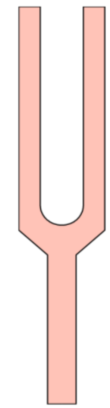
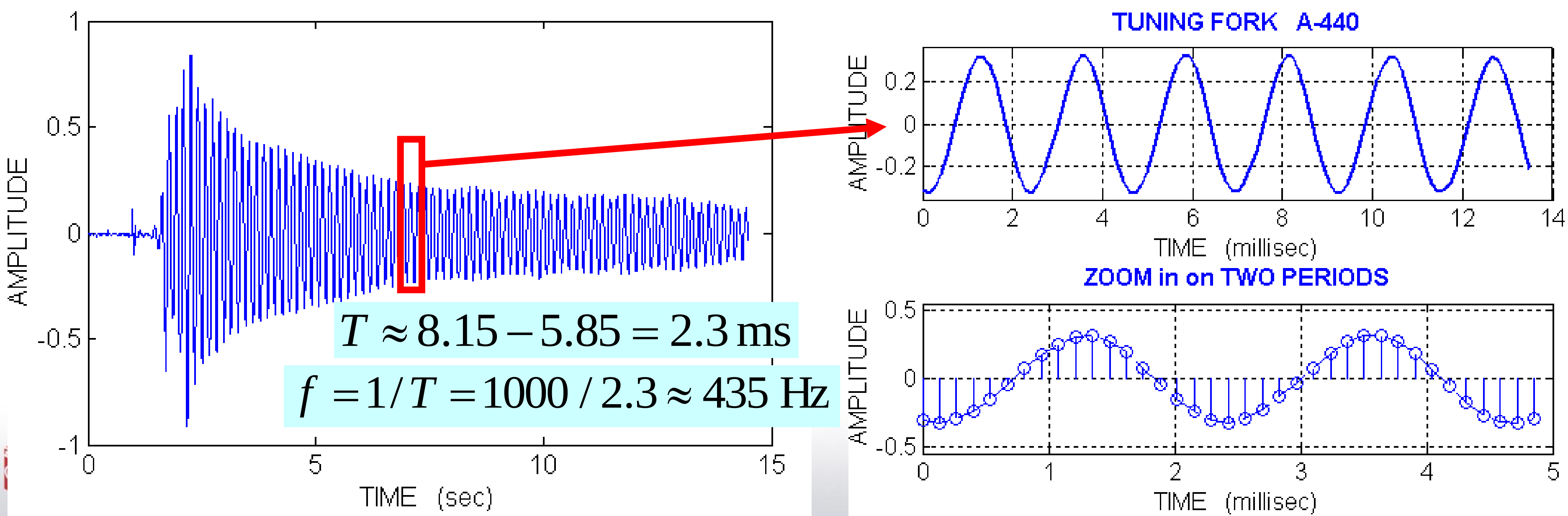
Chapter 2
Sinusoids

Signal

- What is a signal?
  - It's a functions of time, ***x*(*t*)** in the mathematical sense.
- Sound from a tuning fork
  - Waveform is a sinusoidal signal.
  - Computer plot looks like a sine wave.
  - This should be the mathematical formula:

