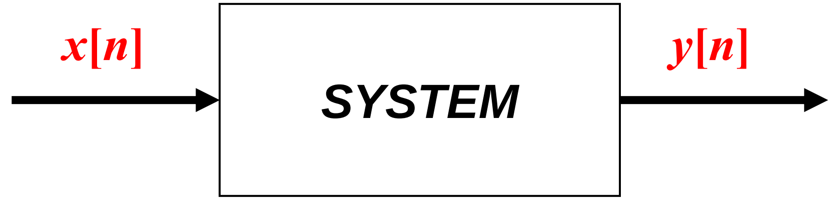
Chapter 5
FIR Filters

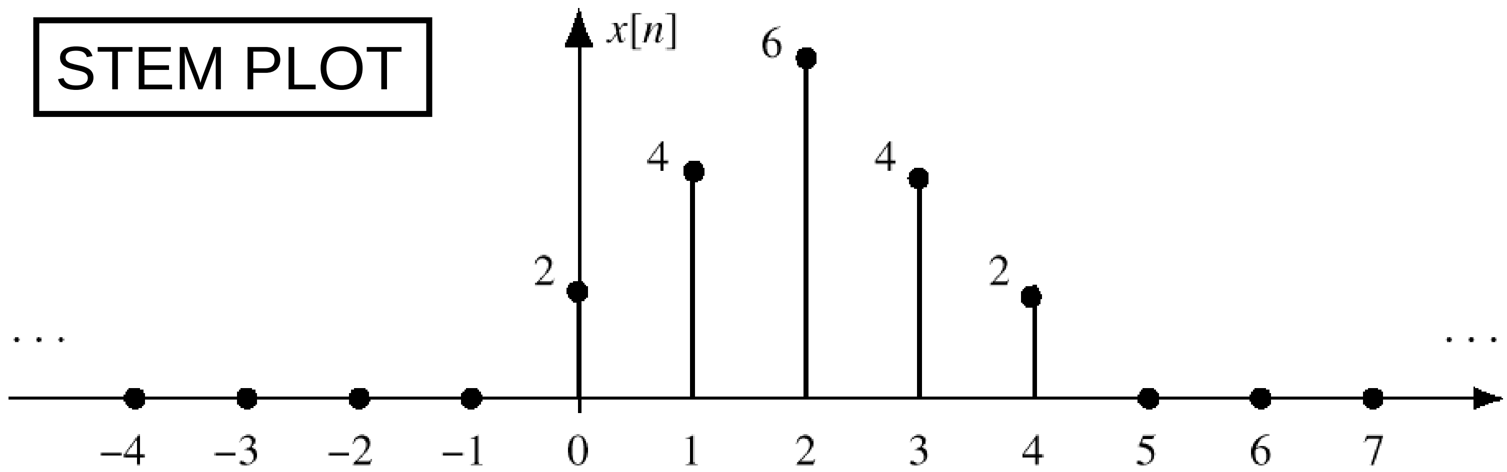
DISCRETE-TIME SYSTEM



- OPERATE on $x[n]$ to get $y[n]$.
- WANT a **GENERAL** CLASS of SYSTEMS.
  - **ANALYZE** the SYSTEM.
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - **SYNTHESIZE** the SYSTEM.
- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: “the output at time n is the average of three consecutive input values.”

DISCRETE-TIME SIGNAL

- $x[n]$ is a LIST of NUMBERS.
  - INDEXED by “n”



- Support of a sequence
  - The set of values over which the sequence is nonzero
  - $x[n]$ is a finite-length signal. Its support is $0 \le n \le 4$.

3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS.
  - Do this for each "*n*".

Make a TABLE.

the following input–output equation

Difference equation

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

n=0 $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

n=1 $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

INPUT AND OUTPUT SIGNALS

INPUT SIGNAL

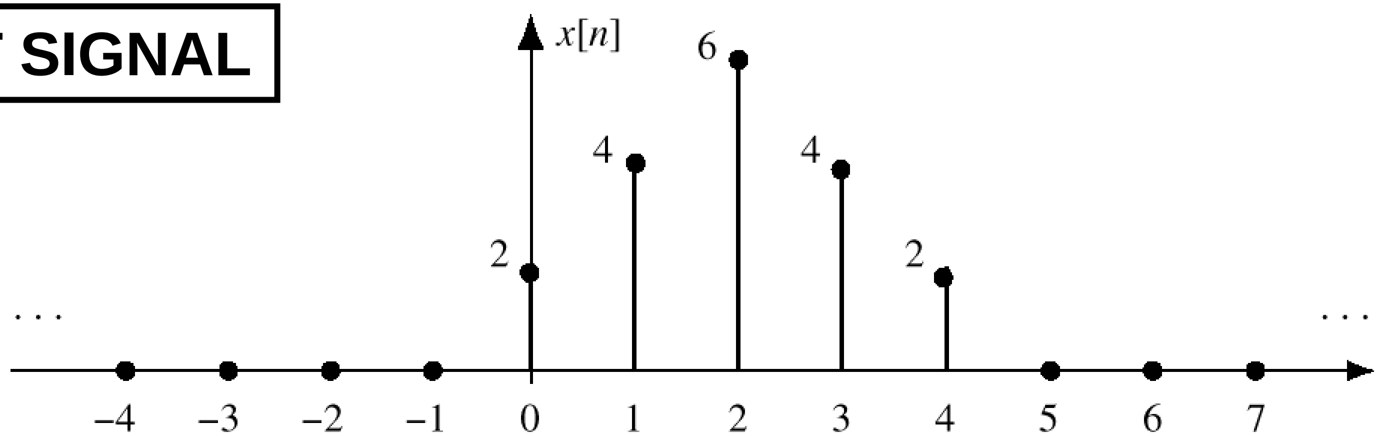


Figure 5.2 Finite-length input signal, $x[n]$.

