The Locus Algorithm

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Abstract

This Paper describes the design, implementation and operation of a new algorithm, The Locus Algorithm; which enables optimised differential photometry. For a given target, The Locus Algorithm identifies the pointing for which the resultant FoV includes the target and the maximum number of similar reference stars available, thus enabling optimised differential photometry of the target. The application of The Locus Algorithm to a target from the Sloan Digital Sky Survey to provide optimum differential photometry for that target is also described. The algorithm was also used to generate catalogues of pointing's to optimise Quasars variability studies and to generate catalogues of optimised pointings in the search for Exoplanets via the transit method.

1. Introduction

Photometric variability studies involve identifying variations in brightness of a celestial point source over time. Such studies are hampered by the Earth's atmosphere, which causes first order and second order extinction [3, 4]. Differential Photometry mitigates the effect of the Earth's atmosphere by comparing the brightness of a target to reference stars in the same Field of View (FoV). Differential photometry can be optimised for the target by choosing a pointing whose Field of View (FoV) includes the target and the maximum number of reference stars of similar magnitude and colour. [1–4]).

The Locus Algorithm enables optimised differential photometry by identifying the pointing for which the resultant FoV includes the target and the best set of similar reference stars available.

2. Conceptual basis to The Locus Algorithm

A locus can be defined around any star such that a FoV centred on any point on the locus will include the star at the edge of the FoV. For fields containing stars close to one another, if one locus intersects with another, they produce Points of Intersection (PoIs) as shown in Figure 1.

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Preprint submitted to Astronomy & Astrophysics *May 14, 2019*

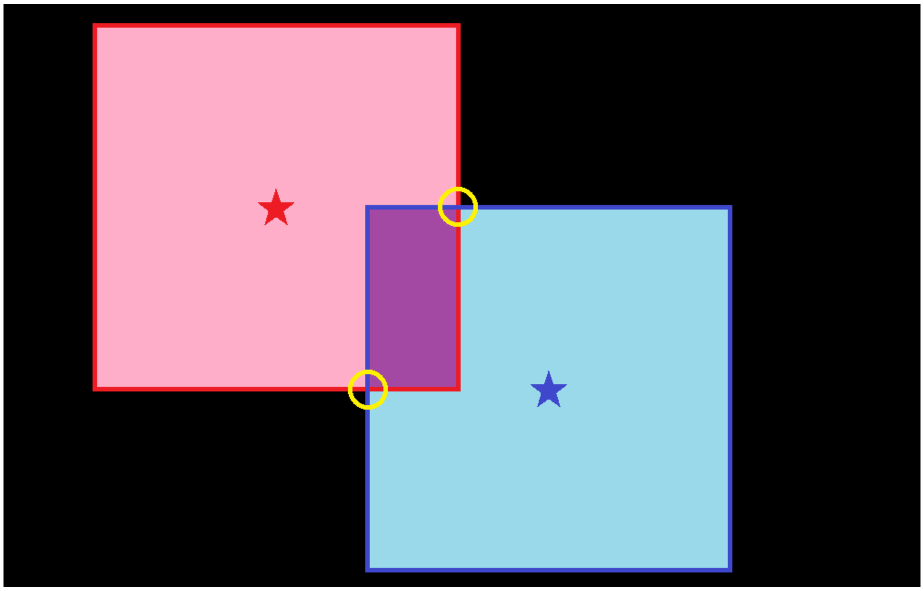


Figure 1: Diagrammatic representation of two stars with loci (red and blue perimeter lines), which intersect and produce two Points of Intersection (PoI's) circled in yellow.

A FoV centred on any such PoI will include both stars associated with creating it. At Points of Intersection the set of stars that can be included in a FoV changes.

The Locus Algorithm considers candidate reference stars in what is termed a Candidate Zone (CZ) - the zone of sky centred on the target within which a FoV can be selected which includes both the reference star and the target. For Candidate Reference Stars within the CZ, loci are determined, and all relevant PoI are identified. Each PoI is assigned a score derived from the number and similarity of reference stars included in a FoV centred on that PoI. The PoI with the highest score becomes the pointing for the target.

3. Locus Algorithm Design

Based on the conceptual outline above, this section provides a mathematical definition of the Locus Algorithm and an explanation of the terms used in it. Section 4 below describes a worked example of this algorithm applied to a sample star, SDSS ID 1237680117417115655.

3.1. Definition of Coordinate System and Locus

For computational efficiency, The Locus Algorithm considers a Field of View to be a rectangular area on the sky orientated such that the edges are aligned with the primary x and y axes of the Cartesian coordinate system. Movement of the field is restricted to x or y translations.

