Multirate Signal Processing

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Multirate Signal Processing

Multirate systems

- ❖ Input rate ➔ Multiple data processing rate ➔ output rate
- ❖ Sampling (or data) rate conversion required
  - Down sampling → Decimation
  - Expansion, up sampling → Interpolation
  - Rate conversion (by an arbitrary ratio)
- ❖ Advantages
  - Better performance: expansion
  - Less data : decimation
  - can accurately support all the devices with different rate requirements
- ❖ Filter bank: Subband processing, data compression

Multirate Signal Processing

Decimation and Expansion

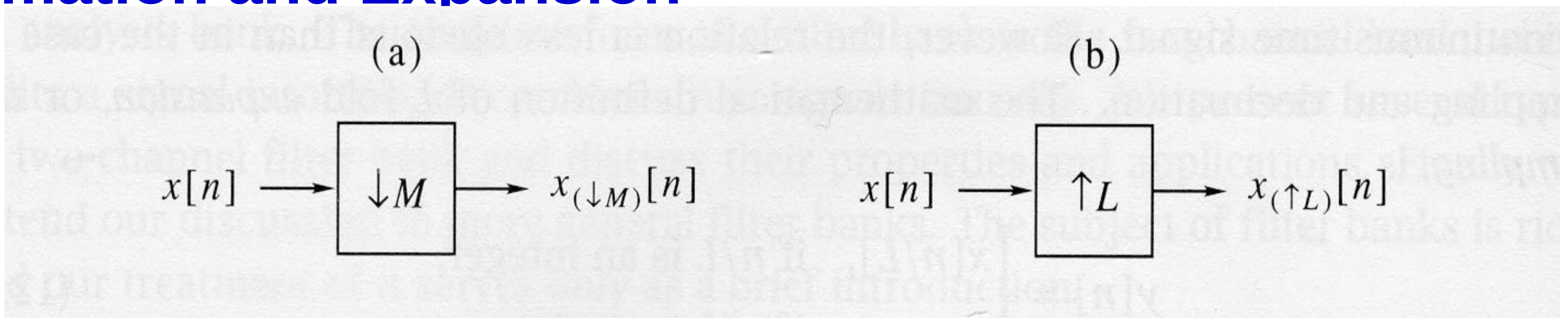


Figure 12.3 Block-diagram notations for (a) M-fold decimation and (b) L-fold expansion.

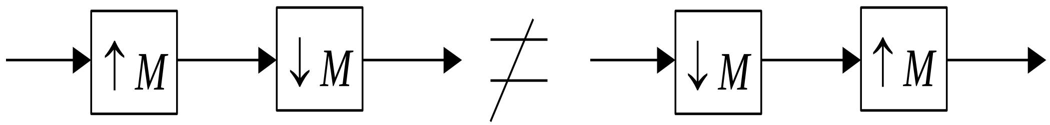
- **Decimation**

$$y(n) = x_D(n) = x_{(\downarrow M)}(n) = x(nM)$$

- **Expansion**

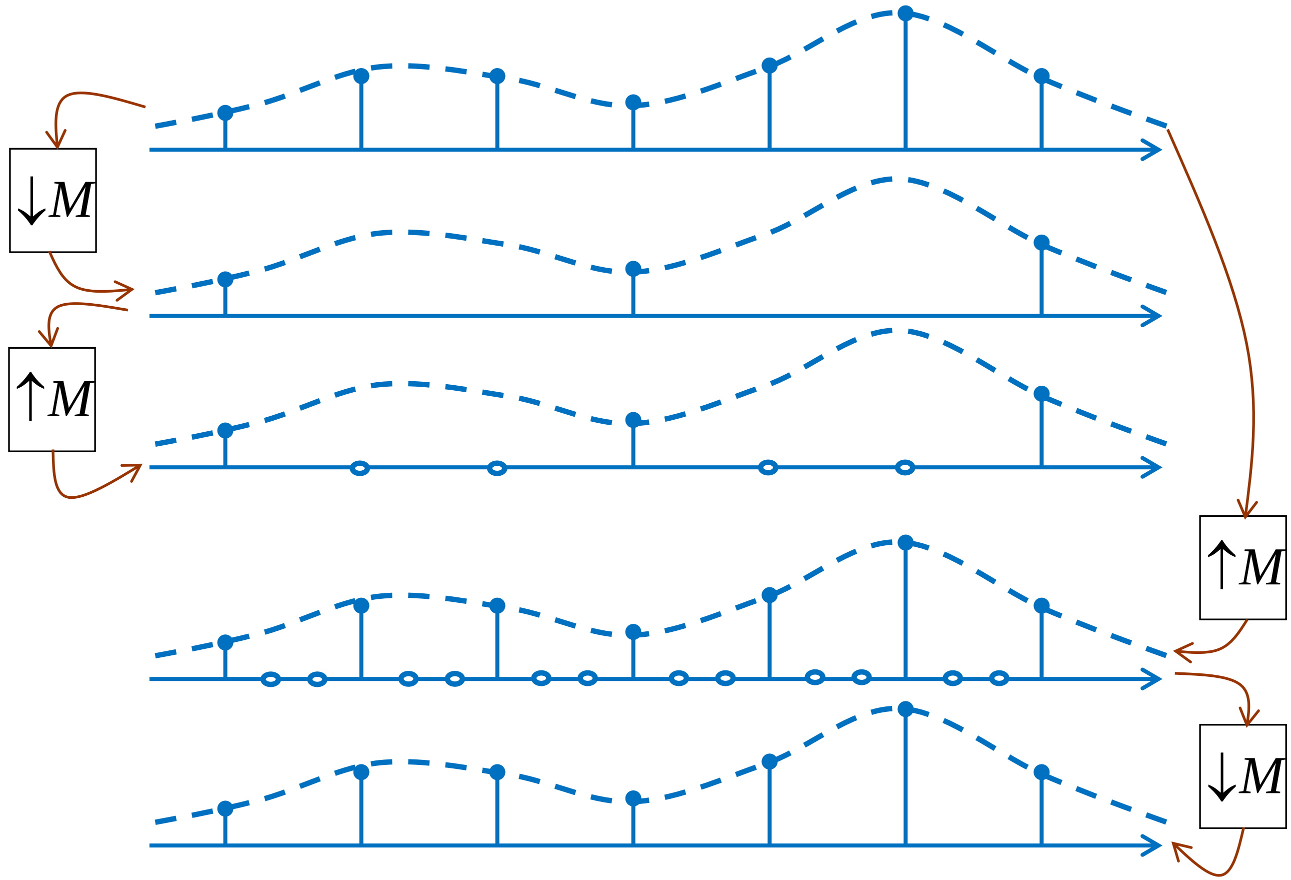
$$y(n) = x_E(n) = x_{(\uparrow L)}(n) = \begin{cases} x(n/L), & n = kL \ (k \text{ integer}) \\ 0, & \text{otherwise} \end{cases}$$

- **Decimation and expansion are both time variant operations**



Decimation and Expansion

- **Example: L=M=3**



Decimation and Expansion

Theorem 12.1 : If M and L are relatively prime, then

$$\longrightarrow \boxed{\uparrow L} \longrightarrow \boxed{\downarrow M} \longrightarrow \quad = \quad \longrightarrow \boxed{\downarrow M} \longrightarrow \boxed{\uparrow L} \longrightarrow$$

❖ **Proof**

$$\{x_{(\downarrow M)}\}_{(\uparrow L)}(n) = \begin{cases} x(nM/L), & nM = kL \\ 0 \quad , & otherwise \end{cases}$$

$$\{x_{(\uparrow L)}\}_{(\downarrow M)}(n) = \begin{cases} x(nM/L), & n = mL \\ 0 \quad , & otherwise \end{cases}$$

Since M and L are coprime, $nM = kL$ only when $n = jL$, which implies that the above two expressions are identical.

Hence,

$$\{x_{(\downarrow M)}\}_{(\uparrow L)}(n) = \{x_{(\uparrow L)}\}_{(\downarrow M)}(n)$$

Decimation and Expansion

Transform domain analyses of decimation and expansion

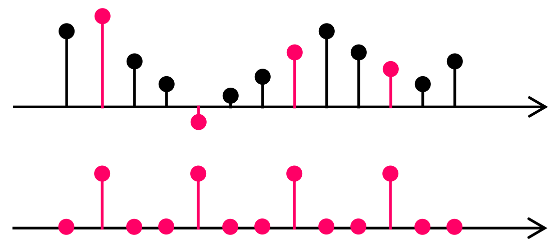
❖ Decimation

- **Let's introduce the comb sequence** IDFT of an M-long sequence of 1

$$c_M(n) = \sum_{-\infty}^{\infty} \delta(n - kM) = \frac{1}{M}\sum_{m=0}^{M-1} W_M^{mn}, \quad where \quad W_M = e^{j2\pi/M}$$

Then,

a sequence of 1



$$x_D(n) = x(nM)c_M(nM)$$

$$X_D(z) = \sum_{n=-\infty}^{\infty} x(nM)c_M(nM)z^{-n} = \sum_{n=-\infty}^{\infty} x(n)c_M(n)z^{-n/M}$$

$$= \frac{1}{M}\sum_{m=0}^{M-1}\sum_{n=-\infty}^{\infty} x(n)\underbrace{W_M^{mn} z^{-n/M}}_{(W_M^{-m} z^{1/M})^{-n}} = \frac{1}{M}\sum_{m=0}^{M-1} X(\underbrace{z^{1/M} W_M^{-m}}_{e^{j\theta/M}e^{-j2\pi m/M}})$$

$$\Leftrightarrow \; X_D(\Theta) = \frac{1}{M}\sum_{m=0}^{M-1} X\left(\frac{\Theta - 2\pi m}{M}\right) = \frac{1}{M}\sum_{m=0}^{M-1} X\left(\frac{\Theta}{M} - \frac{2\pi m}{M}\right)$$

Decimation and Expansion

❖ Decimation

- Aliasing error will occur if sampling rate is lower than the Nyquist rate.
- Anti-aliasing filter is required.
- Input signal or the anti-aliasing filter output should be band-limited to $\theta \in [-\pi/M,\ \pi/M]$

$$M = 3$$

$$X(\theta)$$

$$X\left(\frac{\theta - 2\pi m}{M}\right) = X\left(\frac{\theta}{M} - \frac{2\pi m}{M}\right)$$

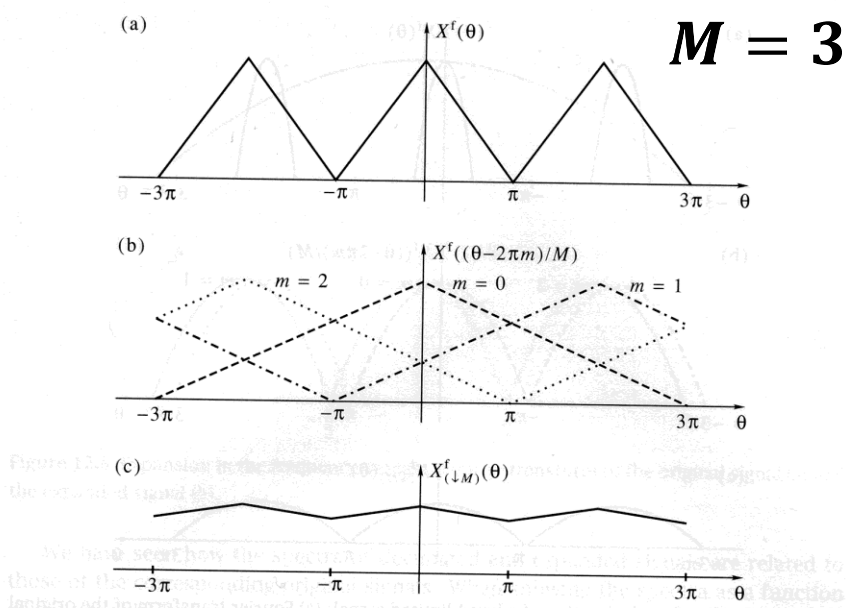


Figure 12.5 Aliasing caused by decimation: (a) Fourier transform of the original signal; (b) shifted and frequency-scaled replicas of part a; (c) Fourier transform of the decimated signal.

$$X\left(\frac{\theta - 2\pi \cdot m + 2\pi M}{M}\right) = X\left(\frac{\theta - 2\pi \cdot m}{M} + 2\pi\right)$$

➔ $Period\ \ in\ \ \theta = 2\pi M = 6\pi$

Decimation and Expansion

❖ Decimation

- **No aliasing case**

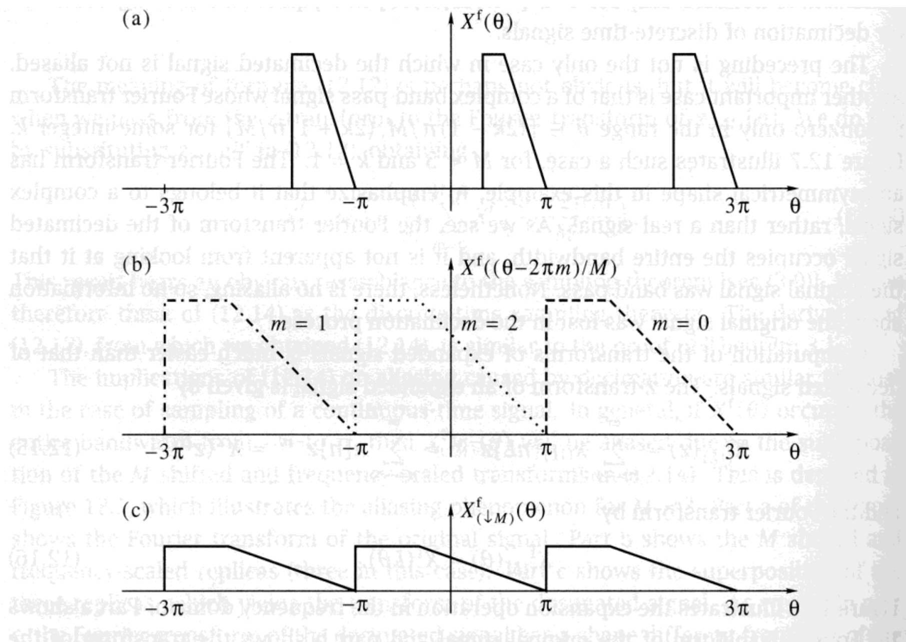
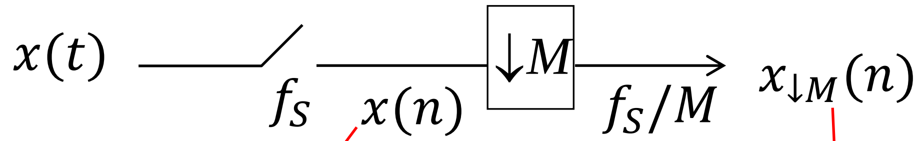


Figure 12.7 Alias-free decimation of a complex band-pass signal: (a) Fourier transform of the original signal; (b) shifted and frequency-scaled replicas of part a; (c) Fourier transform of the decimated signal.



