Chapter 3 : STACKS AND QUEUES

Data Structures Lecture Note

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3.1 STACKS

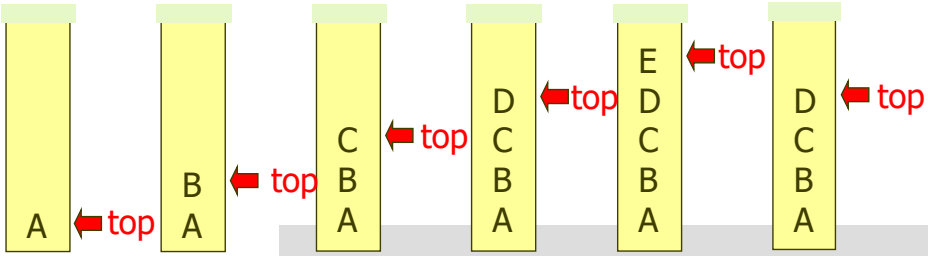
- The *stacks* and *queues* are special cases of the more general data type, *ordered list*.
- A *stack* is an ordered list in which insertions and deletions are made at one end called the *top*.

$S = (a_0, a_1, \ldots, a_{n-1})$

↑ ↑

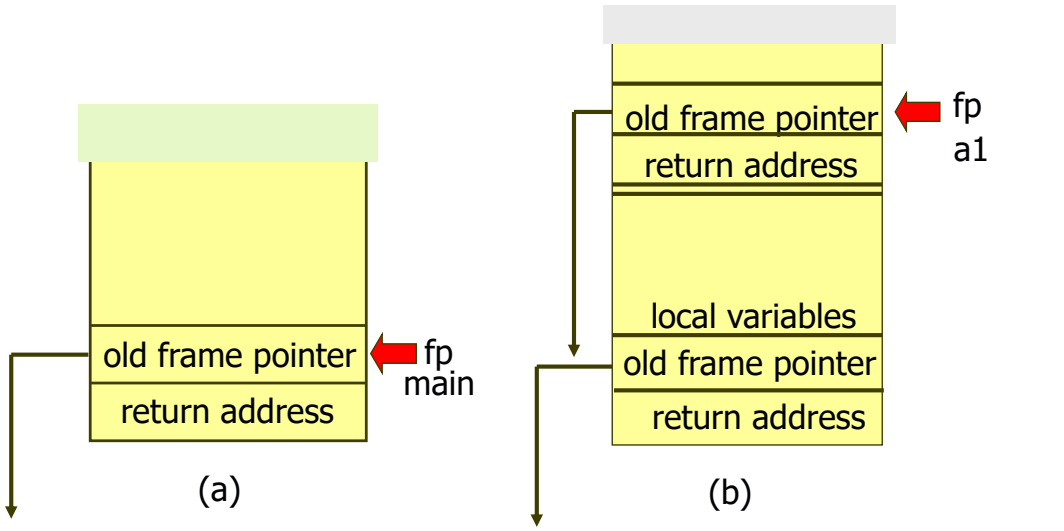
bottom **top**

***Last-In-First-Out (LIFO)* list.**



- **Example 3.1 [System stack] :**

  To process function calls, whenever a function is invoked, the program creates a structure, referred to as *activation record* or *stack frame*



- **[ADT 3.1]: Abstract data type Stack**

ADT *Stack* is

objects : a finite ordered list with zero or more elements.

functions : for all *stack* ∈ *Stack*, *item* ∈ *element*, *maxStackSize* ∈ positive integer

Stack CreateS(*maxStackSize*) ::=
create an empty stack whose maximum size is *maxStackSize*

Boolean IsFull(*stack, maxStackSize*) ::=
If (number of elements in *stack*==*maxStackSize*) **return** *TRUE*
else return *FALSE*

Stack Push(*stack, item*) ::=
if (IsFull(*stack*)) *stackFull*
else insert *item* into top of *stack* and **return**

Boolean IsEmpty(*stack*) ::=
if (*stack*==Create(*maxStackSize*)) **return** *TRUE*
else return *FALSE*

Element Pop(*stack*) ::=
if (IsEmpty(*stack*)) **return**
else remove and return the *item* on the top of the stack.