$$A\cos(2\pi(440)t+\varphi)$$



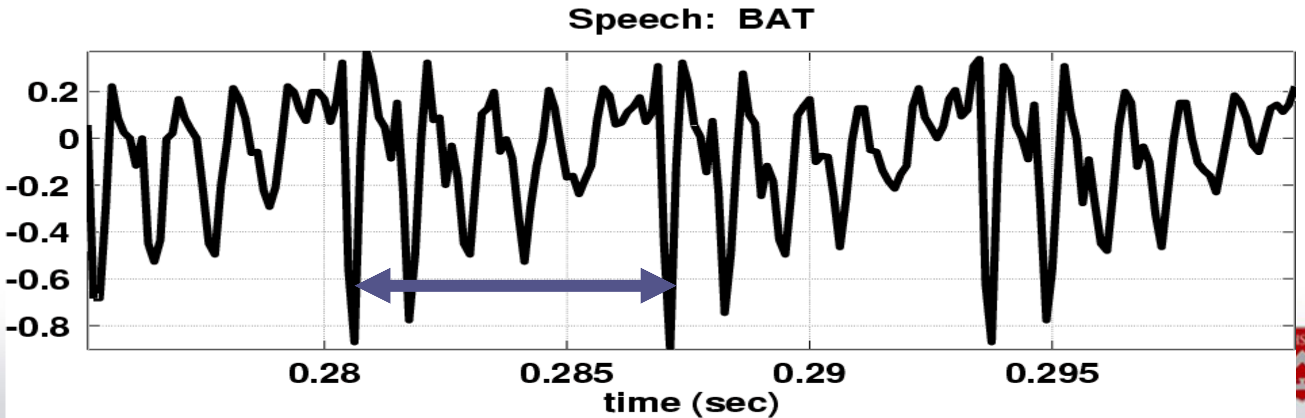


$$T \approx 8.15 - 5.85 = 2.3\ \mathrm{ms}$$

$$f = 1/T = 1000/2.3 \approx 435\ \mathrm{Hz}$$

Speech Signal

- More complicated signal (BAT.WAV)
- Waveform ***x*(*t*)** is NOT a Sinusoid.
- Theory will tell us
  - ***x*(*t*)** is approximately a sum of sinusoids.
  - FOURIER ANALYSIS
    - Break ***x*(*t*)** into its sinusoidal components.
  - Called the FREQUENCY SPECTRUM
- Nearly **<u>Periodic</u>** in Vowel Region
  - Period is (approximately) $T = 0.0065$ sec.



DIGITIZE the WAVEFORM

- ***x*[*n*]** is a SAMPLED SINUSOID.
  - A list of numbers stored in memory
- Sample at 11,025 samples per second
  - Called the SAMPLING RATE of the A/D
  - Time between samples is
    - 1/11025 = 90.7 microsec.
- Storing digital sound
  - The sampling rate of a CD is 44,100 samples per second.
  - 16-bit samples
  - Stereo uses 2 channels.
  - The number of bytes for 1 minute is
    - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes.

SINUSOIDAL SIGNAL

- COSINE FORM

$$A\cos(2\pi(440)t+\varphi)$$

- Relationship

$$\sin(\omega t)=\cos(\omega t-\tfrac{\pi}{2})$$

- Sinusoidal signal

$$A\cos(\omega t+\varphi)$$

  - **FREQUENCY** $\boxed{\omega}$
    - Radians/sec
    - Hertz (cycles/sec)