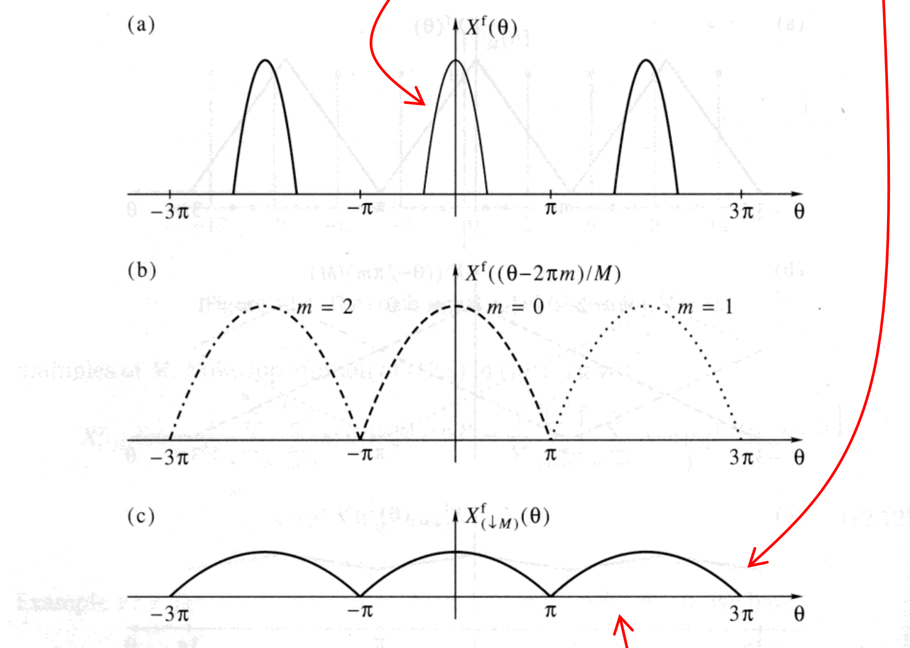
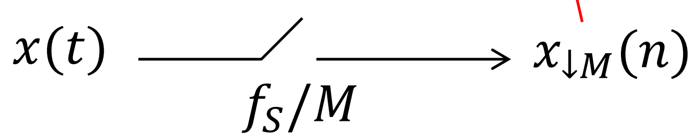


Figure 12.6 Alias-free decimation of a band-limited signal: (a) Fourier transform of the original signal; (b) shifted and frequency-scaled replicas of part a; (c) Fourier transform of the decimated signal.

$$x(n)\ must\ be\ \frac{\pi}{M}\ band\ limited.$$



Decimation and Expansion

❖ **Expansion**

$$X_E(z) = \sum_{n=-\infty}^{\infty} x_E(n) z^{-n} = \sum_{n=\text{multiple of } L} x(n/L) z^{-n} = \sum_{n=-\infty}^{\infty} x(n) z^{-nL} = X(z^L)$$

$$\Leftrightarrow \quad X_E(\Theta) = X(L\Theta)$$

$$\boxed{X_E(\theta) = X\left(e^{jL\theta}\right) = X\left(e^{j(L(\theta + (2\pi/L)k))}\right) = X\left(e^{j(L\theta + 2\pi k)}\right)}$$

$X\left(e^{jL\theta}\right)$ *repeats at regular intervals*

$$\left(\frac{2\pi}{L}\right)k, k = 0,1,2,\dots,L-1$$

Decimation and Expansion

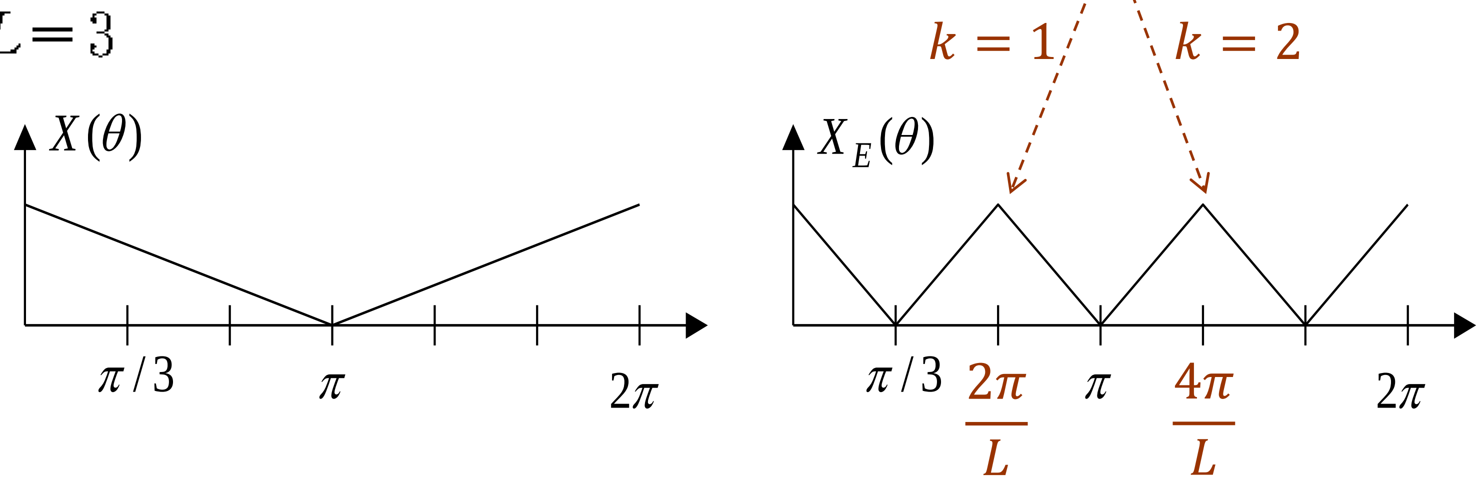
❖ Expansion

$$X_E(z) = \sum_{n=-\infty}^{\infty} x_E(n) z^{-n} = \sum_{n=\text{multiple of } L} x(n/L) z^{-n} = \sum_{n=-\infty}^{\infty} x(n) z^{-nL} = X(z^L)$$

$$\Leftrightarrow \quad X_E(\Theta) = X(L\Theta)$$

$$\boxed{X_E(\theta) = X(e^{jL\theta}) = X(e^{j(L(\theta + \underline{(2\pi/L)k}))}) = X(e^{j(L\theta + 2\pi k)})}$$

- $L = 3$

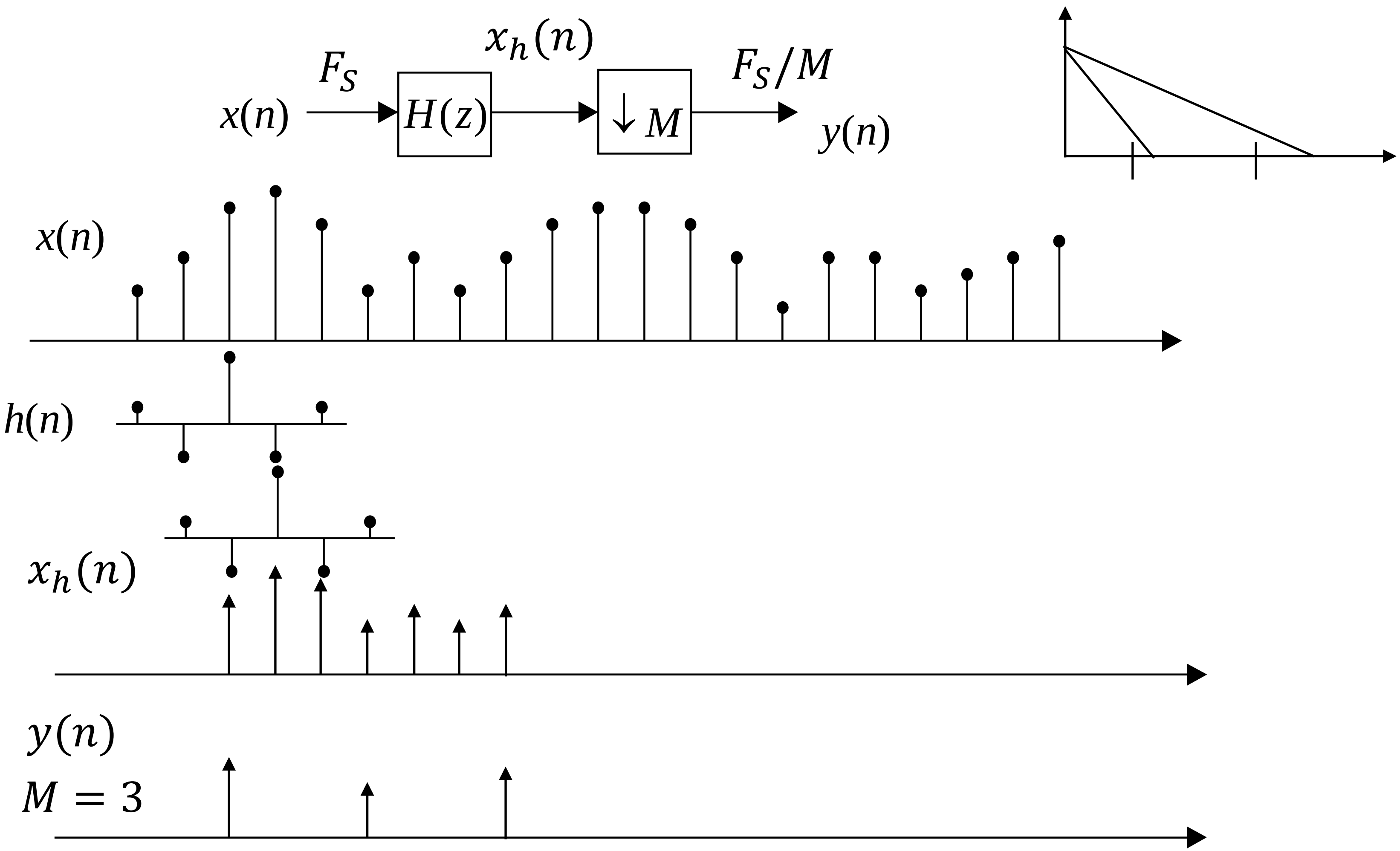


Bandwidth is scaled by $1/L$

$L-1$ images

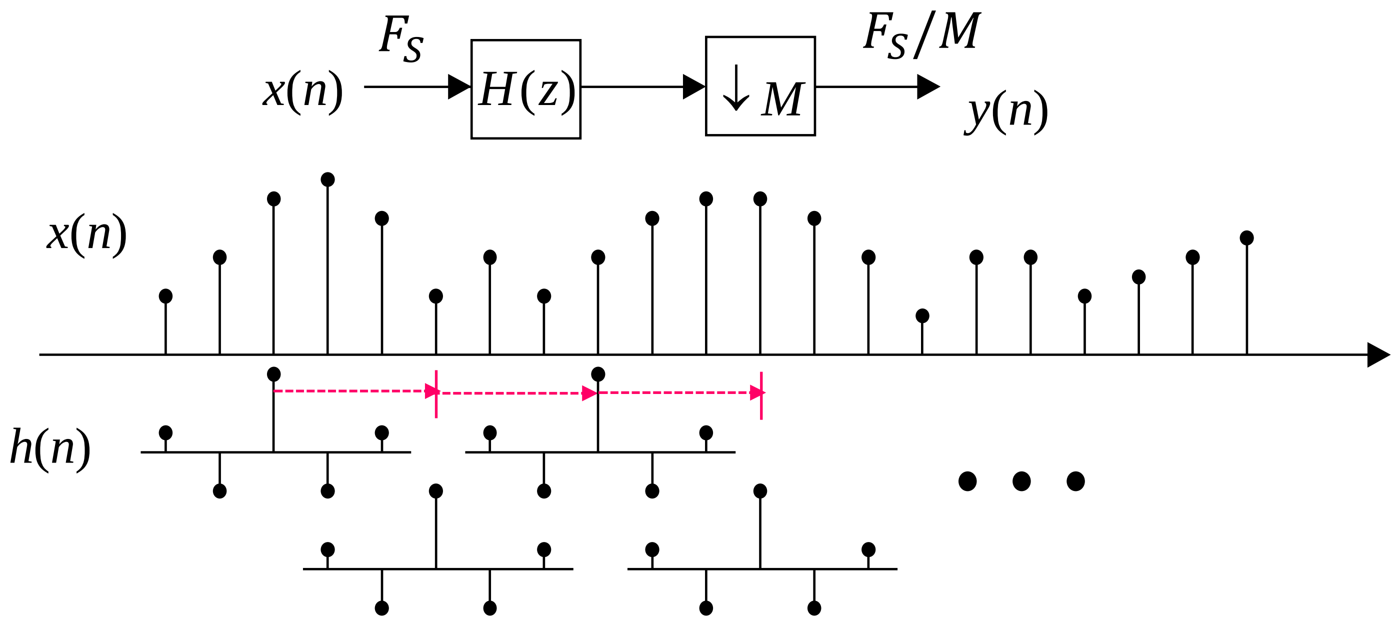
Decimation and Expansion

M-fold decimation



Decimation and Expansion

M-fold decimation

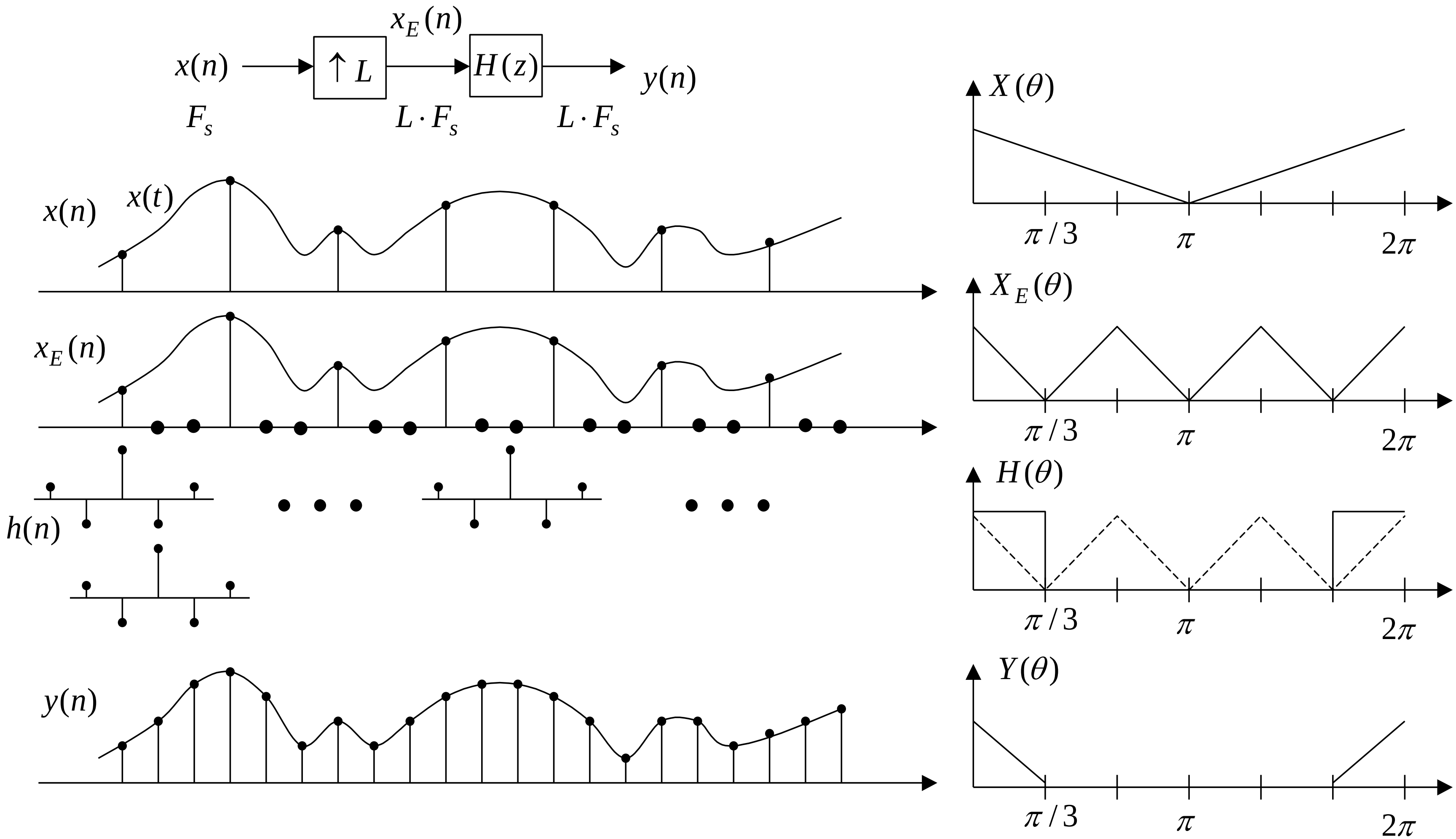


$$y(n) = \sum_{i=-\infty}^{\infty} h(i)x(nM - i) = \sum_{i=-\infty}^{\infty} x(i)h(nM - i)$$