<Implementation of *Stack*>

Using a one-dimensional array.

```
Stack CreateS(maxStackSize) ::=
   #define  MAX_STACK_SIZE 100   /* maximum stack size */
   typedef  struct  {
         int  key;
         /*  other fields  */
   } element;
   element  stack[MAX_STACK_SIZE];
   int  top = -1;

Boolean IsEmpty(stack) ::= top < 0;
Boolean IsFull(stack) ::= top >= MAX_STACK_SIZE-1;
```


[Program 3.1] (Add to a stack)

```
void push(element item)
{
/* add an item to the global stack */
     if (top >= MAX_STACK_SIZE-1)
          stackFull();
     stack[++top] = item;
}
```

[Program 3.2] (Delete from a stack)

```
element pop()
{
/* return the top element from the stack  */
     if (top == -1)
          return stackEmpty();      /* return an error key */
     return stack[top--];
}
```

3.2 STACKS USING DYNAMIC ARRAYS

```
Stack CreateS() ::=
    typedef  struct  {
        int  key;
        /*  other fields  */
    } element;
    element  *stack;
    MALLOC(stack, sizeof(*stack));
    int capacity = 1;
    int top = -1;

Boolean IsEmpty(stack) ::= top < 0;
Boolean IsFull(stack) ::= top >= capacity-1;

void stackFull()
{
     REALLOC(stack, 2*capacity*sizeof(*stack));
     capacity *= 2;
}
```

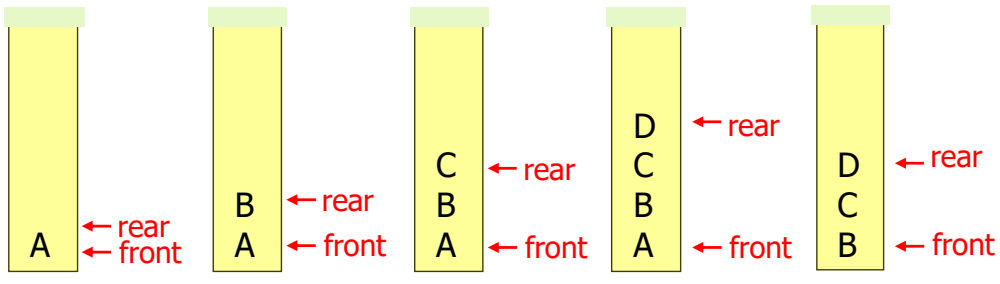

3.3 QUEUES

- A *queue* is an ordered list in which all insertions take place at one end called the *rear* and all deletions take place at the opposite end called the *front*.

$$Q = (a_0, a_1, \ldots, a_{n-1})$$

front ↑ rear ↑

First-In-First-Out (FIFO) list.



[ADT 3.2]: Abstract data type Queue

ADT *Queue* is
  objects : a finite ordered list with zero or more elements.
  functions : for all *queue* ∈ *Queue*, *item* ∈ *element*, *maxQueueSize* ∈ positive integer
    Queue CreateQ(*maxQueueSize*) ::=
      create an empty queue whose maximum size is *maxQueueSize*
    Boolean IsFullQ(*queue, maxQueueSize*) ::=
      if (number of elements in *queue*==*maxQueueSize*) **return** *TRUE*
      else return *FALSE*
    queue AddQ(*queue, item*) ::=
      if (IsFullQ(*queue*)) *queueFull*
      else insert *item* at rear of *queue* and **return**
    Boolean IsEmptyQ(*queue*) ::=
      if (*queue* == CreateQ(*maxQueueSize*)) **return** *TRUE*
      else return *FALSE*
    Element DeleteQ(*queue*) ::=
      if (IsEmptyQ(*queue*)) **return**
      else remove and return the *item* at front of queue.

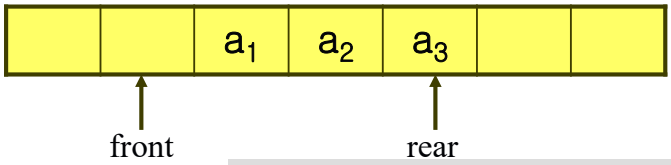
<Implementation of *Queue*>

The simplest scheme employs a one-dimensional array and two variables, *front* and *rear*.

The *front* points the position in front of the first element and the *rear* points the position of the last element.