However, the Celestial coordinate system is defined by the Equatorial coordinate system, with coordinates specified by Right Ascension (RA) and Declination (Dec). Because this is a spherical coordinate system, unit angle in RA is foreshortened, with the degree of foreshortening defined in Expression 1

$$angle \in RA = \frac{TrueAngle}{cos(Dec)} \tag{1}$$

Expression 1: Right Ascension foreshortening with Declination

By using this conversion, it is possible to approximate to a high degree of accuracy a Cartesian coordinate system using RA and Dec; with a small FoV of East-West size R and North-South size S about a target located at point RA_t and Dec_t. Expression 2 defines a corrected angular size in RA direction (R')

$$R' = \frac{R}{cos(Dec_t)} \tag{2}$$

Expression 2: Definition of a corrected angular size along the RA direction (R′)

Given these terms, Expression 3 defines the FoV.

$$\begin{gathered} RA_t - \frac{R'}{2} \leq RA \leq RA_t + \frac{R'}{2} \\ Dec_t - \frac{S}{2} \leq Dec \leq Dec_t + \frac{S}{2} \end{gathered} \tag{3}$$

Expression 3: Definition of a FoV of size R x S centred on a target at (RA_t , Dec_t)

This definition is accurate to approximately 1% for a FoV of area 15′ square outside celestial polar regions as shown in Expression 4.

$$\begin{gathered} Given : R, S = 15', |Dec| \leq 66.5ř \\ \frac{|\frac{R}{cos(Dec-S)} - \frac{R}{cos(Dec+S)}|}{\frac{R}{cos(Dec)}} \leq 0.01 \end{gathered} \tag{4}$$

Expression 4: Evaluation of the accuracy of the R′ for areas away from the celestial pole.

As expresed here, the formula does not consider RA "loop around" from 359.99°to 0.00°; resulting, for example, in the exclusion of 0.23% of the SDSS catalogue. Planned enhancements to The Locus Algorithm will resolve these shortcomings.

We can therefore define the locus about any star on the sky located at RA_t and Dec_t as the values of Right Ascension and Declination as defined in Equation 2.

3.2. Candidate Zone

A Candidate Zone is defined as a region centred on the target, equal to four times the area of the Field of View, within which any reference star can be included in a Field of View with the target and can therefore be considered as a candidate reference star in identifying the optimum pointing. Conversely, stars outside the candidate zone cannot be included in a Field of View with the target and cannot therefore be considered as candidates reference stars. Hence the Candidate Zone is the maximum region of sky centred on the target from which to choose candidate reference stars when identifying an optimum pointing for a given target. For a target positioned at coordinates RA_c and Dec_c the resulting Candidate Zone is defined by Expression 5.

$$\begin{aligned} RA_t - R' \leq RA_r \leq RA_t + R' \\ Dec_t - S \leq Dec_r \leq Dec_t + S \end{aligned} \tag{5}$$

Expression 5: Definition of a Candidate Zone of size 2R x 2S centred on a target with coordinates (RA_t, Dec_t), in which zone reference stars with coordinates (RA_r, Dec_r) can be found.

3.3. Identification and Filtering of Reference Stars

For each target, a list of candidate reference stars in its Candidate Zone is produced based on the following criteria:

- Position: the reference star must be in the Candidate Zone as defined in Expression 5.
- Magnitude: the magnitude of the reference star (mag_r) must be within a user-defined limit (Δmag) of the target's magnitude (mag_t) as shown in Expression 6.
- Colour: the colour index (e.g. $g - r$) of the reference star (col_r) must match the colour of the target (col_t) to within a user-specified limit(Δcol) as shown in Expression 7.
- Resolvability: the reference star must be resolvable, i.e. no other star that would impact a brightness measurements within a user-specified resolution limit.

All stars in the Candidate Zone which pass these initial filters become the list of candidate reference stars for which loci will be identified.

$$mag_t - \Delta mag \le mag_c \le mag_t + \Delta mag \tag{6}$$

Expression 6: Definition of the limits of mag difference between the target and references.

$$col_t - \Delta col \le col_c \le col_t + \Delta col \tag{7}$$

Expression 7: Definition of the limits of colour difference between the target and references.

3.4. Identifying the Effective Locus for each Candidate Reference Star

The locus associated with each candidate reference star must be identified based on Equation 3. For the purposes of identifying Points of Intersection, only the side surrounding a given candidate reference star closest to the target need be considered. Hence, we can define the effective locus for such a candidate reference star as a single line of constant RA and a single line of constant Dec nearest the target star as shown in Figure 2.