$H(z)$: Lowpass anti-aliasing filter with $\theta_c = \pi / M$

Decimation and Expansion

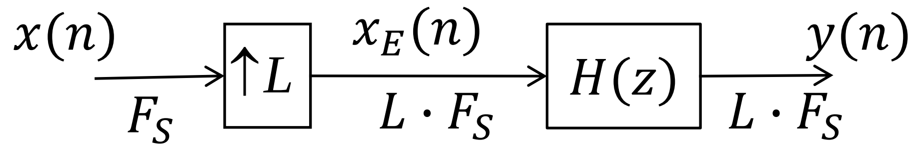
L-fold Interpolation

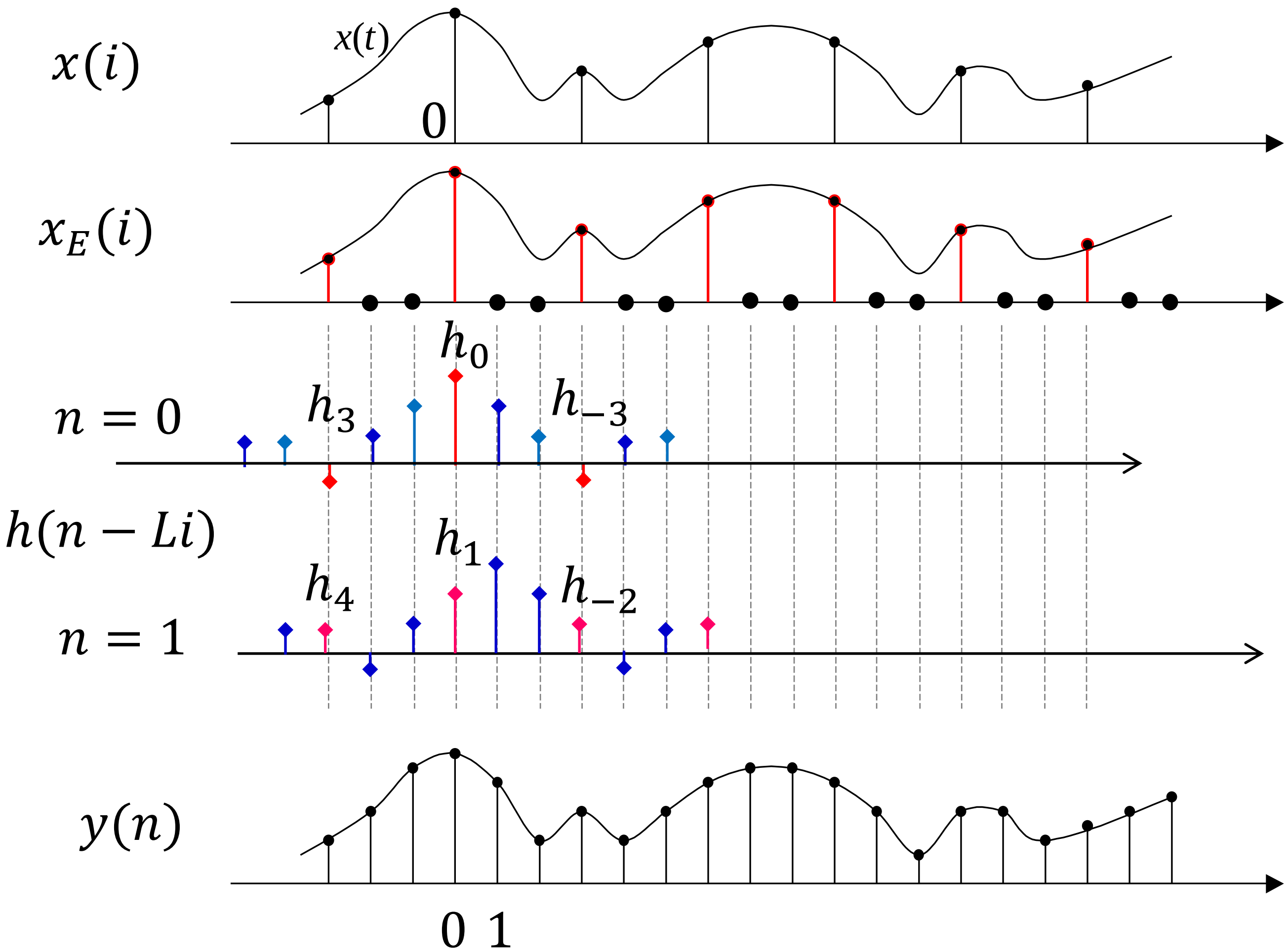


$H(z)$: Lowpass anti-aliasing filter with $\Theta_c = \pi / L$

Decimation and Expansion

L-fold Interpolation





$$y(n) = \sum_{i=-\infty}^{\infty} x_E(i)h(n-i)$$

$$= \sum_{i=-\infty}^{\infty} x(i)h(n-Li)$$

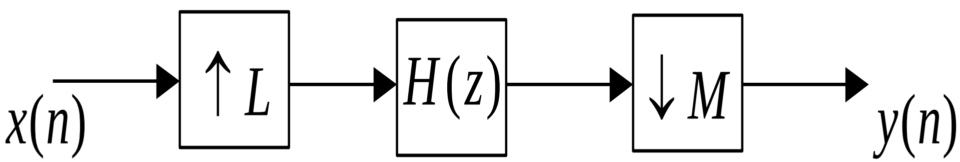
Decimation and Expansion

Sampling rate conversion

❖ **Non-integer conversion rate that can be expressed as L/M.**

Ex) 3/4 = 0.75, 4/3 = 1.25, ...

❖ **Implementation**



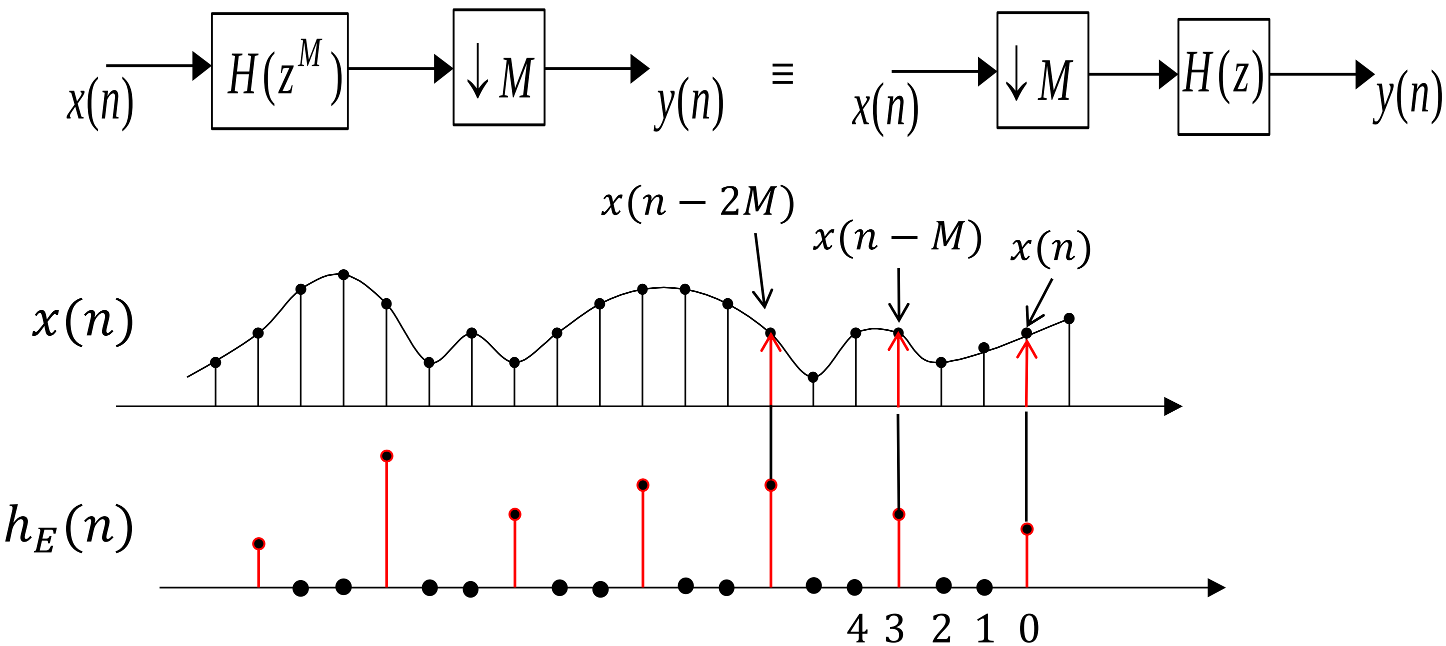
$$H(z):\ \Theta_c = \min[\pi/L, \pi/M]$$

$$y(n) = \sum_{i=-\infty}^{\infty} x(i)h(Mn - Li)$$

Multirate Identities

Decimation identity

❖ **Time domain analysis**



Multirate Identities

Decimation identity

- **Z-domain analysis**

$$x(n) \longrightarrow \boxed{H(z^M)} \longrightarrow \boxed{\downarrow M} \longrightarrow y(n) \quad \equiv \quad x(n) \longrightarrow \boxed{\downarrow M} \longrightarrow \boxed{H(z)} \longrightarrow y(n)$$

$$Y(z) = \{X(z)H(z^M)\}_{\downarrow M} = \frac{1}{M}\sum_{m=0}^{M-1} X(z^{1/M} W_M^{-m}) H\{(z W_M^{-mM})\}$$

$$= \frac{1}{M}\sum_{m=0}^{M-1} X(z^{1/M} W_M^{-m}) H(z) = X_D(z) H(z)$$

Note) $W_M^{-mM} = e^{-j\frac{2\pi}{M}mM} = e^{-j2\pi m} = 1$

Multirate Identities

Expansion identity

$$x(n) \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow L} \rightarrow y(n) \quad \equiv \quad x(n) \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z^L)} \rightarrow y(n)$$

$$Y(z) = \{H(z)X(z)\}_{\uparrow L} = H(z^L)X_E(z) = H(z^L)X(z^L)$$

Polyphase Filters

Polyphase representation of decimation

		●	▼	■	●	▼	■	●	▼	■	●	▼	■	●	▼	■
		$\underline{h_0}$	h_1	h_2	$\underline{h_3}$	h_4	h_5	$\underline{h_6}$	h_7	h_8	$\underline{h_9}$	h_{10}	h_{11}	$\underline{h_{12}}$	h_{13}	h_{14}
	h_0	$\underline{h_1}$	h_2	h_3	$\underline{h_4}$	h_5	h_6	$\underline{h_7}$	h_8	h_9	$\underline{h_{10}}$	h_{11}	h_{12}	$\underline{h_{13}}$	h_{14}	
h_0	h_1	$\underline{h_2}$	h_3	h_4	$\underline{h_5}$	h_6	h_7	$\underline{h_8}$	h_9	h_{10}	$\underline{h_{11}}$	h_{12}	h_{13}	$\underline{h_{14}}$		

$$H(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)$$

●	h_0	h_3	h_6	h_9	h_{12}	:	$p_0(n)$
▼	h_1	h_4	h_7	h_{10}	h_{13}	:	$p_1(n)$
■	h_2	h_5	h_8	h_{11}	h_{14}	:	$p_2(n)$

h_0	0	0	h_3	0	0	h_6	0	0	h_9	0	0	h_{12}	:	$P_0(z^3)$
h_1	0	0	h_4	0	0	h_7	0	0	h_{10}	0	0	h_{13}	:	$P_1(z^3)$
h_2	0	0	h_5	0	0	h_8	0	0	h_{11}	0	0	h_{14}	:	$P_2(z^3)$

$P_0(z^3)$	h_0	0	0	h_3	0	0	h_6	0	0	h_9	0	0	h_{12}			
$z^{-1}P_1(z^3)$		h_1	0	0	h_4	0	0	h_7	0	0	h_{10}	0	0	h_{13}		
$z^{-2}P_2(z^3)$			h_2	0	0	h_5	0	0	h_8	0	0	h_{11}	0	0	h_{14}	

Polyphase Filters

Polyphase representation of decimation

❖ Derivation

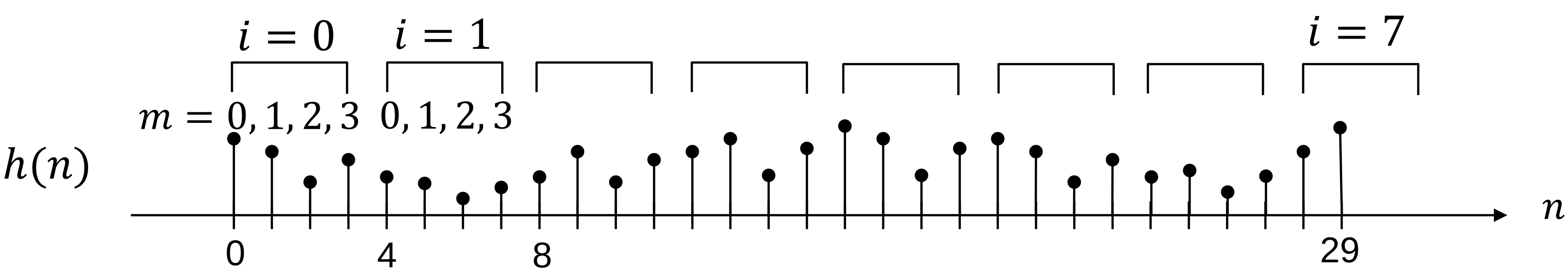
Let $i = i' M + m, \quad 0 \le m \le M-1$

Then we get

$$y(n) = \sum_{i=0}^{N} h(i)x(nM - i) = \sum_{m=0}^{M-1} \sum_{i'=0}^{I'} h(i'M + m)x((n-i')M - m), \quad 0 \le m \le M-1$$

where $I' = \lfloor N/M \rfloor$

Ex) N = 29, M=4 $I' = \lfloor 29/4 \rfloor = 7$



Polyphase Filters

Polyphase representation of decimation

❖ **Derivation**

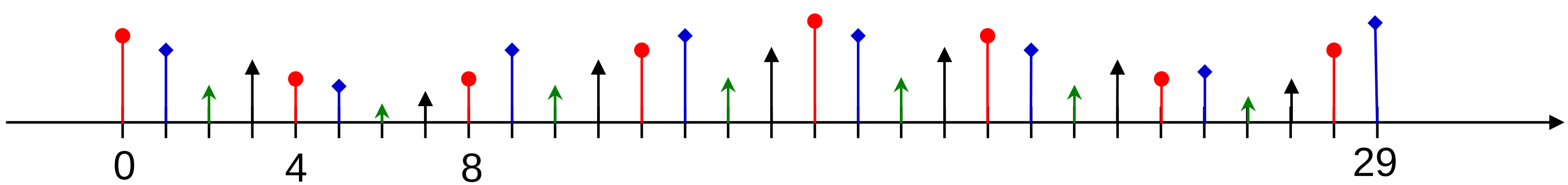
Let's define following sequences

$$p_m(n) = h(nM+m) \text{ and } u_m(n) = x(nM-m) \text{ for } 0 \leqq m \leqq M-1$$

Then, $$y(n) = \sum_{m=0}^{M-1} \{p_m(n) * u_m(n)\}$$

$$y(n) = \sum_{i=0}^{N} h(i)x(nM-i) = \sum_{m=0}^{M-1} \sum_{i=0}^{I} h(iM+m)x((n-i)M-m), \quad 0 \le m \le M-1$$

$p_0(n)$ $p_1(n)$ $p_2(n)$ $p_3(n)$



"For efficient computation and low hardware complexity"

Polyphase Filters

Polyphase representation of decimation

❖ **Poly phase components of $h(n)$**

- ***M*** **FIR filters** $p_m(i)$

$p_m(i)$ **obtained by advancing the $h(n)$ by m and then decimating by *M***

- $$P_m(z) = \sum_i p_m(i) z^{-i} = \sum_i h(iM + m) z^{-i}$$

$u_m(n)$ **obtained by delaying the $x(n)$ by m and then decimating by *M***

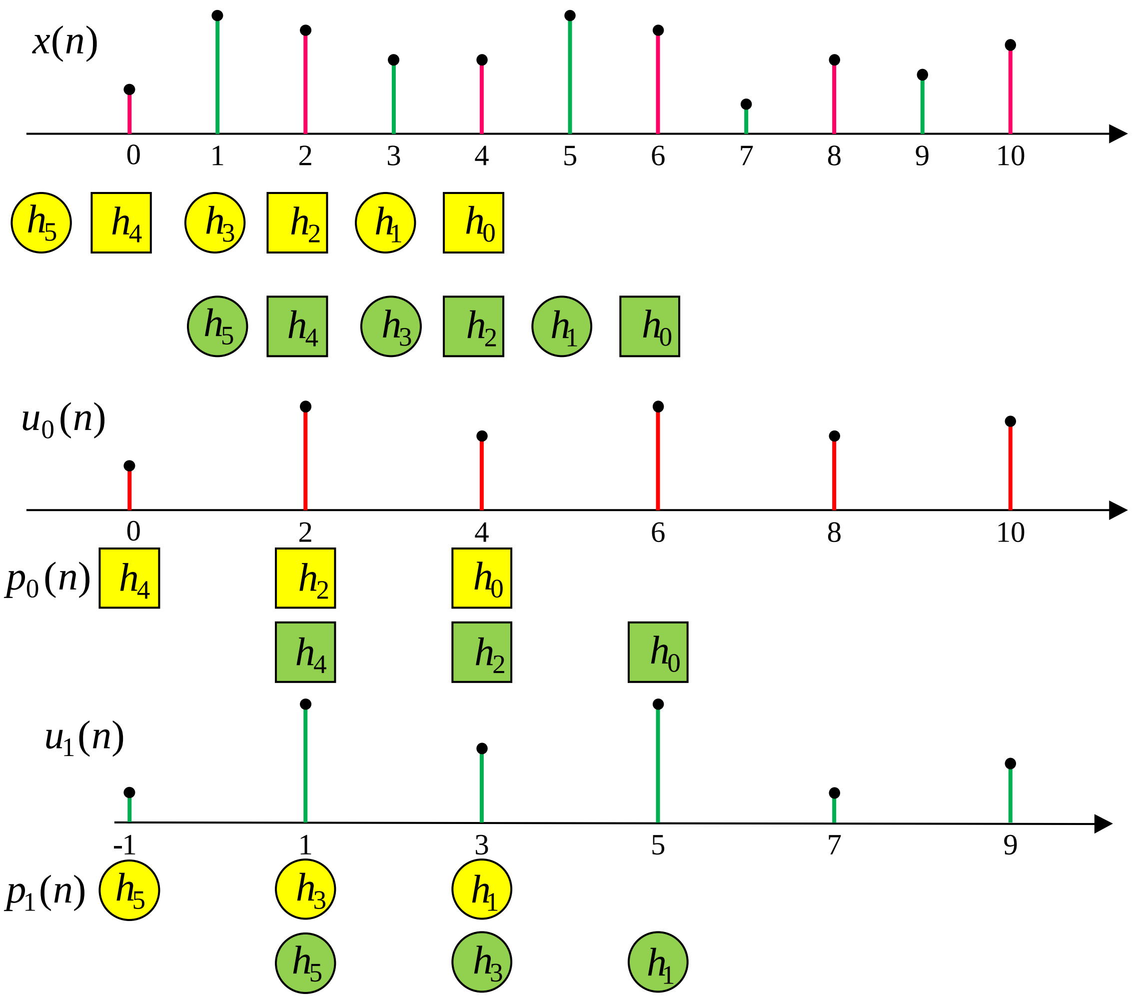
$$u_m(n) = x(nM - m)$$

$u_0(n)$	$\underline{x_0}$	x_1	x_2	$\underline{x_3}$	x_4	x_5	$\underline{x_6}$	x_7	x_8	$\underline{x_9}$	x_{10}	x_{11}	$\underline{x_{12}}$	x_{13}	x_{14}		
$u_1(n)$		x_0	x_1	$\underline{x_2}$	x_3	x_4	$\underline{x_5}$	x_6	x_7	$\underline{x_8}$	x_9	x_{10}	$\underline{x_{11}}$	x_{12}	x_{13}	$\underline{x_{14}}$	
$u_2(n)$			x_0	$\underline{x_1}$	x_2	x_3	$\underline{x_4}$	x_5	x_6	$\underline{x_7}$	x_8	x_9	$\underline{x_{10}}$	x_{11}	x_{12}	$\underline{x_{13}}$	x_{14}