OUTPUT SIGNAL

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

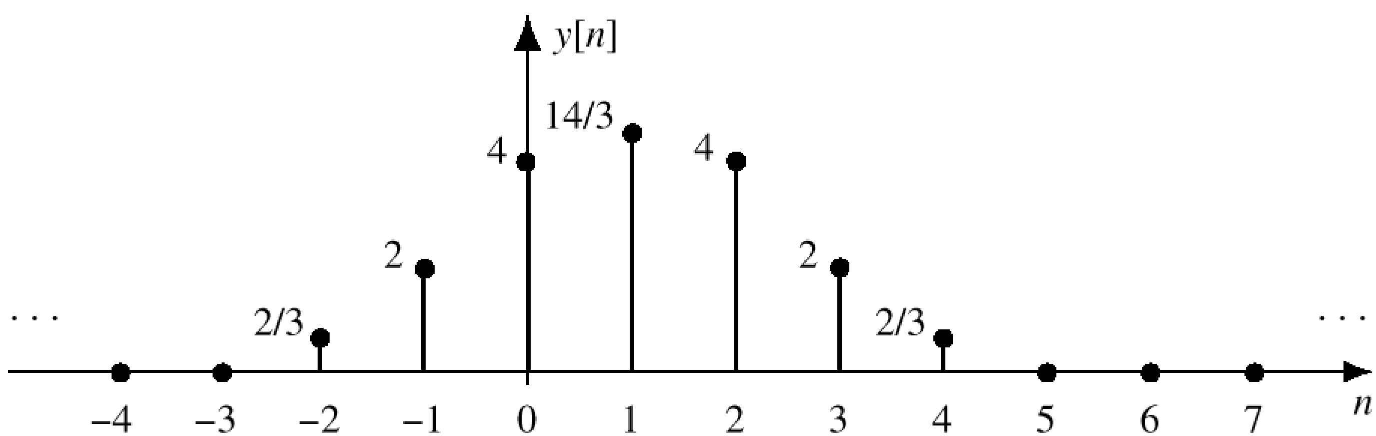


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, AND FUTURE

Sec. 5.2 The Running Average Filter **123**

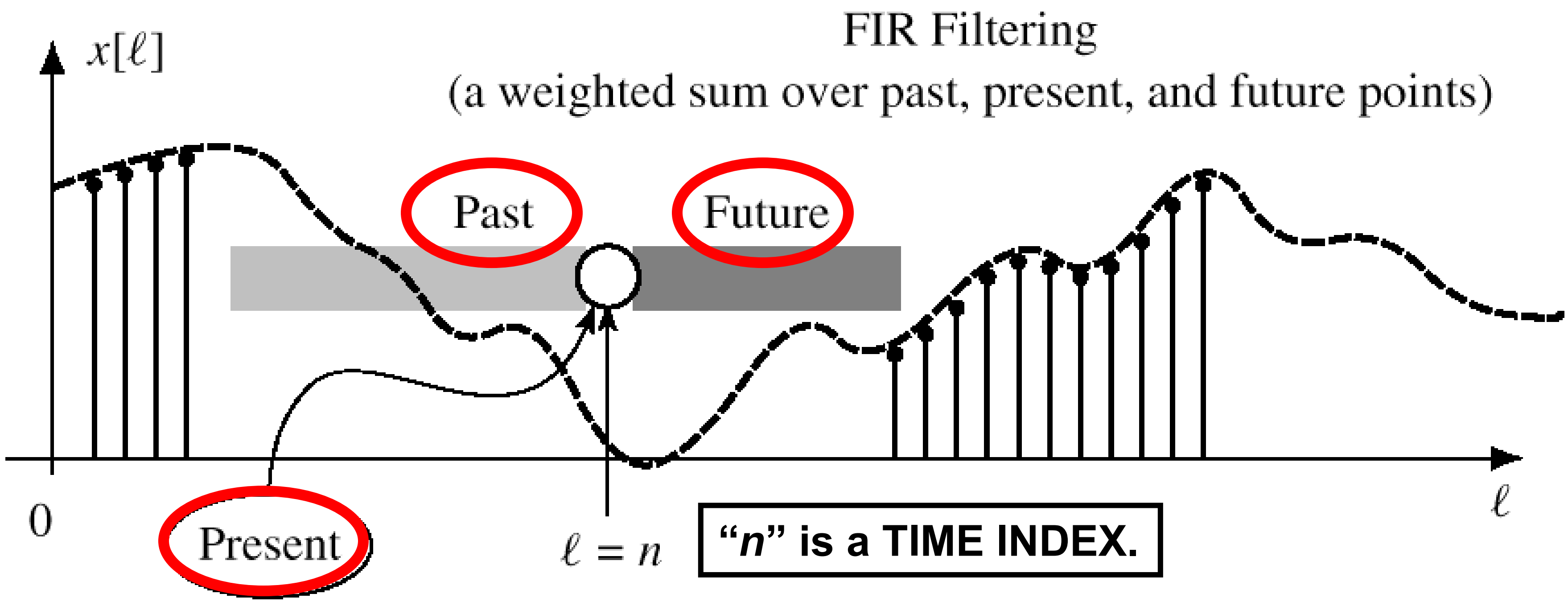


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

ANOTHER 3-PT AVERAGER

- Use “PAST” VALUES of *x*[*n*].
  - IMPORTANT IF “*n*” represents the PRESENT TIME.
    - WHEN *x*[*n*] & *y*[*n*] ARE STREAMS.
  - CAUSAL SYSTEM: Use PRESENT and PAST VALUES

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

  - Noncausal system: Use future values.

GENERAL FIR FILTER (1)

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER.

