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THE PERIOD-LUMINOSITY RELATION. IV. INTRINSIC RELATIONS AND REDDENINGS FOR THE LARGE MAGELLANIC CLOUD CEPHEIDS

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ABSTRACT

Reddening-independent parameters, specifically period, amplitude, and the Wesenheit function, are used to probe the intrinsic calibration of the period-luminosity relation for Cepheids with photoelectric B, V photometry in the Large Magellanic Cloud. These relations can be inverted to determine reddenings to the individual Cepheids confirming earlier indications that detectable absorption exists within the LMC itself and that the longer-period Cepheids are systematically more heavily obscured. Independent of the detailed reddening values it is also shown that amplitude varies systematically across the instability strip such that at all periods amplitude increases toward the cool (faint) edge.

Subject headings: galaxies: Magellanic Clouds — stars: cepheids — stars: pulsation

I. INTRODUCTION

Clearly, our understanding of the Cepheid instability strip is in large part dependent upon the way in which we calibrate its observable properties. Because of the large sample of Cepheids in the Magellanic Clouds these galaxies have often been used in past calibrations, and this study will be no exception.

In the following we have used only the high quality photoelectric B, V observations of Cepheids in the Large Magellanic Cloud as published by Gascoigne and Kron (1965), Gascoigne (1969), Madore (1975), Martin and Warren (1979), Gascoigne and Shobbrook (1978), and Connolly (1980). The error analysis on these data by Martin, Warren, and Feast (1979) indicates that the various sets of photometry are consistent to better than ± 0.02 mag and therefore by adopting a working error of ± 0.04 mag the V and $B-V$ photometry should be useful in determining intrinsic magnitudes, colors, amplitudes, and reddenings to this same level. That is the purpose of this paper.

Some years ago van den Bergh (1975) introduced the Wesenheit function into the study of the period-luminosity relation. Its power as a reddening-free representation was later discussed in a general way by Madore (1976*a*), but it is here that we will delve into its properties more closely and use it as the basis for a reanalysis of the period-luminosity relation, thereby revealing intrinsic properties of the instability strip and detailing the reddenings to individual Cepheids in the Large Magellanic Cloud.

II. THE WESENHEIT FUNCTION

a) *General Remarks*

Only one linearly independent combination of V and $(B-V)$ can be made that is itself independent of reddening. This is W, the Wesenheit function. If R is the ratio of total-to-selective absorption, then

$$\begin{aligned} W &= V - R(B-V) \\ &= V_0 + A_v - R(B-V)_0 - RE(B-V) \\ &= V_0 - R(B-V)_0. \end{aligned} \tag{1}$$

This quantity W is formed from V and $(B-V)$ as observed, and it is numerically equal to the *intrinsic values* of those two magnitudes and colors so combined. W is reddening-free.

If a linear form of the period-luminosity color relation is chosen such that

$$V_0 = \alpha \log P + \beta(B-V)_0 + \gamma + (\mathrm{mod})_0, \tag{2}$$

it then follows that

$$W = \alpha \log P + (\beta - R)(B-V)_0 + \gamma + (\mathrm{mod})_0. \tag{3}$$

Notice that by this both W and V_0 must have exactly the same zero point and period dependence. Only the color-sensitive variation of the two quantities, read at constant

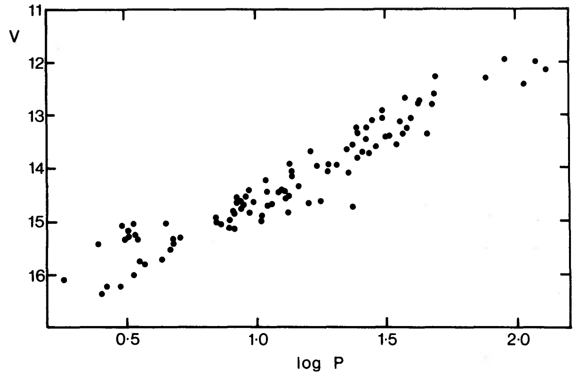

FIG. 1.—Visual period-luminosity relation for all LMC Cepheids with published photoelectric data. Note the highly reddened Cepheid HV 2749 at log $P \sim 1.36$ and the flattening of the relation at long periods.

