MaxCut and Statistical Physics

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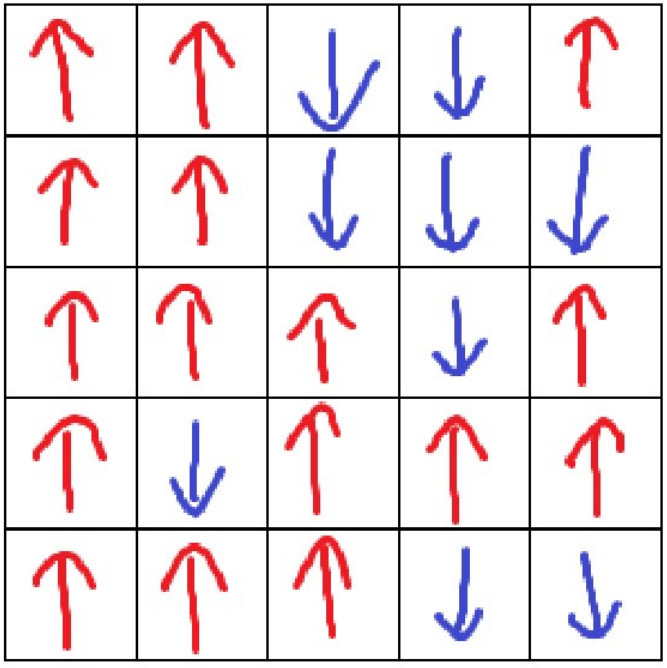
Ising Model

The Ising model is used to model the spin configuration in regular crystals, which gives rise to the magnetic properties of a material.



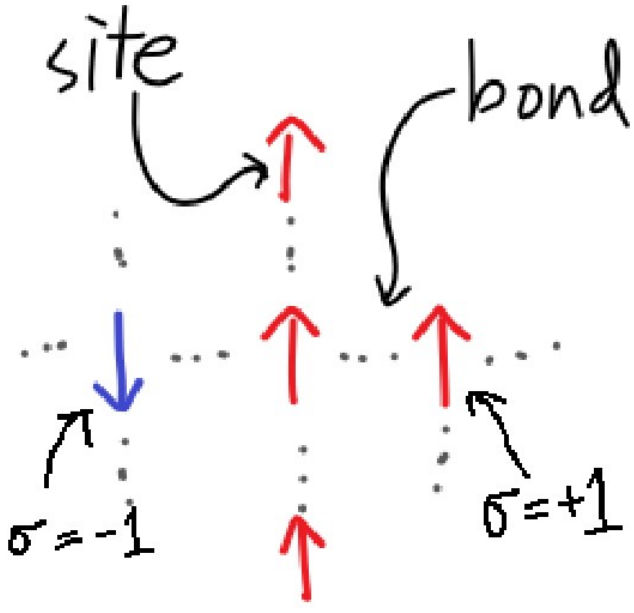


Figure 1: Ising Model Illustraction [1]

Ising Model [2]

The Ising model is used to model the spin configuration in regular crystals, which gives rise to the magnetic properties of a material. The spin configuration of n particles can be defined as:

$$\{\sigma\} = \{\sigma_1, ..., \sigma_n\}$$

Where $\sigma \in \{-1, 1\}$. The energy of the configuration can be defined as follows, where J_{ij} is the coupling constant between spin i and j. H is the constant for the interaction between the spin and magnetic field, and h_i is that with local fields.

$$\mathcal{H}(\{\sigma_i\}) = -\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i H\sigma_i - \sum_i h_i\sigma_i$$

Ising Model Implementation [2]

Abstract the problem with a mathematical graph,
$R = (V \cup \{s, t\}, E \cup F)$ where:

- V: each spin is mapped to a vertex of the graph, $V = \{1, 2, ..., n\}$
- E: to each two spins i and j that interact, associate an edge ij in E of weight $w(ij) = J_{ij}$
- F: to each spin i that $H + h_i > 0$, associate an edge si in F of weight $w(ij) = H + h_i > 0$
- F: to each spin i that $H + h_i < 0$, associate an edge si in F of weight $w(ij) = H + h_i < 0$

Minimize the following Hamiltonian:

$$\mathcal{H}(\{\sigma_i\}) = -\sum_{ij \in E} w(ij)\sigma_i\sigma_j - \sum_{si \in F} w(si)\sigma_i - \sum_{si \in F} -w(it)\sigma_i$$

Ising Model Applications

- The Spin Glass
  - a magnetic state in which the spins are aligned to a single direction at "freezing temperature." [3]
  - Ferromagnetic [2]
  - Antiferromagnetic [4]

Reference

[1] *The ising model*, [Online]. Available: https://stanford.edu/~jeffjar/statmech/intro4.html.

[2] J. A. d'Auriac, M. Preissmann, and A. Sebö, "Optimal cuts in graphs and statistical mechanics," *Mathematical and Computer Modelling*, vol. 26, no. 8-10, pp. 1–11, 1997.

[3] J. A. Mydosh, *Spin glasses: an experimental introduction*. CRC Press, 1993.

[4] A. Coja-Oghlan, P. Loick, B. F. Mezei, and G. B. Sorkin, "The ising antiferromagnet and max cut on random regular graphs," *SIAM Journal on Discrete Mathematics*, vol. 36, no. 2, pp. 1306–1342, 2022.