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $\boxed{b_k = \{3, -1, 2, 1\}}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

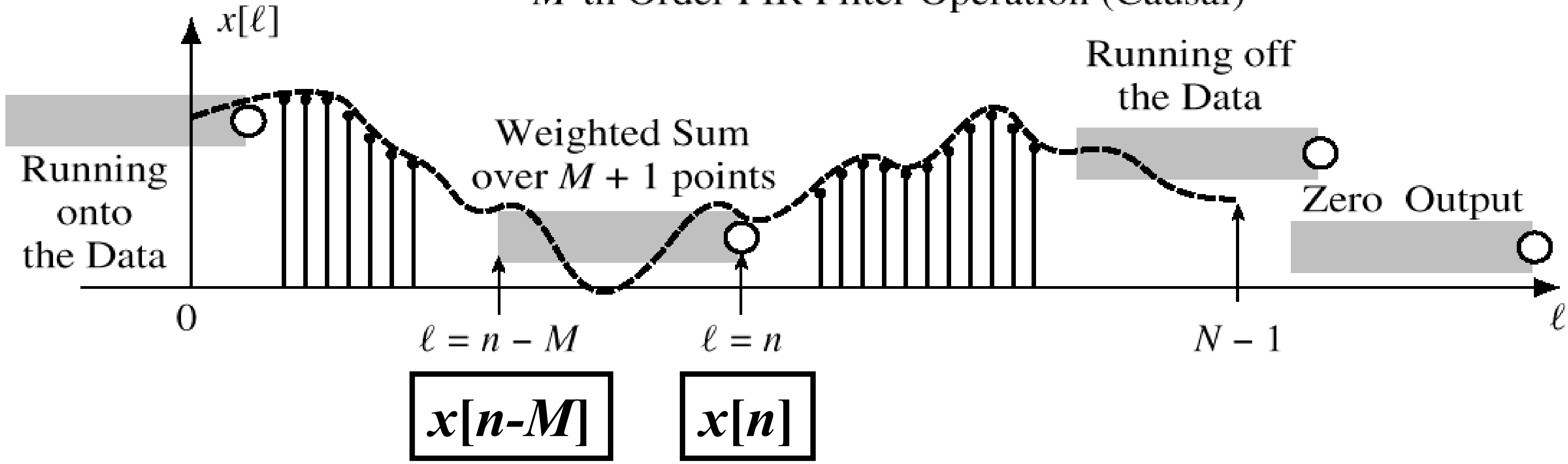
  - FILTER **ORDER** is M.
  - FILTER **LENGTH** is $L = M+1$.
    - The NUMBER of FILTER COEFFICIENTS is L.

GENERAL FIR FILTER (2)

- SLIDE a WINDOW over *x*[*n*].

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

M-th Order FIR Filter Operation (Causal)



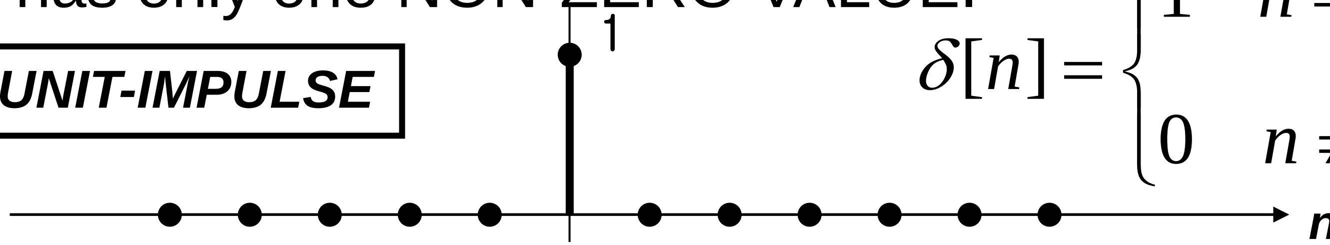
FILTERED STOCK SIGNAL

UNIT IMPULSE SIGNAL $\delta[n]$

- $\delta[n]$ has only one NON-ZERO VALUE.

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



n	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	$\boxed{1}$	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	$\boxed{1}$	0	0	0	0

$\delta[n]$ is NON-ZERO when its argument is equal to ZERO.