$$\omega=(2\pi)f$$

$$T=\frac{1}{f}=\frac{2\pi}{\omega}$$

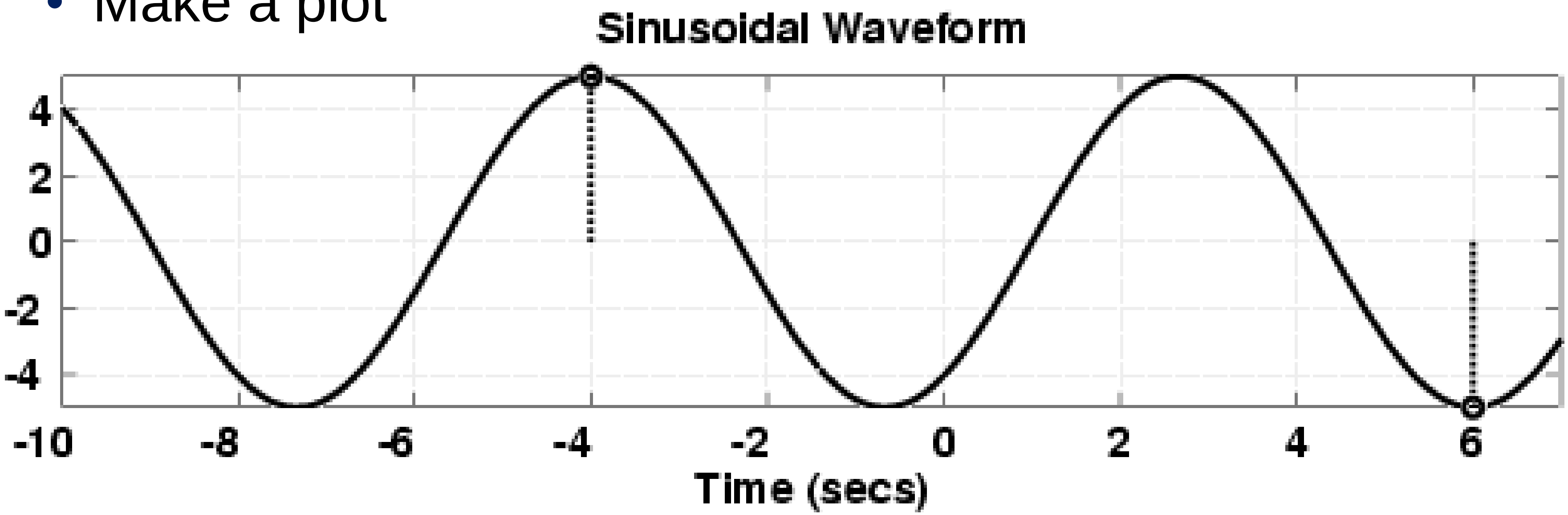
  - **PERIOD** (in sec)
  - **AMPLITUDE** $\boxed{A}$
    - Magnitude
  - **PHASE** $\boxed{\varphi}$

EXAMPLE of a SINUSOID

- Given the Formula

$$5\cos(0.3\pi t + 1.2\pi)$$

- Make a plot



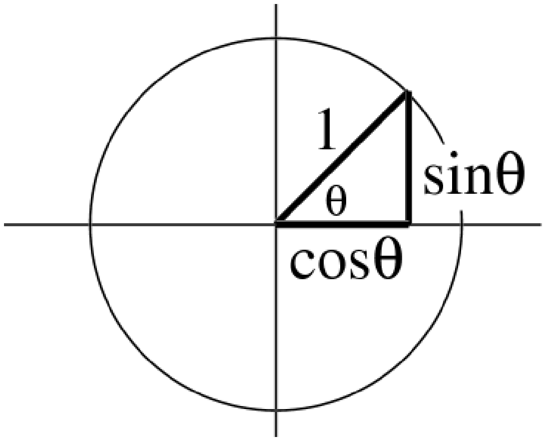
- The formula defines *A*, ω, and ϕ.

$$A = 5,\ \omega = 0.3\pi,\ \varphi = 1.2\pi$$

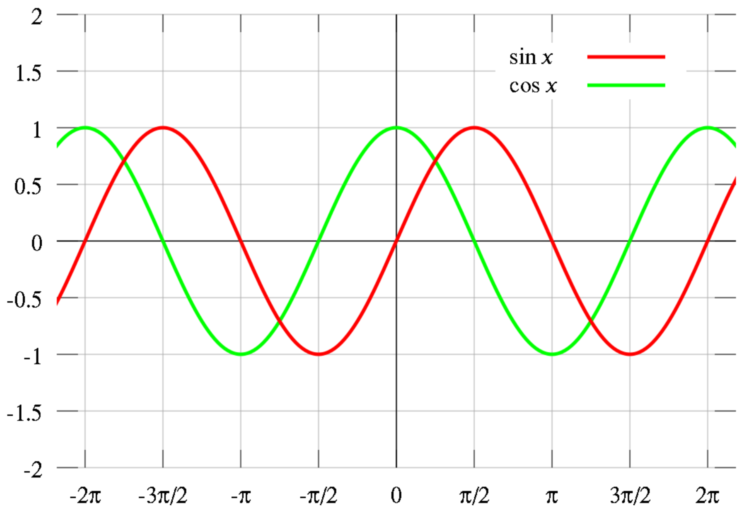
$$\boxed{T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3}$$