Specifically, the effective locus can be defined as a corner point of the locus and two lines: one of constant RA and the other of constant Dec emanating from the corner point.

Using the Equatorial Coordinate System discussed in Section 3.1, with coordinates of the target specified by (RA_t, Dec_t) and coordinates of the candidate reference star defined by (RA_r, Dec_r) and a size of FoV of horizontal length R and vertical length S, the coordinates of the corner point (RA_c, Dec_c) are defined as shown in Expression 8. The directions $DirRA$ (the direction of the

$$\begin{aligned}
RA_t \le RA_r &\Rightarrow RA_c = RA_r - \frac{R'}{2} \\
RA_t > RA_r &\Rightarrow RA_c = RA_r + \frac{R'}{2} \\
Dec_t \le Dec_r &\Rightarrow Dec_c = Dec_r - \frac{S}{2} \\
Dec_t > Dec_r &\Rightarrow Dec_c = Dec_r + \frac{S}{2}
\end{aligned} \tag{8}$$

Expression 8: Definition of the corner point (RA_c, Dec_c) of the effective locus for a FoV of size R x S for a candidate reference star at (RA_r, Dec_r) and a target at (RA_t, Dec_t)

line of constant RA) and $DirDec$ (the direction of the line of constant Dec) of the lines is determined by the RA and Dec of the candidate reference star relative to that of the target are given in Expression 9 and as described below.

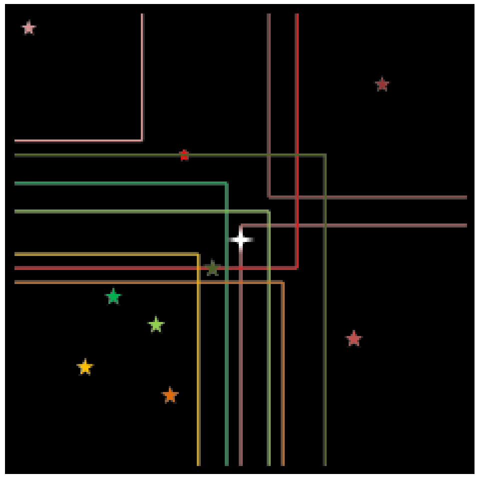


Figure 2: Each effective locus is defined by assigning a pair of RA and Dec coordinates for a corner point and a pair of lines North or South and East or West from the corner point. In this diagram, each candidate reference star is assigned a colour, and the effective locus that corresponds to it is drawn in the same colour.

- If the RA of the candidate is greater than the target, the line of constant Dec is drawn in the direction of increasing RA
- If the RA of the candidate is less than the target, the line of constant Dec is drawn in the direction of decreasing RA
- If the Dec of the candidate is greater than the target, the line of constant RA is drawn in the direction of increasing Dec
- If the Dec of the candidate is less than the target, the line of constant RA is drawn in the direction of decreasing Dec.

$$
\begin{aligned}
RA_t \leq RA_r &\Rightarrow DirDec = +ive \\
RA_t > RA_r &\Rightarrow DirDec = -ive \\
Dec_t \leq Dec_r &\Rightarrow DirRA = +ive \\
Dec_t > Dec_r &\Rightarrow DirRA = -ive
\end{aligned}
\tag{9}
$$

Expression 9: Definition the directions ($DirRA$, $DirDec$) of the lines from the corner point of that define the effective locus for a FoV of size R x S for a candidate reference star at (RA_c, Dec_c) and given a target at (RA_t, Dec_t). In current implementations, these values are encoded as a binary switch, with 1 representing increasing ($+ive$) direction and 0 representing decreasing ($-ive$) direction.

3.5. Identifying and Scoring Points of Intersection and identifying the pointing.

The points where lines from any two loci are identified. This involves comparing the corner point RA and Dec and direction of lines for one locus with the corner point RA and Dec and direction of lines for a second locus. In total eight variable associated with each two loci are checked:

- For Locus 1: RA_{c1}, Dec_{c1}, $DirRA_1$, $DirDec_1$
- For Locus 2: RA_{c2}, Dec_{c2}, $DirRA_2$, $DirDec_2$

Using these parameters, a check as to whether an intersection between the two loci occurs is achieved as follows:

- A line of constant Dec in the positive RA direction from the corner point of locus 1 will intersect with a line of constant RA in the positive Dec direction from the corner point of locus 2 if locus 1 has a lower RA than locus 2 and locus 1 has a higher Dec than locus 2.
- A line of constant RA in the positive Dec direction from the corner point of locus 1 will intersect with a line of constant Dec in the positive RA direction from the corner point of locus 2 if locus 1 has a lower Dec than locus 2 and locus 1 has a higher RA than locus 2.