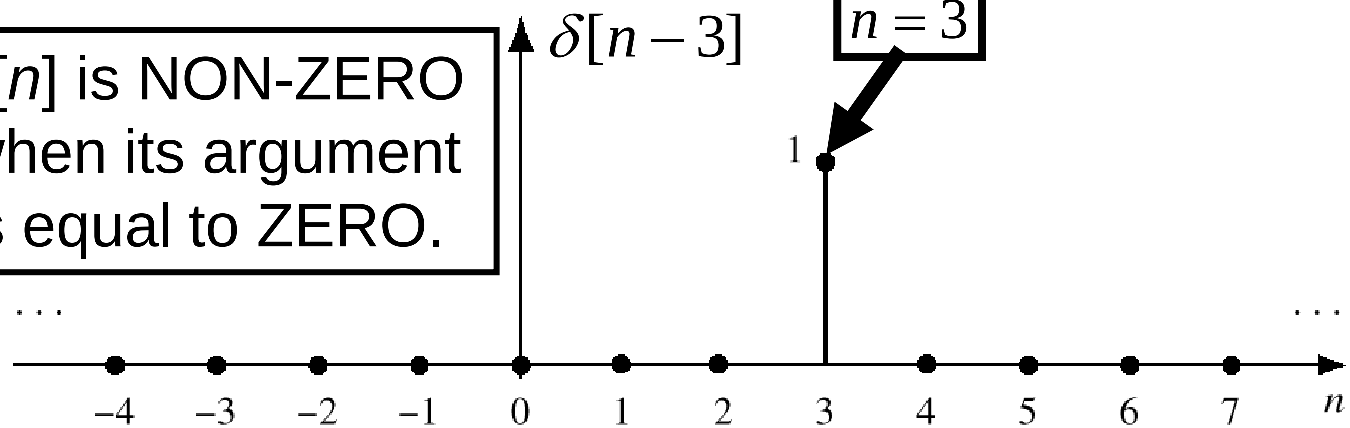
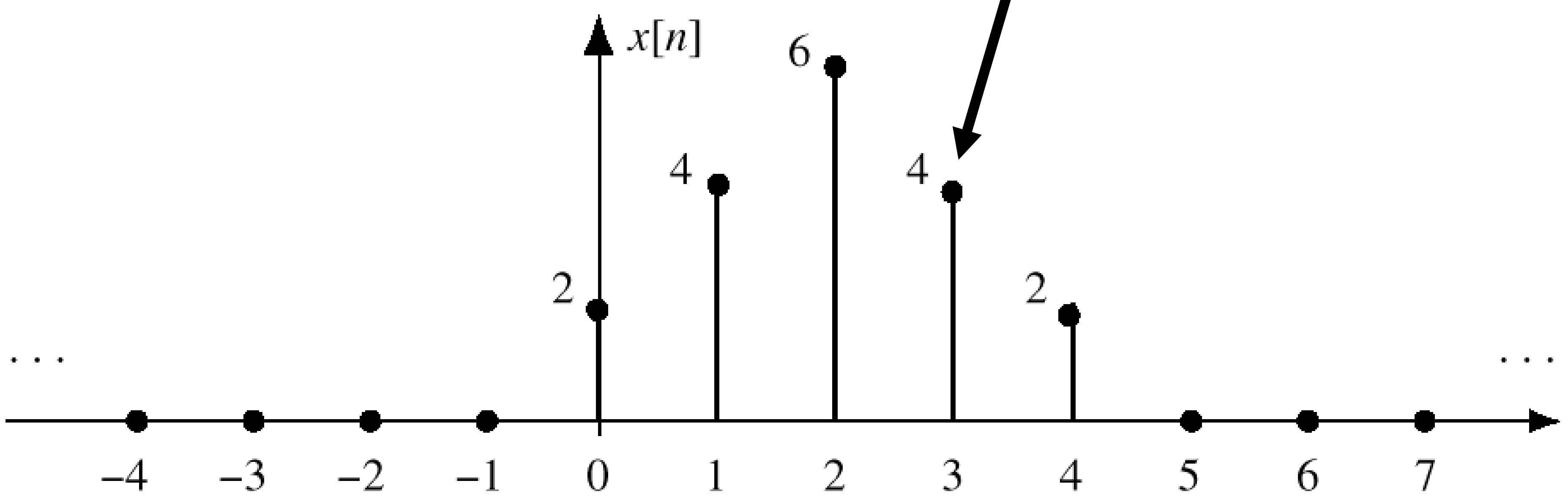


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

MATH FORMULA for $x[n]$

- Use SHIFTED IMPULSES to write $x[n]$.

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$



SUM of SHIFTED IMPULSES

n	$\ldots$	-2	-1	0	1	2	3	4	5	6	$\ldots$
$2\delta[n]$	0	0	0	$\boxed{2}$	0	0	0	0	0	0	0
$4\delta[n-1]$	0	0	0	0	$\boxed{4}$	0	0	0	0	0	0
$6\delta[n-2]$	0	0	0	0	0	$\boxed{6}$	0	0	0	0	0
$4\delta[n-3]$	0	0	0	0	0	0	$\boxed{4}$	0	0	0	0
$2\delta[n-4]$	0	0	0	0	0	0	0	$\boxed{2}$	0	0	0
$x[n]$	0	0	0	2	4	6	4	2	0	0	0

$$x[n] = \sum_{k} x[k]\delta[n-k]$$

This formula ALWAYS works.

$$= \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots \quad (5.3.6)$$

4-pt AVERAGER

- CAUSAL SYSTEM: USE PRESENT and PAST VALUES.

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = $\delta[n]$

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

- OUTPUT is called "IMPULSE RESPONSE".

$$h[n] = \{\ldots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \ldots\}$$

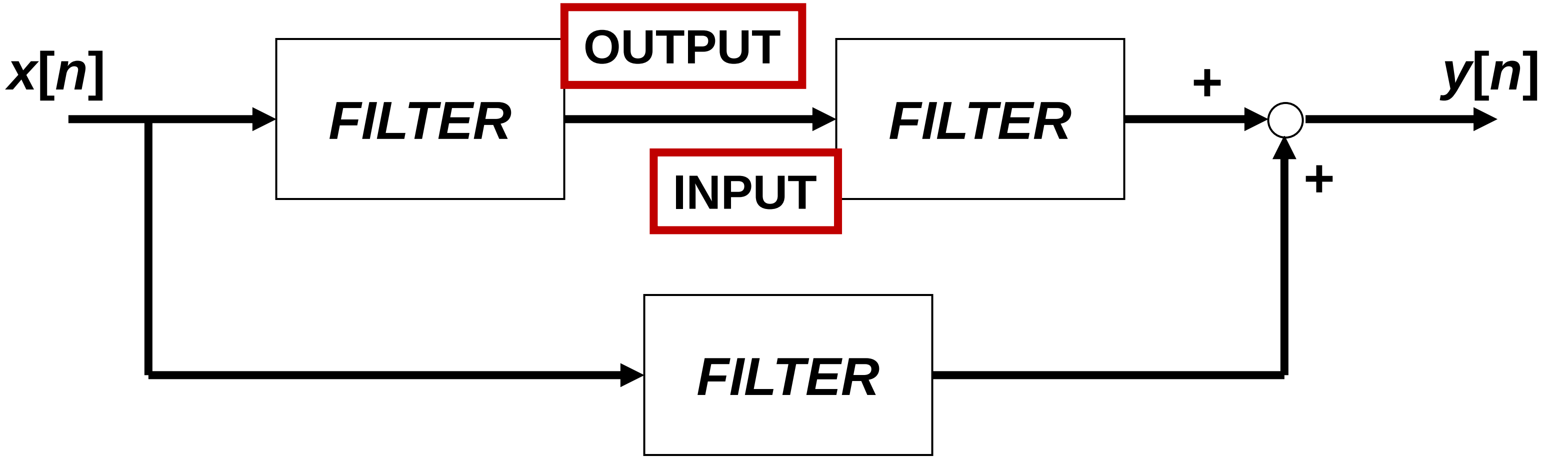
  - Finite impulse response (FIR) system
    - The length of the impulse response is finite.