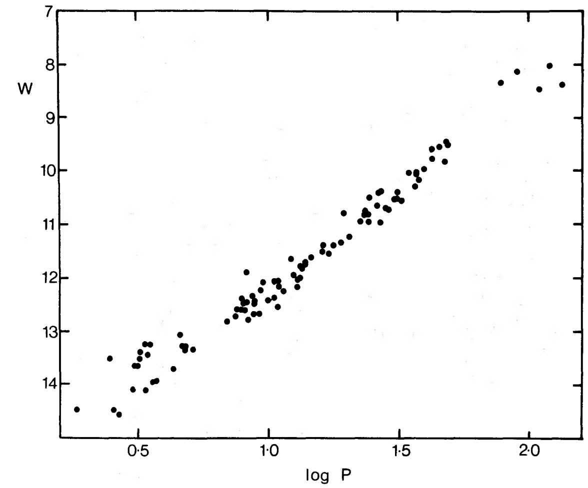

FIG. 2.—The reddening-free Wesenheit function for LMC Cepheids as in Fig. 1. Note that the width of the relation is still appreciable and that the relation is linear over the entire period range.

period, is modified. Thus we would expect the *forms* of the period-luminosity relations to be identical whether expressed as V_0 versus log P or W versus log P. Only the widths of these respective representations of the data will vary, their ratio W to V_0 being $(\beta - R)/\beta$.

In the real world several problems present themselves in discussing just the widths. The parameter β is not independently known, and errors in the photometry can act to create an additive spread. This latter complication makes a derivation of β, from widths alone, far from convincing for many. This aside, V_0 is not directly observable since differential reddening also systematically widens the instability strip. A lower limit on β should still be obtainable with these data, as shown by Madore (1976*a*), but an ambiguity in the sign of the width ratios makes this solution double valued.

Values of β in the literature, based on photoelectric photometry, are typically 2.6 (Sandage and Tammann 1968; Martin, Warren, and Feast 1979). With such values a width ratio W to V_0 of about one quarter (for $R \approx 3.2$) is predicted. The reader may examine Figures 1 and 2 to see whether this prediction is borne out.

Perhaps just as interesting as the prediction of the extremely narrow width to W is the concomitant prediction that the ordering of stars across the strip should reverse. That is, if $\beta < R$ the brightest star in V_0 should become the faintest stars in W, read at constant log P. A casual inspection of the stars at log $P > 1.8$ shows this too not to be the case. We will concern ourselves with the details of this in the next section, but the main point here is that the predictions of the canonical calibration do not appear to be confirmed by W.

Before passing on to the point-by-point correlations one other general observation is worth making. As has been noted by Sandage and Tammann (1968), the long-period end of the period-luminosity relation *appears* to go nonlinear and flatten out beyond log $P > 1.8$. If this is an intrinsic nonlinearity (and linearity is by no means demanded by nature), it too should be preserved in the transformation from V to W. As can be seen from Figures 1 and 2 this apparent curvature is not preserved. The most obvious explanation is that this small sample of long-period Cepheids is systematically reddened to fainter apparent magnitudes (an effect cancelled by the transformation to W). This conclusion anticipates later conclusions and sets the stage for the quantified description of individual reddening derived below.

b) *Detailed Correlations*

To investigate the structure of the instability strip, magnitude residuals read at constant period have been computed for both V and W. These residuals ΔV and ΔW must be read from the same fiducial line, as can be seen from the fact that both V_0 and W have the identical period dependence in equations (2) and (3). All 96 LMC Cepheids for which photoelectric $\langle B \rangle$ and $\langle V \rangle$ magnitudes have been published have been used, and these residuals are plotted in Figure 3.

As can be seen from Figure 3 there is a great deal of scatter, much more than would be expected from photometry good to ± 0.04 magnitudes. An eye fit to the data indicates that the brightest Cepheids in V are still the brightest Cepheids transformed to W. A value of $\Delta W/\Delta V_0 = (\beta - R)/\beta \sim 1/2$ (i.e., $\beta \sim 6$) appears to fit the trend in the data. On no account is a fit corresponding to $\beta = 2.6$ acceptable since it would have a negative slope if plotted in Figure 3.

