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THE CEPHEID DISTANCE SCALE[1]

BARRY F. MADORE

NASA/IPAC Extragalactic Database, Infrared Processing and Analysis Center, Jet Propulsion Laboratory
California Institute of Technology, Pasadena, California 91125

AND

WENDY L. FREEDMAN

The Observatories, Carnegie Institution of Washington, 813 Santa Barbara Street, Pasadena, California 91101

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ABSTRACT

Developments concerning of the period-luminosity (PL) relation for classical Cepheids are discussed with particular emphasis on the fact that Cepheids continue to provide the most homogeneous and reliable set of distances to many nearby galaxies. Observational programs relating to extragalactic Cepheids are also reviewed, and new distances and reddenings to individual galaxies are determined from these data using multiwavelength PL relations, based for the first time on self-consistent calibrating data sets. Three recent promising secondary extragalactic distance indicators (Tully-Fisher, planetary nebula luminosity functions, and surface-brightness fluctuations) tie their zero points to the Cepheid distance scale and give globally consistent values for the Hubble constant of about 80 km sec^{-1} Mpc^{-1}. Tying the Faber-Jackson relation and the supernova distance scales to the Leo and Virgo clusters yields similar results. Such a value for the Hubble constant is marginally inconsistent with new estimates of the ages of globular clusters, and it may prove to be a serious problem for standard cold dark-matter models of the universe.

Key words: galaxies: distances–stars: photometry–stars: Cepheids

1. Introduction

Cepheids are high-luminosity, radially pulsating, variable stars. Their intrinsic brightnesses range from ($-2 > M_V > -6$) making them ideal candidates for distance indicators on Galactic and extragalactic scales. Due to their variability they are easily isolated, identified, and classified. But perhaps most importantly, detailed stellar-pulsation models of these objects indicate that we understand the basic physics underlying their luminosities, colors, and periods. In this regard, Cepheids appear to stand alone in the extragalactic distance scale.

This review will concentrate on our developing understanding of the very basic principles characterizing the variability seen in individual Cepheids, as well as the systematics and trends observed for Cepheids as an ensemble. The emphasis will be on Cepheids as accurate extragalactic distance indicators and will not enter the debate concerning the exact value of the zero point for the period-luminosity (PL) relation since this is adequately covered in recent commentaries by Walker 1988 and Schmidt 1991 which now suggest that there is a convergence of opinion on the zero point of the Galactic Cepheid period-luminosity relation, at the level of about $\pm$ 0.10 mag. As optimistic as these reports are, readers are still referred to the cautioning remarks by Turner 1990.

In Section 2 we present a physical paradigm within which Cepheids, as distance indicators, are embedded. (However, for dissenting views on the whole process see Clube & Dawe 1980; Stift 1982, 1990). Section 3 is a discussion of the difficulties and uncertainties caused by reddening and metallicity differences. Section 4 puts into perspective the explicit determination of reddening made

[1]One in a series of invited review articles currently appearing in these *Publications*.

possible by employing panoramic and long-wavelength detectors. Sections 5 and 6 then review the status of the Cepheid-based distances to Local Group galaxies and those beyond the Local Group, respectively. In Section 7 we report the influence of the new Cepheid distance scale calibration on the recent determination of the Hubble constant. Finally, Section 8 gives a preview of what can be achieved with a refurbished Hubble Space Telescope. Appendix A looks at the differences between the period-luminosity-color (PLC) relation and the PL relation. Appendix B discusses an implicit method for dealing with reddening. And finally, Appendix C critically examines the prospects for determining reddenings to individual Cepheids.

In order to place this review into historical context the interested reader is referred to the monographs by Fernie 1969, Sandage 1972, 1988a, Stothers 1983, Freedman 1988a, Walker 1988, Feast & Walker 1987, and Madore 1985, 1986 and references therein. A review by Hodge 1981 on the extragalactic distance scale is itself especially relevant inasmuch as many of the major topics of concern raised by him regarding the Cepheid PL relation have now been addressed at least in part by direct observations. The evolutionary status of Cepheids is most recently reviewed by Chiosi 1990, while Simon 1990 gives a detailed look at the convergence of techniques used to calibrate the Galactic PL relation and the confrontation of these observations with pulsation theory.

2. Simple Physical Considerations

The basic physics connecting the luminosity and color of a Cepheid to its period is well understood. Using Stefan's law

$$L = 4\pi R^2 \sigma T_e^4 \ ,$$

the bolometric luminosities L *of all stars* (including Cepheids) can be derived. The radius R is a geometric term, parameterizing the total emitting surface area $4\pi R^2$, and the effective temperature T_e is a thermal term, used to parameterize the areal surface brightness, given by σT_e^4. Expressed in magnitudes, Stefan's law becomes

$$M_{BOL} = -5 \log R - 10 \log T_e + C \ ,$$

and it is schematically shown in Figure 1. It should be noted that *the entire* M_{BOL}, $\log T_e$ *plane is mapped by* equation (2), and that within the context of this relation alone, for any value of $\log T_e$, an unbounded range of radius R is independently possible. However, once R *and* T_e are each specified, M_{BOL} is uniquely defined. Additional constraints, outside of Stefan's law itself, must be involved if bounds on permitted values of the independent geometric and thermal variables are to be considered. An important constraint is provided by stellar evolution. In terms of time scales and allowed equilibrium configurations, the core-hydrogen-burning main sequence is one of the most striking and well-known examples of such a "constraint" on populating the H-R diagram. Hydrostatic equilibrium can be achieved for long periods of time along the hydrogen-burning main sequence, and as a result we are constrained to observe most of the stars there most of the time. There are of course other constraints.

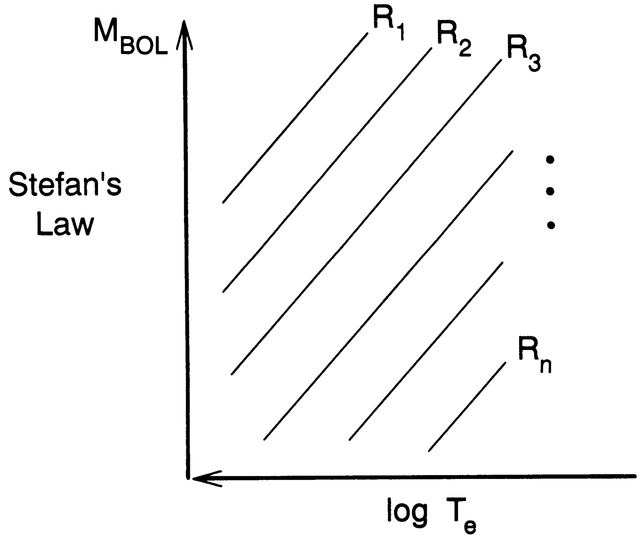

FIG. 1–Stefan's law expressed in graphical form projected onto the theoretical M_{BOL}, $\log T_e$ plane where loci of constant radius are indicated by upward sloping lines. The entire plane is mapped by Stefan's law.

For mechanical systems it is well-known that $P\ \rho^{1/2} = Q$, where Q is a structural constant, and P is the natural free pulsation period, determined by gravity through ρ, the mean density of the system, in turn defined by $\mathfrak{M} = 4/3\pi\ R^3\rho$, where $\mathfrak{M}$ is the total mass of the system. If it is assumed that mass is predominantly a function of R and T_e, then the pulsation period can be used as the second observable parameter instead of requiring the radius to be observed directly.

If we then linearly map $\log (T_e)$ into an observable intrinsic color (i.e., $(B-V)$), and map radius into an observable period, we thereby predict a new two-parameter description of the luminosity of (pulsating) stars. This is precisely the physical basis for the PLC relation for Cepheids, as was so elegantly introduced and explained over a quarter of a century ago (Sandage 1958; Sandage & Gratton 1963; Sandage & Tammann 1968). In its linearized form for pulsating variables, Stefan's law takes on the following form of the PLC: $M_V = \alpha \log P + \beta\ (B-V)_0 + \gamma$. In analogy to plotting Stefan's law in the theoretical $M_{BOL} - \log T_e$ above, Figure 2 shows the PLC mapped into the observational $M_V, (B-V)_0$ color-magnitude diagram. Again the entire plane is mapped. To see how the PLC relates to the more commonly referred to relations (the PL and PC relations) the reader is referred to a more detailed discussion in Appendix A, the caption to Figure 3, and the Cepheid section in Jacoby et al. 1992 where an

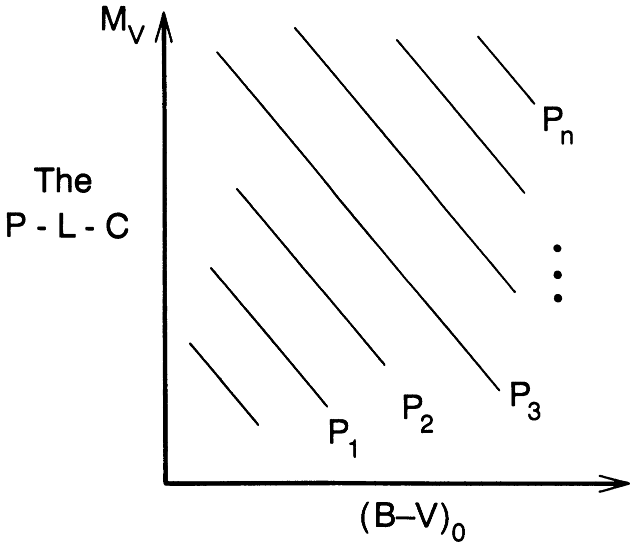

FIG. 2–The PLC relation expressed in graphical form as projected onto the observational M_V, $(B-V)$ plane, where loci of constant period are downward sloping lines. As in Figure 1 the entire plane is mapped by the PLC relation.

empirical analog of Figure 3 is given.

Stars can be found evolving across many parts of the color-magnitude diagram. However, only in very narrowly defined regions do they become pulsationally unstable. Cepheid pulsation in particular occurs because of the changing atmospheric opacity with temperature in the helium ionization zone. This zone acts like a heat engine and valve mechanism, alternately trapping and then releasing energy, thereby periodically forcing the outer layers of the star into motion against the restoring force of gravity. Not all stars are unstable to this mechanism. The cool (red) edge of the Cepheid instability strip is thought to be controlled by the onset of convection, which then prevents the helium ionization zone from driving the pulsation (see Baker & Kippenhahn 1965; Deupree 1977 for references). For hotter temperatures a blue edge is encountered when the helium ionization zone is found too far out in the atmosphere for significant pulsations to occur. Further details and extensive references can be found in the monograph on stellar pulsation by Cox 1980.

3. Observational Considerations

3.1 *General Considerations*

By the 1960s the instability strip had been observed to have the following general properties: periods for Cepheids ranged from several days to a few hundred days; at constant period, the B-magnitude total width of the PL relation was about 1.2 mag; the V-magnitude width was measured to be about 0.9 mag; and the $(B-V)$ color width was found to be about 0.4 mag, with the reddest Cepheids being the faintest at any given period. In a practical sense this meant that, in estimating distances, any individual Cepheid could deviate from the statistical ridge line by up to ± 0.6 mag in B; and such an error (if applied to one Cepheid) would translate into an equivalent error of about 30% in distance. Large samples can decrease the error on the apparent modulus inversely with the square root of the number, a formal error of only 10% being possible with a sample containing as few as a dozen Cepheids.

The discussion in the preceding section concerned an idealized PLC relation, expressed in its linearized form. Some of the difficulties encountered in the empirical calibration of this relation will be discussed in the present section. We concentrate on extragalactic studies. And so we do not discuss the issue of how best to determine independent distances and independent reddenings for the Galactic population of classical Cepheids: as plentiful as they are Galactic Cepheids in the field are problematic, while only a handful of (short-period) Cepheids are contained in open clusters, which are generally sparsely populated and often heavily obscured. The interested reader is, however, referred to the recent papers by Fernie 1990, Fernie & McConegal 1983, Feast & Walker 1987, and Jacoby et al. 1992 for a pathway into the literature on this Galactic approach to the calibration.

Because they are nearby, and because of the large numbers of Cepheids cataloged in them (Payne-Gaposchkin 1971; Payne-Gaposchkin & Gaposchkin 1966), the Magellanic Clouds have long been the test-bed for calibrations of the period-luminosity and period-luminosity-color relations. Indeed, the original discovery of these relations was made in the Magellanic Clouds (Leavitt 1906). These same large samples allowed the first estimates of the slope of the PL relation and first approximations to the period and color dependences of the PLC relation. Now with a geometric expansion parallax to the LMC via the remnant of Supernova 1987A there is a geometric distance modulus estimate of 18.57 ± 0.10 mag for the LMC (Panagia & Gilmozzi 1991). The precision of this measurement should improve with time as the expansion continues to be monitored. Nevertheless, it is reassuring that this expansion method agrees extremely well with existing estimates of the LMC distance modulus, and that an independent check of our zero point is provided by the measurement of RR Lyrae distances, which also are in very good agreement with the Cepheids (for a recent review see Westerlund 1990 and references therein).