HARDWARE STRUCTURES

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- INTERNAL STRUCTURE of “FILTER”
  - WHAT COMPONENTS ARE NEEDED?
  - HOW DO WE “HOOK” THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

BUILDING BLOCKS

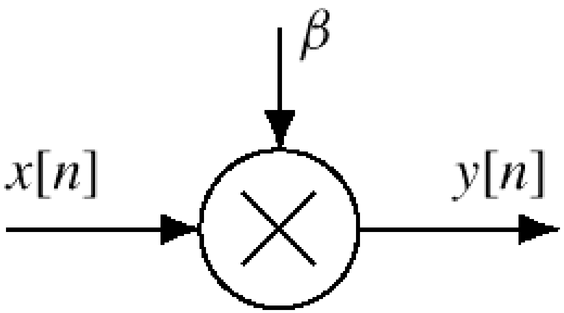


- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE A 3-PT FIR FILTER.

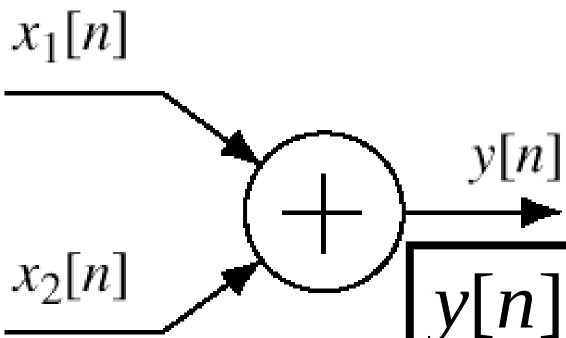
HARDWARE ATOMS

- Add, Multiply, & Store

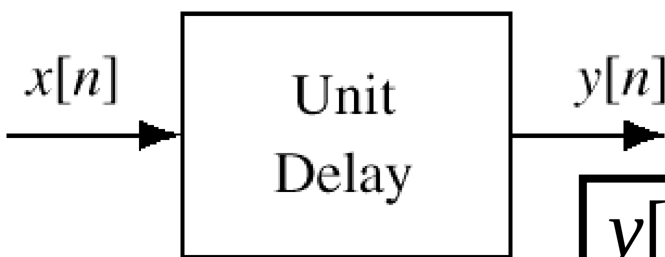
$$\boxed{y[n] = \sum_{k=0}^{M} b_k x[n-k]}$$



$$\boxed{y[n] = \beta x[n]}$$



$$\boxed{y[n] = x_1[n] + x_2[n]}$$

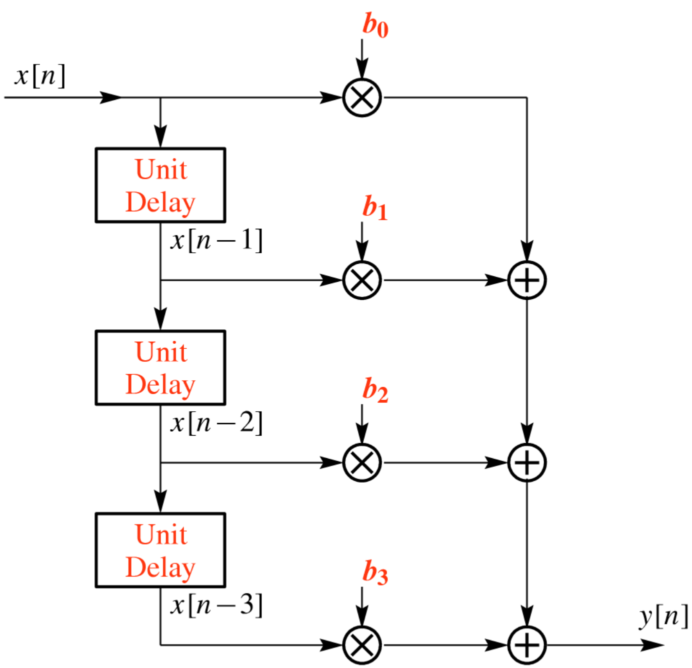


$$\boxed{y[n] = x[n-1]}$$

FIR STRUCTURE

$$\boxed{y[n] = \sum_{k=0}^{M} b_k x[n-k]}$$

- Direct Form



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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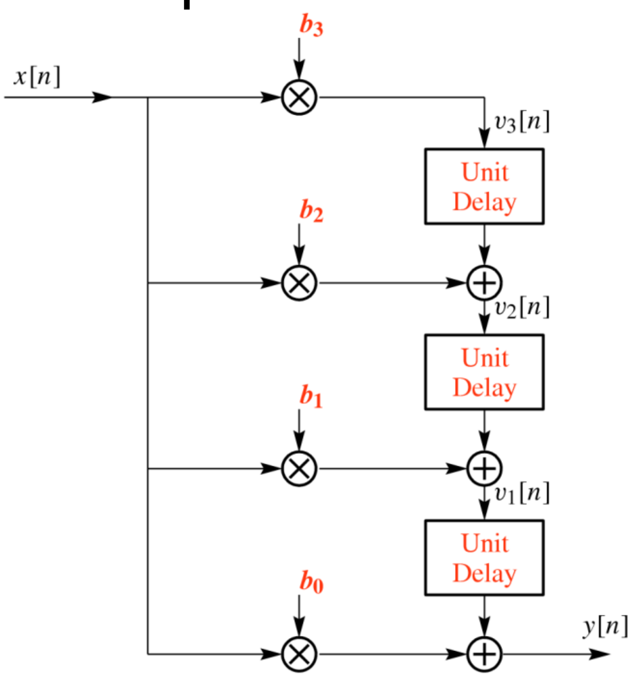
- Transposed Form