This last estimate of β is essentially the width argument revisited with the sign ambiguity removed and the derived value of β still a lower limit, since we have not explicitly considered the effects of differential reddening. Equations (2) and (3) demand a unique mapping of

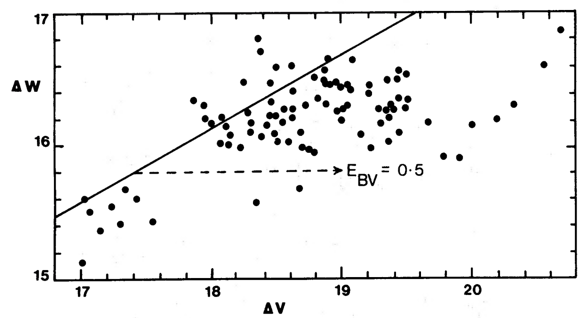

FIG. 3.—Residuals from the blue edge of the period-luminosity relations as plotted in Figs. 1 and 2. The parameter ΔW is an intrinsic strip penetration parameter measured from the blue (bright) edge of the instability strip; ΔV is an apparent penetration parameter being the sum of the true visual penetration away from the blue edge plus a random interstellar absorption component A_v. Both parameters are given with respect to the same fiducial line, (defined by $-4.0 \log P$), as stressed in the text.

ΔV_0 onto ΔW; however, we have plotted ΔV which will scatter away from this relation systematically toward larger values. At best we can hope that a "blue envelope" of least reddened Cepheids will be preserved for large enough samples.

The slanting solid line in Figure 3 is our best estimate of the intrinsic ΔV_0 versus ΔW relation, trying to allow for observational error and small sample statistics. In fact what follows is quite insensitive to the choice of the β within a large range (5–20). The line plotted has the equation

$$\Delta V_0 = 1.85 \Delta W - 11.90, \quad (4)$$

and corresponds to $\beta = 7$. Also shown in Figure 3 is an arrow giving the magnitude and direction of displacement due to 0.5 mag of selective absorption. Remembering that ΔW is independent of reddening this arrow must be horizontal.

III. INDIVIDUAL REDDENINGS FOR LMC CEPHEIDS

Assuming that the scatter in Figure 3 is dominated by differential reddening within and across the face of the LMC and accepting the intrinsic blue envelope, it is now possible to derive individual reddenings for each and every Cepheid with B, V photometry. Using equations (1) through (4) it is straightforward to show that for the LMC Cepheids

$$E(B-V) = -0.26V - 1.05 \log P + 1.84(B-V) + 3.62. \quad (5)$$

Figure 4 shows a histogram of the reddenings obtained using equation (5). The distribution is markedly skew symmetric. This is unlike an error-induced (Gaussian) distribution but more Poisson in nature as might be expected from the random interception of absorbing clouds along the line of sight. The median value of the reddening is $E(B-V) = 0.13$ mag and the mode is 0.11 mag not too dissimilar from the galactic foreground reddening of 0.08 mag suggested by Gascoigne (1969).

What is striking however is that some of the reddenings are quite appreciable, getting as high as 0.71 mag for HV 2827. Not that these reddenings are totally surprising: Feast (1974) has already noticed a color anomaly in the longest period Cepheids based on classification spectra in the sense that these stars were too red for their spectral type. And Madore (1976*b*), using *UBV* photoelectric photometry and a reddening formula derived by Tammann (1968), also found appreciable reddenings for these Cepheids. The results for those stars in common and where more than one spectroscopic determination is available show good agreement, with the *UBV* determinations being somewhat low compared to the present study.

Table 1 shows the mean reddenings of the LMC Cepheids and their associated standard errors as a function of period. Up to a period of about 10 days the reddenings are consistently low and average to about $E(B-V) = 0.08$ mag. However toward longer periods these younger Cepheids do show a systematic increase in their average reddenings, although, as expected by the Poisson nature of the reddening, their dispersions increase as well.