For work on the distance scale, the existence of a statistical relation between period and luminosity is of such great utility by itself that it is of little wonder that concern about second-order effects in the calibration (i.e., those aspects above and beyond establishing the slope and zero point) were **not** of immediate import in the earliest studies. Some of these issues are: the origin of the scatter in the PL relation; the systematic effects of red-

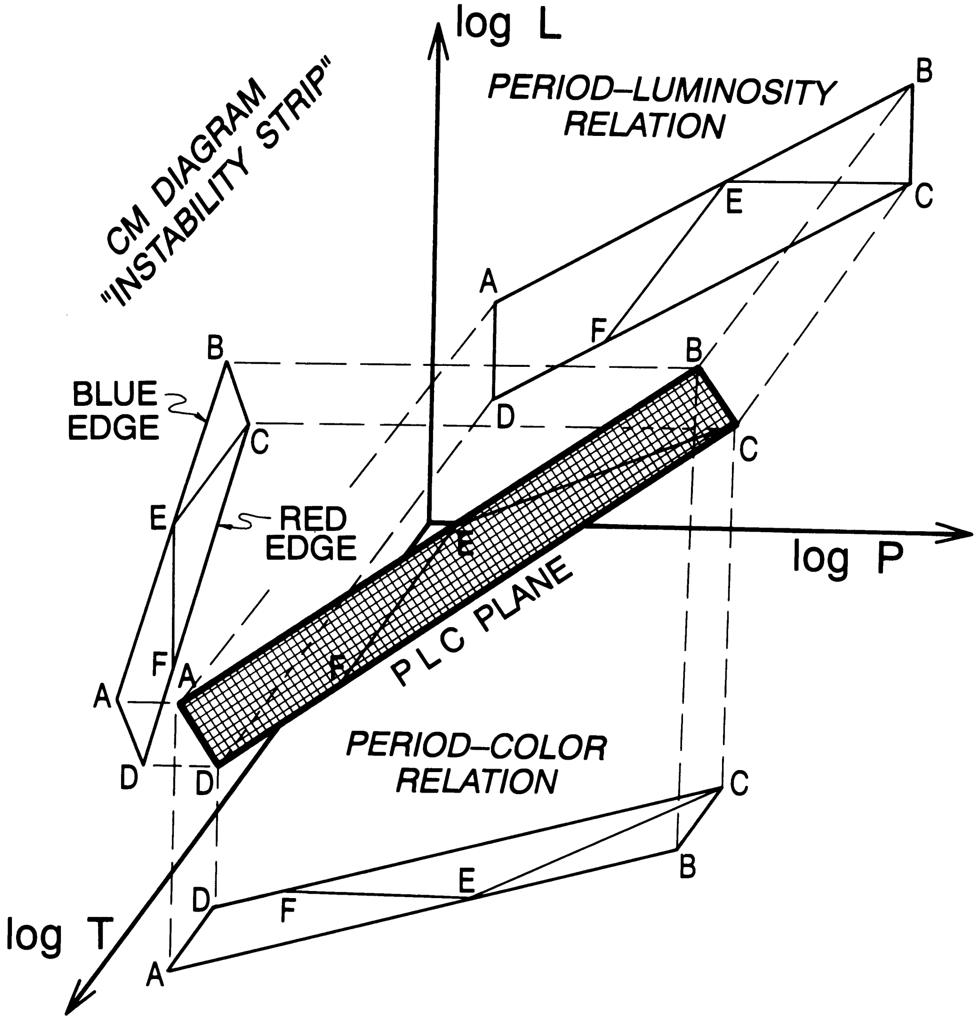

FIG. 3–The Cepheid Manifold: Projections of the PLC plane (shown shaded) onto the three principal coordinate systems (luminosity [log L] increasing up, period [log P] increasing to the right, and color [log T] becoming bluer/hotter to the lower left). The backward projection onto the (log L, log P) plane gives the period-luminosity relation. Projecting to the left gives the position of the instability strip within the color-magnitude diagram. Projecting down gives the period-color relation. EC is a line of constant luminosity. EF is a line of constant color. AD and BC are lines of constant period.

dening; the systematic effects of metallicity (e.g., Gascoigne 1974; Stothers 1983; Freedman & Madore 1990), companions (e.g., Madore 1977; Coulson, Caldwell & Gieren 1986); CNO abundance, helium abundance, and mass loss (e.g., Lauterborn, Refsdal & Weigert 1971; Lauterborn & Siquig 1974; Cox, Michaud & Hodson 1978; Becker & Cox 1982); magnetic fields (e.g., Stothers 1982); the possibility of curvature in the PL relation (e.g., Fernie 1967; Sandage & Tammann 1968, 1969); the relative disposition and the slopes of the red and blue edges of the instability strip (Fernie 1990); and the physical origin of these constraints (e.g., Iben & Tuggle 1972a,b, 1975; Chiosi et al. 1991), etc. Unfortunately, several of the corrections to be considered are probably manifesting themselves simultaneously and at the same level of numerical significance.

Before we can approach an empirical determination of the coefficients in the PLC relation (or any determination of their variation with metallicity) we must solve the reddening problem. While theory predicts a finite width to the instability strip (with temperature/color being the controlling parameter), and while metallicity is a quantity that is known to be different from galaxy to galaxy (and it is known to vary systematically within individual galaxies), only when reddening has been accounted for can we go on to look for meaningful correlations of luminosity residuals with intrinsic color and/or metallicity, for instance. To decouple and solve for the effects of metallicity, reddening, and intrinsic temperature variations, high-precision photometry is a prerequisite, and at least as many independently measured quantities are needed as there are parameters to be determined.

3.2 *Reddening*

Interstellar grains, within our Galaxy, along the line of sight to a nearby galaxy, or within the galaxy being studied, will each result in light being selectively scattered and absorbed. If any one of these components of extinction is not accounted for, a Cepheid in an external galaxy will appear fainter and more distant than it actually is, and at the same time it will appear redder and cooler than it truly is. Systematic errors will thereby creep into the distance scale.

Accounting for the Galactic foreground component associated with dust in the plane of our own Milky Way is relatively straightforward and will not be discussed here in any detail. The use of foreground stars and/or reddening maps, generated from galaxy counts in combination with neutral hydrogen studies (see Burstein & Heiles 1984), appears to be quite reliable and is widely used. Since most external galaxies subtend an angular size small in comparison to expected variations of extinction across the line of sight, and since most extragalactic studies are also done at fairly high Galactic latitudes, these foreground Galactic extinction corrections are relatively small and of low variance.

Dealing with the reddening internal to the parent galaxy itself is more problematic. In the earliest studies it was simply ignored. Even if this simplification has proven true for the first few specific cases, there is no reason to believe that it would have been obtained in general.

To illustrate the systematic effects of reddening on the observed PL and PLC relations, we consider the following example. Suppose for the moment that the instability strip is intrinsically very narrow both in color and in magnitude at fixed period. Now consider a sample of Cepheids drawn from this strip in a nearby galaxy, where on average the reddening is $E(B-V) = 0.2$ mag, with a standard deviation of $\pm$ 0.1 mag. These stars, differentially obscured, would be observed to have a period-color relation with a full ($\pm$ two-sigma) color width of $\sim$ 0.4 mag, a B period-luminosity relation with a magnitude width of 1.7 mag, and a V period-luminosity relation with a width of 1.3 mag (for a ratio of $A_V/E(B-V) = 3.3$). As "predicted" from a general consideration of the theoretical PLC relation, the deviations in the period-luminosity relation would be found to be correlated, with residuals from the period-color relation, apparently confirming the theory. But none of this correlation would be intrinsic, of course, despite it being very well-defined. Unfortunately, too, solutions for distances would be systematically in error since the ridge line of the data (defining the mean PL relations) would be displaced from the intrinsic strip by $A_B = 4.3\,E(B-V)$, always toward larger apparent distances.

The above example is extreme, but it illustrates the point that any attempt to disentangle the effects of differential reddening and true color deviations within the instability strip must rely first on a precise and *thoroughly independent* determination of the intrinsic structure of the period-luminosity-color relation. In order to achieve that calibration high-quality, independent reddenings and distances to individual calibrator Cepheids must be available. The uncertainty involved in undertaking this first step will affect all future results based on those assumptions. In Section 4 and in Appendix C we discuss old methods that have been adopted to deal with the reddening problem and emphasize new methods that have been brought to bear with the introduction of panoramic, digital detectors operating at optical and now at near-infrared wavelengths.

3.3 *Metallicity Sensitivity of the PL Relation*

The chemical composition of a star plays a role in setting the rate of energy generation, it affects the evolution off of the main sequence, and it determines the wavelength distribution of the emergent flux. The role of metallicity in the evolution of individual Cepheids in specific, and for the forms of PL, PC, and PLC in general, has been a matter of conjecture and debate for several decades now. Stothers 1988 has reviewed the theoretical aspects of this problem in great detail, concluding that at blue wavelengths the effect could be of concern to the distance scale.

Freedman & Madore 1990 have published the first empirical test of the effects of metallicity on the Cepheid PL relation, using a multiwavelength technique (see Section 5.2) applied to groups of Cepheids at various radial distances from the center of M 31. If the Cepheids are participating in the radial metallicity gradient in M 31 (traced by two other Population I objects, H II regions, and supernova remnants), then any apparent shift in the true moduli (corrected for Galactic foreground reddening and for reddening internal to M 31) could plausibly be interpreted as due to the metallicity sensitivity of the PL relation. PL relations based on new CCD light curves were measured for samples of Cepheids in the inner and outer regions of M 31. The result is that after correcting for reddening effects, to within the limits of the data, no statistically significant differences in distance moduli were measured for the three fields. (See Sec. 5.5 and Fig. 6.) And this result stands whether or not the expected more metallicity-sensitive B-band data are used in the analysis. That is, for metallicities covering the range typical for the central regions of giant spirals down to Magellanic Cloud-type systems (twice to one-half solar) the effect on the derived true modulus is less than 0.13 mag. If confirmed, this result means that Cepheids can provide extinction-corrected distances that are independent of metallicity uncertainties at levels that are systematically good to better than 10%. Near-infrared data are being obtained for the M 31 Cepheids to extend this study and allow a direct comparison of bolometric magnitudes with theoretical predictions.

4. Recent Advances Due to New Technology

4.1 *Application of Near-Infrared Techniques*

Although the first near-infrared observations of Galactic Cepheids were made more than 20 years ago (Wisnewski & Johnson 1968) their applicability to the distance scale was not appreciated until quite recently. In the first of a series of papers McGonegal et al. 1982 unambiguously demonstrated that, once periods were measured from optical data, near-infrared observations of Cepheids provided a number of advantages.

It was anticipated that by going to the infrared any concerns about total and/or differential reddening would be significantly reduced. For instance, a blue extinction of $A_B = 0.32$ mag (typical for Cepheids in the LMC, for instance) would translate to a total correction of $A_K = 0.05$ mag at 2.2 microns. The magnitude of such a correction in the near-infrared is comparable to the uncertainty alone associated with most optical extinctions!

But it was also immediately clear from the outset (see, for example, Fig. 4) that the infrared had other advantages directly applicable to the establishment of the Cepheid distance scale. First of all, the decrease in observed width of the PL relation was dramatic. Even single observations of the Magellanic Cloud Cepheids (uncorrected to mean light) produced a PL relation with remarkably small scatter ($\pm$ 0.2 mag). This narrowing of the width is due to two effects: a decreased sensitivity to differential reddening, but, more significantly, due to a much-decreased sensitivity of the infrared surface brightness to the temperature width of the instability strip. For exactly the same reason, the amplitudes of individual Cepheids (shown in Fig. 5, as plotted earlier by Wisnewski & Johnson 1968) also decrease with the wavelength of the observations. Thus, Cepheids observed at long wavelengths and at random points in their cycle are (1) closer to their time-averaged mean magnitudes than the equivalent observation in the blue, and (2) the mean magnitudes themselves are in fact coming from a narrower projection of the instability strip into the infrared plane than the equivalent blue PL relation. From B to K, a typical Cepheid amplitude drops from 1.0 mag to 0.2 mag, while the width of the PL relation decreases from 1.6 mag to 0.5 mag. As a result, for distance determinations even single, random-phase observations of known Cepheids, when made in the near-infrared, are comparable in accuracy to complete time-averaged magnitudes (derived from a dozen or more observations) in the blue.

The periods for many extragalactic Cepheids had already been determined and made available from photographic studies at blue wavelengths in many long-term studies by Hubble, Baade, Sandage, Swope, Payne-Gaposchkin, Gaposchkin, and others. As a result, in only a matter of nights, it was therefore possible to reobserve the entire galaxy sample in the near-infrared and thereby provide new accurate distances, almost unaffected by absorption effects.

However, until recently, with the advent of near-infrared arrays, there have been limitations in the application of the IR to the Cepheid distance scale, even for nearby Local Group galaxies. Single-channel infrared detectors, available at the outset, were aperture devices which had to be "chopped", "on" and "off" the source in order to continuously monitor the intense and fluctuating terrestrial sky contribution to the signal. For Galactic Cepheids, and even for those in the Magellanic Clouds, a typical aperture of 5 arc sec could be placed over a star and chopped to a nearby reference region a few arc seconds away with relatively small uncertainty. However, for Cepheids at larger distances, the near-infrared observations of Cepheids in M 31 (Welch et al. 1986) and M 33 (Madore et al. 1985) show much more noise than could be attributed to photon statistics alone. The most likely sources of error are crowding and confusion; that is, contamination in one of the two comparison fields where "skies" were being measured, or contamination in the object aperture itself. Although no systematic error is expected from this contamination, the random errors are appreciable. New observations are being obtained with near-infrared InSb and HgCdTe arrays, and two-dimensional image analysis will allow more accurate infrared magnitudes to be measured in M 31 and M 33.

4.2 *CCDs and Multiwavelength Coverage*

Much was learned from the early near-infrared observations that could be applied to optical observations, given accurate data gathered over a sufficiently broad wavelength range. CCDs provided just that opportunity, bridging the development gap slowly being filled by the relatively small-area IR arrays. Given a wavelength sensitivity running from the B band (0.45 μm) to the I band (0.7 μm), the CCDs afford the opportunity to gather seeing-limited, panoramic, digital data, which can subsequently be reduced using local sky subtraction and point-spread-function fitting techniques, to derive accurate magnitudes and colors. Crowding and confusion errors can be dealt with at the one-square-arc-second level. Use of CCDs and near-IR arrays decreases the areal confusion by about an order of magnitude over the near-infrared apertures.