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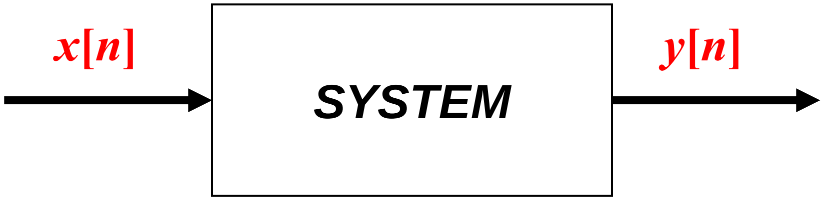
SYSTEM PROPERTIES



- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
  - “No output prior to input”

TIME-INVARIANCE

- IDEA:
  - “Time-shifting the input will cause the same time-shift in the output.”
  - EQUIVALENTLY, we can prove that the time origin (n=0) is picked arbitrarily.

$$y[n-n_0]=T(x[n-n_0])$$

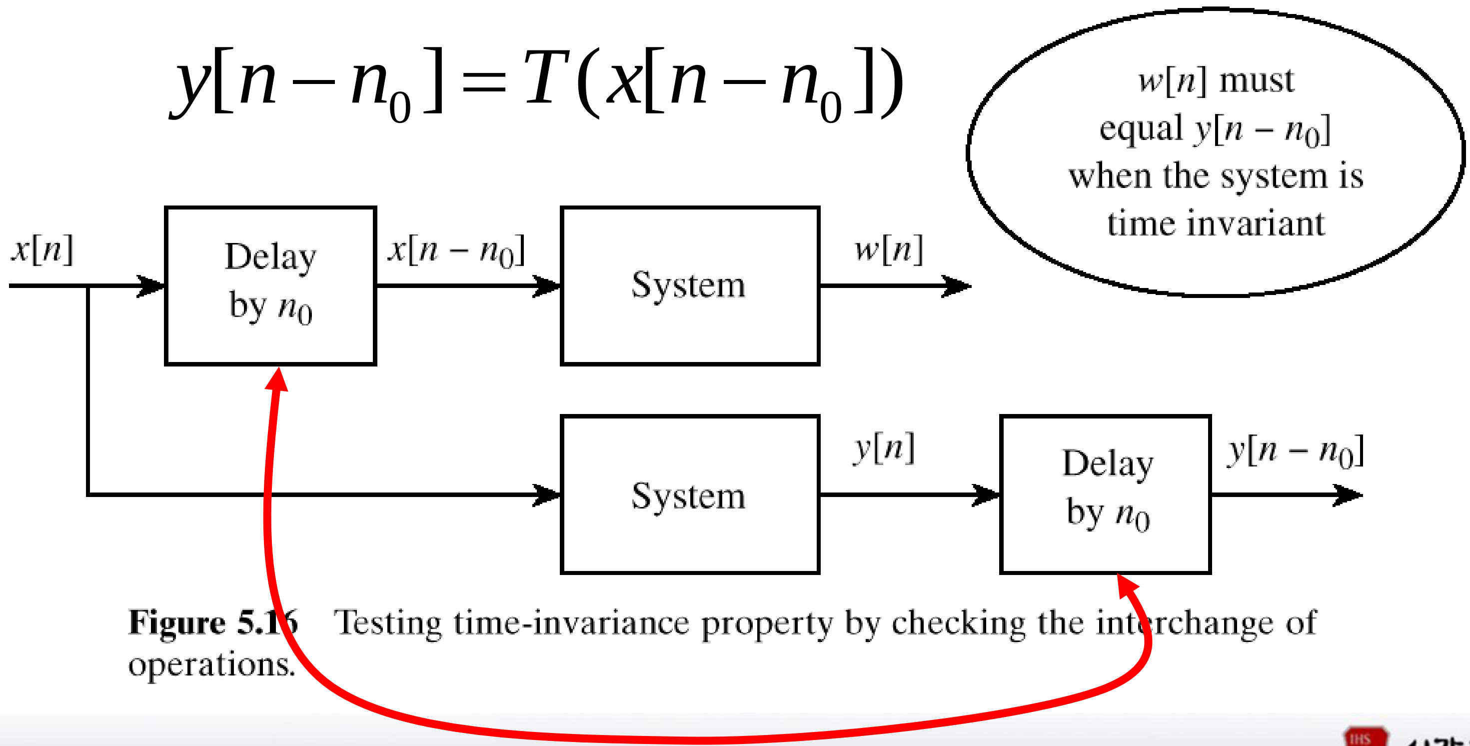


Figure 5.16 Testing time-invariance property by checking the interchange of operations.

LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
  - “Doubling $x[n]$ will double $y[n]$.”
- SUPERPOSITION
  - “Adding two inputs gives an output that is the sum of the individual outputs.”

$$\alpha y_1[n] + \beta y_2[n]$$
$$= T(\alpha x_1[n] + \beta x_2[n])$$

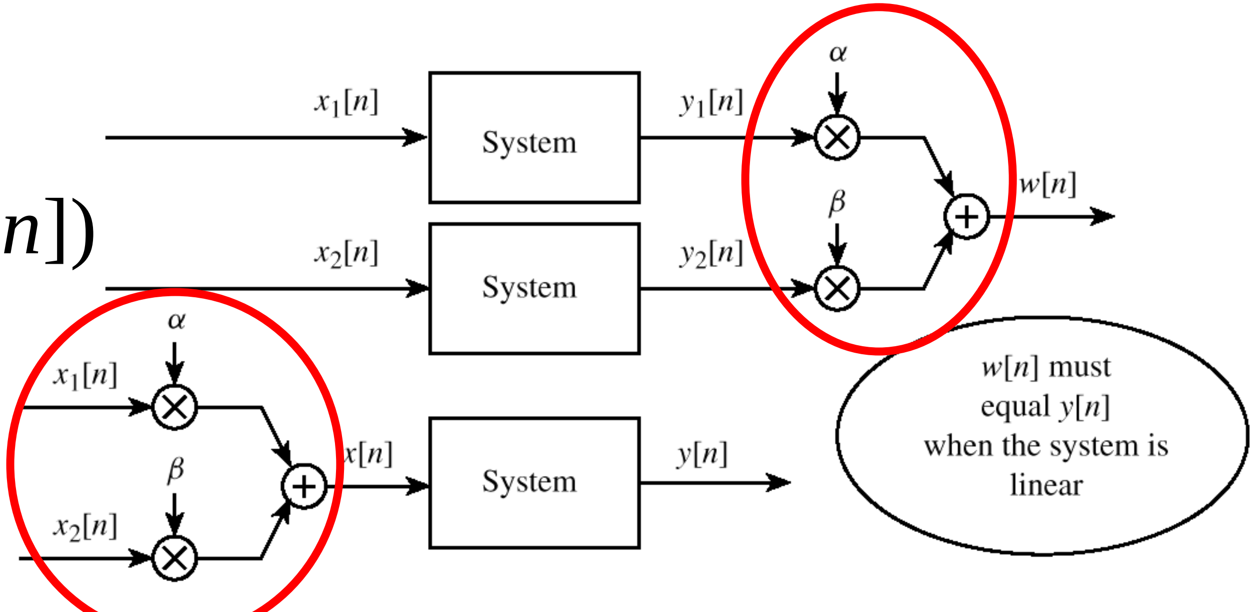


Figure 5.17 Testing linearity by checking the interchange of operations.

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **IMPULSE RESPONSE** $h[n]$
  - **CONVOLUTION**: $y[n] = x[n]*h[n]$
    - The "rule" defining the system can ALWAYS be written as convolution.
- Example (FIR filter): $h[n]$ is the same as b_k.

FIR IMPULSE RESPONSE

- Filter output

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

n	$n<0$	0	1	2	3	$\ldots$	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	$\ldots$	b_M	0	0

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

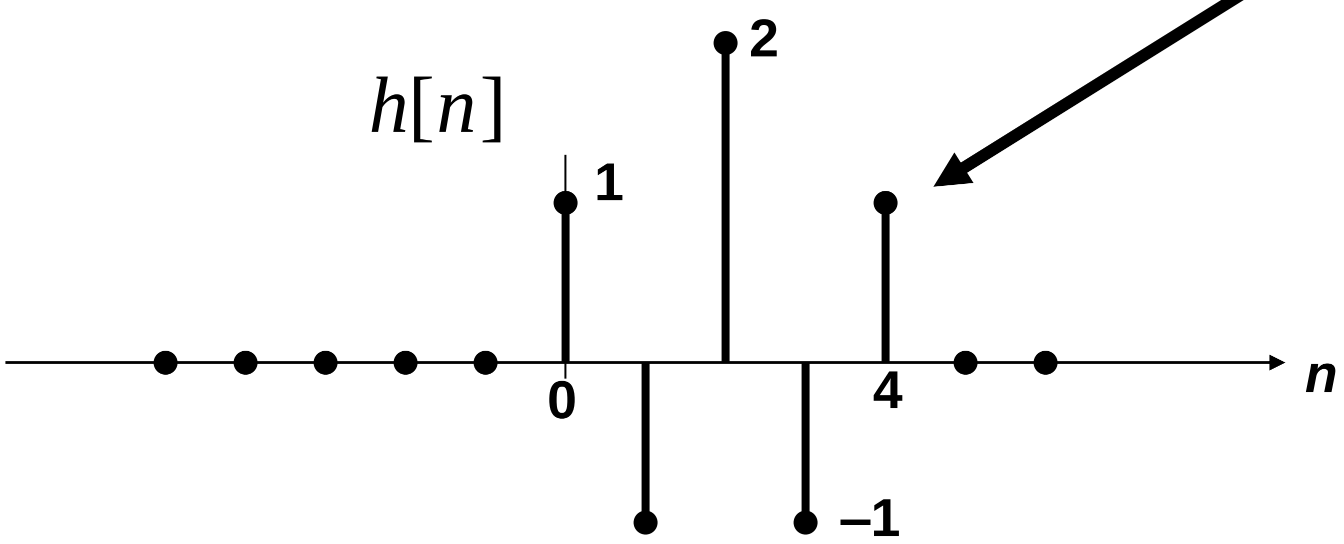
CONVOLUTION

MATH FORMULA for *h*[*n*]

$$b_k = \{\, 1, -1, 2, -1, 1 \,\}$$

- Use SHIFTED IMPULSES to write ***h***[***n***]

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



Convolution Sum

- Output = Convolution of $x[n]$ & $h[n]$
  - NOTATION: $y[n] = h[n] * x[n]$
  - Here is the FIR case:

FINITE LIMITS

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Same as b_k

FINITE LIMITS

CONVOLUTION Example

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$

$$x[n] = u[n]$$

n	-1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	-1	2	-1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	-1	-1	-1	-1	-1	-1	-1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	-1	-1	-1	-1	-1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

MATLAB for FIR FILTER

- `yy = conv(bb,xx)`
  - VECTOR **bb** contains Filter Coefficients.