Polyphase Filters

Polyphase representation of decimation

❖ **Type 1 polyphase:** **Polyphase implementation of a M-fold decimator**

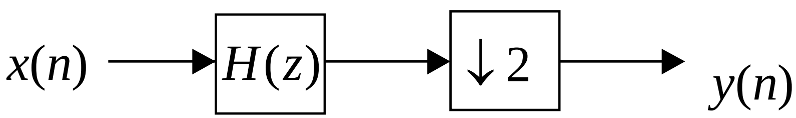


M=2 case

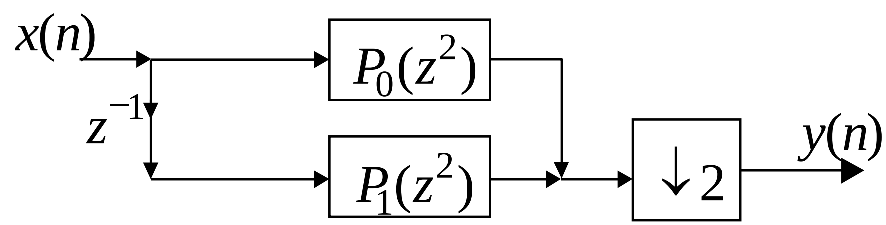
Polyphase Filters

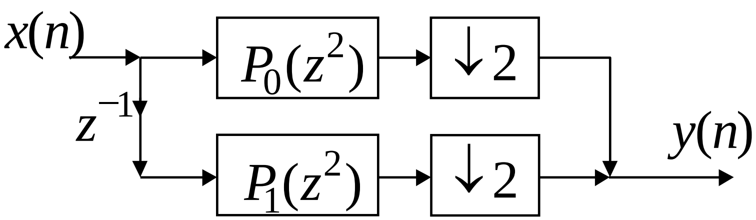
Polyphase representation of decimation

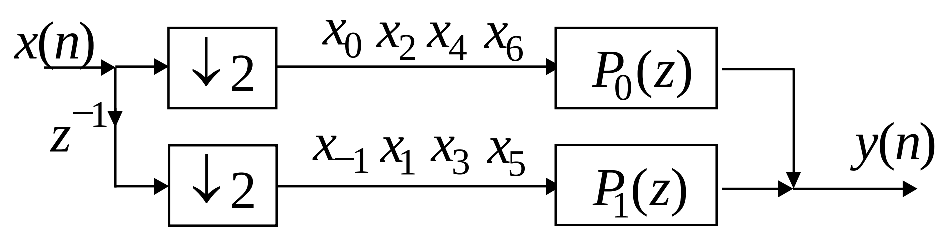
❖ **Type 1 polyphase: Polyphase implementation of a M-fold decimator**



$$H(z) = P_0(z^2) + z^{-1}P_1(z^2)$$







Ex1) $H(z) = 0.1 + 0.2z^{-1} + 0.3z^{-3} + 0.4z^{-4} + 0.5z^{-5} + 0.7z^{-7} + 0.8z^{-8} + 0.9z^{-9}$

$p_0(n) = \{$ 0.1, 0, 0.4, 0, 0.8 $\}$

$p_1(n) = \{$ 0.2, 0.3, 0.5, 0.7, 0.9 $\}$

Polyphase Filters

Polyphase representation of decimation

❖ **Type 1 polyphase:** **Polyphase implementation of a M-fold decimator**

Ex2) $H(z) = 0.1 + 0.2z^{-1} + 0.3z^{-3} + 0.4z^{-4} + 0.5z^{-5} + 0.7z^{-7} + 0.8z^{-8} + 0.9z^{-9}$

$$x(n) \longrightarrow \boxed{H(z)} \longrightarrow \boxed{\downarrow 3} \longrightarrow y(n)$$

$p_0(n) = \{\quad 0.1,\ 0.3,\ 0,\ 0.9 \quad\}$

$p_1(n) = \{\quad 0.2,\ 0.4,\ 0.7 \quad\}$

$p_2(n) = \{\quad 0,\ 0.5,\ 0.8 \quad\}$

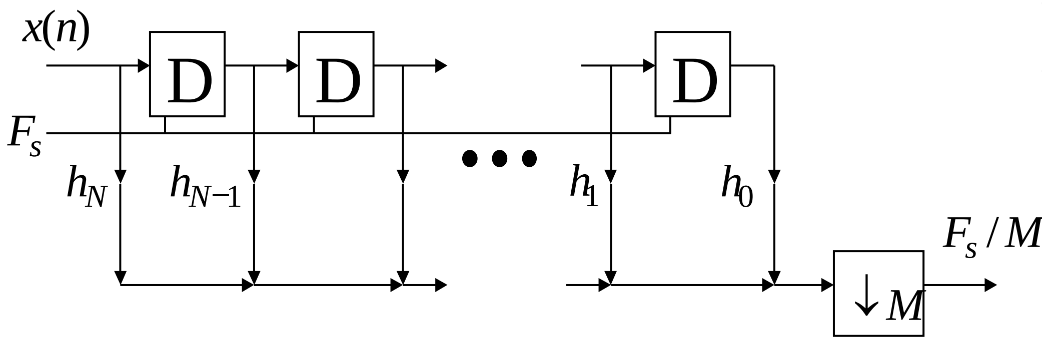
$P_0(z) = \quad 0.1 + 0.3z^{-1} + 0.9z^{-3}$

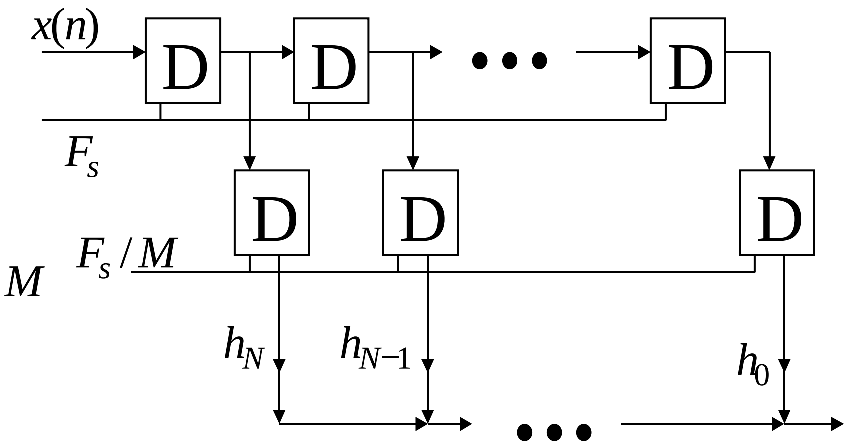
$P_1(z) = \quad 0.2 + 0.4z^{-1} + 0.7z^{-2}$

$P_2(z) = \quad 0.5z^{-1} + 0.8z^{-2}$

Polyphase Filters

Cf) Direct implementation of decimation





- # memories = N
- # multipliers = N+1
- Multiplier speed = F_s

memories = 2N

multipliers = N+1

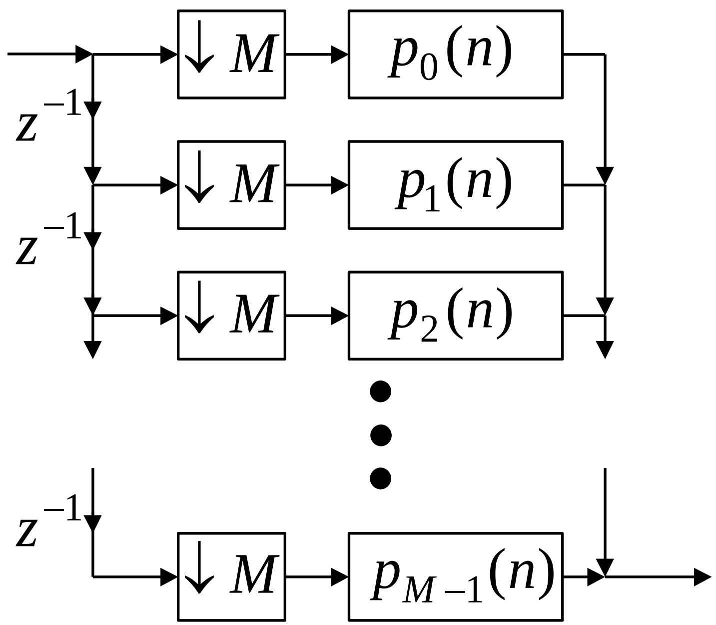
Multiplier speed = F_s/M

Polyphase Filters

Polyphase representation of decimation

- ❖ **Polyphase implementation of a M-fold decimator**

  Compare the # of memories, # of multipliers, and multiplication speed



Polyphase Filters

Polyphase representation of decimation

❖ **Commutator model**

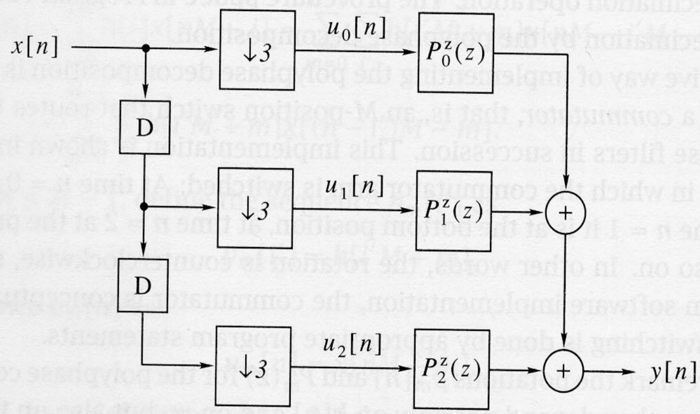


Figure 12.16 Polyphase decomposition of filtering and M-fold decimation (shown for $M = 3$).

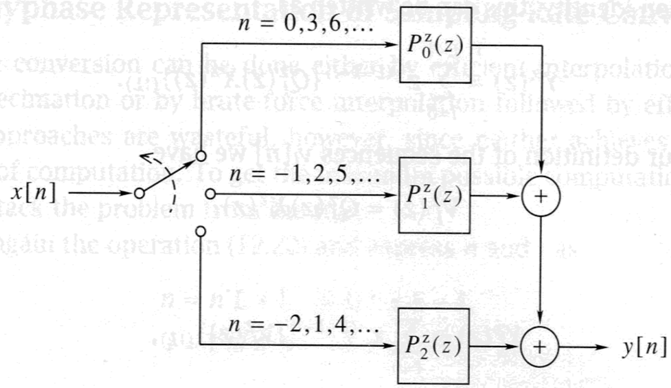


Figure 12.17 Polyphase decomposition of filtering and M-fold decimation with a commutator instead of delays and decimation (shown for $M = 3$).

Polyphase Filters

Polyphase representation of expansion (interpolation)

- **Type 2 polyphase**

$$H(z) = \sum_{l=0}^{L-1} z^{-(L-1-l)} Q_l(z^L)$$

$$Q_l(z) = P_{L-1-l}(z)$$

$$q_l(n) = h(nL + L - 1 - l]$$

$$v_l(n) = y(nL + L - 1 - l)$$

$$H(z) = \sum_{l=0}^{L-1} z^{-l} P_l(z^L)$$

$$\boxed{H(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)}$$

Polyphase Filters

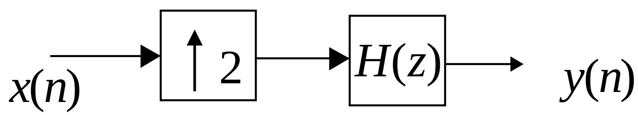
Polyphase representation of expansion

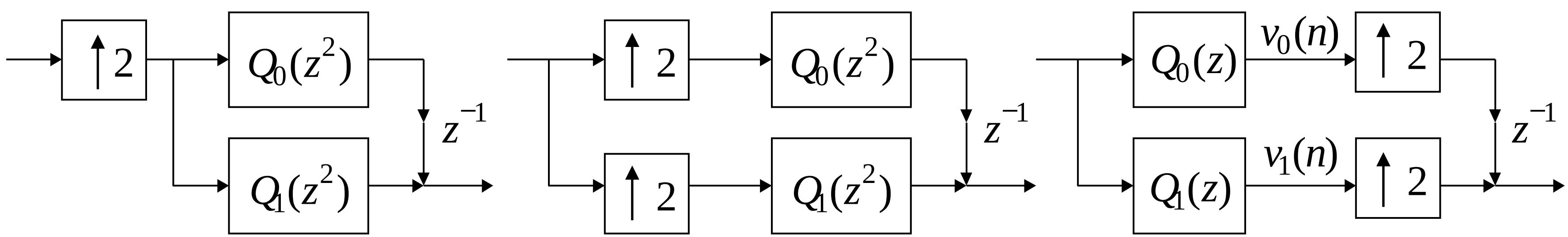
❖ **Expander**

$$Y(z) = H(z)\cdot X_E(z) = \sum_{l=0}^{L-1} z^{-(L-1-l)} Q_l(z^L) X(z^L)$$

$$= \sum_{l=0}^{L-1} z^{-(L-1-l)} \{Q_l(z) X(z)\}_{\uparrow L} = \sum_{l=0}^{L-1} z^{-(L-1-l)} \{V_l(z)\}_{\uparrow L}$$

where $V_l(z) = Q_l(z) X(z)$

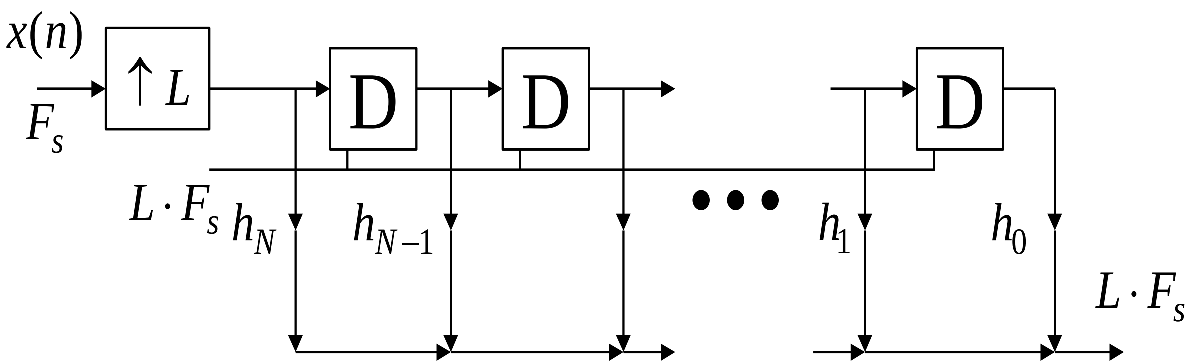




Polyphase Filters

Polyphase representation of expansion

❖ Direct implementation

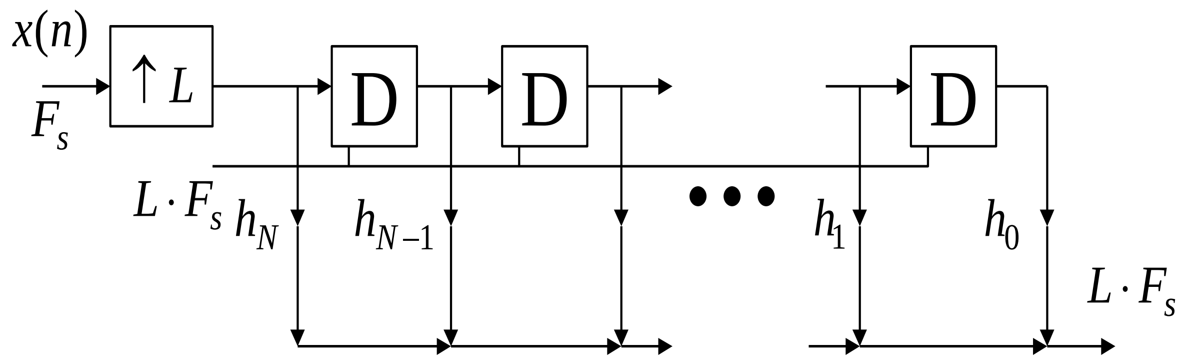


What are the problems with this structure ??