Thus, we can use a simple condition front == rear to check if the queue is empty.

```
Queue CreateQ(maxQueueSize) ::=
        #define  MAX_QUEUE_SIZE 100   /* maximum queue size */
        typedef  struct  {
                 int  key;
                 /*  other fields  */
                 } element;
        element  queue[MAX_QUEUE_SIZE];
        int rear = -1;
        int front = -1;

  Boolean IsEmptyQ(queue) ::=  front == rear;
  Boolean IsFullQ(queue) ::=  rear == MAX_QUEUE_SIZE-1;
```



- **[Program 3.5] Add to a queue**

```
void addq(element item)
{
/* add an item to the queue */
     if (rear == MAX_QUEUE_SIZE-1)
          queueFull();
     queue[++rear] = item;
}
```

- **[Program 3.6] Delete from a queue**

```
element deleteq()
{
/* remove element at the front of the queue  */
     if (front == rear)
          return queueEmpty();  /* return an error key */
     return queue[++front];
}
```

- **Example 3.2 [Job scheduling] :**

A job queue by an operating system.

If the operating system does not use priorities, then the jobs are processed in the order they enter the system.

- **[Figure 3.5] : Insertion and deletion from a sequential queue**

front	rear	Q[0]	Q[1]	Q[2]	Q[3]	Comments
-1	-1					queue is empty
-1	0	J1				Job1 is added
-1	1	J1	J2			Job2 is added
-1	2	J1	J2	J3		Job3 is added
0	2		J2	J3		Job1 is deleted
1	2			J3		Job2 is deleted

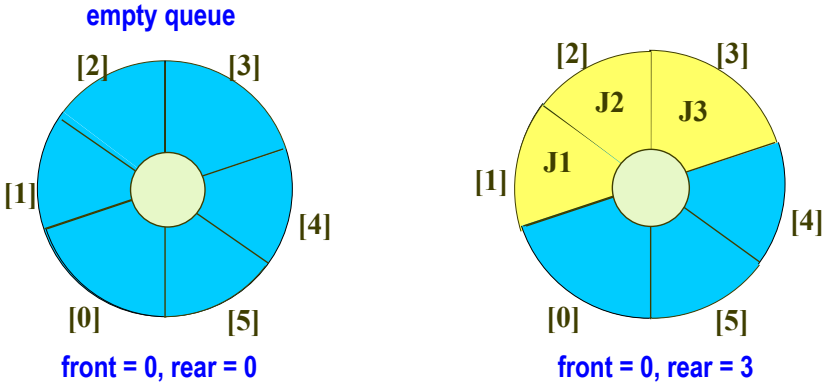
- **Handling of *queueFull* :**
  - As jobs enter and leave the system, the queue gradually shift to the right.
  - Eventually the rear index equals MAX_QUEUE_SIZE - 1, suggesting that the queue is full.
  - In this case, *queueFull* should move the entire queue to the left so that the first element is again at *queue*[0] and *front* is at -1.
  - The *rear* is also recalculated.
  - Shifting an array is very time-consuming.

- **A more efficient implementation :**

  By regarding the array *queue*[MAX_QUEUE_SIZE] as **circular**.

  The *front* index always points one position counterclockwise from the first element in the queue.

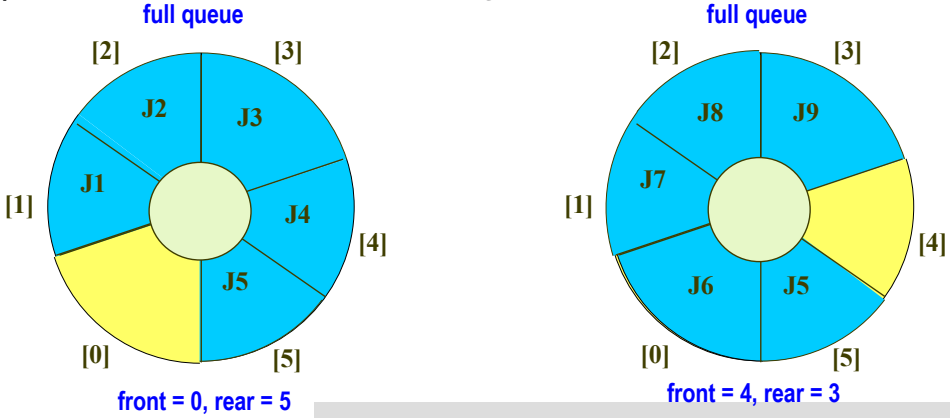
  The *rear* index points to the current end of the queue.



- The queue is empty iff *front == rear*.