CCDs also offer the advantage of a large wavelength coverage, thereby allowing an explicit determination of the reddening from the optical data themselves. In this case, it is not necessary to rely purely on foreground estimates and/or on assuming that additional reddening is of negligible importance. Given a knowledge of the interstellar extinction law as a function of wavelength it is possible to fit all of the data simultaneously. Freedman 1988b introduced this new approach to determining true moduli for extragalactic Cepheids using multiwavelength

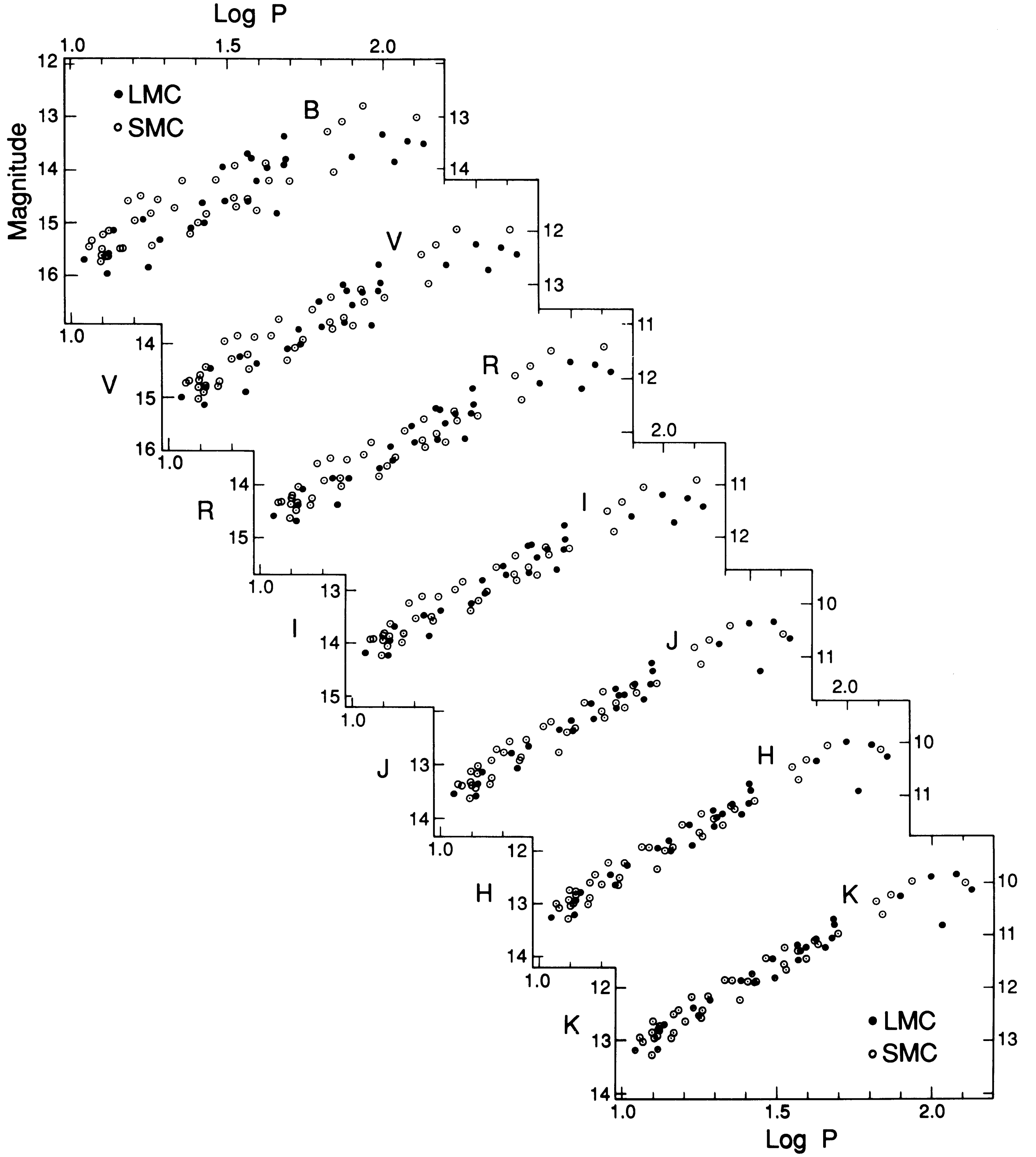

FIG. 4–Magellanic Cloud Cepheid period-luminosity relations at seven wavelengths, from the blue to the near-infrared, constructed from a self-consistent data set (Freedman & Madore 1992). LMC Cepheids are shown as filled circles; SMC data, shifted to the LMC modulus, are shown as open circles. Note the decreased width and the increased slope of the relations as longer and longer wavelengths are considered.

data, applying it first to single-epoch observations of Cepheids in IC 1613 and later refining it and expanding its application to data obtained for M 31, M 33, and NGC 300 as cited below. For a detailed discussion of the technique and its implementation the interested reader is referred to those papers. Briefly stated, one determines differential apparent moduli, scaled against the corresponding LMC PL relations. By assuming that all difference as a function of inverse wavelength can be attributed to selective absorption, fitting an interstellar extinction

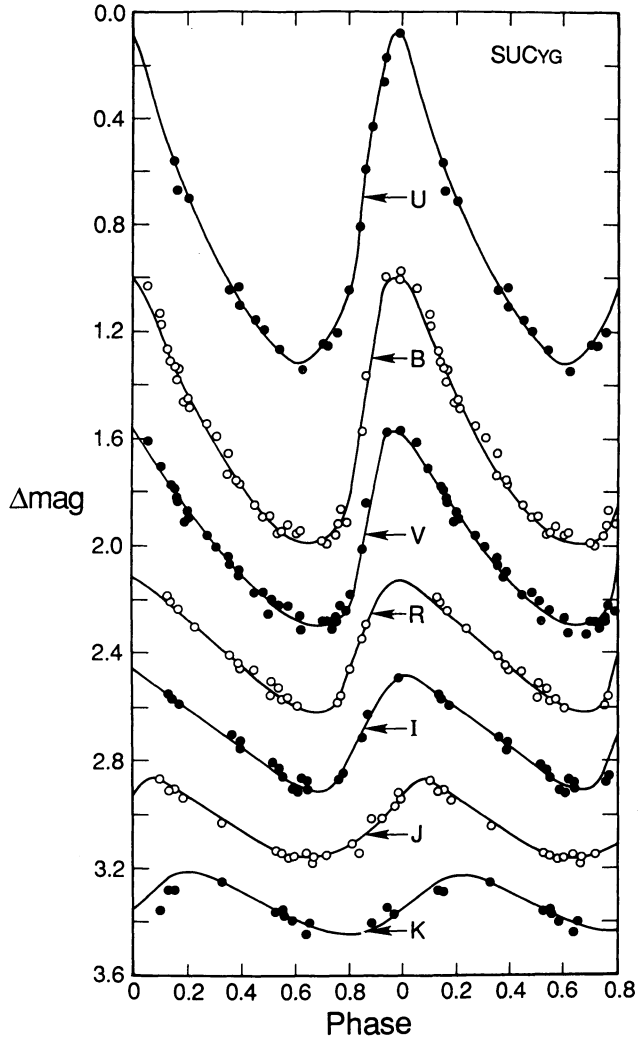

FIG. 5–Variations of amplitude and phase of maximum seen in the light curve of a typical Galactic Cepheid as a function of increasing wavelength. Note the monotonic drop in amplitude, the progression toward more symmetric light variation, and the phase shift of maximum toward later phases, all with increasing wavelength. Upper light curves are for short wavelengths (ultraviolet, blue, and visual); lower light curves are for long wavelengths (red and near-infrared out to $K = 2.2$ microns).

law to the data simultaneously estimates the total (foreground plus internal) absorption and the true distance modulus relative to the LMC.

In the following we briefly review the systematics of the PL relation as observed at wavelengths ranging from the blue to the near-infrared. To do this we concentrate on self-consistent data sets assembled for this purpose by Freedman & Madore 1992 and plotted in Figure 4. The stars included in their compilation are Cepheids in the LMC and SMC for which there are time-averaged mean magnitudes at all seven wavelengths (excluding R for the LMC sample which largely lacks this bandpass in published Cepheid observations). Figure 6 shows how the slope and the dispersion of the PL relation change systematically as a function of the effective wavelength of the filter bandpass. These impressions are quantified in the equations presented at the end of Section 4. In any case, as the longer-wavelength data are considered, it is clear that both quantities (slope and dispersion) have already begun to converge on an asymptotic value (set by the period-radius relation, which because of its geometrical nature is largely wavelength independent). From this point of view it makes no practical sense to observe Cepheids at wavelengths much beyond 2 microns if the aim is to decrease the dispersion in the observed PL relation. Fortunately, this regime is still accessible from the ground and is not far into the thermal IR where background effects become extremely large.

For comparison Cardelli, Clayton, and Mathis 1989 show in their Figure 3 the rate of falloff in the monochromatic extinction as a function of wavelength scaled to the visual extinction. As is well appreciated, the extinction does continue to decrease with increasing wavelength, making it sensible to extend the observations as far into the infrared as is practical. Of course, a decrease in extinction by a factor of over 50 is realized at K in comparison to B, so for most purposes this too is a reasonable wavelength at which to stop the effort.

It is generally true that the sense of the trends with wavelength that are observed for individual Cepheids during their pulsation cycle are also true for the ensemble of Cepheids reduced to mean light. That is, the temperature-induced variations in brightness (either around a pulsation cycle or across the instability strip) decrease with increasing wavelength. Similarly, the effects of obscuration decrease with wavelength regardless of whether there is an instantaneous phase point or a time-averaged one. Given the independent information on the extinction law, it is possible to combine the observations of extragalactic Cepheids in search of a simultaneous solution for the true distance modulus and reddening which are consistent with data obtained at many wavelengths. Some of these observations will be more sensitive to reddening than others; some will have larger uncertainties in the mean, others will have less; in the end an average over the ensemble should provide a good estimate of the derivative quantities, in addition to an estimate of their uncertainties.

The availability of panoramic linear detectors, such as the CCD, on the study of extragalactic Cepheids has been significant. It is now possible to obtain from the ground high-quality light curves of Cepheids in galaxies 2 Mpc away, ranging from blue and visual wavelengths, as well as reaching out to nearly one micron with CCD detectors and then extending to 2.2 microns with the newly available HgCdTe and InSb infrared areal detectors.

4.3 *Obtaining Accurate Cepheid Distances*

Once new extragalactic Cepheids are found at least four issues need to be adequately addressed, all of which are tightly coupled to common sets of observations: (1) Periods

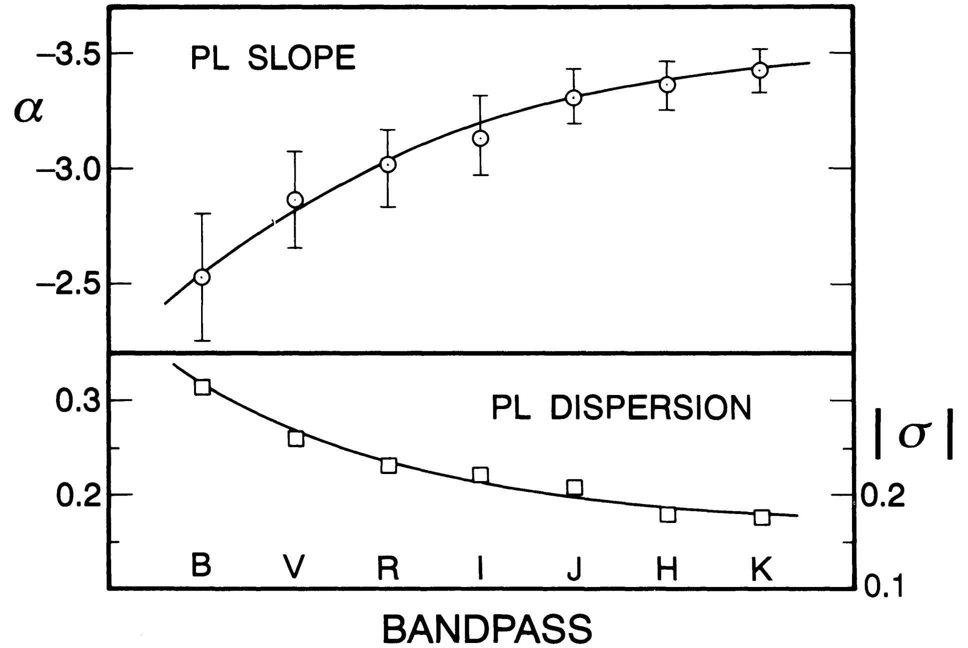

FIG. 6–Variations of the slope (circles) and dispersion (squares) of the Cepheid period-luminosity relation as a function of bandpass, sequentially ordered by effective wavelength.

have to be determined. (2) Complete light curves have to be delineated. (3) Mean magnitudes must be derived. (4) Accurate colors are required for reddening determinations. Needless to say it would be hard to derive (1) the periods (or prove that a star is, in fact, a Cepheid) without (2) the light curves, and vice versa. Similarly, (3) and (4), accurate mean magnitudes and colors depend on correct period phasing and proper light-curve fitting. But the requirements for accurate periods are in fact quite different from the requirements for accurate magnitudes. The number of data points required to yield a time-averaged magnitude (of specified precision) increases as the square of the light-curve amplitude. This makes colors and mean magnitudes based on short-wavelength observations more costly in observing time than their longer-wavelength counterparts. On the other hand, for fixed photometric uncertainties, periods increase in accuracy almost linearly with the time interval over which the observations are spaced. Furthermore, periods good to a few percent can be obtained using only moderately accurate photometry after only a dozen or so cycles, thereby making the time constraint a minimal one.

Finally, one must contend with the intrinsic width of the instability strip as projected into the PL relation. Increasing N, the numbers of Cepheids, is the most obvious solution here. For the B band the equivalent dispersion in magnitude in the Cepheid PL relation is ± 0.35 mag. The error in the mean apparent distance modulus decreases like $\sqrt{N}$. In the absence of reddening then it would appear that, for *apparent* distance moduli alone, a dozen Cepheids will give the requisite accuracy in the mean. But of course the real problem, once again, comes when trying to deal with reddening. And an example using two bandpasses only illustrates this graphically. In such a case the ensemble-averaged extinction essentially comes from differencing the mean apparent moduli found at two different wavelengths. Multiplying this difference by the ratio of total-to-selective extinction appropriate to those two wavelengths and subtracting the product from the mean apparent modulus gives the final true modulus. If 10% in distance is the goal (0.2 mag in true modulus), then, for the filter combination $V\ I$, simple arithmetic shows that two to three dozen Cepheids are required to establish the mean moduli to such a degree that the reddening-corrected modulus has a final error of less than 0.2 mag. Of course either increasing the number of wavelengths and/or increasing the wavelength baseline will each reduce the final error on the mean without demanding an increase in sample size.