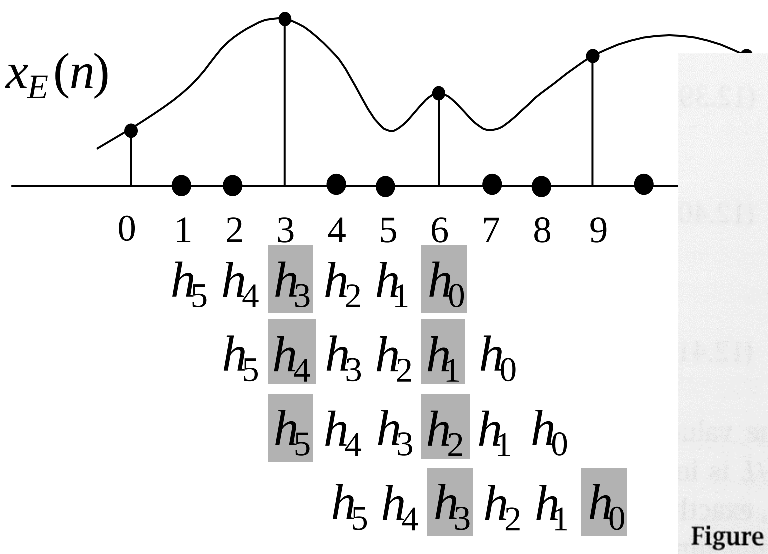
Polyphase Filters

Polyphase representation of expansion

❖ Direct implementation



❖ Polyphase implementation



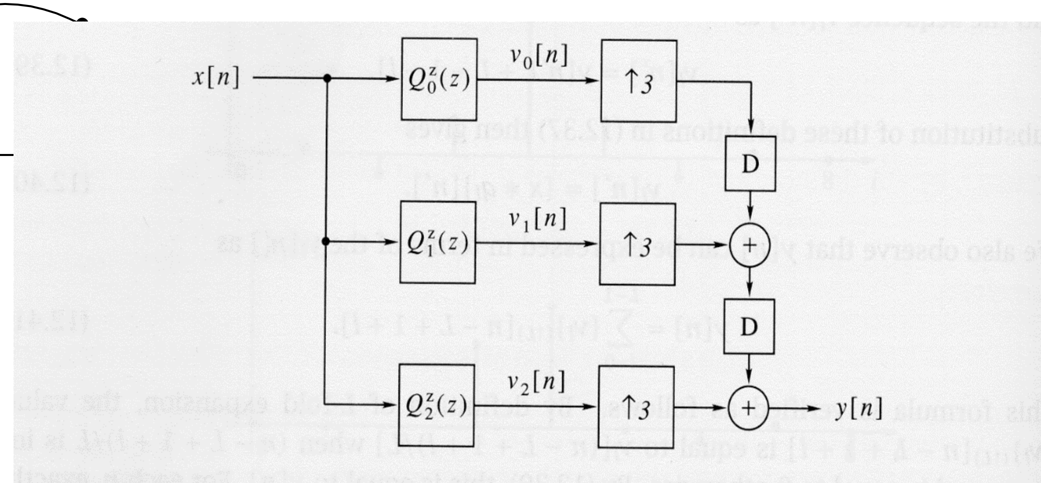


Figure 12.18 Polyphase decomposition of L-fold expansion and filtering (shown for $L = 3$).

Polyphase Filters

Polyphase representation of expansion

❖ Polyphase implementation

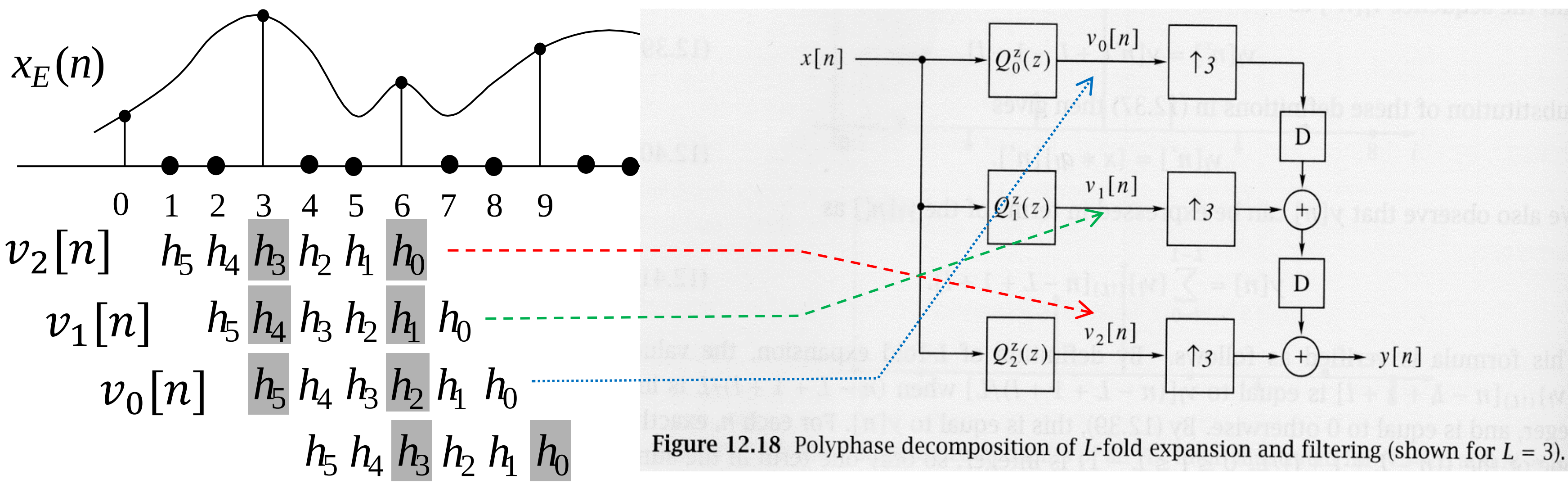


Figure 12.18 Polyphase decomposition of L-fold expansion and filtering (shown for $L = 3$).

$v_2[0]$	0	0	$v_2[1]$	0	0	$v_2[2]$	0	0	$v_2[3]$
$v_1[0]$	0	0	$v_1[1]$	0	0	$v_1[2]$	0	0	$v_1[3]$
$v_0[0]$	0	0	$v_0[1]$	0	0	$v_0[2]$	0	0	$v_0[3]$

$v_2[0]$	0	0	$v_2[1]$	0	0	$v_2[2]$	0	0	$v_2[3]$		
	$v_1[0]$	0	0	$v_1[1]$	0	0	$v_1[2]$	0	0	$v_1[3]$	
		$v_0[0]$	0	0	$v_0[1]$	0	0	$v_0[2]$	0	0	$v_0[3]$

Polyphase Filters

Polyphase representation of expansion

❖ **Polyphase implementation: Commutator model**

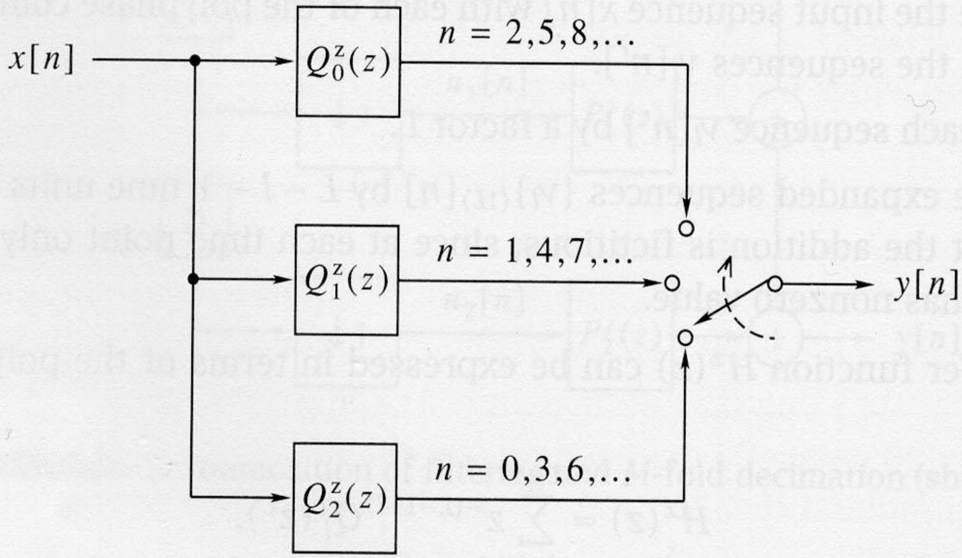


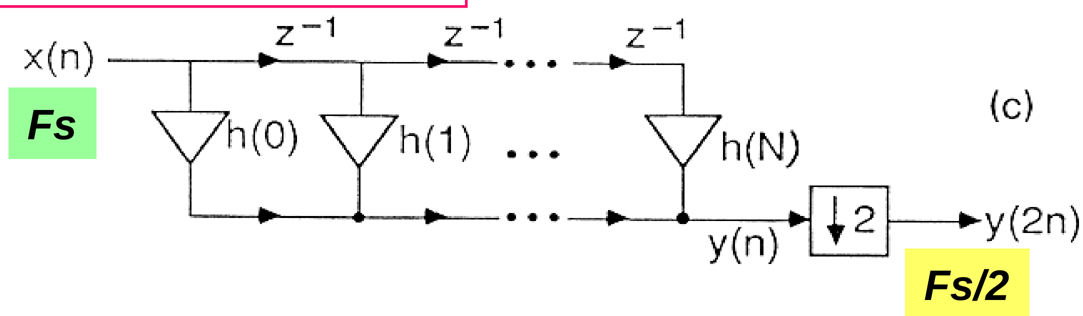
Figure 12.19 Polyphase decomposition of L-fold expansion and filtering with a commutator instead of expansion and delays (shown for $L = 3$).

Summary

- *Efficient structures for decimation and interpolation filters*

Decimation filter: M=2

Direct implementation

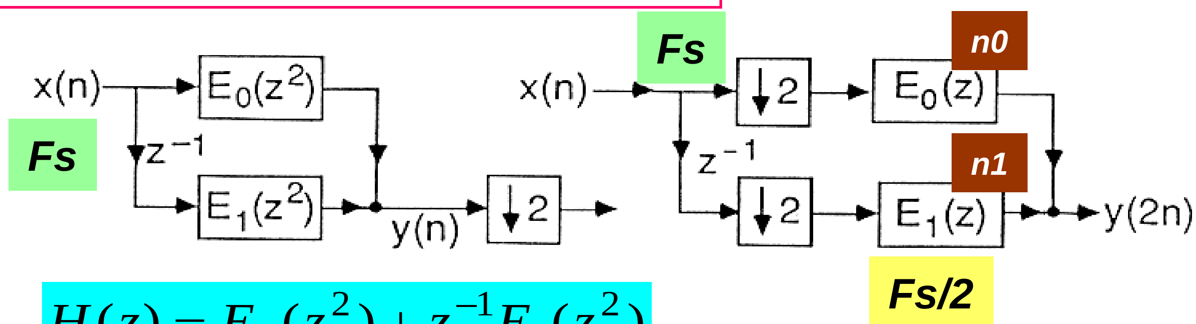


N+1 multiplications and N additions at Fs

(N+1) MPUs and N APUs

During the odd clock cycles, resting

Polyphase implementation: Type 1



$$H(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

N+1=n0+n1+2 multiplications and N additions at Fs/2
= (N+1)/2 multiplications and N/2 additions at Fs

(N+1)/2 MPUs and N/2 APUs

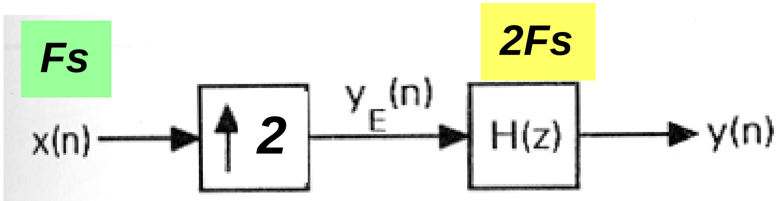
✓ ***Due to noble identity***

Summary

- *Efficient structures for decimation and interpolation filters*

Interpolation filter: L=2

Direct implementation

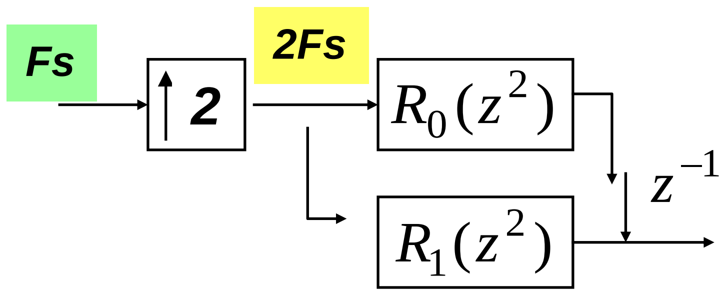


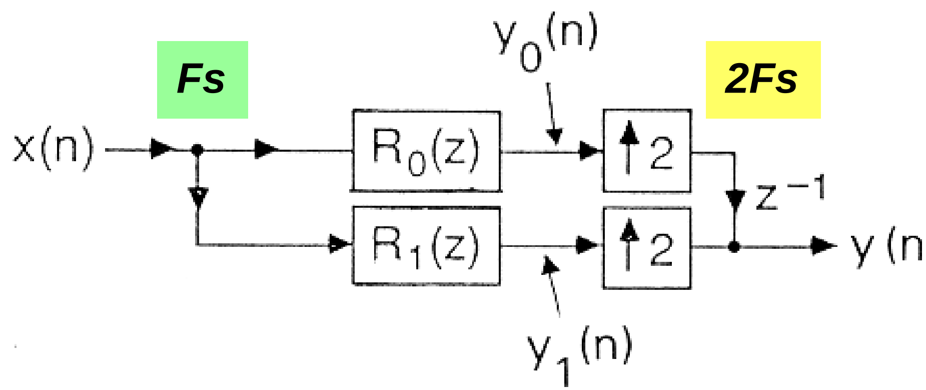
2(N+1) MPUs and 2N APUs

Half of the samples are zero

Polyphase implementation: Type 2

$$H(z) = z^{-1}R_0(z^2) + R_1(z^2)$$



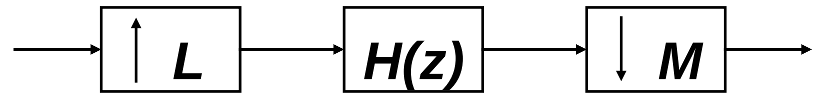


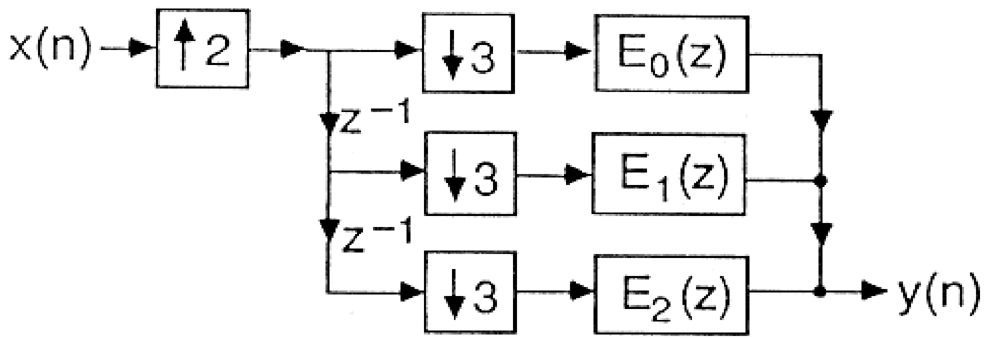
(N+1) MPUs and (N-1) APUs

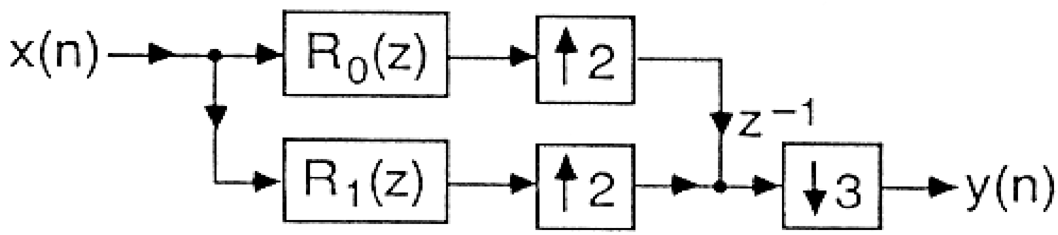
$y_1(0)\ y_0(0)\ y_1(1)\ y_0(1)\ y_1(2)\ y_0(2)$