- **When is the queue full?**

  To distinguish between an empty and a full queue, we adapt the convention that a circular queue of size MAX_QUEUE_SIZE will be permitted to hold at most MAX_QUEUE_SIZE-1 elements.



- To implement *addq* and *deleteq* for a circular queue, we must assure that a circular rotation occurs.
- Using the modulus operator :

  rear = (rear + 1) % MAX_QUEUE_SIZE;

  front = (front + 1) % MAX_QUEUE_SIZE
- Initial values:

  rear = 0;

  front =0;

- **[Program 3.7] : Add to a circular queue**

```
void addq(element item)
{   /* add an item to the queue */
    rear = (rear + 1) % MAX_QUEUE_SIZE;
    if (front == rear)
        queueFull(); /* print error and exit */
    queue[rear] = item;
}
```


- **[Program 3.8] : Delete from a circular queue**

```
element deleteq()
{   /* remove element at the front of the queue  */
    if (front == rear)
        return queueEmpty();  /* return an error key */
    front = (front + 1) % MAX_QUEUE_SIZE;
    return queue[front];
}
```

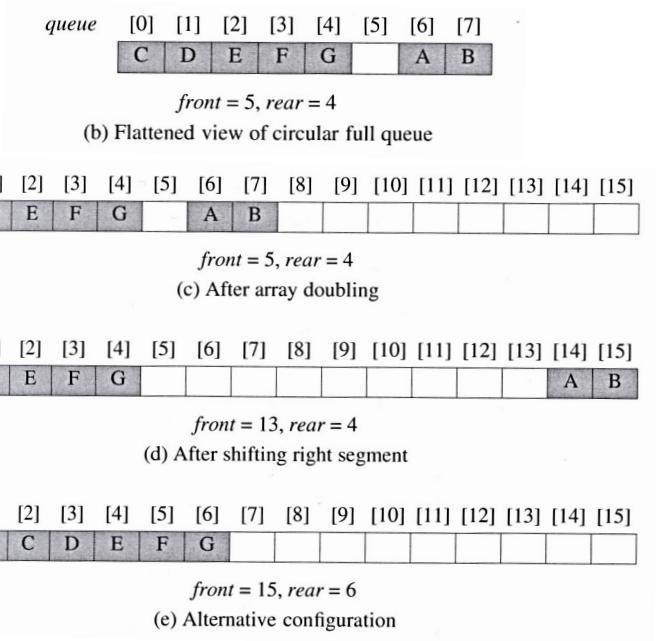
3.4 CIRCULAR QUEUES USING DYNAMICALLY ALLOCATED ARRAYS

- To add an element to a full queue, we must first increase the size of this array using a function such as *realloc*.
- As with dynamically allocated stacks, we use array doubling.

- **< Doubling queue >**

Let *capacity* be the number of positions in the array *queue* .

(1) Create a new array *newQueue* of twice the *capacity*.
(2) Copy the second segment to positions in *newQueue* beginning at 0.
(3) Copy the first segment to positions in *newQueue* beginning at *capacity-front-1*.



[Program 3.9] : Add to a circular queue

```
void addq(element item)
{ /* add an item to the queue */
    rear = (rear + 1) % capacity;
    if (front == rear)
        queueFull(); /* double capacity */
    queue[rear] = item;
}
```


[Program 3.10] : Doubling queue capacity

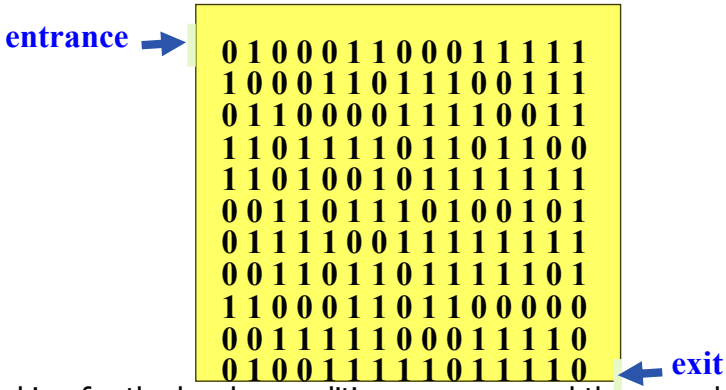
```
void queueFull()
{
    /* allcoate an array with twice the capacity */
    element* newQueue;
    MALLOC(newQueue, 2 * capacity * sizeof(*queue));

    /* copy from queue to newQueue */
    int start = (front +1) % capacity;
    if (start < 2)  /* no wrap around */
        copy(queue+start, queue+start+capacity-1, newQueue);
    else
    { /* queue wraps around */
        copy(queue+start, queue+capacity, newQueue);
        copy(queue, queue+rear+1, newQueue+capacity-start);
    }

    /* switch to newQueue */
    front = 2 * capacity -1;
    rear = capacity – 2;
    capacity *= 2;
    free(queue)
    queue = newQueue;
}
```

copy(*a*,*b*,*c*): copy elements from locations *a* thru *b*-1 to locations beginning at *c*