In closing this section we present the multiwavelength PL relations used by us in determining the reddenings and distances to Cepheids in external galaxies. We emphasize that these relations are fundamentally different from those published by other workers up to this point, because they are derived from sets of data which are now totally self-consistent. Specifically, all of the PL relations are based on the same stars in order to eliminate sample-dependent variations in the solutions. Furthermore, Cepheids with $\log P > 1.8$ are excluded from the least-squares fits due to uncertainties in their reddenings and their evolutionary status. The LMC data set (scaled and dereddened as outlined in the next section) has been chosen as fiducial because of its large sample size and large wavelength coverage and because the LMC is very close to being face-on, thereby minimizing the effects of back-to-front geometry on the solutions. The relations are centered on $\log P = 1.0$, the midpoint of the range of periods considered here. Errors on the quoted coefficients are given after each of the values. Following each of the PL relations, the quantity in square brackets is the rms dispersion about the mean for that relation.

$$M_B = -2.43(\pm 0.14)(\log P - 1.00) - 3.50(\pm 0.06)\ [\pm 0.36]$$
$$M_V = -2.76(\pm 0.11)(\log P - 1.00) - 4.16(\pm 0.05)\ [\pm 0.27]$$
$$M_R = -2.94(\pm 0.09)(\log P - 1.00) - 4.52(\pm 0.04)\ [\pm 0.22]$$
$$M_I = -3.06(\pm 0.07)(\log P - 1.00) - 4.87(\pm 0.03)\ [\pm 0.18]$$

(Note that the *RI* magnitudes are on the Cousins system, while our *JHK* magnitudes are on the CIT/CTIO system. There are 32 LMC Cepheids for which *BVI* photoelectric photometry is available in the range $0.2 < \log P < 1.8$. *R* photometry is not available for many of the stars used above; however, *R* magnitudes were derived using the methodology set out in Freedman 1988b.)

Finally, we give below consistent PL solutions, based on a smaller set consisting of only 25 LMC stars, each of which has *BVRIJHK* photometry available (given the same conditions outlined above).

$$M_B = -2.53(\pm 0.28)(\log P - 1.00) - 3.46(\pm 0.12)\ [\pm 0.40]$$
$$M_V = -2.88(\pm 0.20)(\log P - 1.00) - 4.12(\pm 0.09)\ [\pm 0.29]$$
$$M_R = -3.04(\pm 0.17)(\log P - 1.00) - 4.48(\pm 0.08)\ [\pm 0.25]$$
$$M_I = -3.14(\pm 0.17)(\log P - 1.00) - 4.84(\pm 0.06)\ [\pm 0.21]$$
$$M_J = -3.31(\pm 0.11)(\log P - 1.00) - 5.29(\pm 0.05)\ [\pm 0.16]$$
$$M_H = -3.37(\pm 0.10)(\log P - 1.00) - 5.65(\pm 0.04)\ [\pm 0.14]$$
$$M_K = -3.42(\pm 0.09)(\log P - 1.00) - 5.70(\pm 0.04)\ [\pm 0.13]$$

The effective wavelengths for each of the seven bandpasses were chosen to be appropriate for a G-star spectrum (see, for example, Bessell 1979) where, for future reference, we have adopted *B* (0.44 μm), *V* (0.550 μm), *R* (0.653 μm), *I* (0.789 μm), *J* (1.25 μm), *H* (1.60 μm), and *K* (2.17 μm).

In closing this section it should be noted that there are external checks on the Cepheid calibration and distance scale derived from it that can and have been applied successfully to galaxies within the Local Group. An extensive review of these methods (including the use of RR Lyrae stars, red-giant luminosity functions, novae, and long-period variables, to name just a few) confirms (conservatively at the ± 0.2 mag level) the basic solidity of the Cepheid calibration (van den Bergh 1989; de Vaucouleurs 1991). For details the interested reader is referred to those reviews and the many references cited therein. Because many of these independent methods that provide checks on the Cepheid distance scale use intrinsically fainter stars, it is unlikely that galaxies significantly beyond the Local Group will be of much use in further refining the agreement (or disagreement) between the various estimators. However, more extensive and more precise observations of those same (faint) distance indicators within these and other Local Group galaxies will be crucial for fine-tuning the calibration and may be especially helpful in establishing the level at which metallicity corrections are needed in Population I and Population II distance indicators alike.

5. Local Group Galaxies

In this section we rapidly review the status of the individual galaxies in the Local Group for which (nonphotographic) digital data on known Cepheids have been obtained in the last few years. This discussion and that in the next section update and supersede similar overviews published earlier by Madore 1985 and Walker 1987b. The results on distances and reddenings are summarized in Table 1.

Rather than tackling the whole question of the Galactic calibration and its application to extragalactic Cepheids (which are generally of longer period and have differing metallicities) we prefer to continue with a tradition of securing the extragalactic distance scale to an adopted distance and reddening for the Cepheids in the Large Magellanic Cloud. Accordingly, for the LMC we adopt hereafter 18.5 mag for its true modulus and 0.10 mag for $E(B-V)$, and scale all other Cepheid-based distances assuming a value for the total-to-selective absorption of $R_V = A_V/E(B-V) = 3.3$ (appropriate for the later spectral types of Cepheids). All period-luminosity fits are done over the range $0.2 < \log P < 1.8$. Furthermore, as discussed in Freedman 1988b, all fits are carried out for a set of stars in the LMC defined by the simultaneous availability of photometry at *B*, *V*, *and I* wavelengths. Although the sample of Cepheids with *B* and *V* photometry alone is about a factor of two larger, the comparison of inconsistent data sets can lead to erroneous results. It should be

TABLE 1

Cepheid Distances to Nearby Galaxies

Galaxy	A_B (mag)	$(m-M)_o$ (mag)	D (Mpc)	Bands
NGC 6822	1.21	23.59	0.52	I
IC 1613	0.10	24.42	0.77	B V R I
M 31	0.00-0.99	24.44	0.77	B V R I
M 33	0.41	24.63	0.84	B V R I
WLM	[0.09]	24.89	0.95	I
Sextans A	[0.06]	25.75	1.41	B
Sextans B	[0.05]	25.80	1.44	B
NGC 3109	[0.14]	25.94	1.54	B
NGC 2403	[0.16]	27.51	3.18	I
M 81	[0.15]	27.59	3.30	I
M 101	[0.00]	29.38	7.52	R

noted that, although we quote the sources of original data throughout the next two sections, *the distances and reddenings given for each galaxy here are based on a new and homogeneous application of the multiwavelength fitting procedures* discussed earlier in this review (and described in detail in Freedman, Wilson & Madore 1991). In this procedure we have adopted the reddening law as determined by Cardelli, Clayton & Mathis 1989. By definition, our multiwavelength fitting procedure does not yield information about the distance and/or reddening to the LMC, as they are adopted ab initio. For completeness, we also review data for Cepheids in the Magellanic Clouds.

5.1 *The Magellanic Clouds*

For a relatively complete bibliography of modern photographic and photoelectric observations of Magellanic Cloud Cepheids the reader is referred to Tables 2 and 4 in Madore 1985; they will not be repeated here. Moreover, Caldwell & Laney 1991 have reviewed progress in the Southern Hemisphere on calibrating the Cepheid PL relations. They use data available in Madore 1985, in addition to more recently published near-infrared *JHK* data available for many dozens of Cepheids observed by Welch et al. 1987 and also Laney & Stobie 1986a,b. Following the lead by Welch & Madore 1984 these latter papers concentrate, among other things, on using the near-infrared data to determine the back-to-front geometry of the two Magellanic Clouds based on the ability of near-infrared observations of Cepheids to give extremely precise distances to individual stars. In this regard, it should be noted that Visvanathan 1989 has calibrated the Cepheids PL relation at 1.05 micron and applied it to Cepheids in the SMC (Mathewson, Ford & Visvanathan 1986, 1988), also aiming to probe the structure of the SMC in the context of a tidal encounter/disruption model.

5.2 *IC 1613*

Sandage 1971 published photographic light curves and periods for 25 of the confirmed Cepheids discovered, but never published, by Baade. Both single-phase, near-infrared *H*-band observations of ten of those Cepheids (McAlary, Madore & Davis 1984) and single-phase, multiwavelength *BVRI* observations (Freedman 1988b) of eleven of them have now been published. Freedman's work incorporated the near-infrared data. Using the new fitting procedure (rather than an earlier adopted linear extinction approximation), and excluding the lower-accuracy *H*-band data, yields a total mean reddening of $E(B-V) = 0.02$ mag, and a true modulus of 24.42 $\pm$ 0.13 mag, corresponding to a distance of 765 kpc. To maintain homogeneity for comparison of relative distances, this value supersedes the distance modulus 0.12 mag lower, quoted by Freedman 1988b.

It is also worth noting at this point that Freedman's 1988b conclusions concerning the universality of the slope of the PL relation (once brought into some degree of doubt because of the uncertainties in the faint-magnitude calibration of the photographic data on the IC 1613 Cepheids) has subsequently been bolstered by the additional analysis of 16 newly confirmed Cepheids in IC 1613, as discussed in Carlson & Sandage 1990.

5.3 *NGC 6822*

One of the most extensive modern studies of the dwarf irregular Local Group galaxy NGC 6822 is the photographic work of Kayser 1967. This study built on the original work of Hubble 1925 who found several Cepheids and a number of bright irregular variables in this galaxy. Unfortunately, NGC 6822 is fairly close to the Galactic plane ($b = -18$ degrees), resulting in large, and still somewhat uncertain, foreground reddening estimates. Published estimates for the reddening to the Cepheids in NGC 6822 range from $E(B-V) = 0.19$ to 0.42 mag.

Modern results on the Cepheid distance to NGC 6822 have been appearing slowly. Both Hodge 1977 and then van den Bergh & Humphreys 1979 have reported photoelectric *BV* observations of the 65-day Cepheid, V7 in NGC 6822. Multiwavelength *BVRI* CCD data have been obtained by us but as yet are unpublished; while Schmidt, Spear & Simon 1986, Schmidt & Spear 1987, Schmidt & Simon 1987, and Schmidt & Spear 1989 have published some CCD observations of Cepheids in NGC 6822, which were obtained for other reasons. Of late, the only directed study of the Cepheid distance to NGC 6822 is the paper by McAlary et al. 1983 on near-infrared *H*-band aperture photometry of nine Cepheids. Unfortunately, an independent determination of the foreground/internal reddening was not attempted because only one wavelength was involved in the new study. Visvanathan 1989 observed three Cepheids in NGC 6822 once each at 1.05 μm and derived a true modulus of 23.26 mag (scaled to an LMC true modulus of 18.5 mag). On the other hand, random-phase *I*-band CCD data (Lee, Freedman & Madore 1992) yield a true modulus of 23.59 mag.

Pending complete publication of the new CCD data, we can use the photographic observations of Kayser 1967 in combination with the *H*-band observations of McAlary et al. 1983 to provide a multiwavelength fit and solve for the true distance modulus and reddening to NGC 6822. In this application we have rederived all apparent moduli with respect to our internally self-consistent set of LMC Cepheid data. Excluding Kayser's variable No. 30 (which falls many sigma above the mean *B* PL relation), we find $(m-M)_B = 24.66 \pm 0.06$ mag, $(m-M)_V = 24.50 \pm 0.08$ mag, and $(m-M)_H = 23.77 \pm 0.17$ mag, resulting in $(m-M)_0 = 23.66$ mag with $E(B-V) = 0.26$ mag. The reddening is well within the range of previous estimates quoted above and is remarkably close to Kayser's original estimate of 0.27 mag. Despite this formal solution, it is clear that the Cepheids in NGC 6822 could profit from a modern investigation at several wavelengths, since the true modulus is probably still uncertain at the ± 0.2 mag level. A study of this nature is now underway at Palomar.

5.4 *M 33*

Hubble 1926 discovered 35 Cepheids in the inner regions of M 33 and determined their periods. These photographic data were later recalibrated by Sandage 1983. Sandage & Carlson 1983a added identifications, periods, and photographic mean magnitudes for 13 new Cepheids in an outer region of this galaxy, while Kinman, Mould & Wood 1987 then surveyed the main body of M 33 using photographic plates taken at the prime focus of the KPNO 4 m in search of long-period variables and, in the process, discovered 54 new Cepheids. All four of these publications include finder charts for their variables.

Building on these discovery papers, single-phase H-band observations (Madore et al. 1985) were made of 15 Cepheids in M 33. These were then augmented by and incorporated into a multiwavelength study of 19 Cepheids (Freedman et al. 1991) using CCD observations to obtain light curves and, therefore, time-averaged magnitudes and colors. It was the H-band study of the M 33 Cepheids that first noted the basic limitation due to crowding and confusion on the application of aperture techniques to the infrared Cepheid distance scale. *JHK* array imaging of individual Cepheids in M 33 and M 31 is being done by the authors using the Palomar 200-inch (5-m) telescope, so some of those early limitations should soon be lifted. Until those data are available the CCD observations alone indicate that, for M 33, $A_B = 0.41$ mag and the true modulus is 24.63 ± 0.09 mag, corresponding to a linear distance of 840 kpc.

5.5 *M 31*

Baade & Swope 1963, 1965 and Gaposchkin 1962 have each used 200-inch (5-m) plate material to catalog Cepheid variables in M 31. With the exception of the outermost Field IV (for which there are both *B* and *V* data) only blue photographic magnitudes are available, making reddening estimates for the Cepheids both circumstantial and rather unreliable. Nearly a quarter of a century later Welch et al. 1986 obtained single-phase H-band observations of 22 Cepheids in M 31. And even more recently Freedman & Madore 1990 have published multiwavelength PL relations for 38 Cepheids in three of the Baade and Swope M 31 fields resulting in independent reddenings and true distance moduli for each of the regions. A consistent true distance modulus of 24.44 ± 0.10 mag (corresponding to 770 kpc) is determined here with mean reddenings ranging from $E(B-V) = 0.00$ mag in Field IV to 0.25 mag in Field III. Those data are shown plotted in Figure 7.

