TP3 : Harris Corners and Matching

The goal of this lab is to implement an harris corner detector and match the harris corners of two images using the sum of square differences.

1 A reformulation of the harris detector

— Given some σ, we start by defining w_σ the gaussian with standard deviation σ with truncated support outside the square of size 6σ :

$$w_\sigma(x, y) = \frac{1}{Z} \exp\left(-\frac{(x^2 + y^2)}{2\sigma^2}\right) \text{ if } max(|x|, |y|) < 3\sigma, 0 \text{ otherwise} \tag{1}$$

and Z a normalization coefficient such that $\sum_{x,y} w_\sigma(x, y) = 1$

— Using such a gaussian with we compute a smoothed image I_{smooth} that corresponds to a smoothed version of the image I using a Gaussian filter with standard deviation $\sigma_1 = 2$

$$I_{smooth} = I * w_{\sigma_1}$$

— We compute I_x and I_y the gradient of the smoothed image I_{smooth}

— Given an image, we compute the harris score for each pixel (i, j) of the image a matrix M_{ij} using

$$M_{ij} = \sum_{x,y} w_{\sigma_2}(x - i, y - j) \begin{bmatrix} I_x(x, y)^2 & I_x(x, y)I_y(x, y) \\ I_x(x, y)I_y(x, y) & I_y(x, y)^2 \end{bmatrix}$$

With the parameter $\sigma_2 = 3$ controlling the size of the window $(6\sigma_2)$ used as context when computing the harris score. We introduce four images m_{11}, m_{12}, m_{21} and m_{22} such that :

$$M_{ij} = \begin{bmatrix} m_{11}(i, j) & m_{12}(i, j) \\ m_{21}(i, j) & m_{22}(i, j) \end{bmatrix}$$

We get

$$m_{11} = \sum_{x,y} w_{\sigma_2}(x - i, y - j)I_x(x, y)^2$$

m_{11} is the convolution of the image $I_x(x, y)^2$ by a gaussian kernel i.e

$$m_{11} = w_{\sigma_2} * I_x^2$$

we also have $m_{12} = m_{21} = w_{\sigma_2} * (I_x I_y)$ and $m_{22} = w_{\sigma_2} * I_y^2$. The Harris score writes

$$R(i, j) = det(M_{ij}) - k(trace(M_{ij}))^2$$

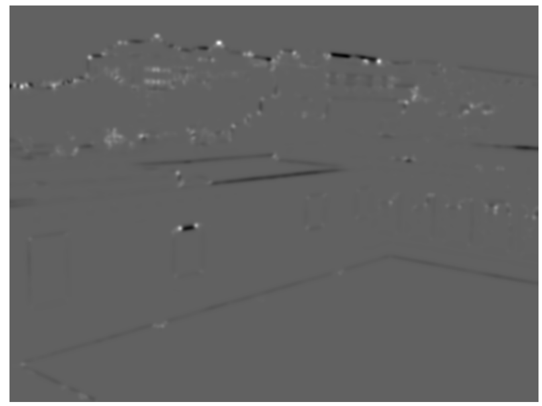
with $k = 0.06$. We have $det(M_{ij}) = m_{11}(i, j)m_{22}(i, j) - m_{12}(i, j)m_{21}(i, j)$ and $trace(M_{ij}) = m_{11}(i, j) + m_{22}(i, j)$ therefore using element-wise operators on arrays we get

$$R = m_{11}m_{22} - m_{12}m_{21} - k(m_{11} + m_{22})^2$$

2 Exercise

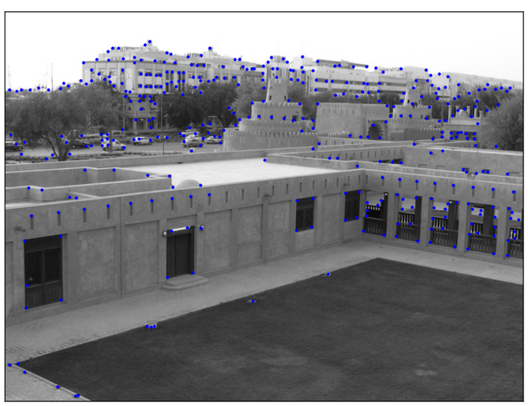
1. Implement a function `smoothedGradient` that computes the smoothed gradients I_x and I_y given an image I and a standard deviation σ
2. Implement a function `HarrisScore` that computes the harris score image R given an image I, σ_1, σ_2 and k

image I

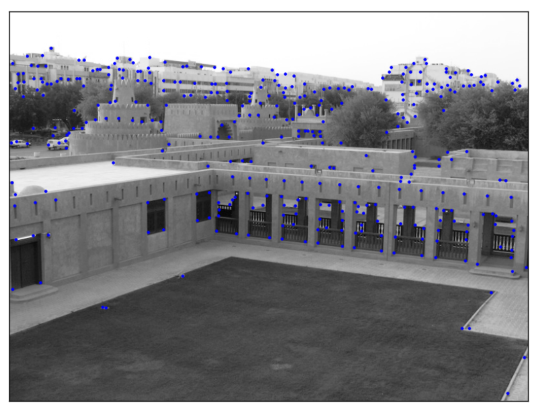


R

3. Implement a function `HarrisCorners` that calls `HarrisScore` to get the Harris score image R , find local maximums that are above 0.005 times the global maximum of R and that are separated by at least 2 pixels using `skimage.feature.peak.peak_local_max` and return the list of peaks. If you display the list of peaks you should get :



corners image 1



corners image 2

4. Implement a function `SSDTable` that takes two array of patches respectively of size $M_1 \times N \times N$ and $M_2 \times N \times N$ and computes a matrix D with D_{ij} the sum of square differences between the intensities of patch i in the first list of patches and j in the second list of patches
5. Implement a function `NCCTable` that takes two array of patches respectively of size $M_1 \times N \times N$ and $M_2 \times N \times N$ and computes a matrix D with D_{ij} equal to one minus the cross correlation between patch i in the first list of patches and j in the second list of patches. In order to avoid removing the mean and divising by the norm of the match for each pair of patches, compute first centered and normalized patches and then you only need to compute scalar product between patches

6. Using the function `extractMatches` and `displayMatches2` display the matches between the two images using either the score given by `SSDTable` or `NCCTable`. You should get something similar to the image next page