3.5 A MAZING PROBLEM

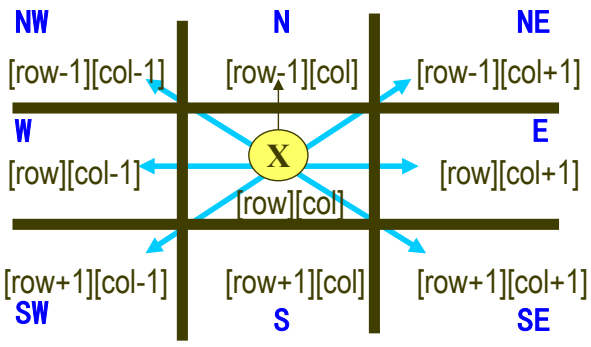
Representation of a maze :

- Two-dimensional array in which 0's represent the open paths and 1's the barriers.



- To avoid checking for the border conditions we surround the maze by a border of 1's. Thus an m×p maze will require an (m+2)×(p+2) array.
- The entrance is at position [1][1] and the exit at [m][p].

- Predefine the possible directions to move in an array, *move*.




```
typedef struct {
    short int vert;
    short int horiz;
    } offsets;

offsets move[8];    /*array of moves for each direction*/
```

- **[Figure 3.10]: Table of moves**

Name	Dir	move[dir].vert	move[dir].horiz
N	0	-1	0
NE	1	-1	1
E	2	0	1
SE	3	1	1
S	4	1	0
SW	5	1	-1
W	6	0	-1
NW	7	-1	-1

- If we are at position, *maze*[*row*][*col*], we can find the position of the next move, *maze*[*nextRow*][*nextCol*], by setting
  nextRow = row + move[dir].vert;
  nextCol = col + move[dir].horiz;
- To record the maze positions already checked, we maintain a second two-dimensional array, *mark*.
- A stack is used to store the positions on the path from the entrance to the current position.

- **[Program 3.11] : Initial maze program**

```
initialize a stack to the maze's entrance coordinates and direction to north;
while (stack is not empty)  {
      /* move to position at the top of stack  */
      <row,col,dir> = delete from top of stack;
      while (there are more moves from current position)  {
            <nextRow, nextCol> = coordinates of next move;
             dir = direction of move;
             if ((nextRow==EXIT_ROW) && (nextCol==EXIT_COL))
             success;
             if ((maze[nextRow][nextCol]==0) &&(mark[nextRow][nextCol]==0)) {
                   /* legal move and haven't been there */
                    mark[nextRow][nextCol] = 1;
                  /* save current position and direction */
                   add <row,col,dir> to the top of the stack;
                  row = nextRow;  col = nextCol;  dir = north;
            }
      }
}
printf("No path found");
```

- **Defining a stack:**

```
#define MAX_STACK_SIZE 100
typedef struct  {
short int row;
short int col;
short int dir;
}  element;
element stack[MAX_STACK_SIZE];
```

- Need to determine a reasonable bound for the stack size.

[Figure 3.11] Simple maze with a long path

0	0	0	0	0	0	1
1	1	1	1	1	1	0
1	0	0	0	0	0	1
0	1	1	1	1	1	1
1	0	0	0	0	0	1
1	1	1	1	1	1	0
1	0	0	0	0	0	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0