6. Beyond the Local Group

To discover Cepheids in galaxies, and then confidently determine their periods, light curves, mean magnitudes, and colors, is an extremely demanding task. So it is perhaps not too surprising that, although Cepheids in galaxies as far away as M 81 were known to Baade in the early 1960s, up until the 1980s the only major galaxy outside of the Local Group for which a definitive study had been published of its variable-star content was the solitary,

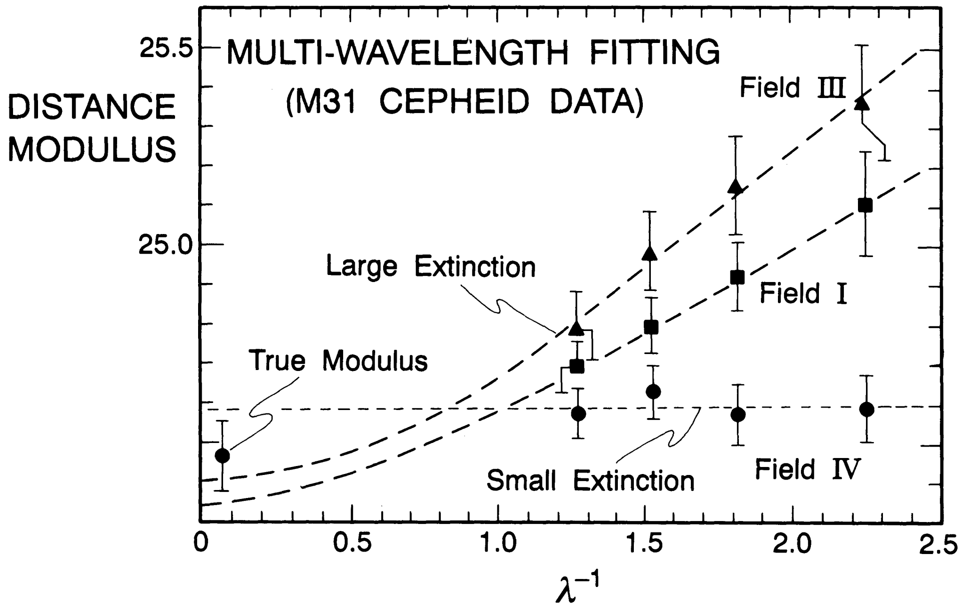

FIG. 7–Multiwavelength fits of normal reddening lines to *BVRI* apparent distance moduli for M 31 Cepheids (Freedman & Madore 1990). The three fields in M 31 have distinctly different average reddenings associated with them; however, the true modulus as given by the intercept at $1/\lambda = 0$ is quite stable, indicating little residual sensitivity of the Cepheid distance scale to metallicity.

late-type spiral galaxy NGC 2403 (Tammann & Sandage 1968). Only in the last decade have CCD surveys begun to come to the fore, as the last of the photographic surveys for Cepheids in galaxies easily resolved from the ground are now being published. CCDs have small fields of view, but they have significantly higher quantum efficiency than photographic emulsions, especially at longer wavelengths. Accordingly, they are in fact allowing significant progress to be made in searching for Cepheids in the few remaining galaxies near enough to be resolved from the ground. Progress on those fronts is now reviewed.

6.1 *NGC 2403*

Morphologically very similar to M 33 (and its southern counterpart NGC 300) NGC 2403 is a highly resolved late-type spiral galaxy often associated with M 81 and a handful of lower luminosity spiral and irregular galaxies (e.g., de Vaucouleurs 1975). Many dozens of photographic plates taken by Mount Wilson and Palomar observers were laboriously searched for variables by Tammann & Sandage 1968 resulting in the discovery of 17 Cepheids. Due to plate limitations, only the brightest Cepheids in the outer parts of the galaxy were found. Even so only the brightest parts of the blue light curves were securely calibrated and only fragmentary information was available in the visual bandpass. Nevertheless, good periods were determined by these authors and (assuming no extinction within NGC 2403 itself) an apparent modulus was estimated from the brightness of the apparently least-reddened Cepheid seen at maximum light. These data have been reanalyzed by Madore 1976 and then by Hanes 1982 and again by Rowan-Robinson 1985 with different sets of assumptions and, consequently, diverging conclusions. New data were obviously required before significant progress could be made beyond arcane arguments concerning methodology.

I-band CCD observations of eight of the known Cepheids in NGC 2403 are discussed by Freedman & Madore 1988 who derive a tentative true modulus of 27.51 mag. Their adopted error of ± 0.24 mag includes the uncertainty for the still-undetermined amount of reddening internal to NGC 2403 itself. Given what we now know about the difficulties inherent in doing near-infrared aperture photometry in confusion-limited cases, the modified *J*-band observations reported by McAlary & Madore 1984 and the conclusions drawn therein are thus superseded.

6.2 *M 81*

Baade 1963 was aware that Cepheids had been detected in M 81 but it was another 20 years before any results were published. Because of the high surface brightness of the disk of M 81, photometry of individual stars is very difficult, making the detection of variable stars such as Cepheids extremely taxing, especially near the plate limit. Nevertheless, Sandage 1984 succeeded in determining the periods for two Cepheids in M 81 using *B*-band photographic plate material. Freedman & Madore 1988 obtained *I*-band CCD photometry of these stars (based on finder charts and periods made available by Sandage 1987, private communication) and derived a true modulus of 27.59 ± 0.31 mag adopting a foreground Galactic extinction of $A_B = 0.15$ (Burstein & Heiles 1984). This modulus corresponds to a distance of 3.30 Mpc which is still uncertain at the 15% level inasmuch as the sample is small and no corrections for reddening internal to M 81 itself have yet been determined. Multicolor CCD observations have already been obtained by the authors and should shortly clarify this point.

6.3 *M 101*

Sandage & Tammann 1974 attempted to find Cepheids in their 200-inch (5-m) photographic plate material on M 101. Their failure led them to conclude that M 101 has a distance modulus in excess of 29.3 mag. Later, using CCD detectors Cook, Aaronson & Illingworth 1986 did succeed in detecting at least two Cepheids at $R \sim 23$ mag (with preliminary periods of 37 and 47 days) in a relatively uncrowded outer region of this galaxy. Given the uncertainties in the absolute zero point of the calibration of the Cepheid PL relation and the unknown extinction internal to M 101 itself, we find an apparent *R*-band modulus of 29.38 mag. Other fields in M 101 also observed by this group (Cook, Aaronson & Illingworth 1989) have yet to be fully analyzed, and, too, color information useful in determining total extinction is not yet available.

6.4 *NGC 7793 and NGC 247*

Using the prime-focus CCD at the CTIO 4-m telescope, Freedman et al. 1988a,b have discovered a dozen variables of the expected magnitude and color for Cepheids in NGC 247 and eight such variables in NGC 7793. The main problem confronting this study is the uncertainty in period determinations resulting from the limited number of observations. Observations aimed at breaking the aliasing ambiguity are continuing at the Dupont 2.5-m telescope.

6.5 *NGC 300*

Graham 1984 conducted a photographic study of NGC 300 using the Tololo 4-m reflector and was able to discover 18 Cepheids, determine their periods, and estimate mean *B* and *V* magnitudes. Madore et al. 1987 subsequently observed two of these Cepheids at *H*. Walker 1988 later criticized Graham's photoelectric calibration sequence and suggested a correction to the photographic photometry of the Cepheids. Visvanathan 1989 observed three Cepheids in NGC 300 at 1.05 μm and derived a true modulus of 25.80 mag (scaled to our adopted LMC modulus). Finally, Freedman et al. 1991 present *BVRI* CCD observations of 16 Cepheids in NGC 300 giving revised periods and time-averaged magnitudes and colors. A true distance modulus of 26.67 mag, corresponding to a distance of 2.16 Mpc, is derived from a multiwavelength analysis of the CCD data of the ten Cepheids with complete light curves.

6.6 *Wolf-Lundmark-Melotte*

A photographic study of the Wolf-Lundmark-Melotte (WLM) galaxy has been published by Sandage & Carlson 1985a where 15 Cepheids are identified and periods determined (ranging from 3.3 to 9.6 days). The CCD study of WLM published by Ferraro et al. 1989 unfortunately included none of the Sandage and Carlson Cepheids, but the CCD photometry does indicate that the photographic photometry is too bright by +0.4 mag in both B and V (as derived from the intercomparison of the two studies given in their Table 2).

Applying the B offset to the photographic data for the Cepheids in WLM and rederiving the apparent modulus as discussed above we find $(m - M)_B = 25.19$. Adopting $A_B = 0.09$ mag (Burstein & Heiles 1984) gives a true modulus of 25.10 mag. From preliminary reductions of I-band CCD observations of five WLM Cepheids, Lee et al. 1992 find a true modulus of 24.89 ± 0.15 mag, corresponding to a linear distance of 0.95 Mpc.

6.7 *NGC 3109*

From photographic material Demers, Kunkel & Irwin 1985 report discovering five Cepheids with periods in the range 10 to 23 days in the nearly edge-on, Magellanic-type irregular galaxy NGC 3109. Sandage & Carlson 1988, also using photographic material but with better plate scale, were able to observe deeper into the luminosity function and determine the periods for 29 Cepheids. The periods of their Cepheids ranged from 6 to 31 days, confirming only two out of the five earlier-reported periods. Using the data from Sandage and Carlson, and adopting a foreground extinction of $A_B = 0.14$ mag from Burstein & Heiles 1984, we derive a true modulus of 25.94 mag for NGC 3109, corresponding to 1.54 Mpc. No correction for extinction internal to NGC 3109 itself has been applied.

6.8 *Sextans A and Sextans B*

In a photographic survey of Sextans A, Sandage & Carlson 1983b discovered five Cepheids whose periods ranged from 15 to 25 days. In a later paper Sandage & Carlson 1985b recalibrated the Sextans A photometry and added data on the adjacent dwarf-irregular galaxy Sextans B, in which they discovered four Cepheids having periods ranging from 7 to 28 days. However, CCD data for stars in Sextans A (Hoessel, Schommer & Danielson 1983) after transforming from the original Gunn system to the BV system give a zero-point difference between the two data sets amounting to 0.2 mag. Walker 1987a then used a CCD with standard Johnson filters to study this problem further and concluded that the Sandage and Carlson B-band data are too faint by 0.16 mag in the color range appropriate to Cepheids. With the original Sextans A data transformed to the Sextans B photometric scale as described by Sandage & Carlson 1985b, and then further corrected using Walker's offset, we derive true moduli of 25.59 and 25.64 mag for Sextans A and B, respectively, after adopting foreground extinctions of $A_B = 0.06$ and 0.05 mag (Burstein & Heiles 1984). The above true moduli correspond to 1.31 and 1.34 Mpc, respectively. Finally, it should be noted that, for Sextans A, Visvanathan 1987 reports a true modulus of only 25.35 mag, based on single-phase 1.05-μm observations of three Cepheids.

6.9 *M 83 (= NGC 5236), IC 5152, and the Phoenix Dwarf*

Caldwell & Schommer 1988 are now monitoring the luminous southern spiral M 83 for Cepheids, while Caldwell, Schommer & Graham 1988 report actually having discovered Cepheids in two southern dwarf irregular galaxies, IC 5152 and the Phoenix dwarf, as well as in M 83. No details have yet been published.

6.10 *The Centaurus Group*

Finally, it should be reported that at least two groups are attempting to discover Cepheids in the Centaurus group. Walker (private communication) has been obtaining CTIO prime-focus CCD frames of galaxies in this cluster, but his coverage is presently insufficient to securely identify variables, and therefore no periods have yet been reported. On the other hand, Tammann et al. 1991 have a Key Project underway at ESO which has nine half-nights used in early 1991 to begin a search for Cepheids in the Centaurus group late-type galaxies NGC 5236, NGC 5264, and AM 1324-411.

7. The Hubble Constant

7.1 *Preamble*

The extragalactic distance scale, by its very nature, requires a foundation of well-calibrated nearby galaxies for the extrapolation of their own properties, or the properties of their subcomponents, out into the general field. The need to have a secure zero point in any distance indicator is self-evident. But, the need to accurately know the dispersion in that same distance indicator is perhaps less obvious. While the two components of the solution are not disconnected, the root of much of the present controversy in the extragalactic distance scale revolves as much around adopted values of the dispersion in various distance indicators as it does around their zero points. The reason for this situation is simple: small samples can in principle give rise to large systematic errors (as, for instance, when a few calibrators are drawn from a parent population with a large dispersion which can then give rise to a large deviation in the adopted mean).

A significant and continuing problem is that only a few galaxies are available to work with in any secondary calibration for use in determining the extragalactic distance scale. From the ground, seeing limitations restrict our attempts at finding Cepheids in galaxies not much beyond 5 Mpc. That volume of space contains few galaxies of utility in calibrating the Hubble constant. The tenfold increase in resolution promised by the Space Telescope, once it is retrofitted with the correcting optics, will make it practical to push the Cepheid-discovery phase out possibly as far as the Virgo cluster, thereby involving a volume of space sufficient to provide several-dozen interesting new calibrators (in the field and in groups) which are sure to advance this endeavor in a major way.

Unfortunately, it is impractical to use Cepheids to obtain distances to galaxies sufficiently far away that they directly and unequivocally measure the expansion of the universe. Even if attempts to gauge the distances to a few select galaxies in the Virgo cluster using the Space Telescope are eventually successful, deeper probes will still be required because of the local velocity perturbations from the cluster itself. Of course, a secure distance to the Virgo cluster would go a long way toward settling the continuing controversy over the distance scale, the divergence of opinion having recently shifted away from the Local Group to the Virgo cluster and beyond. But the continuing debate over the mean velocity of the Virgo cluster, its back-to-front geometry, the disposition of the spiral galaxies with respect to the elliptical component, and the effects of shot noise in any distance based on less than a dozen Cepheid-calibrated galaxies in the cluster will leave the issue open and the data amenable to a variety of interpretations.