Summary

Ex: M=3, L=2 ; M/L = 1.5





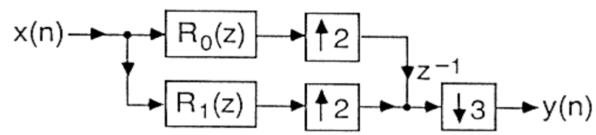


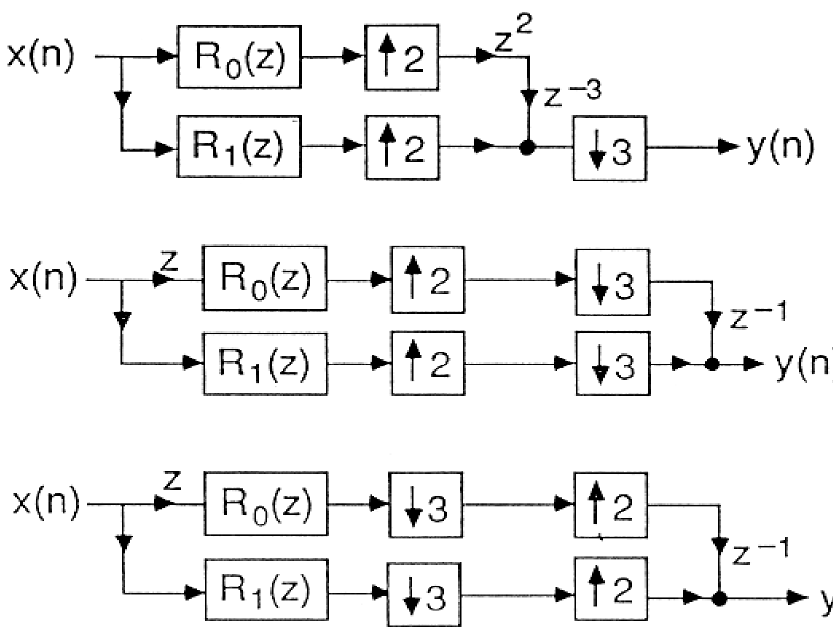
$$R_0(z) = R_{00}(z^3) + z^{-1}R_{01}(z^3) + z^{-2}R_{02}(z^3)$$

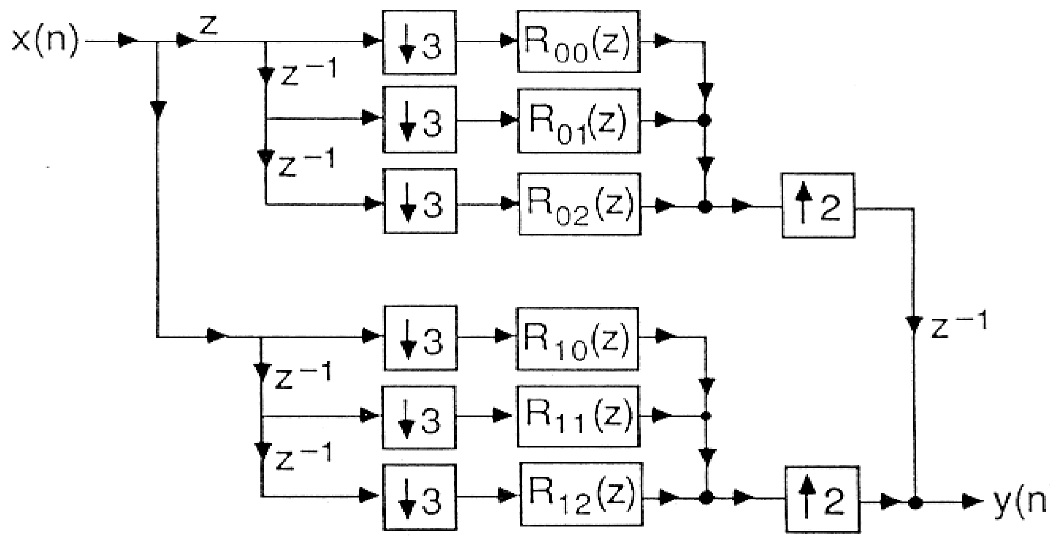
$$R_1(z) = R_{10}(z^3) + z^{-1}R_{11}(z^3) + z^{-2}R_{12}(z^3)$$

Summary

Ex: M=3, L=2 ; M/L = 1.5





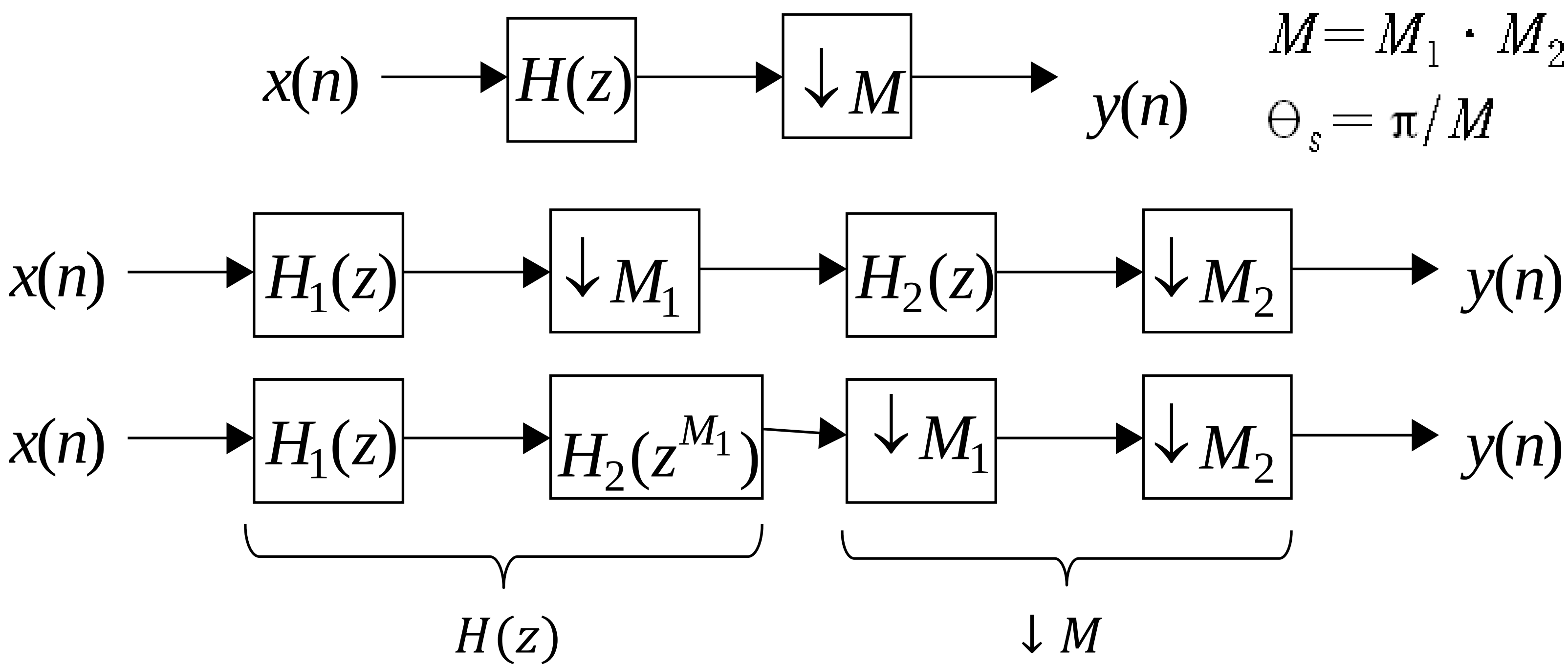


Multistage Implementations

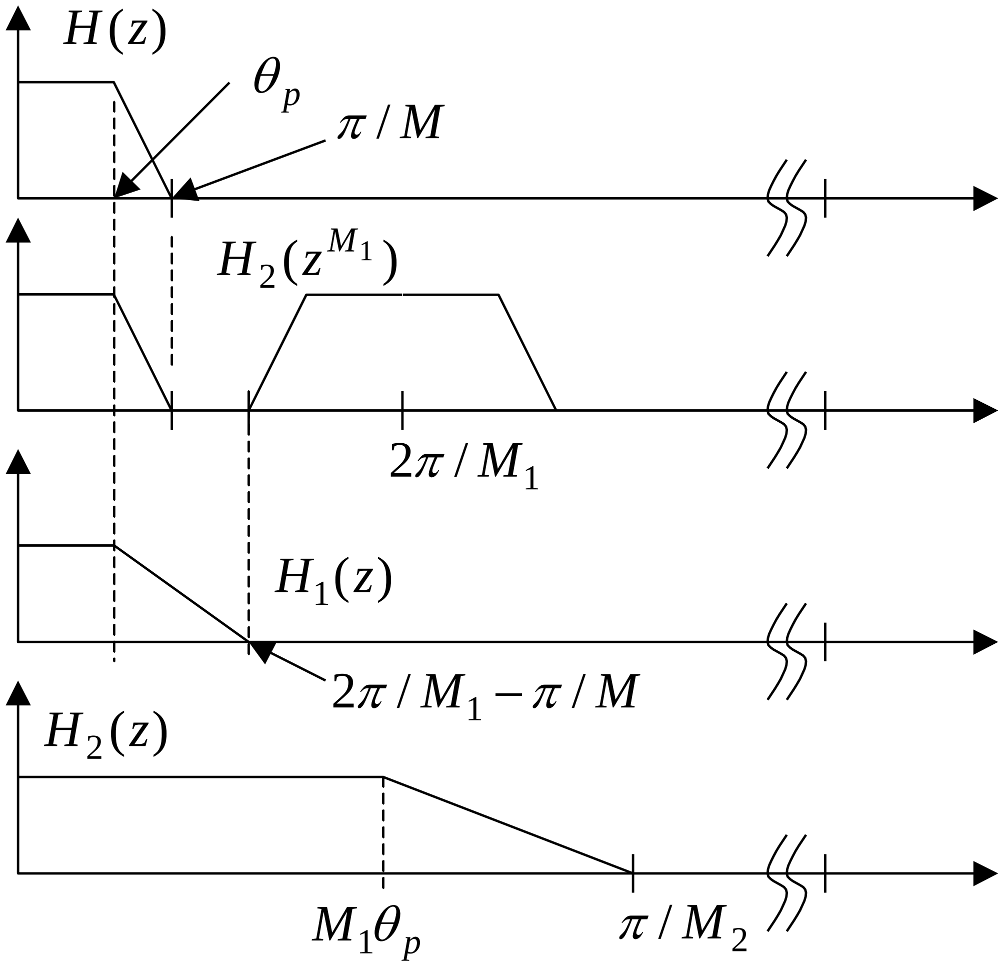
Multistage schemes

- ❖ Lower rate operation
- ❖ Lower computation amount: Simple filter spec

Multistage decimation



Multistage Implementations



$H_1(\Theta)$

$$\Theta_{p,1}=\Theta_p$$

$$\Theta_{s,1}=2\pi/M_1-\pi/M$$

$H_2(\Theta)$

$$\Theta_{p,2}=M_1\Theta_p$$

$$\Theta_{s,2}=M_1\cdot\Theta_s=M_1\cdot\pi/M$$

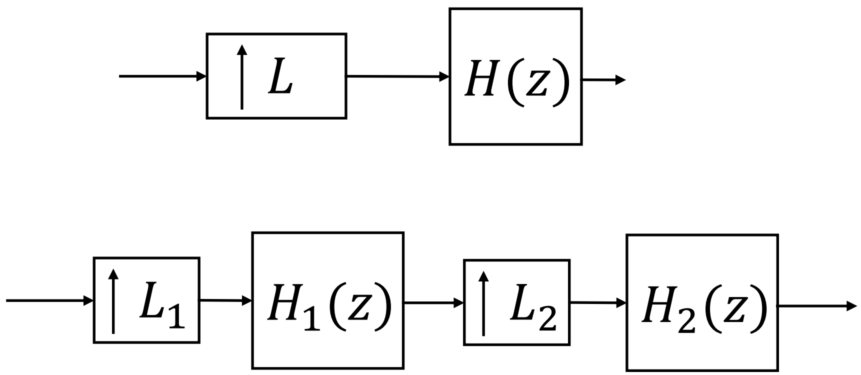
$$=\pi/M_2$$

$$\delta_{p,1}=\delta_{p,2}=0.5\delta_p$$

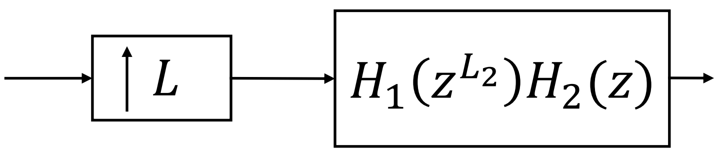
$$\delta_s=\delta_{s,1}=\delta_{s,2}$$

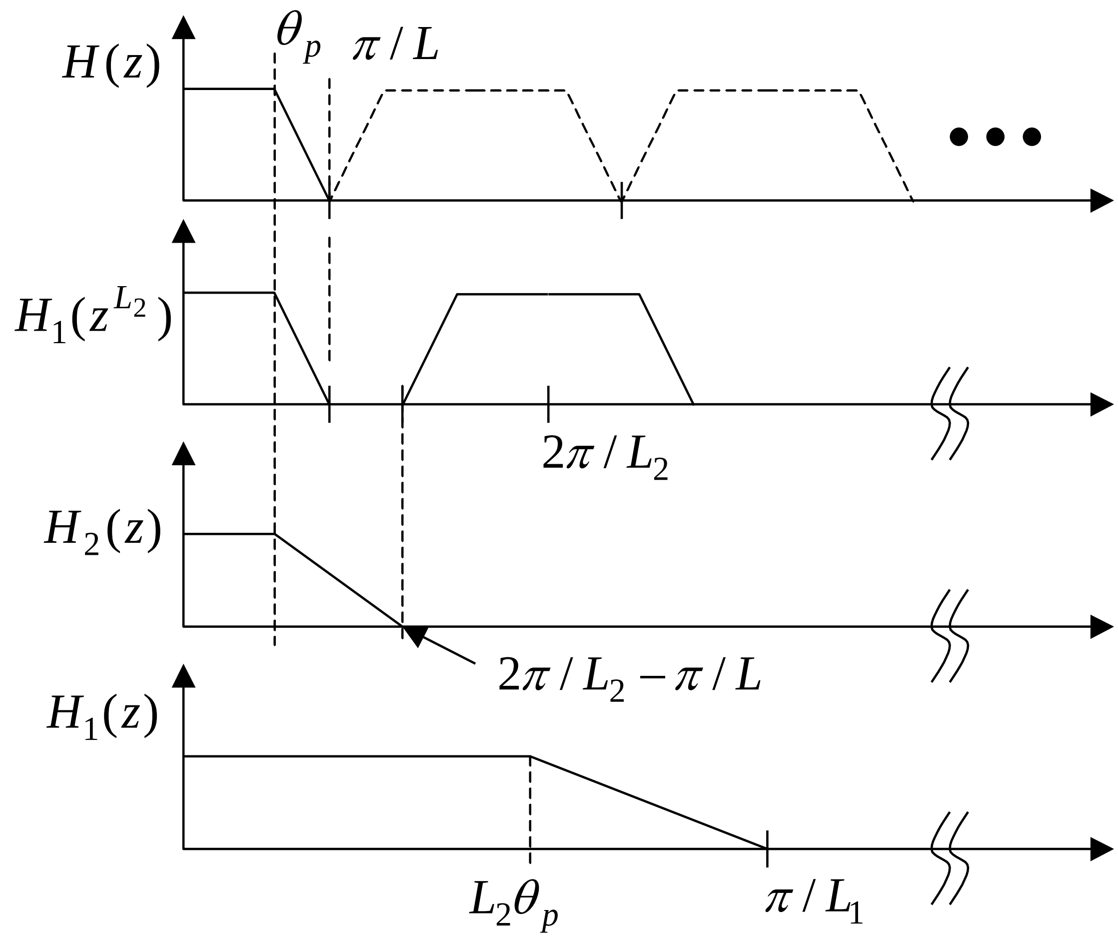
Multistage Implementations

Multistage Expansion (or interpolation)



$$L = L_1 L_2$$





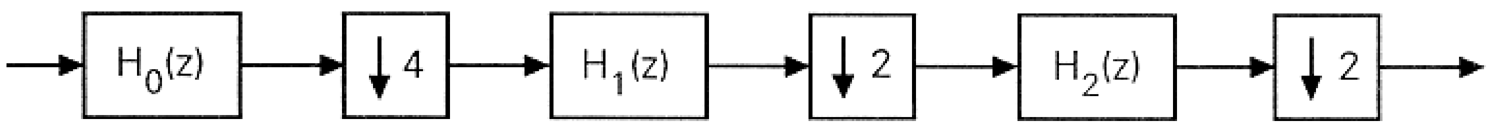
Multistage Implementations

Motivation of multistage implementation

- **Length of a linear phase FIR filter**

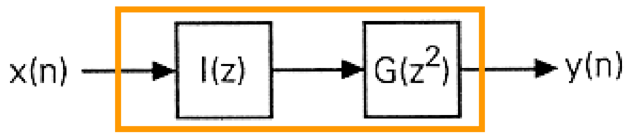
$$N \approx \frac{D(\delta_1, \delta_2)}{\Delta f}$$