- **[Program 3.12] : Maze search function**

we assumed that arrays, *maze, mark, move, stack*, and constants *EXIT_ROW, EXIT_COL, TRUE, FALSE*, and variable, *top*, are declared as global.

```
void path(void)
{
 /*  output a path through the maze if such a path exists  */
      int i, row, col, nextRow, nextCol, dir, found = FALSE;
      element position;
      mark[1][1] = 1;  top = 0;
      stack[0].row = 1; stack[0].col = 1; stack[0].dir = 0;
      while  (top > -1 && !found)  {
            position = pop();
            row = position.row;  col = position.col, dir = position.dir;
            while  (dir < 8 && !found)  {
                  /*  move in direction dir  */
                  nextRow = row + move[dir].vert;
                  nextCol = col + move[dir].horiz;
                  if  (nextRow == EXIT_ROW && nextCol == EXIT_COL)
                        found = TRUE;
```

```
                else  if  (!maze[nextRow][nextCol] && !mark [nextRow][nextCol])  {
                        mark [nextRow][nextCol] = 1;
                        position.row = row;  position.col = col;
                        position.dir = ++dir;
                        push(position);
                        row = nextRow;  col = nextCol;  dir = 0;
                }
                else  ++dir;
            }
        }
        if  (found)  {
            printf("The path is : \n");
            printf("row  col \n");
            for  (i = 0; i <= top; i++)
                printf("%2d%5d", stack[i].row, stack[i].col);
            printf("%2d%5d\n", row, col);
            printf("%2d%5d\n", EXIT_ROW, EXIT_COL);
        }
        else  printf("The maze does not have a path \n");
}
```


- Analysis of *path* :

The size of the maze determines the computing time of *path*.

Since each position within the maze is visited no more than once, the worst case time complexity of the algorithm is O(*mp*) where *m* and *p* are, respectively, the number of rows and columns of the maze.

3.6 EVALUATION OF EXPRESSIONS

3.6.1 Expressions

- The representation and evaluation of expressions is of great interest to computer scientists.

- For examples :

  ((*rear*+1 == *front*) || ((*rear* == MAX_QUEUE_SIZE-1) && ! *front*))

  x = a/b-c+d*e-a*c

  Tokens in expressions :

  operators,
  operands, and
  parentheses

- For x = a/b-c+d*e-a*c, when a=4, b=c=2, d=e=3, what is the value of x?

  ((4 / 2) - 2) + (3 * 3) - (4 * 2)
  = 0 + 9 - 8
  = 1

  or

  (4 / (2 - 2 + 3)) * (3 - 4) * 2
  = 4 / 3 * (-1) * 2
  = -2.666…

- If we wanted the second answer, we would have written it as follows,
  x = (a / (b - c + d)) * (e - a) * c.

- Within any programming language, there is a precedence hierarchy that determines the order in which we evaluate operators.
- Parentheses are used to override precedence, and the expressions are always evaluated from the innermost parenthesized expression first.
- See Figure 3.12 for precedence hierarchy for C

cf. associativity

- **Ways of writing expressions :**

Infix notation - a * b
Prefix notation - * a b
Postfix notation - a b *

〉 parenthesis-free notations

Although *infix* notation is the most common ways of writing expressions, it is not the one used by compilers to evaluate expressions.

Compilers typically use a parentheses-free notation referred to as *postfix*

Infix	Postfix
2+3*4	234*+
a*b+5	ab*5+
(1+2)*7	1 2+7*
a*b/c	ab*c/
((a/(b-c+d))*(e-a)*c	abc-d+/ea-*c*
a/b-c+d*e-a*c	ab/c-de*+ac*-

[Figure 3.13] Infix and postfix notation

3.6.2 Evaluating Postfix Expressions

- To evaluate a postfix expression, we make a single left-to-right scan of it.

  1) Place the operands on a stack until we find an operator.
  2) Remove, from the stack, the correct number of operands for the operator.
  3) Perform the operation
  4) Place the result back on the stack.

- To simplify our task, we assume that the expression contains only the binary operators

  +, -, *, /, and %

  and that the operands in the expression are single digit integers.

- **The complete declarations :**

```
#define MAX_STACK_SIZE 100 /*maximum stack size*/
#define MAX_EXPR_SIZE 100    /*max size of expression*/
typedef enum {lparen, rparen, plus, minus, times, divide, mod, eos, operand} precedence;
int stack[MAX_STACK_SIZE];    /* global stack */
char expr[MAX_EXPR_SIZE];          /* input string */
```

Token	Stack			Top
	[0]	[1]	[2]	
6	6			0
2	6	2		1
/	6/2			0
3	6/2	3		1
−	6/2−3			0
4	6/2−3	4		1
2	6/2−3	4	2	2
*	6/2−3	4*2		1
+	6/2−3+4*2			0

