\cos\left(\varphi_2-\varphi_1-\varphi_{ext}\right)-\cos\left(\varphi_2-\varphi_3+\eta\varphi_{ext}\right)]. \tag{4}$$

II. REPRESENTATION OF HAMILTONIAN IN THE CHARGE BASIS

We represent the system Hamiltonian $H = T+U$ in the charge basis. Taking into account the phase operator form $e^{\pm i\varphi} = |n \pm 1\rangle \langle n|$ and omitting constants in the potential energy, the Hamiltonian is transformed to

$$\begin{aligned} H = E_C C \vec{n}^T \mathbf{C}^{-1} \vec{n} - \frac{E_J}{2}\big(&\alpha \left|n_2+1\right\rangle \left\langle n_2\right| + \left|n_1+1\right\rangle \left\langle n_1\right| + \left|n_3+1\right\rangle \left\langle n_3\right| + \\ &+ e^{-i\varphi_{ext}} \left|n_1-1, n_2+1\right\rangle \left\langle n_1, n_2\right| + e^{+i\eta\varphi_{ext}} \left|n_2+1, n_3-1\right\rangle \left\langle n_2, n_3\right| + c.c.\big) \end{aligned} \tag{5}$$

Physically the penultimate term corresponds to a CP exchange between island 1 and island 2. An example of a term could be

$$\frac{1}{2} e^{-i\varphi_{\text{ext}}} \left|-1,1,0\right\rangle \left\langle 0,0,0\right| + \frac{1}{2} e^{+i\varphi_{\text{ext}}} \left|0,0,0\right\rangle \left\langle -1,1,0\right|, \tag{6}$$

which would be a pair of symmetrical off-diagonal elements in the matrix of Fig. 2.

To decide on the number of states for the simulation, we took a sufficiently complete system state of 19 interacting CPs, and methodically "switched off" interactions between the high-CP-number states. Further increase, of the number of states does not improve accurate of calculations. We logged the deviation of the energy spectra of Fig. 3, where it shows that 9-CP-states describes the system almost as good as with the complete system state.

III. TRANSITION MATRIX ELEMENTS

Here we derive the transition matrix element used in the main paper

$$\left\langle g\right| \hat{V}_2 \left|e\right\rangle \equiv \frac{E_C}{2e(1+\alpha)} \left\langle g\right| \left[\hat{n}_1 + 2\hat{n}_2 + \hat{n}_3\right] \left|e\right\rangle. \tag{7}$$

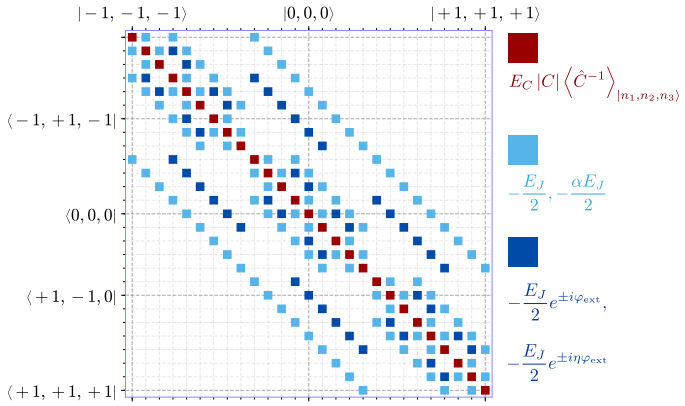

FIG. 2. **Hamiltonian in the CP-basis representation for 3-CP-states-per-island (27 system states):** Purple square denote the kinetic terms that all fall on the main diagonal. Light blue squares denote simple off-diagonal terms distributed symmetrically about the main diagonal, arising from e.g. $\cos(\varphi_2)$. Dark blue squares are have an additional flux dependence $e^{i\varphi_{\rm ext}}, e^{i\eta\varphi_{\rm ext}}$, arising from e.g. $\cos(\varphi_2 - \varphi_1 - \varphi_{\rm ext})$.

Microwaves in the transmission line, with voltage $V_{\rm mw} = |V_{\rm mw}|\cos(\omega_{21}t)$ are coupled via the coupling capacitor $C_{\rm c}$ to the qubit. Transitions $|1\rangle \leftrightarrow |2\rangle$ stimulated by this driving generate a qubit voltage of $V_2 = \langle 1|\,\hat{V}_2\,|2\rangle$

Expressing the voltage on the different islands:

$$\begin{aligned}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \vec{V} = \vec{Q}C^{-1} = 2eC^{-1}\vec{n} &= \frac{2e}{|C|}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2+\alpha & -1 \\ 0 & -1 & 2 \end{pmatrix}^{-1}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \\ &= \frac{2e}{|C|}\frac{1}{4+4\alpha}\begin{pmatrix} 3+2\alpha & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3+2\alpha \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}\end{aligned} \tag{8}$$

one can read off

$$\hat{V}_2 = \frac{e}{C(1+\alpha)}\left[\hat{n}_1 + 2\hat{n}_2 + \hat{n}_3\right]. \tag{9}$$

Thus, the transition matrix element

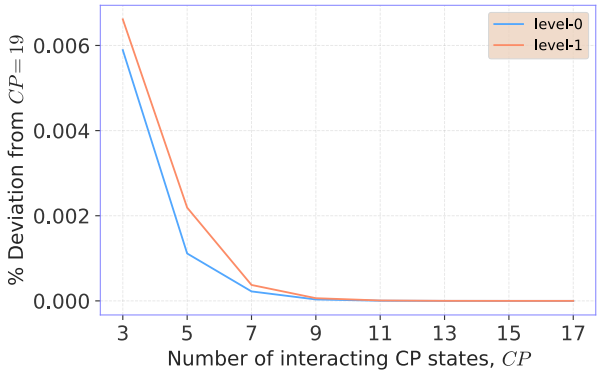

FIG. 3. **Choosing the lowest number of interacting CP, that would capture the nature of perturbation effects:** The energy levels for state $|0\rangle$ (blue) and $|1\rangle$ (red) were simulated for different numbers of interacting CP and compared to the simulation with 19CP (which we assume to be the most accurate and most intense simulation for the system). Consistency was reached for simulations with 9CP.

$$d_{i,j} = \langle i| \hat{V}_2 |j\rangle \equiv \frac{E_C}{2e(1+\alpha)} \langle i| [\hat{n}_1 + 2\hat{n}_2 + \hat{n}_3] |j\rangle . \tag{10}$$