- **Multistage implementation helps to reduce the overall filter length.**



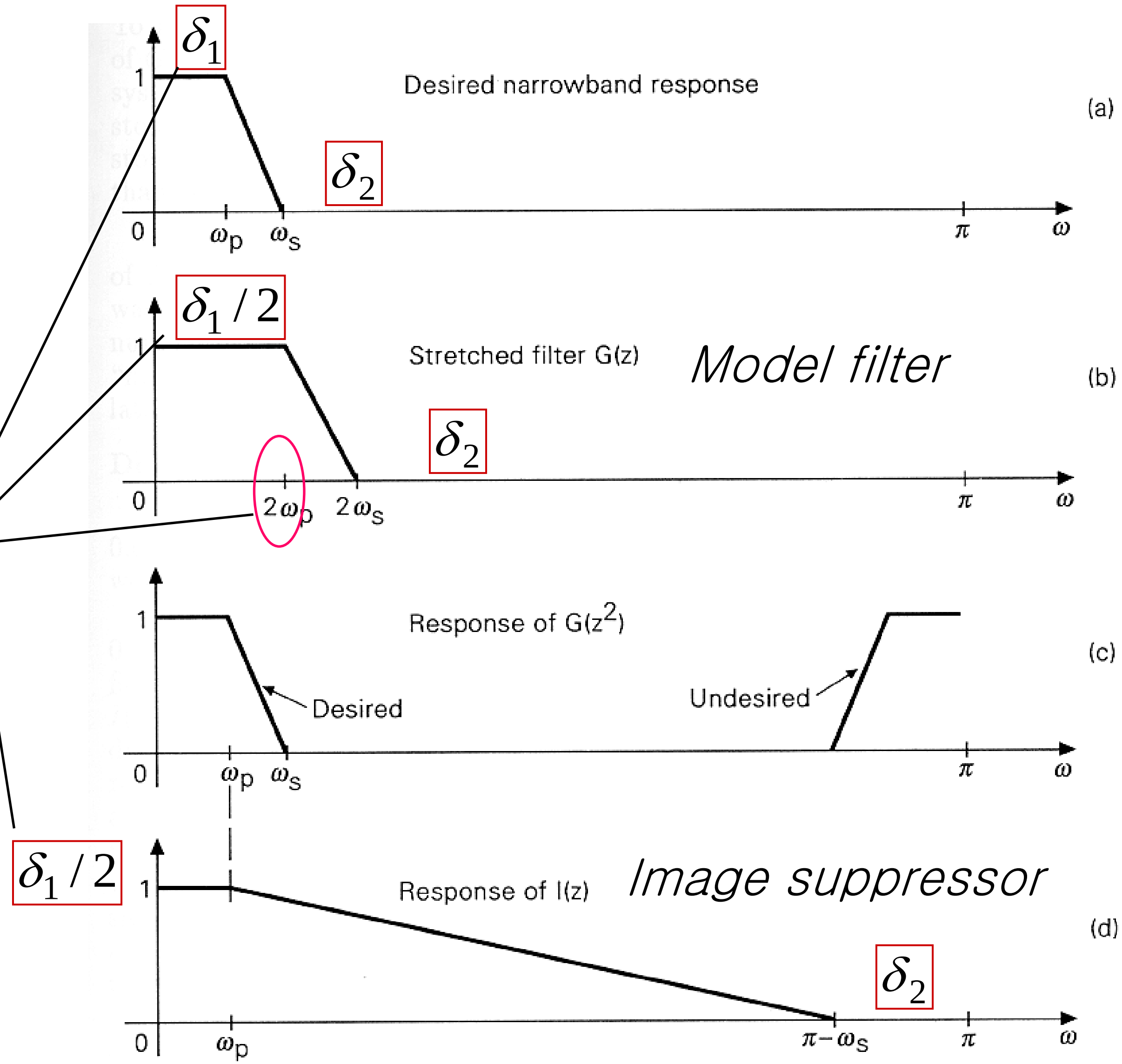
Multistage Implementations

IFIR (Interpolated FIR) approach



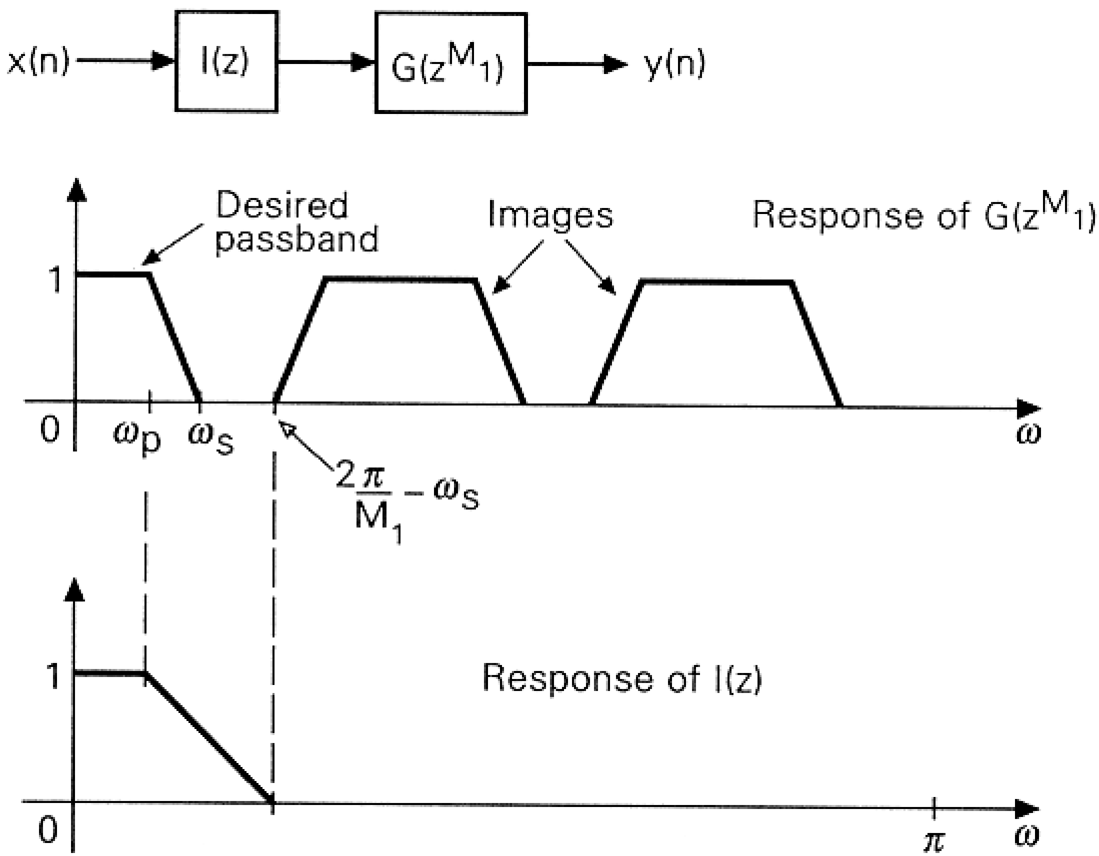
H(z)

$$N > N_g + N_i$$
$$N_g \approx N/2$$



Multistage Implementations

IFIR (Interpolated FIR) approach

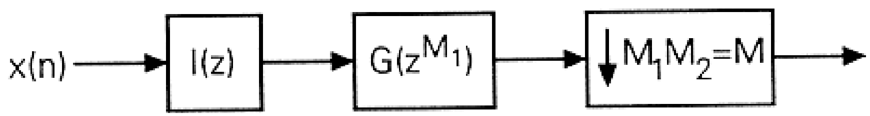


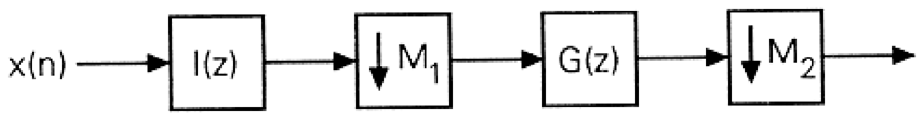
Optimum # of stages ?

Neuvo. Et al. [1984]

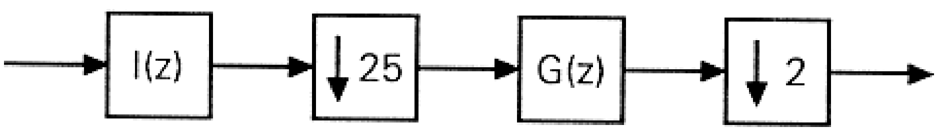
Multistage Implementations

Multistage design of decimation filter





Design example 4.4.2

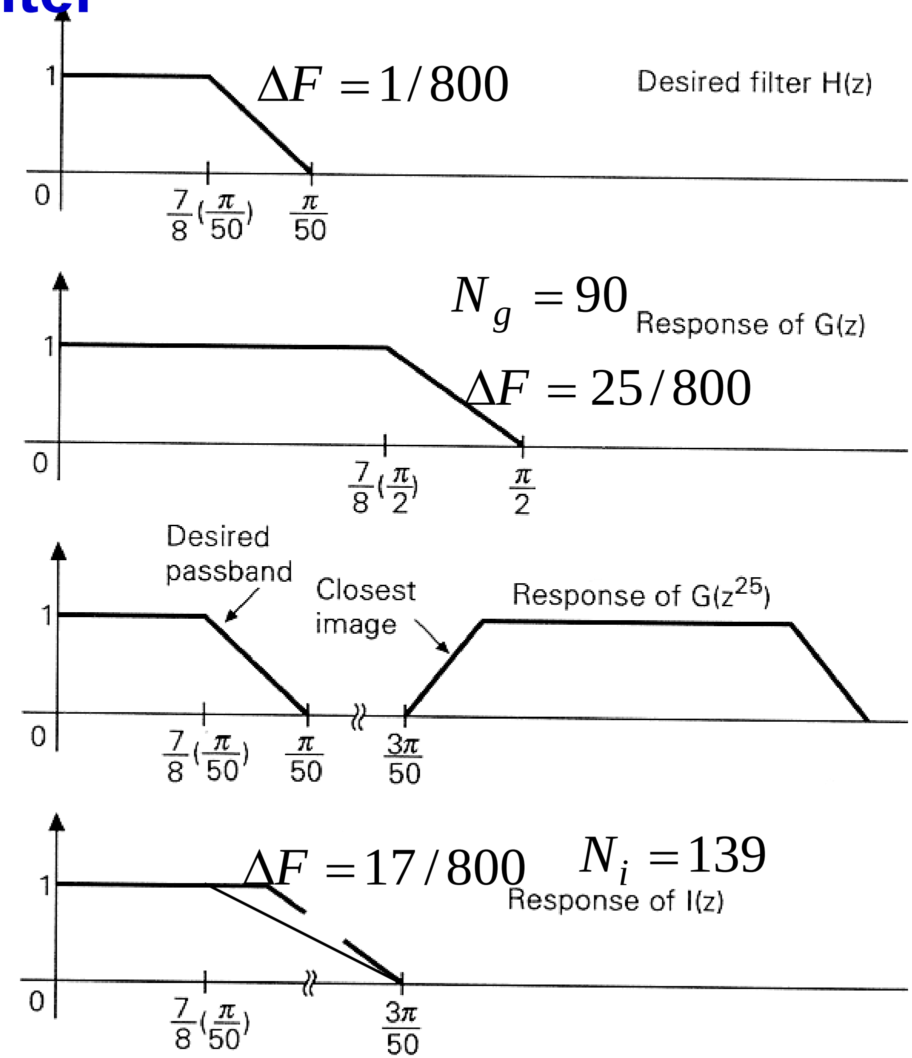


$$F_s = 8KHz$$

$$\delta_1 = 0.01, \quad \delta_2 = 0.001$$

$$F_1 \approx 70Hz, \;\; F_2 \approx 80Hz$$

$$N = 2028 \qquad 2\pi\frac{80}{8K} = \frac{\pi}{50}$$



Multistage Implementations

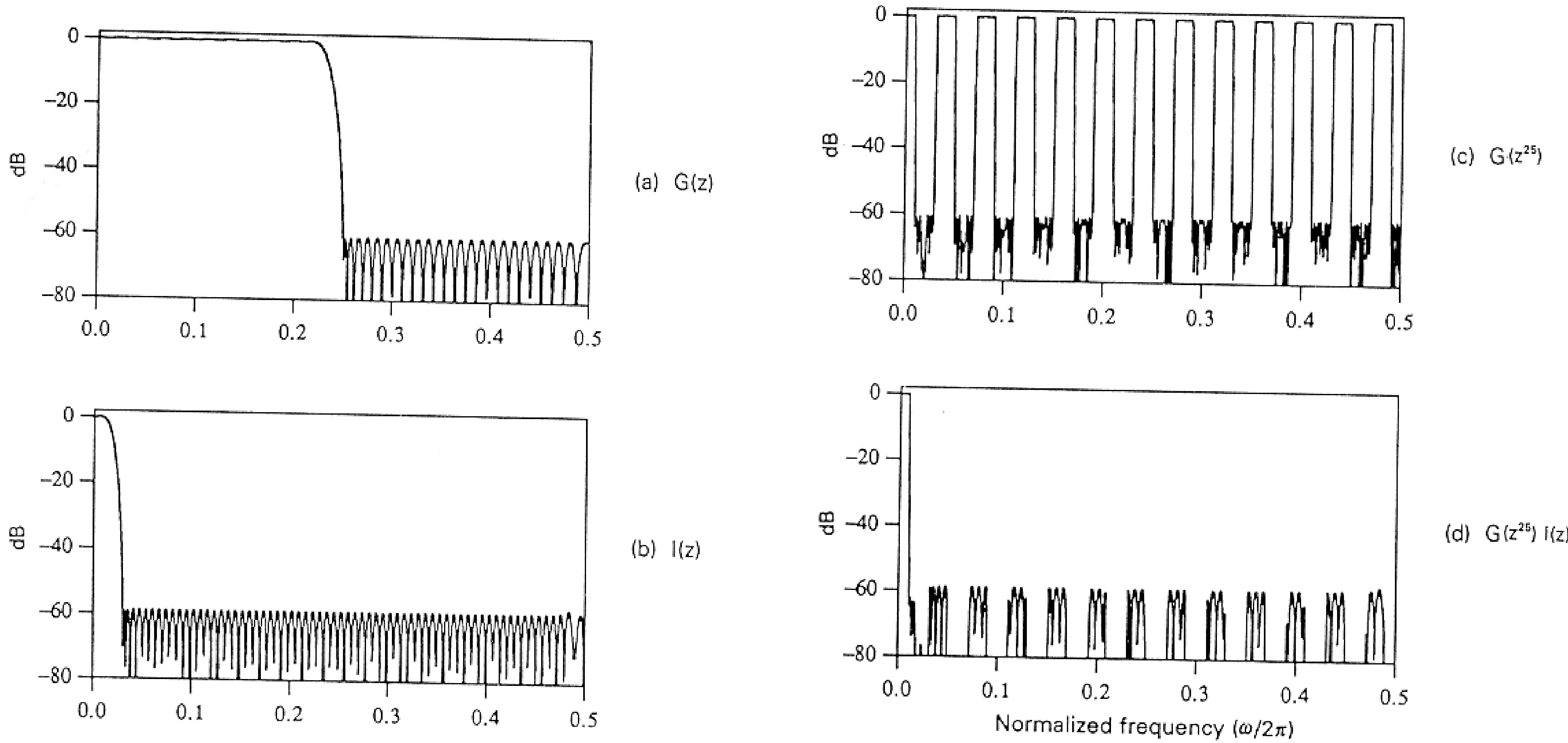
Multistage design of decimation filter

Design example 4.4.2

	Direct design $H(z)$	Multistage Design		
		$G(z)$	$I(z)$	Total
Filter order	2,028	90	139	2,389
MPUs	$\approx$21	0.92	2.8	3.72
APUs	$\approx$41	1.8	5.56	7.36
Mul per sec (8 kHz)	168,000			29,760
Add per sec (8 kHz)	328,000			58,880