Accordingly, one or more secondary distance indicators are needed. It is generally accepted that these secondary distance indicators are best calibrated by the Cepheid PL relation. At the moment some of the most promising indicators for confidently extending the distance scale, with the intent of determining the value of the Hubble constant and placing a credible uncertainty on that value, are: the Tully-Fisher relation, the planetary nebula luminosity function (PNLF), the surface brightness fluctuation technique, and the Faber-Jackson relation. The latter three, because they apply primarily to early-type spirals, S0 galaxies, or ellipticals, might seem to be outside of the sphere of influence offered by the Cepheid calibration, but this is not strictly true. With the availability of the Space Telescope a volume of space possibly one-thousand times larger than is regularly available to ground-based telescopes will be made accessible. This volume includes several groups of galaxies that are known to contain both early-type systems and late-type spirals. Confirmed group membership will then allow Cepheid distances to be brought into the calibration of galaxies that do not themselves contain Cepheids. An extended calibration of this nature will be useful for several applications. Specifically, M 31 (with a Cepheid-based distance and significant bulge luminosity) has been used to calibrate the PNLF (Ciardullo et al. 1989) and (along with M 81) a recent application of the Faber-Jackson relation (Dressler 1987). For the surface-brightness-fluctuation technique the apparent association of M 32 again with M 31 has been used as a calibrating path. For the future, obvious groups with a mix of Hubble types, worthy of detailed distance determinations, are the Centaurus (NGC 5128) group, and the Leonis I (M 96 = NGC 3368) group; with the NGC 2841 group, the NGC 1023 group, and the NGC 2997 group offering some interesting potential as well. The Fornax cluster in the south is expected to be almost as much of a challenge for the Space Telescope as observing Virgo in the north, although that does not dismiss it from consideration by any means; its more compact structure suggests that back-to-front uncertainties may be less than in the Virgo region, thereby commending it for detailed study.

7.2 *The Cepheid Calibration of the Hubble Constant*

A thorough discussion of the determination of the Hubble constant and its uncertainties is beyond the scope of this review, and those details will be covered in a companion paper (Freedman & Madore 1992). Here we briefly note the recent evidence for a value of $H_0 \sim 80$ km sec^{-1} Mpc^{-1} that has been rapidly accumulating.

The Tully-Fisher relation and its calibration deserve special attention for at least two compelling reasons. First of all it can be (and has been) calibrated directly by Cepheids. Second of all it has been subjected to a wide variety of consistency checks verifying the universality of the slope and confirming its very low dispersion. That is not to say that there is universal agreement over these conclusions (e.g., see Kraan-Korteweg, Cameron & Tammann 1988 and Sandage 1988b for very different views). Larger numbers of Cepheid-based calibrators are still needed to convincingly settle the dispute, but even at this early stage it can be concluded that if the Cepheid-calibrated galaxies form a fair and representative sample then the dispersion in Tully-Fisher is small (on the order of ± 0.3 mag) and the Hubble constant is large (on the order of 80 km sec^{-1} Mpc^{-1}; Freedman 1990; Freedman & Madore 1992). However, if the local sample is biased and the dispersion in Tully-Fisher is really larger than indicated, then the derived value of the Hubble constant depends critically on the disposition of the few calibrating galaxies with respect to the true mean of the relation (Sandage 1988b).

The luminosity function of planetary nebulae has been shown to be a very promising distance indicator. A recent series of papers (e.g., Jacoby 1989; Ciardullo et al. 1989; Jacoby et al. 1989; Ciardullo, Jacoby & Ford 1989; Jacoby, Ciardullo & Ford 1990; Jacoby, Walker & Ciardullo 1990) have applied the method to galaxies of different morphological types and colors, to galaxies in the same groups, and to galaxies with independent Cepheid-based distances. All tests to date indicate that the method is reliable. The value for the Hubble constant determined from the planetary nebulae when calibrated with the new Cepheid distance scale is $H_0 \sim 80$ km sec^{-1} Mpc^{-1}.

Measuring the I-band surface-brightness fluctuations seen in the Population II component of nearby galaxies, Tonry & Schneider 1988, Tonry, Ajhar & Luppino 1989, 1990, and Tonry 1991 have been measuring distances to galaxies in the Centaurus, Virgo, Fornax, and Eridanus clusters. Tonry 1991 finds excellent agreement between these estimates and those given by the Tully-Fisher relation and by the planetary nebula luminosity function; somewhat poorer agreement is found in a comparison with the $D_n - \Sigma$ relation. Using the new M 31 distance presented here a value of the Hubble constant of $H_0 \sim 80$ km sec^{-1} Mpc^{-1} again is found.

Recently, Pierce 1990 has calibrated the $D_n - \Sigma$ relation using the distances to elliptical galaxies in the Virgo and Fornax clusters, with distances measured using planetary nebulae (via Cepheids). The uncertainty in these disance measurements is of course increased as more intermediate steps are added, but Pierce derives a value of H_0 in the range 80–90 km sec^{-1} Mpc^{-1}.

The calibration of Type Ia supernovae is problematic. Their very high intrinsic luminosities make them potentially very powerful probes out to cosmologically significant distances; but the low absolute frequency of these events makes their discovery difficult and makes their calibration extremely uncertain. No Type Ia supernovae have been observed in galaxies with Cepheid determined distances. However, Fukugita & Hogan 1991 have attempted a calibration using the planetary nebula distances to galaxies in Virgo which have had supernovae observed in them. They obtain a value of the Hubble constant in the range $H_0 = 75$–100 km sec^{-1} Mpc^{-1}.

8. The Future

A high value for the Hubble constant comes with some well-known interesting consequences: the corresponding Hubble time is potentially in conflict with age estimates for globular clusters, and it is troublesome for the standard cosmological model based on cold dark matter. A value of the Hubble constant of $H_0 = 80$ km sec^{-1} Mpc^{-1} corresponds to an age of the universe of 8×10^9 years if $\Omega = 1$, and 12×10^9 if $\Omega = 0$. Recent age estimates for globular clusters have been decreasing, but they still lie in the range 13–15(± 3) Gyr (e.g., Renzini 1991; VandenBerg 1990). With large values of the Hubble constant (i.e., short time scales) cold dark-matter models for the evolution of the universe do not build up structures on large scales in time to be consistent with present-day observations (Davis et al. 1985; White et al. 1987). The Space Telescope Key Project on the extragalactic distance scale will, it is hoped, resolve these issues once and for all.

For the near term at least, progress at infrared wavelengths will most likely come from ground-based facilities. For instance, the NICMOS infrared array camera, if used to replace one of the instruments in the Space Telescope, will have the added advantage of low background emission, and it should be able to follow up on Cepheids discovered at shorter wavelengths by the Space Telescope earlier in its mission. But, as explained earlier, infrared detectors (large or small, ground based or in space) are not the way to discover Cepheids, because of their intrinsically lower amplitudes at infrared wavelengths. Discovery is a task better suited to shorter wavelength panoramic detectors. On the other hand, reddening is best handled by a large wavelength baseline extending as far into the near-infrared as is possible. Efficiently coordinating the needed short- and long-wavelength observations will be the key to future work in this field. Some additional thoughts on these matters are contained in Madore & Freedman 1985.

Any efficient ground-based attempts to detect Cepheids in external galaxies must carefully consider the question of how the periods are going to be unambiguously determined. The rapid success enjoyed by the recalibration of the extragalactic distance scale using infrared detectors was in no small part due to the fact that enormous amounts of telescope time had already been invested in discovering those Cepheids and determining their periods, to a degree that is unsurpassed in recent studies of similar galaxies. Fortunately, with space observations there is the added advantage that the exposures can be optimally scheduled to minimize aliasing effects, while constraints imposed by weather and time-scheduling difficulties are virtually eliminated.

To briefly summarize, there is presently no quick (and accurate) route to the extragalactic distance scale but, with some patience and effort, Cepheids can provide a reliable calibration. With good periods, accurate photometry at two or more wavelengths, and adequate sample sizes, it is possible to obtain Cepheid distances, corrected for reddening, that are good to better than ± 0.2 mag in true distance modulus (i.e., good to $\pm 10\%$ in distance). With *BVRI* CCD data, new distances to several galaxies beyond the Magellanic Clouds have now been obtained, corrected for the first time for the effects of interstellar reddening. Surveys are in progress to find Cepheids in additional galaxies. If the scatter in the Tully-Fisher relation is less than ± 0.45 mag, then Cepheid distances to 20 galaxies will yield an uncertainty in the zero point of only 0.1 mag which is 5% in distance. Thus, if the Space Telescope optical problems are eventually overcome, as anticipated, a Cepheid calibration of the extragalactic distance scale accurate to 10% can be achieved and the current dispute over the value of the Hubble constant resolved.

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APPENDIX A

Contrasting Aspects of the PL and PLC Relations

A point of some confusion which still needs clarification in the context of discussing Cepheids as distance indicators is in the operational difference between the PLC relation and the PL relation. As discussed in Section 2 the PLC is simply a restatement of Stefan's law, and it is therefore applicable to stars on an individual basis. The PL and PC relations, on the other hand, are statistical relations for ensembles of stars. These statistical relations are the result of constraints on Stefan's law, a strip in which stars are unstable to pulsation. In this Appendix we expand on and illustrate these statements of difference.

A PLC relation exists for all stars. That is, for all combinations of temperature and luminosity one can calculate a period, since the PLC relation embodies exactly the same (universally applicable) principles as does Stefan's law, from which it is derived. To use the PLC formalization one must of course observe a color, correct it to an intrinsic color, and then independently determine a fundamental period in order to calculate a luminosity. But while all stars may have mathematically and physically well-defined fundamental-mode periods, not all stars are unstable to these oscillations, and so their pulsational periods often are not manifest directly.

Much as nature has provided us with a useful and powerful constraint on Stefan's law, through the hydrogen-burning main sequence, nature has also provided a different constraint on Stefan's law (i.e., a constraint on the ubiquitous PLC relation) by defining a narrow zone in which stable pulsation can and does occur. This alternate "constraint" manifests itself as the Cepheid instability strip. *The Cepheid instability strip itself should not be confused with the period-luminosity-color relation.* The Cepheid instability strip defines a range of luminosities, colors, and periods over which pulsation is a stable mode for the star and is therefore an observable. But this constraint does not control the detailed correlations between period, temperature, and luminosity, nor does it control the interdependence of the observed parameters for individual stars. In much the same way, the main sequence is a strip of stable hydrogen burning, where stars are constrained to spend a large fraction of their luminous lifetimes. While the gross details of the main sequence are controlled by a single parameter (the mass of the star) the individual stars in this narrowly defined range (the main sequence) still obey the two-parameter Stefan's law.

The Cepheid constraint, in the form of an instability strip, controls the *statistical* properties of the *ensemble* of Cepheid variable stars. As such, physical laws *external to Stefan's law* are responsible for the now-famous group statistical trends of the period-luminosity and period-color relations. But these trends are incomplete (and even sometimes misleading) descriptors for individual stars: *The properties of individual stars can never be accurately defined by the constraints on the PLC relation, but only by the PLC relation itself.*

To illustrate how fundamentally different these two concepts (of the underlying equations and the overlaying constraints) are, the interested reader is referred to a companion paper on the subject (Madore & Freedman 1992). However, the flavor of the argument can be had from considering Figure 8. The solid slanting lines in each part of the figure represent the lines of constant period for the same underlying PLC relation. The heavy pair of lines cutting the constant-period lines represents two extreme examples of hypothetical boundaries to the instability strip inside of which stars are allowed to pulsate. If we now project each of these diagrams into alternate representations of the data, using the period as the abscissa, we get the corresponding period-luminosity and period-color relations as given in Figures 9 and 10. In the period-luminosity representation of the data contained in Figure 9 (where the instability strip is essentially vertical in the *C-M* diagram), there is a strong *statistical* trend of luminosity with period. However, in the corresponding period-color plot (Fig. 9), there is no correlation at all, in a statistical sense; yet for individual stars the period, luminosity, and color are in any case perfectly correlated through the period-luminosity-color relation. Conversely, in Figure 10, there is no statistical period-luminosity relation; there is a very strong period-color relation, and yet again precisely the same PLC relation is generating the data. Only the constraints have changed.

The distribution of observed data points (drawn from the underlying PLC) may be constrained both by observational selection effects and/or by different physics, without necessarily violating the two-parameter period-luminosity-color relation. Accordingly, differences in the

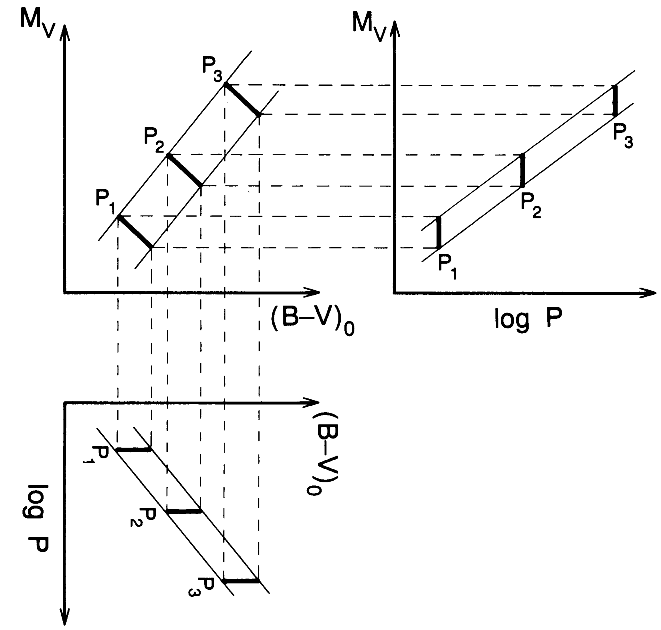

FIG. 8–Projections of the instability strip. The Cepheid instability strip in the color-magnitude plane is shown plotted in the upper left-hand panel. Lines of constant period (P_1, P_2, P_3, etc.) are shown as thick lines between the red and blue edges of the instability strip which act as constraints on the underlying PLC relation shown in its entirety in Figure 2. The broken horizontal lines show how the instability trips and the lines of constant period map over into the period-luminosity plane, while the vertical lines show the mapping down into the period-color plane (which can be seen in its normal orientation by rotating the diagram by 90 degrees counterclockwise).