TRIGONOMETRIC FUNCTIONS

- Trigonometric functions



- Common values
  - $\sin(k\pi) = 0$
  - $\cos(0) = 1$
  - $\cos(2k\pi) = 1$, $\cos((2k+1)\pi) = -1$
  - $\cos((k+0.5)\pi) = 0$



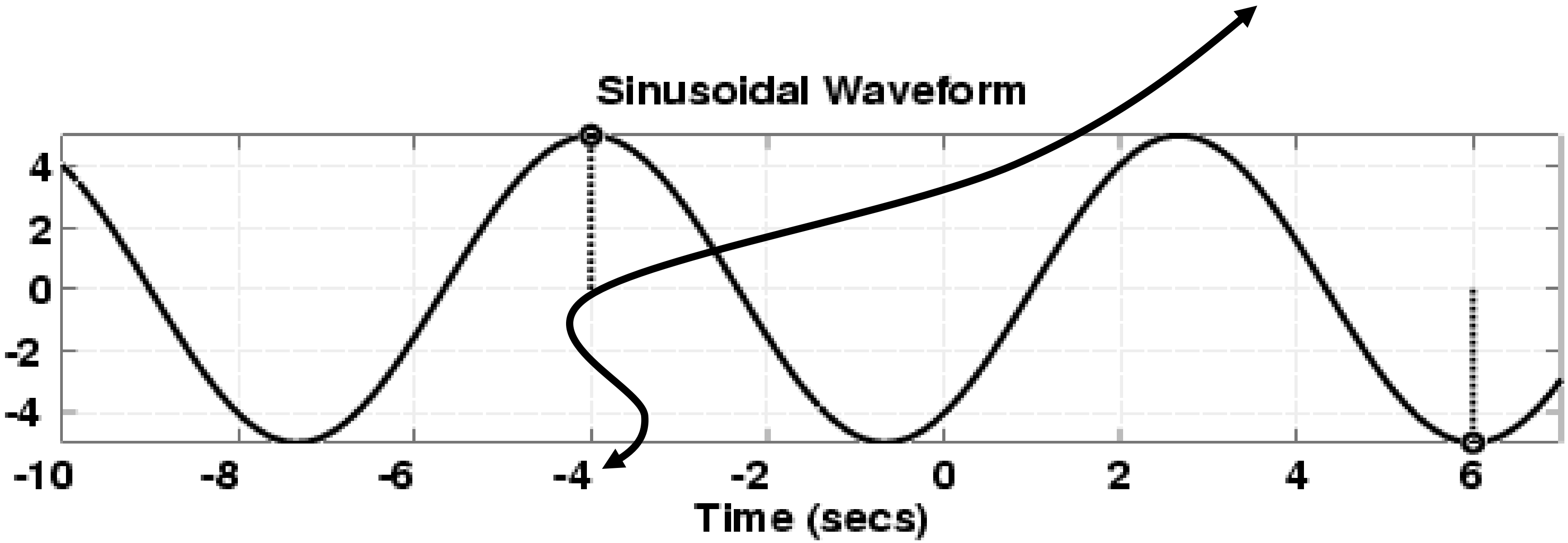
TIME-SHIFT

- In a mathematical formula we can replace *t* with $t\text{-}t_m$.

$$x(t - t_m) = A\cos(\omega(t - t_m))$$

- Then the t=0 point moves to $t=t_m$.
- Peak value of $\cos(\omega(t\text{-}t_m))$ is now at $t=t_m$.
- Time-shifted sinusoid

$$x(t+4) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



PHASE <–> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_m)) = A\cos(\omega t+\varphi)$$