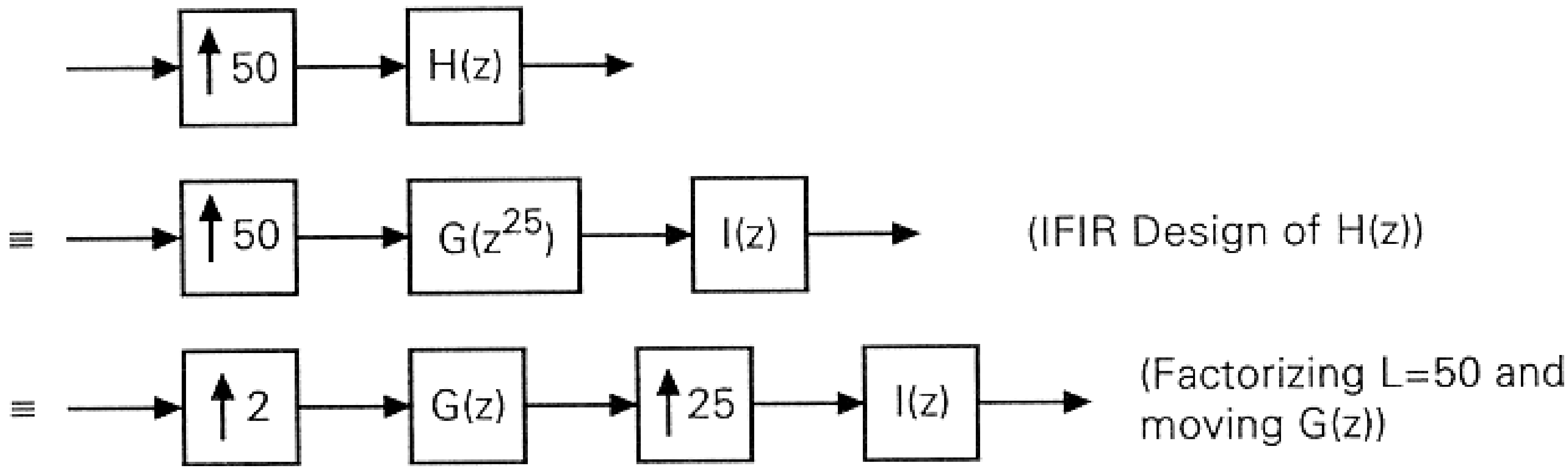
Multistage Implementations

Multistage design of decimation filter



Multistage Implementations

Multistage design of interpolator



Applications of Multirate systems

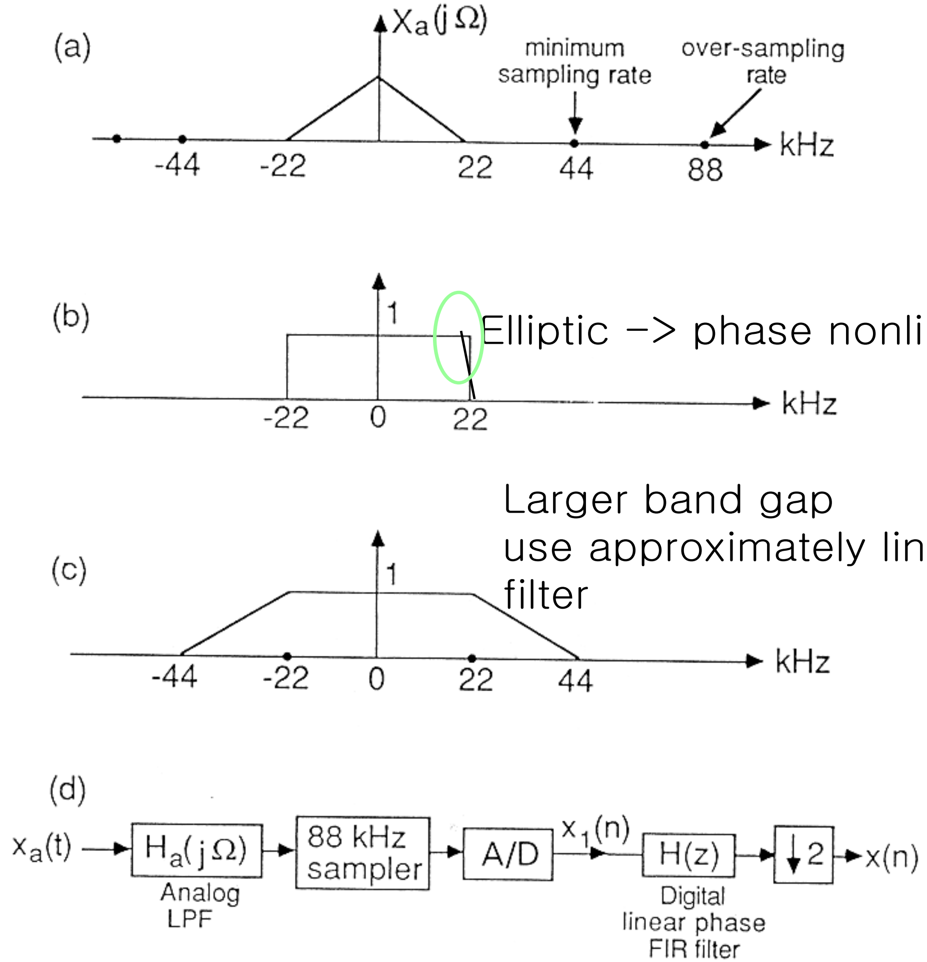
Digital audio

- **A/D**
- **D/A**
- **Fractional sampling rate conversion**

Studio: 48KHz
CD mastering: 44.1KHz

Broadcasting: 32KHz

48K -- 44.1K :
L=441, M=480

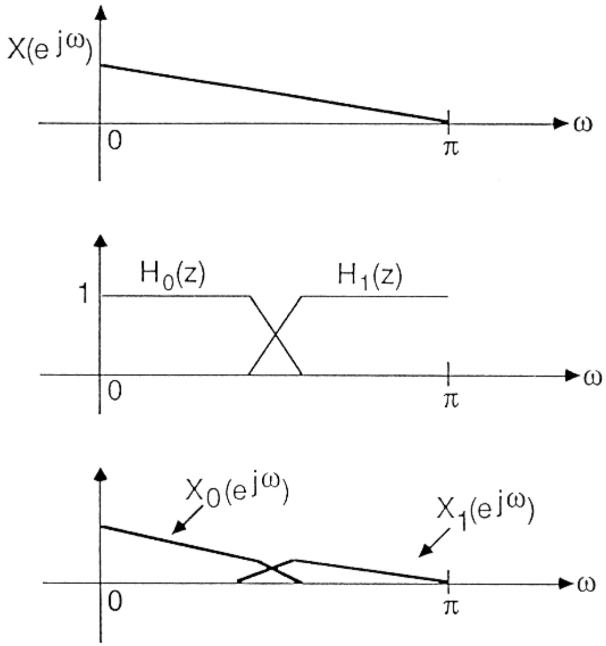


Elliptic -> phase nonlinearity

Larger band gap
use approximately linear phase filter

Applications of Multirate systems

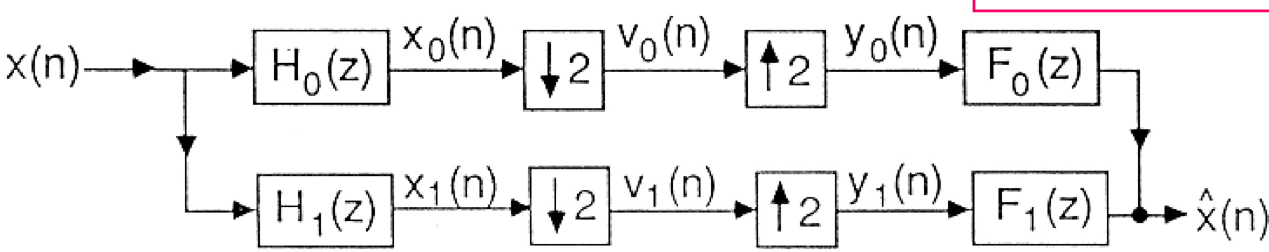
Subband coding of speech and image signals



DPCM, ADPCK: Jayant and Noll, 1984

Image compression: subband coding
Woods and O'Neil, 1986
Smith and Eddins, 1990
Woods, 1990

Music signals: DCC
Veldhuis, et al., 1989
pp. 3597-3620, ICASSP, 1991
Fettweis, et al., 1990



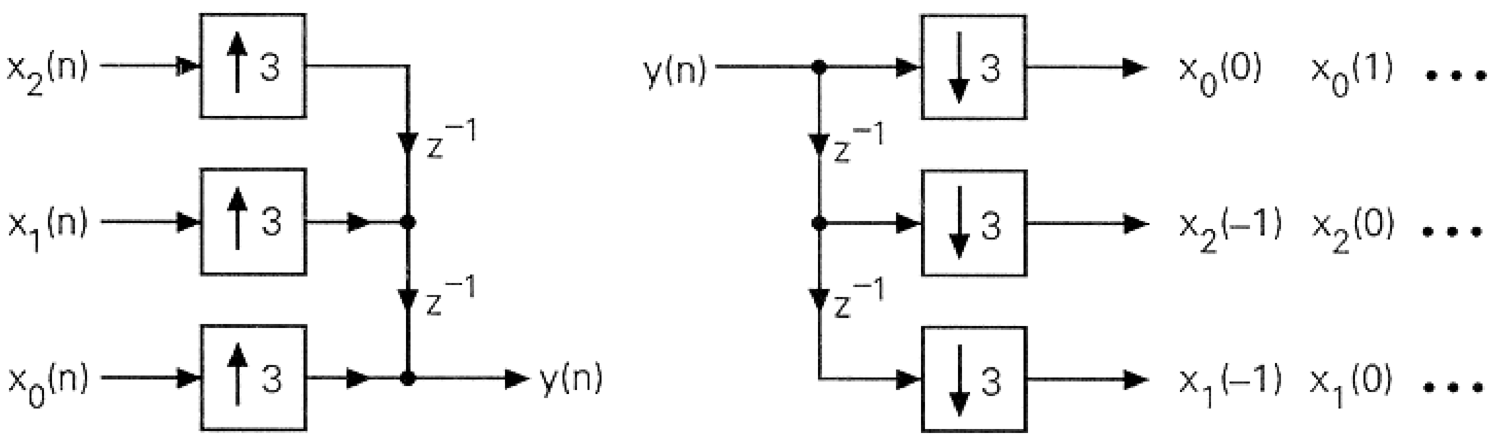
Applications of Multirate systems

Analog voice privacy systems

Multirate adaptive filters

Shynk [1992]

Transmultiplexers

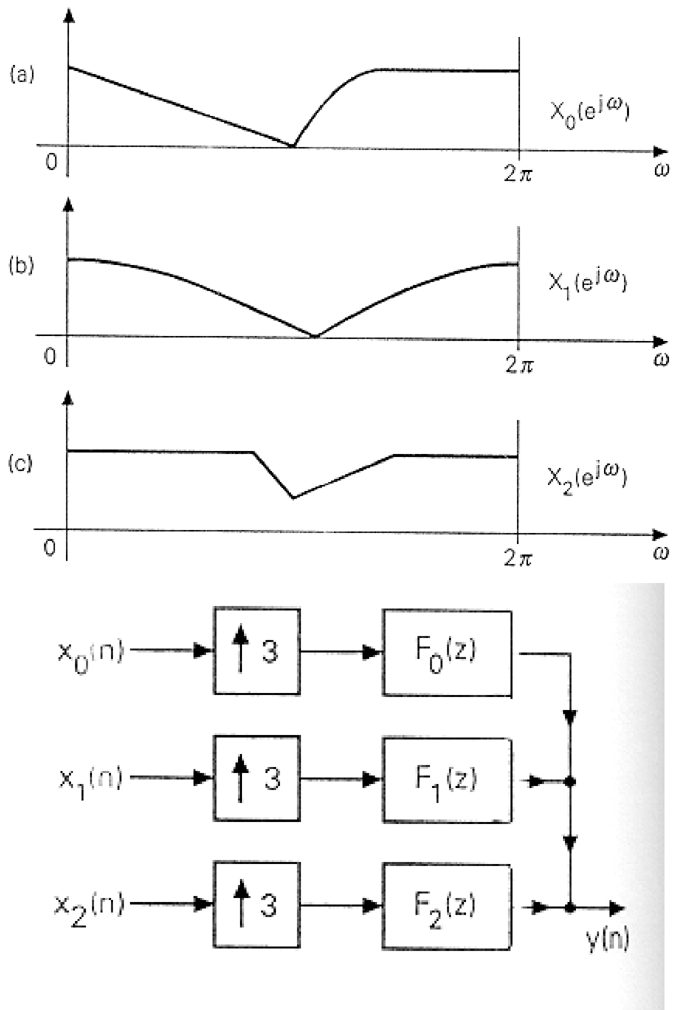


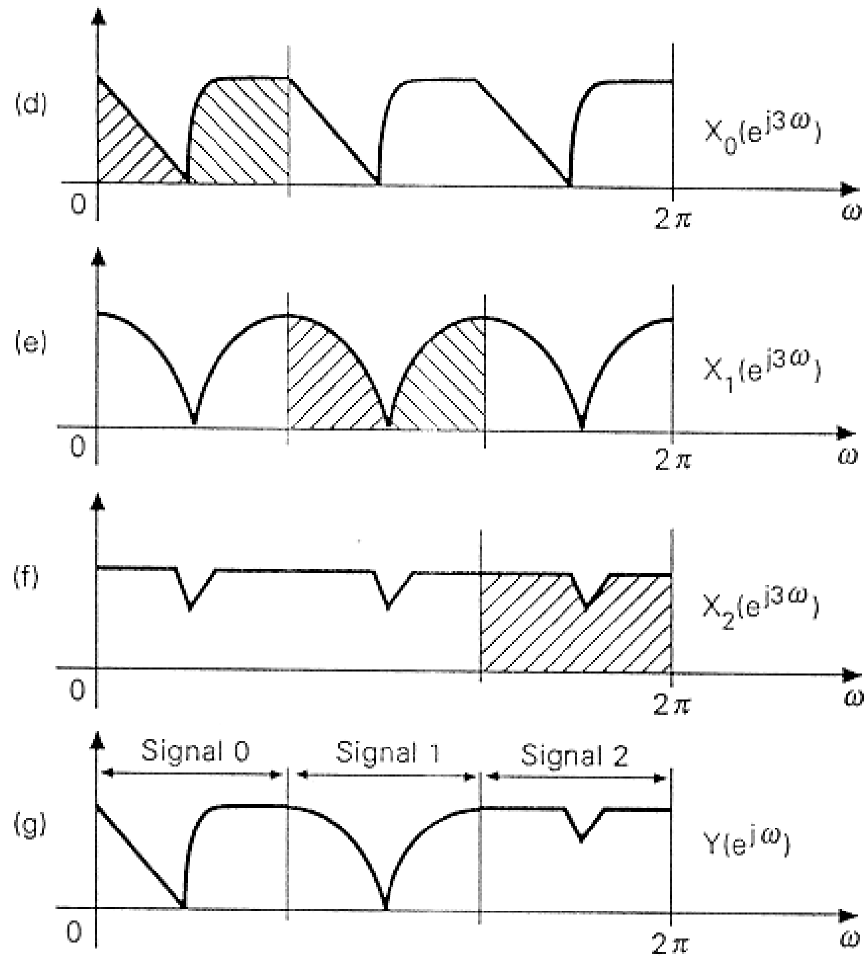
(a) Multiplexing (b) Demultiplexing

Time domain multiplexing *Demultiplexing*

Applications of Multirate systems

Transmultiplexer structure





Applications of Multirate systems

Transmultiplexer structure