[Figure 3.14] Postfix evaluation

- **[Program 3.13] : Function to evaluate a postfix expression**

```
int eval(void)
{
/*  evaluate a postfix expression, expr, maintained as a global  variable.  '\0' is the end of the
     expression. The stack and  top of the stack are global variables.  getToken is used to
     return the tokentype and the character symbol.  Operands are assumed to be single
     character digits  */
     precedence  token;
     char  symbol;
     int  op1, op2;
     int  n = 0;  /*  counter for the expression string  */
     int  top = -1;
     token = getToken(&symbol, &n);
```



```
     while (token != eos)     {
          if (token == operand)
               push(symbol-'0');    /*  stack insert  */
          else    {
          /* remove two operands, perform operation, and return result to the stack   */
               op2 = pop();    /*  stack delete  */
               op1 = pop();
               switch (token)   {
                    case plus : push(op1+op2);      break;
                    case minus : push(op1-op2);    break;
                    case times : push(op1*op2);    break;
                    case divide: push(op1/op2);     break;
                    case mod : push(op1%op2);
               }
          }
          token = getToken(&symbol, &n);
     }
     return  pop(&top);     /*  return result  */
}
```

- **[Program 3.14] : Function to get a token from the input string**

```
precedence getToken(char *symbol, int *n)
{
/*  get the next token, symbol is the character representation, which is returned, the
token is represented by its enumerated value, which is returned in the function name  */
     *symbol = expr[(*n)++];
     switch  (*symbol)   {
          case '(' : return  lparen;
          case ')' : return  rparen;
          case '+' : return  plus;
          case '-' : return  minus;
          case '/' : return  divide;
          case '*' : return  times;
          case '%' : return  mod;
          case ' ' : return eos;
          default : return  operand;  /* no error checking, default is operand */
     }
}
```


3.6.3 Infix To Postfix

- An algorithm for producing a postfix expression from an infix one :
  1) Fully parenthesize the expression.
  2) Move all binary operators so that
     they replace their corresponding right parenthesis.
  3) Delete all parentheses.

- For example, a/b-c+d*e-a*c
  After step 1,
  ((((a/b)-c)+(d*e))-(a*c))
  Performing step 2 and 3,
  ab/c-de*+ac*-

- Although this algorithm works well when done by hand,
  it is inefficient on a computer because it requires two passes.

- Note that the order of operands is the same in infix and postfix.
- Thus we can form the postfix equivalent by scanning the infix expression left-to-right.

  However, the order in which the operators are output depends on their precedence.
- Since we must output the higher precedence operators first, we save operators until we know their correct placement.

  A stack is one way of doing this, but removing operators correctly is problematic.

- Method:

  Operators with higher precedence must be output *before* those with lower precedence. Therefore, we stack operators as long as the precedence of the operator at the top of the stack is *less than* the precedence of the incoming operator.
- Parenthesized expression:

  Stack "(" and operators till we reach ")". At this point we unstack till we reach the corresponding "(". Then delete "(".

- **[Figure 3.15] Translation of a+b*c to postfix**

Token	Stack [0]	[1]	[2]	Top	Output
a				-1	a
+	+			0	a
b	+			0	ab
*	+	*		1	ab
c	+	*		1	abc
eos				-1	abc*+

- **[Figure 3.16] Translation of a*(b+c)/d to postfix**

Token	Stack [0]	[1]	[2]	Top	Output
a				-1	a
*	*			0	a
(	*	(		1	a
b	*	(		1	ab
+	*	(	+	2	ab
c	*	(	+	2	abc
)	*			0	abc+
/	/			0	abc+*
d	/			0	abc+*d
eos	/			0	abc+*d/

- **A+(B*(C+(D-F)))**
- **A B CDF-+* +**

- The analysis of the two examples suggests a precedence-based scheme for stacking and unstacking operators.
- We need two types of precedence, an *in-stack precedence* (*isp*) and an *incoming precedence* (*icp*).
  → Remove an operator from the stack only if its *isp >= icp* of the new operator.

```
precedence stack[MAX_STACK_SIZE];
 /* isp and icp arrays -- index is value of precedence
  lparen, rparen, plus, minus, times, divide, mod, eos  */
static int isp[] = {0, 19, 12, 12, 13, 13, 13, 0};
static int icp[] = {20, 19, 12, 12, 13, 13, 13, 0};
```