- and we obtain: $\boxed{-\omega t_m = \varphi}$

- or, $\boxed{t_m = -\frac{\varphi}{\omega}}$

SINUSOID from a PLOT

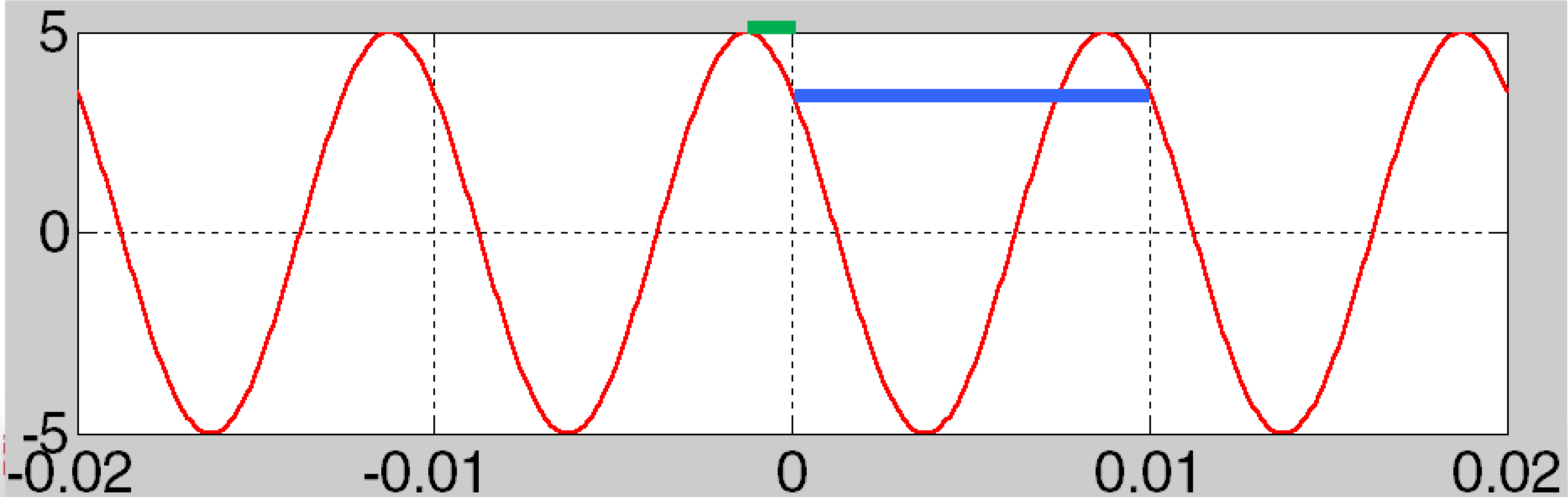
- Measure the period, *T*.
  - Between peaks or zero crossings
  - Compute the frequency: $\omega = 2\pi/T$.
- Measure the time of a peak: t_m.
  - Compute the phase: $\phi = -\omega t_m$.
- Measure the height of a positive peak: *A*.

$$T = \frac{0.01\text{sec}}{1\,\text{period}} = \frac{1}{100}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

$$t_m = -0.00125\ \text{sec}$$

$$\varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$



PHASE is AMBIGUOUS.

- The cosine signal is periodic.
  - The period is 2π.

$$A\cos(\omega t+\varphi+2\pi)=A\cos(\omega t+\varphi)$$

  - Thus, adding any multiple of 2π leaves $x(t)$ unchanged.

$$\text{If}\quad t_m=\frac{-\varphi}{\omega},\quad \text{then}$$

$$t_{m_2}=\frac{-(\varphi+2\pi)}{\omega}=\frac{-\varphi}{\omega}-\frac{2\pi}{\omega}=t_m-T$$

  - Principal value of the phase shift
    - The value of phase shift that falls between $-\pi$ and $+\pi$.

Thank you

- Reading assignment
  - ~ Section 2-5