... and so on. By checking all such possible combinations, all pairs of loci in the field which result in a Point of Intersection are identified and their RA and Dec noted.

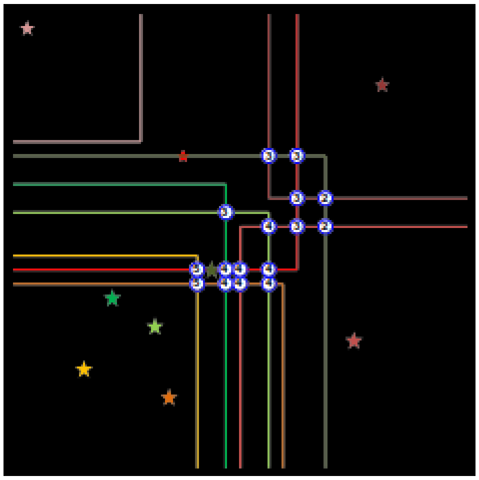


Figure 3: Points of Intersection (PoI), and their associated score. In this diagram each star has a rating of 1, hence the score associated with each PoI is equal to the number of reference stars within a FoV centred at that PoI.

Subsequent to identification, each Point of Intersection is then scored. This is achieved as follows:

- The number of reference stars in the Field of View centred on the Point of Intersection is counted.

- Each reference star is assigned a *rating* value between 0 and 1 based on its similarity in colour to the target.
- The ratings from all counted reference stars in the Field of View are combined into one overall score for the field (Figure 3).
- The Point of Intersection with the highest score becomes the pointing for the target (Figure 4).

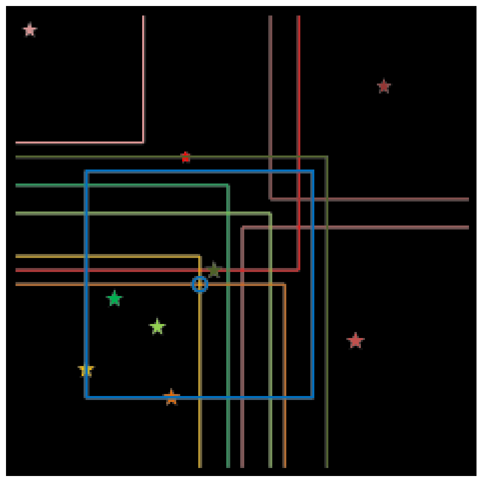


Figure 4: Locus Algorithm. Target: white star. Pointing & FoV: blue. Reference stars and their loci: Fully in the FoV: greens. On the edge of the FoV: yellows. Outside FoV: reds

Scenarios can arise which result in an inability to identify an optimum pointing for a given target for example if there are no, or a maximum of one reference stars in the candidate zone; and if no points of intersection arise – a scenario which can arise if two (or more) reference fall in one quadrant of the candidate zone resulting in concentric loci, or where reference stars are too far apart in different quadrants of the candidate zone in order for their loci to intersect. All four of these scenarios are considered in practical implementations of the locus algorithm aimed at identifying the optimum pointings for a set of targets in a catalogue or list of targets.

In summary, the Locus Algorithm successfully identifies the RA and Dec coordinates of the optimum pointing for a given target, where optimum means a field of view with the maximum number of reference stars which are similar in magnitude and colour to the target.

4. Example Implementation of the Locus Algorithm

To illustrate the workings of the Locus Algorithm, a worked example is given here. The star with SDSS ID 1237680117417115655 (RA = 346.65 and DEC = -5.04) is used as the example. This star has SDSS magnitudes as given in the table below.

Band	SDSS_Magnitude
u	17.20
g	15.38
r	14.65
i	14.40
z	14.28

The telescope system considered has parameters given in the table below:

Parameters	Values
Field of View in minutes	10.00
Resolution Limit in minutes	0.18
Dynamic Range in magnitudes	2.00

The potential reference stars are selected as follows:

- Position: Within the Candidate Zone, SDSS records 1345 separate objects.
- Magnitude: the reference star must be within the dynamic range, 2, of the target's magnitude of 14.648. This leaves 41 potential references.
- Colour: the reference star must match the colour of the target to within a user-specified limit. In this case this means $g - r$ between 0.634 and 0.834 and $r - i$ between 0.149 and 0.349 This leaves 15 potential references.
- Resolvability: the reference star must be resolvable, i.e. no other star that would impact a brightness measurements within a user-specified resolution limit, in this case 11 arc seconds. Any object this close to a potential reference star and with an r-band magnitude which is 5 magnitudes greater than the potential reference or brighter will pollute the light from the potential reference star. This leaves 14 potential references.