- FILTER COEFFICIENTS $\{b_k\}$

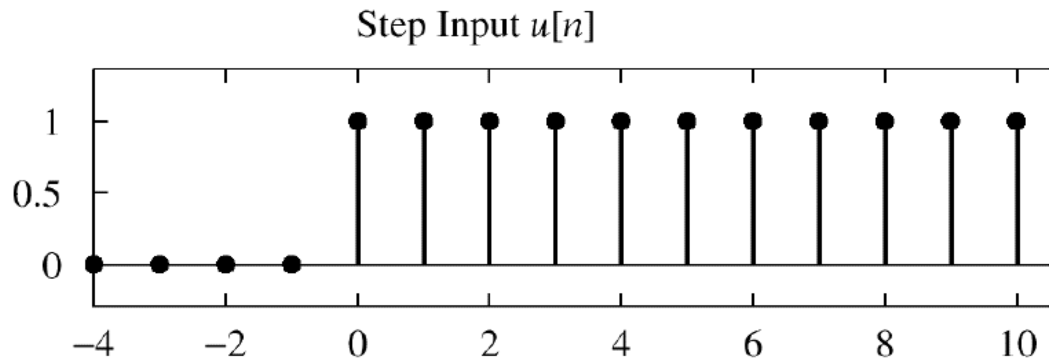
$$\boxed{y[n] = \sum_{k=0}^{M} b_k x[n-k]}$$

conv2()
for images

Example

- An FIR Filter is "FIRST DIFFERENCE".
  - *y*[*n*] = *x*[*n*] - *x*[*n*-1]
- An INPUT is "UNIT STEP".

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



- Find ***y***[***n***].

$$y[n] = u[n] - u[n-1] = \delta[n]$$

Example

- An FIR Filter is “FIRST DIFFERENCE”.
  - ***y*[*n*] = *x*[*n*] - *x*[*n*-1]**
- Write output as a convolution.
  - Need impulse response.

$$h[n] = \delta[n] - \delta[n-1]$$

  - Then, another way to compute the output:

$$y[n] = \left(\delta[n] - \delta[n-1]\right) * x[n]$$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
- Find "overall" $h[n]$ for a cascade.

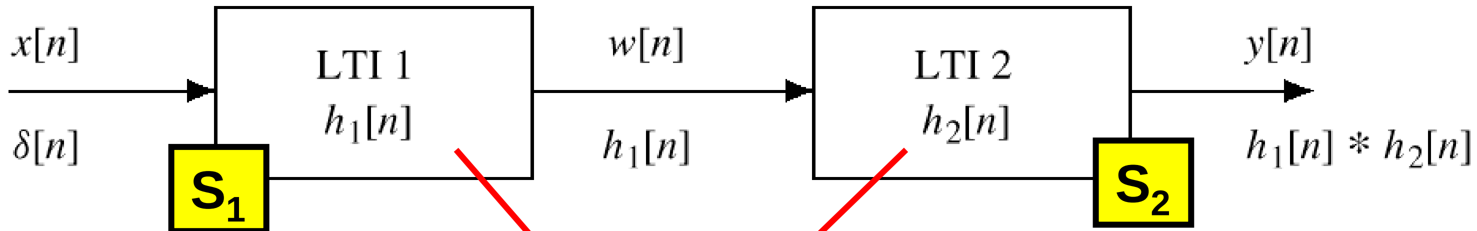


Figure 5.19 A Cascade of Two LTI Systems.

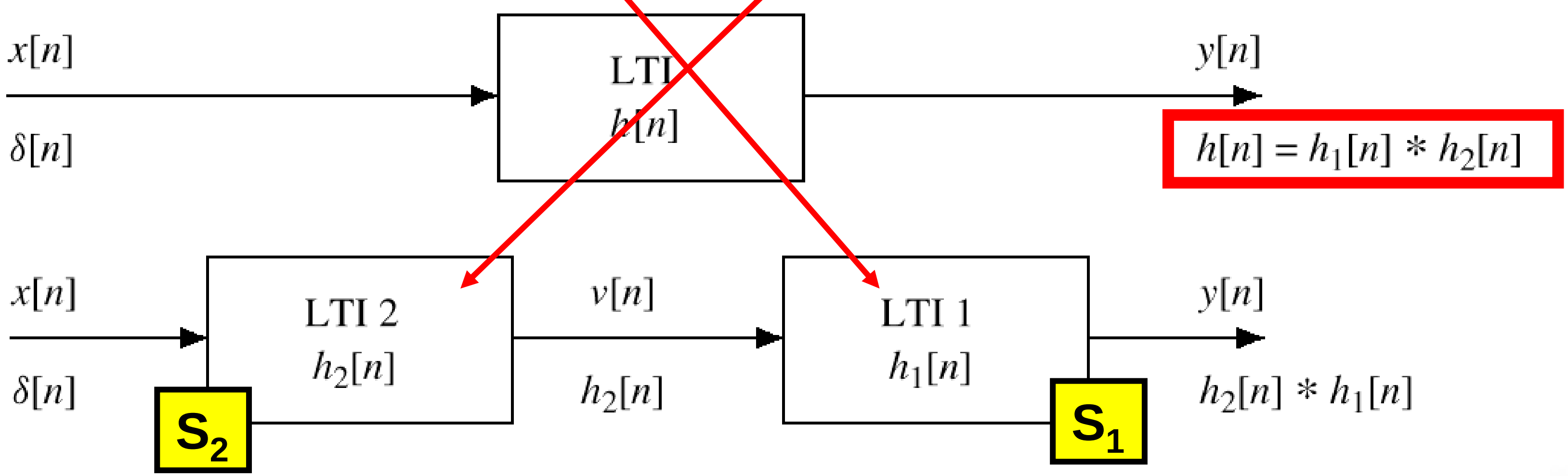


Figure 5.20 Switching the order of cascaded LTI systems.

Thank you

- Homework
  - P-5.2, 3, 5, 6, 7, 9(a,d), 11, 12, 14(a,b), 15(c), & 17
- Reading assignment
  - Section 6-1~5