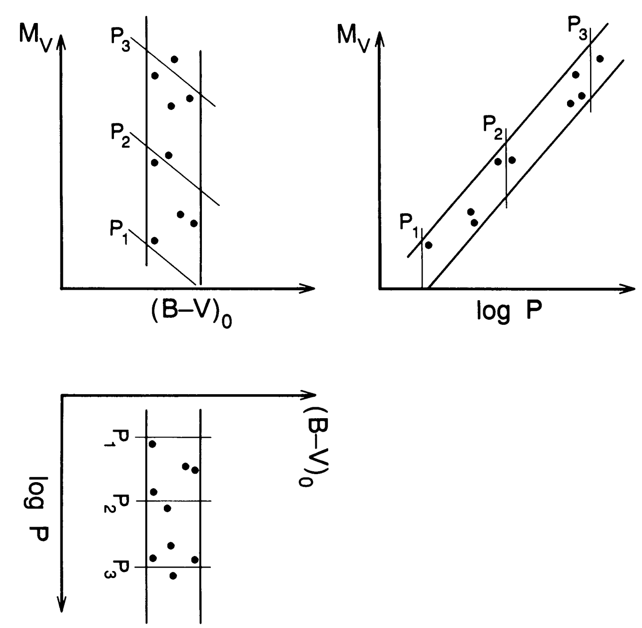

FIG. 9–The same as Figure 8 except now an alternative instability strip is imposed on the underlying "universal" PLC relation. This instability strip is vertical in the C-M diagram resulting in a definite statistical correlation of luminosity with period (right-hand panel), but showing no trend of color with period (lower panel, rotated by 90 degrees).

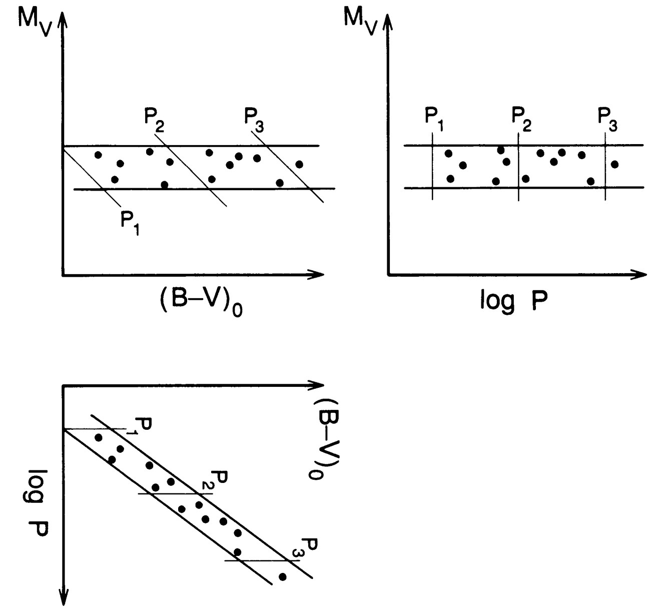

FIG. 10–The same as Figure 8 except now a different instability strip is imposed on the same underlying ("universal") PLC relation. This instability strip is horizontal in the C-M diagram resulting in a definite statistical correlation of color with period (lower panel, rotated by 90 degrees), but no trend of luminosity with period (right-hand panel). For illustrative purposes a sampling of fictitious data points are shown as solid dots.

statistical correlations (PL, PC, etc.) do not, of necessity, signal deeper-seated differences in the properties of the individual stars. On the other hand, differences in the bulk properties of an ensemble may be taken as fair warning that the detailed physics is changing, too. In fact, it is difficult to understand why and how the two properties could be decoupled.

It is of course historically very important that the instability strip is naturally oriented in such a way as to give not only a statistical PL relation but also a statistical PC relation, for it was in attempts to utilize both of these relations (and to understand the "scatter" about their means) that ultimately led to the empirical formulation of a PLC relation (Sandage 1958).

APPENDIX B

A Reddening-Free Formulation of the PL Relation

The absolute calibration of colors and/or magnitudes of Cepheids is obviously wrought with many traps and uncertainties in methodology and procedure. One very appealing, but little used, way to proceed is in an *implicit* formulation and calibration of the problem. Rather than explicitly solving for the reddening/extinction star by star, one can begin quite differently: First adopt a standard reddening line for the color system being employed, then form linear combinations of the magnitudes and colors so that the numerical quantity formed cancels the reddening (star by star) without ever explicitly determining it. Then this formulation can be used both for the calibration objects and for the target Cepheids. Forming this reddening-free magnitude directly parallels the more common reddening-free application to colors, such as $Q = (U-B) - X\,(B-V)$ as defined by Johnson 1963. An example follows.

Suppose that the observed magnitude of the Cepheid is V and its apparent color is $(B-V)$. As in the standard procedure, where individual reddening corrections are made, we assume that the ratio of total-to-selective absorption is known from independent determinations and that its value is $R = A_V/E(B-V)$. Then it naturally follows that a reddening-free quantity W (called the Wesenheit function by Madore 1982) can be formed where $W \equiv V - R(B-V)$. A simple expansion of the relevant terms shows that for small amounts of reddening the numerical value of W is independent of extinction and equal to the value that would be calculated if the intrinsic magnitudes and colors were known (which they now need not be). That is, by definition,

$$W = V_0 + A_V - R\,[(B-V)_0 + E(B-V)]$$

$$W = V_0 + R\,E(B-V) - R\,(B-V)_0 - R\,E(B-V)$$

$$W = V_0 - R\,(B-V)_0 \equiv W_0\ .$$

There exists some confusion in the literature as to the real motivation and physical justification for creating W for Cepheids. W in fact is a reddening-free quantity for stars in general, not exclusively for Cepheids. It is strictly defined by the properties of the interstellar medium, not by any properties of the stars to which it is to be applied. One is not at liberty, for example, to change the value of R from its normal value of 3.3, unless that correction derives from an independent study of the extinction law itself. Accordingly, with the exception of the small known dependence of R on $(B-V)$ and $E(B-V)$ itself (manifest as nonlinear curvature terms at large values of these quantities; true for all applications of reddening corrections, not just W) one cannot manipulate the form of W at all.

The reason for some of the confusion dates back to a similar realization of the PLC introduced by van den Bergh 1975, also called W. In that first appearance of a linear combination of V and $(B-V)$ the complete reddening independence of the function was not fully implemented, as will become clear now. In the following discussion we will use W_{BFM} and W_{VDB} to distinguish between the differing definitions used by Madore 1982 and van den Bergh 1975, respectively.

For Cepheids one can begin to see what W_{BFM} is, both in terms of its parent relationship, the period-luminosity-color relation, and in terms of its own definition. Adopting a linearized form of the period-luminosity-color relation we have

$$V = V_0 + A_V$$

$$V = M_V + \mathrm{mod}_0 + A_V$$

$$V = \alpha \log P + \beta\,(B-V)_0 + \gamma + \mathrm{mod}_0 + A_V$$

where mod_0 is the true distance modulus. By definition,

$$W_{\mathrm{BFM}} \equiv V - R\,(B-V)$$

so, by substituting the period-luminosity-color relation into the definition of W_{BFM} we get

$$W_{\mathrm{BFM}} = \alpha \log P + \beta\,(B-V)_0 + \gamma + \mathrm{mod}_0 + A_V - R\,(B-V)$$

or, expanding the reddening terms, regrouping, and canceling, we get

$$W_{\mathrm{BFM}} = \alpha \log P + [\beta - R]\,(B-V)_0 + \gamma + \mathrm{mod}_0$$

since, by definition, A_V equals $R\,E(B-V)$. For Cepheids, the W_{BFM} can be reformulated as a period-luminosity-color relation. Here it must be emphasized that the zero point γ and the slope of the period dependence α in the W_{BFM} formulation of the period-luminosity-color relation are identical to their counterparts in the V formulation; only the coefficient multiplying the intrinsic color changes from β to $[\beta - R]$.

Here now is the point of confusion. van den Bergh 1975 noted the numerical similarity of $R = 3.2$ to the then-espoused value of $\beta = 2.7$, and at that point he chose his definition of $W_{\mathrm{VDB}} \equiv V - \beta(B-V)$ which is quite a different approach. W_{BFM} has the advantage of being a strictly defined, reddening-free magnitude adopting a well-determined quantity R, which does not attempt to anticipate the a priori unknown value of β. Furthermore, residual scatter in the W_{VDB}, $\log P$ relation is also thereby distinctly of interest in its own right. Unlike scatter in the observed PL or PC relations, scatter in the W_{BFM}, $\log P$ relation cannot be due to reddening effects, be they differential or total. One is not at liberty to adjust the coefficient R for various samples of Cepheids (to minimize the scatter in the residual, for example) unless there is *independent* evidence that the ratio of total-to-selective absorption is different for that particular galaxy. The remaining scatter in a W_{BFM}, $\log P$ plot will be due to a combination of intrinsic (color?) correlations and photometric errors. As is so often the case it will be an understanding of the photometry and its quality that will ultimately limit our understanding of the intrinsic interrelations.

In order to illuminate the differences between the W_{BFM} and W_{VDB}, Figure 11 portrays the mapping of the PLC from its projection onto the traditional color-magnitude diagram (as an instability strip) and then into the three "period-luminosity" planes.

The upper left-hand panel shows a portion of the Cepheid instability strip in a color-magnitude diagram. The upwardly slanting solid lines give the red and blue edges of the strip, while the heavy downward-slanting lines labeled P_1 and P_2 are representative lines of constant

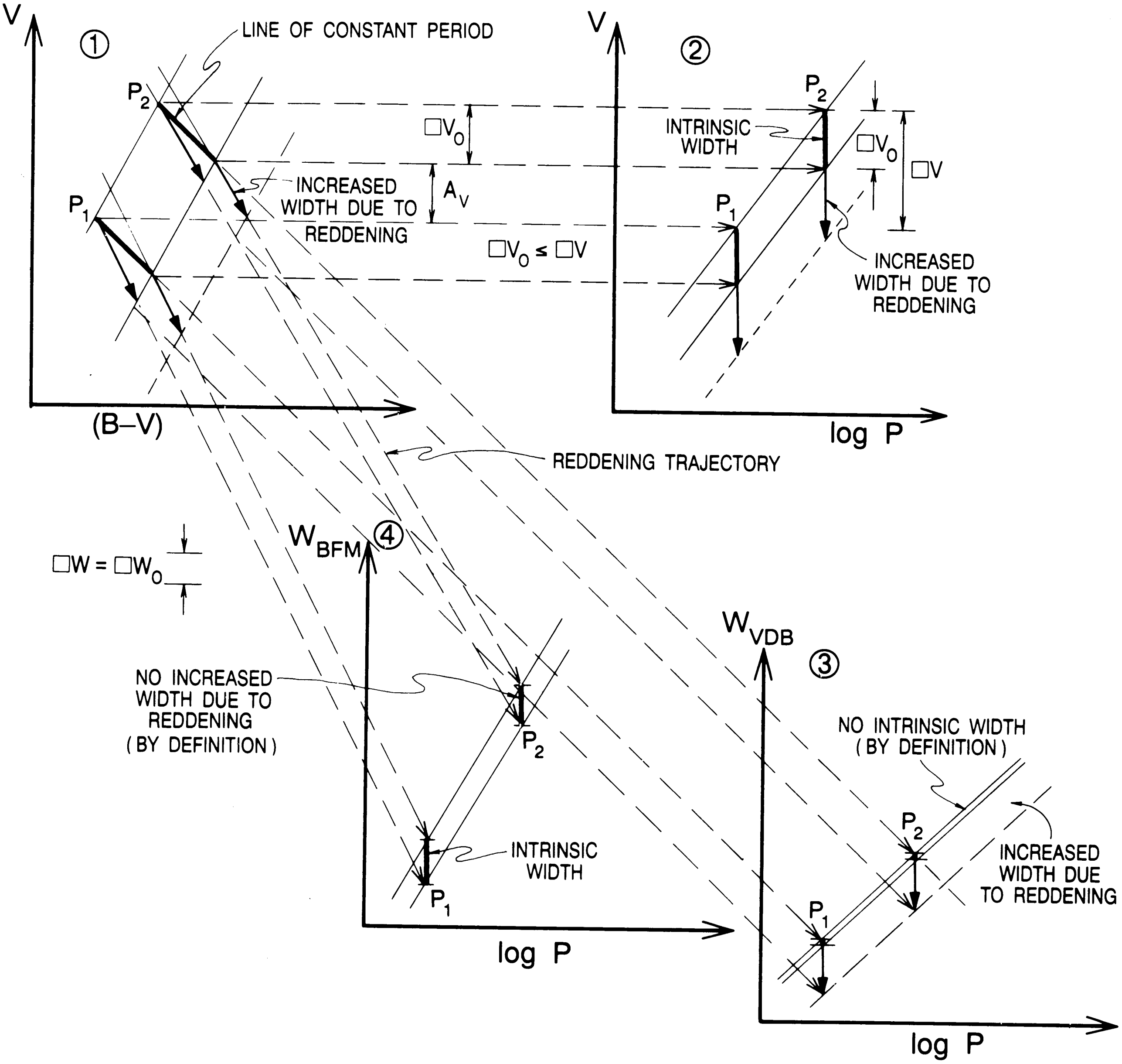

FIG. 11–Projecting the observed instability strip, in the presence of reddening. (1) In the upper-left panel the instability strip superimposed on the PLC is shown projected into the color-magnitude plane. Lines of constant period (P_1 and P_2) are shown as thick solid lines slanting down to the right. The blue and red edges of the instability strip are shown as thinner solid lines sloping down to the left. Arrows indicate the magnitude and direction of reddening which acts to increase the apparent width of the distribution by systematically scattering points to redder colors and to fainter magnitudes. A dashed line parallel to but fainter than the red edge of the instability strip illustrates the bounds of this effect. (2) The upper-right panel is constructed from the first panel by projecting the instability strip and the reddening vectors into the period-luminosity plane. There is a systematic increase in the width of the apparent PL relation due to extinction, as in the *C-M* diagram. (3) If the slopes of the lines of constant period are known a priori then the projection of the instability strip into the W_{VDB},log *P* plane can be performed. By definition the intrinsic width goes to zero in this projection, but because the slope of the lines of constant period are not exactly parallel to the reddening trajectory, extinction does project into this plane and will widen the relation and systematically shift the ridge line. (4) Since reddening trajectories are, in general, already determined a priori it is possible to project the instability strip into the W_{BFM},log *P* plane where (by definition) extinction effects are eliminated. In this case, however, the intrinsic width does still contribute to the dispersion, but the disposition of the individual stars within this strip is not affected by reddening.

period. Arrows indicate the slope of a reddening/extinction line. The upwardly sloping broken line indicates the apparent red edge of the instability strip as defined by stars reddened away from the intrinsic line.