A correlation of reddening with period is perhaps not too distasteful when it is recalled that the longest period

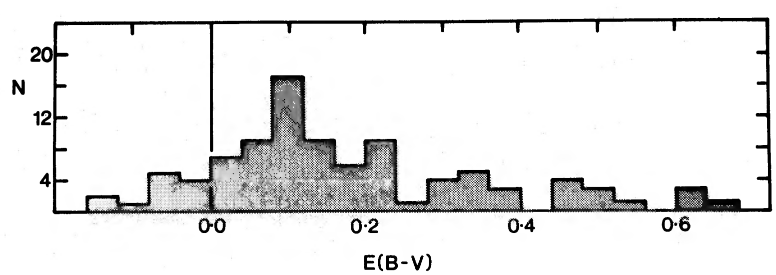

FIG. 4.—Frequency distribution of reddening values for LMC Cepheids

TABLE 1

MEAN REDDENING AS A FUNCTION OF PERIOD

Range in log P	Mean $E(B-V)$	Standard Deviation	Number of Cepheids
<0.8	0.06	±0.08	21
0.8–1.1	0.11	±0.14	27
1.1–1.4	0.22	±0.17	21
1.4–1.7	0.28	±0.16	22
1.7–2.3	0.56	±0.12	5

Cepheids are also the youngest and are therefore more likely to be still associated with the gas and dust from which they were formed. In addition the longest period Cepheids are the most luminous by virtue of their larger radii and, correspondingly, they have lower surface gravities. It is possible that their tenuous supergiant atmospheres are sufficiently volatile so as to later enshroud the long-period Cepheids with their own dust. In this overall regard it is worth recalling that RS Pup, one of the longest period galactic Cepheids, is seen to be in a reflection nebula of its own. (See the fine photograph by van den Bergh 1977.)

IV. THE INTRINSIC STRIP

While no convincing firm value of β can be derived from the few points defining the blue envelope in Figure 3, it is clear that low values are ruled out. If $\beta \sim 7$ or more it is equally clear from Figure 3 that, while the width of the instability strip in V, read at constant period, is relatively fixed at about 1.5 to 2.0 mag, the width in $(B-V)_0$ must be quite small, being on the order of 0.2 mag–0.3 mag.

The intrinsic luminosity-period relation after correcting for reddening is plotted in Figure 5. It has a width of about 1.5 mag and is linear over the entire period range in keeping with the $(W, \log P)$-relation. To further illustrate the consistency of these reddenings in preserving the form and ordering of points within the strip, Figure 6 shows the Cepheids in an exemplary period range orderd in W, tangled in V, and reordered by V_0 using reddenings from equation (5).

If W were dominated by random errors in V and $B-V$, it is unlikely that position within the W instability strip would correlate with any other *independently determined* parameter that is expected to vary across the instability strip also. We have exhausted our discussion of reddening-dependent quantities such as color and magnitude; however we have one test and potential correlation open to us. The amplitude of the light variation is both independent of distance and reddening and is yet another free parameter.

For those Cepheids with published amplitudes Figure 7 shows a plot of the blue amplitude $B_{\rm ampl}$ as a function of ΔW, the penetration into the instability strip measured from the blue edge toward the red. Although the scatter is considerable the correlation is obvious: Those Cepheids with the largest amplitudes are found deep into the instability strip (large ΔW) while low-amplitude Cepheids are found toward the blue edge of the instability strip (small ΔW). An eye fit to the data gives $\Delta W \approx 0.8\, B_{\rm ampl}$.

This correlation indicates that W is not overwhelmingly dominated by errors and that previously reported correlations (or lack thereof) of amplitude with apparent position within the instability strip have been obscured by reddening effects—something that W definitely does not suffer from. That this correlation is also period

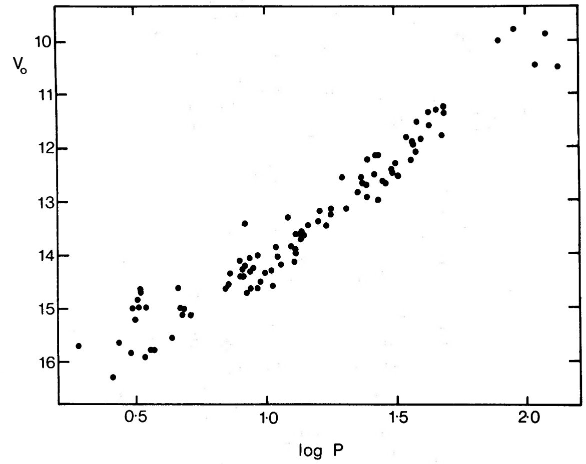

FIG. 5.—Corrected visual period-luminosity relation for LMC Cepheids using reddenings calculated by eq. (5) and $R = 3.2$.