After checking different fields of view, a pointing with RA = 346.65 and DEC = -5.12 included both the target and 7 reference stars. These numbers are presented in the table below.

filters	numbers
Position, in Field of View	1345
Correct Magnitude	41
Correct Colour	15

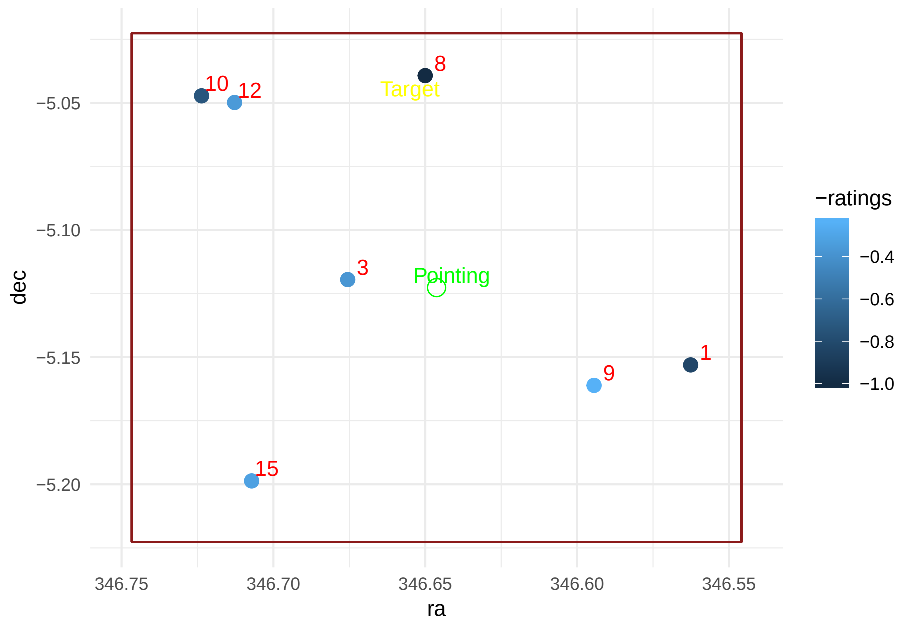
filters	numbers
Resolvable	14
In Final Field of View	7

The SQL query to download potential reference stars from SDSS is given below. This SQL query is run on the CAS database, release DR15, of SDSS. Note the flags to give clean photometry (Aguado et al. (2018))

SELECT objID, ra, dec, psfmag_u, psfmag_g, psfmag_r, psfmag_i, psfmag_z
FROM photoObj
WHERE (ra between (346.48270496969) AND (346.817331746246)
OR ra BETWEEN (706.48270496969) AND (706.817331746246)
OR ra BETWEEN (-13.5172950303096) AND (-13.1826682537544))
AND dec BETWEEN (-5.20597532982638) AND (-4.87264199649304)
AND psfmag_r BETWEEN 12.64849 AND 16.64849
AND (psfmag_g - psfmag_r) BETWEEN (0.633989999999999) AND (0.833989999999999)
AND (psfmag_r - psfmag_i) BETWEEN (0.149080000000001) AND (0.349080000000001)
AND clean = 1
AND (calibStatus_r & 1) != 0

A table with the reference stars in the final field of view is given below:

objID	ra	dec	mag_u	mag_g	mag_r	mag_i	mag_z	ratings
1237680117417050120	346.563	-5.153	18.460	16.498	15.771	15.533	15.397	0.830
1237680117417050133	346.594	-5.161	16.702	14.825	14.068	13.887	13.648	0.241
1237680117417115655	346.650	-5.039	17.199	15.382	14.648	14.399	14.281	1.000
1237680117417115762	346.676	-5.120	18.920	17.022	16.282	15.974	15.851	0.380
1237680065348435996	346.707	-5.199	16.704	14.699	13.905	13.676	13.515	0.322
1237680117417115683	346.713	-5.050	17.585	15.782	15.109	14.867	14.798	0.361
1237680117417115692	346.724	-5.047	18.362	16.576	15.843	15.568	15.464	0.734



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