The horizontal broken lines leading from Panel 1 to Panel 2 show the first mapping of the instability strip into the *V*, log *P* plane. The sloping lines of constant period in Panel 1 now become vertical, with downward extensions due to reddening increasing the apparent dispersion at fixed period.

Panel 3 to the lower right shows the effect of forming W_{VDB} given a priori knowledge of the value of β, the slope of the lines of constant period in the PLC. As can be seen the projected width in the W_{VDB}, $\log P$ plane is nonzero in the presence of reddening. While the intrinsic width of the instability strip does project to zero (by definition, once β is known from independent sources) there is residual widening due to reddening.

Panel 4 shows the effect of forming W_{BFM}. Since the slope of a reddening line is well-known from independent studies, differential reddening has no effect on the width at constant period. The only factor contributing to the width in W_{BFM} is the intrinsic width of the PLC (and, as mentioned earlier, photometric errors in the magnitudes and colors, which broaden all of the above relationships and projections).

Given Figure 11 it is perhaps worth noting that in the presence of differential reddening it is clearly inappropriate to determine the value of β by minimization of residuals, as would be the case in an unrestricted application of a multilinear regression fit to PLC data, for instance. For the example at hand, one can see immediately by inspection that a slope somewhere between the true value of β and the reddening slope R would give the minimum residuals plotted as a function of period. The numerical value of that parameter for any given set of Cepheids is, however, of no universal significance.

APPENDIX C

Comments on Reddening Determinations

It should, by now, go without saying that the removal of reddening is essential to any study that aspires to a calibration of the intrinsic PLC relation for Cepheids. What is all too often forgotten in the process is that the systematics of this first step will not only effect any subsequent "intrinsic" calibration, but the dereddening steps may also well predetermine it. Some authors have claimed that color-color plots for Cepheids can be accurately calibrated and confidently used to determine reddenings to Galactic and extragalactic Cepheids. We examine in some detail now one such case, that of Feast and his collaborators (hereafter referred to as the South African Group). Their calibration is used widely, and its implications are important, so below we discuss it here in some detail.

The South African calibration of the reddening determination begins with Galactic Cepheids (Dean, Warren & Cousins 1978). It has then been applied to Magellanic Cloud Cepheids by (Martin, Warren & Feast 1979) where they derive a PLC relation. And then they have further generalized the dereddening procedure for all extragalactic Cepheids for arbitrary metal abundance (Caldwell & Coulson 1985). Beginning with the specific case of the Galactic calibration the South African Group accept at least five crucial assumptions. Assumption (1): There exists a relation between $(B-V)_0$ and $(V-I)_0$ that is *dispersionless*. Assumption (2): A shape for this relation is obtained by following single stars through their pulsation cycle. Assumption (3): Since no single star cycles through all of the intrinsic colors expected for Cepheids, in practice a variety of trajectories need to be superposed by moving the narrow color-color loops formed by individual stars (of differing periods) back along reddening lines until they overlap and form an *empirically nonlinear* but continuous (*imposed minimum dispersion*) sequence. Assumption (4): A zero point can be fixed by placing stars of independently known reddening (e.g., Galactic cluster Cepheids and very nearby field objects) into this relation. And, finally, Assumption (5): Metallicity corrections can be found that consist of simple zero-point shifts on the colors as deduced by the authors from published theory. This formalism and calibration has been adopted in one form or another by the South African Group for all of their subsequent analyses of Magellanic Cloud data and their calibration of the PLC relation as they have applied it to the extragalactic distance scale.

We now illustrate the dangers inherent in these assumptions. We start with the generally accepted, linear form of the PLC relation. It is written twice below: once with the color $(B-V)_0$, and then again with the color $(V-I)_0$ as the temperature indicator:

$$M_V = \alpha \log P + \beta(B-V)_0 + \gamma \quad \ldots . \quad \text{(C1)}$$

$$M_V = \delta \log P + \epsilon(V-I)_0 + \eta \quad \ldots . \quad \text{(C2)}$$

We then equate these two expressions and regroup like terms, eliminating M_V, thereby leading to the following equation, applicable to the color-color plane:

$$(B-V)_0 = \left[\frac{\delta-\alpha}{\beta}\right] \log P + \left[\frac{\epsilon}{\beta}\right] (V-I)_0 + \left[\frac{\eta-\gamma}{\beta}\right] \quad \ldots . \quad \text{(C3)}$$

By claiming that for Cepheids there is a unique and dispersionless relation between any two intrinsic colors (despite empirical evidence to the contrary, Cousins 1978a,b) the South African Group is explicitly ignoring the period dependence in equation (C3). Their tacit assumption then is that both colors are insensitive to gravity effects (but again see Cousins), and therefore the combination of the equations given above is forced by them to be degenerate in period (i.e., $\alpha = \delta$). The consequences of such an assumption are twofold: not only are the derived reddenings wrong at the level that surface-gravity difference effects from star to star are manifest in the time-averaged colors, but also the inferred intrinsic colors are deceptive. If the South African calibration tracks the ridge line of the true instability strip in the color-color plane then the following statements can be made: obviously the stars residing on the constant period lines intrinsically to the blue of the mean will systematically have their reddenings underestimated by this method (while stars whose intrinsic colors have them on the red side of the instability strip will have their reddenings overestimated). But moreover, as Figure 12 illustrates, should the relative slope of the constant-period line be less than the slope of the reddening line in the color-color plot, then the inferred intrinsic position of the stars within the instability strip will be totally inverted from their true mapping!

While ignoring the period dependence in a second PLC relation may be a convenient and simplifying first approximation to a difficult problem, continuing on to add higher-order nonlinear (curvature) terms, and also use theoretical displacements of the origin (attributed to metallicity effects) seems to be, at the very least, somewhat premature. Furthermore, there is an additional practical problem because the reddening trajectory is *closely parallel to the adopted ridge-line* $(B-V),(V-I)$ *relation*, especially at short periods. This unfortunate coincidence makes their reddening solutions extremely vulnerable to random photometric errors propagating into very large reddening errors.

In fact, degeneracy between many of the important physical parameters controlling the observed properties of Cepheids is a common theme. Decreasing the temperature of a Cepheid makes it redder and fainter; theory predicts that increasing the metallicity should also make a Cepheid redder and fainter; and increased obscuration is known to make any star redder and fainter. Without a doubt some bandpass combinations cannot distinguish between these effects. This is both a blessing and a curse. If they are indistinguishable, then "correcting" for one automatically corrects for the other. But, if the aim is to explicitly identify and evaluate each of these effects, then "correcting" for one may very well obliterate the effect of another. Caveat emptor.

Continuing, the validity of Assumption (2) is questionable. It takes the instantaneous, time-dependent behavior (of luminosity and color) of an *individual* Cepheid of fixed mass, period, and dynamically evolving surface gravity (as it cycles through radius, luminosity, and temperature with phase) and explicitly equates them with the time-averaged properties of *an ensemble of* stars, each having different masses, different mean surface gravities, temperatures, and periods. However, there is no a priori justification for assuming that the behavior of a single star during its cycle is anything more than qualitatively indicative of the way in which the (time-averaged properties of the) instability strip are mapped in luminosity and color when changing from star to star. For example, no one seriously considers determining the slope of the color term β in the PLC relation by looking at how an individual star changes its luminosity with color (as a function of phase). Likewise, it would seem inadvisable to use the color-color trajectory of an individual Cepheid to calibrate the complex mapping of a variety of stars and their PLC relations into the color-color plane.

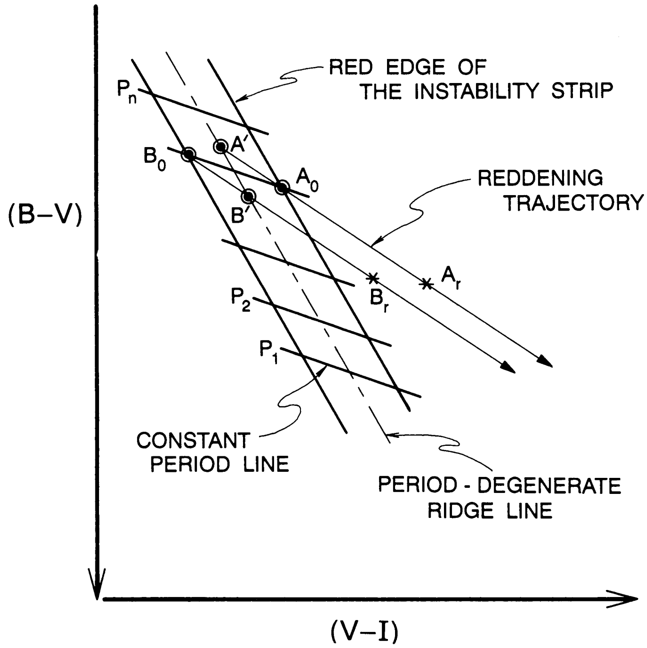

FIG. 12–Projection of the PLC into the $(B-V),(V-I)$ color-color plane. As shown by equation (C3) the projection of the Cepheid PLC relation into the color-color plane results in a finite region crossed by lines of constant period and bounded by the blue and red edges of the instability strip. Representative reddening trajectories are shown passing through the ends of a typical constant-period line such that their projection back onto the central ridge line illustrates the reddening errors made if one erroneously adopts a dispersionless relation between these two colors. Notice how a Cepheid that intrinsically resides on one (red/blue) edge of the instability strip gets forced to the opposite (blue/red) edge by this incorrect procedure.

Despite the small formal uncertainties in the resulting PLC fit determined by this methodology (see Caldwell & Laney 1991 for a recent example) it should be emphasized that almost all methods which "correct" for differential absorption within the PL relation will give rise to a PLC relation of smaller (apparently intrinsic) scatter. Furthermore, the determination of the color term β in equation (C1) depends sensitively and explicitly on there being accurate individual reddenings for each of the stars entering the calibration. For the reasons outlined above, we do not believe that an accurate value of β has yet been determined precisely because the systematics of determining reddenings have yet to be fully appreciated or adequately addressed.

Because of its inherent complexity we are not in a position to solve for individual Cepheid reddenings at this time; however, the impact on the distance scale can be minimized by accepting that reddening is a systematic problem and realizing that it can be dealt with effectively by at least three available means, which involve (1) moving as far to the infrared as is practical, so as to reduce the extinction problem to the level of other systematic and random errors; (2) combining multiwavelength (visual to near-infrared) data for significant numbers of Cepheids in a given galaxy, and determining the ensemble-averaged extinction (and true modulus) using an independently calibrated wavelength-dependent extinction law; or (3) using reddening-free formulations which are designed to cancel out any and all extinction on a star-by-star basis, without ever attempting to determine the amount explicity. Each of these alternatives are dealt with in some detail in this review and its other appendices.

It is clear, however, that if an *explicit* solution for individual reddenings is ever to be found, it will most likely come through an investigation of extragalactic, not Galactic, Cepheids. The LMC has acted as the focal point for calibration purposes for some years now, primarily because it has a large population of Cepheids with known periods and because it is sufficiently close that accurate photometry can be obtained for its stars with relative ease. However, the LMC Cepheids do individually suffer from some degree of extinction internal to the LMC itself and also varying amounts of foreground Galactic extinction. The SMC has a similarly large population of Cepheids (although it is somewhat further away than the LMC). But while it is generally accepted that in comparison to the LMC the extinction is less, internal to and in front of the SMC, it is now known (primarily from studies of the Cepheids! See Welch et al. 1987.) that the back-to-front geometry of the SMC is such that appreciable differential modulus residuals are affecting the magnitudes. Any empirical correlations between reddening-corrected colors and extinction-corrected magnitudes will have this (geometrically induced) noise to contend with.

We suggest that the best place for future work on the intrinsic calibration problem is not the Magellanic Cloud system but the Local Group galaxy IC 1613. The foreground reddening to IC 1613 is, by all estimates, very low and probably quite uniform, considering the high Galactic latitude and small angular size of this galaxy as compared to either of the Magellanic Clouds. In a like way to the SMC, the extinction internal to IC 1613 also appears to be quite small. With the surveys of Sandage 1971 and Carlson & Sandage 1990 now complete, there is also a sizable population of Cepheids in IC 1613 to work with. Of course, crowding is more of a problem for photometry of individual Cepheids in IC 1613 as compared to the LMC, for example; but considering the success had with photoelectric photometers using aperture sizes in excess of 10 arc sec when working on LMC/SMC Cepheids, it is realistic to expect that point-spread-function fitting routines (effectively working on one-arc-sec scales) will be able to do at least as well. And the quality of *BVRI* light curves obtained for Cepheids in NGC 300 (Freedman et al. 1991), at a distance about three times further than IC 1613, seems to bear out this expectation.

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