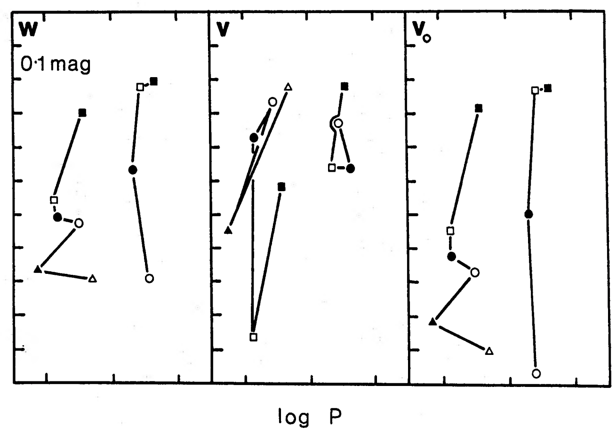

FIG. 6.—A detailed slice of the period-luminosity relation at about 25 days. The left panel illustrates the ordering of points across the intrinsic instability strip as defined by W. This ordering is scrambled by random differential reddening as shown by the apparent visual magnitudes illustrated in the middle panel. By applying the reddenings derived from eq. 5 the ordering of the points in the visual is restored, as shown in the right panel.

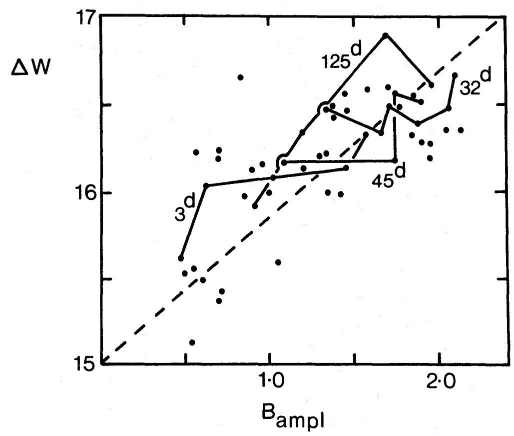

FIG. 7.—The amplitude mapping across the instability strip. Penetration into the instability strip as measured from the blue edge to the red is given by ΔW. The blue light amplitude $B_{\rm ampl}$ is seen to increase toward the red edge of the strip. Several cuts at constant period are also illustrated to show that the mapping is in the same sense (and range) at all periods.

independent is shown by the ordered connecting of four subsets of data drawn from the period ranges around 3, 32, 45, and 125 days, respectively. Each subset has the same sense of the correlation and spans as large a fraction of the range as the data permit.

V. CONCLUSIONS

As foreshadowed by the general discussions of Brodie and Madore (1980) and Clube and Dawe (1980), any differences between this study and previous investigations are due solely to our explicit inclusion of the reddening of the LMC Cepheid sample and our belief that it must be dealt with on a star-by-star basis. W deals with the reddening exactly but implicitly; our reddening formula offers an explicit but somewhat less exact solution. Both solutions are mutually consistent and physically realistic. The consequences are (1) the Cepheids in the Large Magellanic Cloud suffer variable, and sometimes large [$E(B-V)>0.5$ mag] amounts of reddening, (2) the longest-period, brightest Cepheids are most affected by this reddening (the consequences of which are obvious for the distance scale), and finally (3) amplitude is a monotonic function of position in the instability strip, increasing from blue to red at all periods.

Extensions of these methods and conclusions will be made in a forthcoming paper, in which the rest of the Local Group members with Cepheids will be discussed (Madore and Anderson 1982), and J, H, K photometry will be used to confirm the linearity of the period-luminosity relation and the systematic reddening of the longest period Cepheids (McGonegal *et al.* 1982).

This work has been underway for some years now and has profited from distance and isolation, as well as animated discussions with many colleagues. Financial support from the Natural Sciences and Engineering Research Council of Canada, the University of Toronto, and the Science Research Council, England is warmly acknowledged.

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