[program 3.15] : Function to convert from infix to postfix

```
void postfix(void)
{
/*  output the postfix of the expression. The expression string, stack, and the top are global  */
     char symbol;
     int n = 0;
     int top = 0;  /*  place eos on stack  */
     stack[0] = eos;
     for  (token = getToken(&symbol, &n); token != eos; token = getToken(&symbol, &n))  {
          if (token == operand)
               printf("%c", symbol);
          else  (token == rparan)  {
               /*  unstack tokens until left paranthesis  */
               while (stack[top] != lparen)
                    printToken(pop(&top));
               pop();  /* discard the left paranthesis  */
          }
```



```
          else  {
               /*  remove and print symbols whose isp is greater
                   than or equal to the current token's icp  */
               while (isp[stack[top]] >= icp[token])
                    printToken(pop());
               push(token);
          }
     }
     while ( (token = pop()) != eos)
          printToken(token);
     printf("\n");
}
```

- **Analysis of *postfix* :**

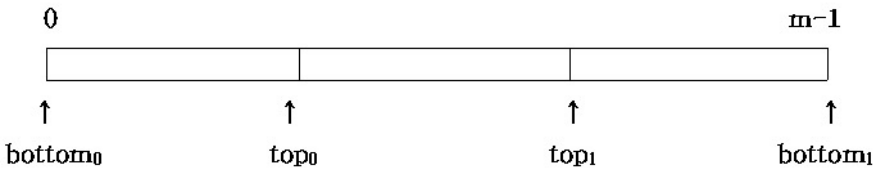
  Let *n* be the number of tokens in the expression.

  Θ(n) time is spent extracting tokens and outputting them.
  Θ(n) time is spent in two *while* loops
  as the number of tokens that get stacked and unstacked is linear in *n*.

  So, the time complexity of function *postfix* is Θ(n).

3.7 MULTIPLE STACKS AND QUEUES

- Implementing multiple stacks (queues) which are mapped sequentially into an array, *memory*[MEMORY_SIZE].
- If we have only two stacks to represent, the solution is simple.



IsEmpty (stacki)
IsFull (stacki)
Push (stacki, item)
Pop (stacki)

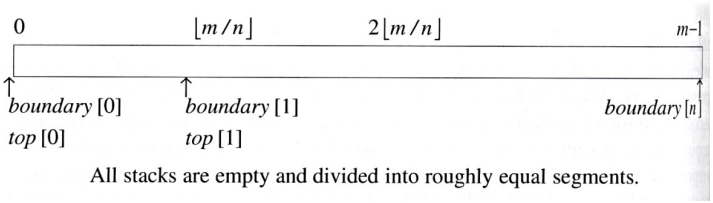
- **More than two stacks :**

Assuming *n* stacks, initially we divide the available memory into *n* segments.

Let *stack_no* refers to the stack number of one of *n* stacks.

The bottom element, *boundary*[*stack_no*], 0≤*stack_no*<MAX_STACKS, always points to the position immediately to the left of the bottom element, while *top*[*stack_no*], 0≤*stack_no*<MAX_STACKS, always points to the top element.


```
#define  MEMORY_SIZE 100       /* size of memory */
#define  MAX_STACKS 10     /* max number of stacks plus 1 */
/*  global memory declaration */
element memory[MEMORY_SIZE];
int  top[MAX_STACKS];
int  boundary[MAX_STACKS];
int  n;     /* number of stacks entered by the user  */

top[0] = boundary[0]  = -1;
for (i =1; i < n; i++)
      top[i] = boundary[i] = (MEMORY_SIZE/n)*i;
boundary[n] = MEMORY_SIZE-1;
```

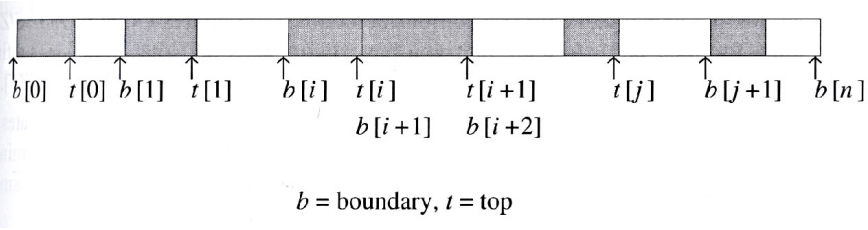


All stacks are empty and divided into roughly equal segments.

- What do we do if a stack is full?

  We create an error recovery function, *stackFull*, which determines if there is any free space in memory.

  If there is space available, it should shift the stacks so that space is allocated to the full stack



b = boundary, t = top