CSEG601 & CSE5601:
Spatial Data Management & Application

R-tree and its variations

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R-tree

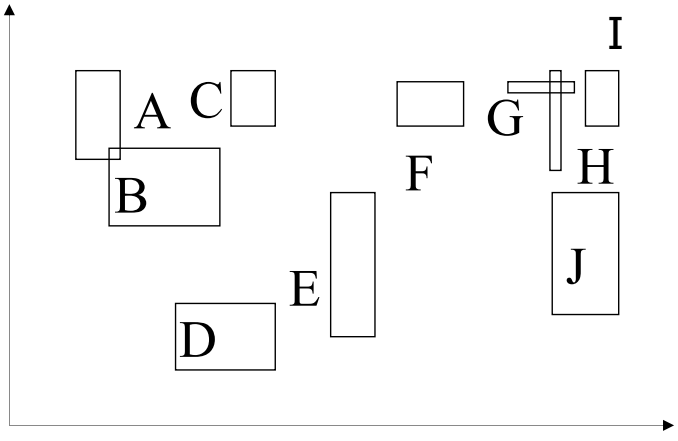
- [Guttman 84] Main idea: allow parents to overlap!
  - => guaranteed 50% utilization
  - => easier insertion/split algorithms.
  - (only deal with Minimum Bounding Rectangles - **MBR**s)

R-tree

- A multi-way external memory tree
- Index nodes and data (leaf) nodes
- All leaf nodes appear on the same level
- Every node contains between m and M entries
- The root node has at least 2 entries (children)

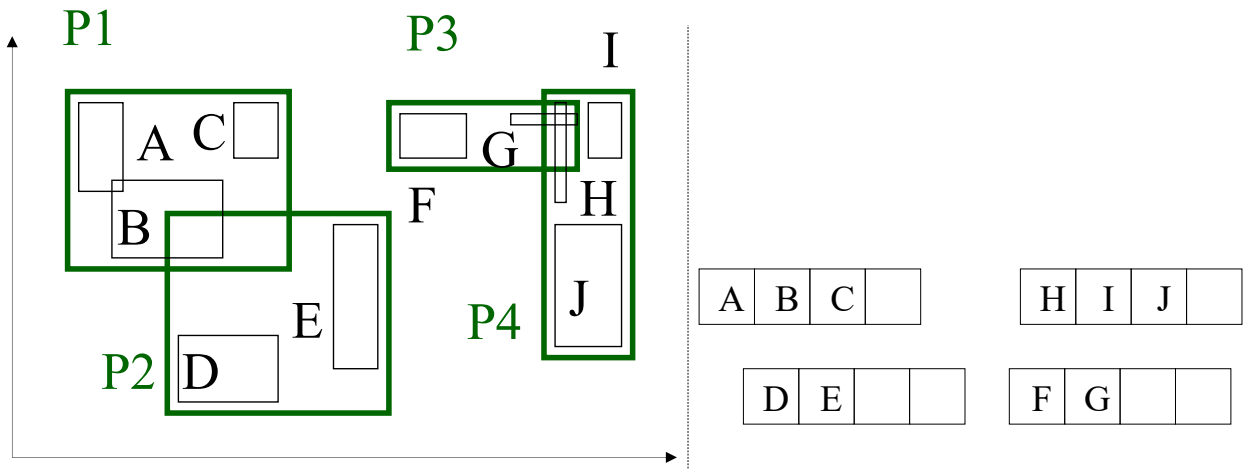
Example

- eg., w/ fanout 4: group nearby rectangles to parent MBRs; each group -> disk page



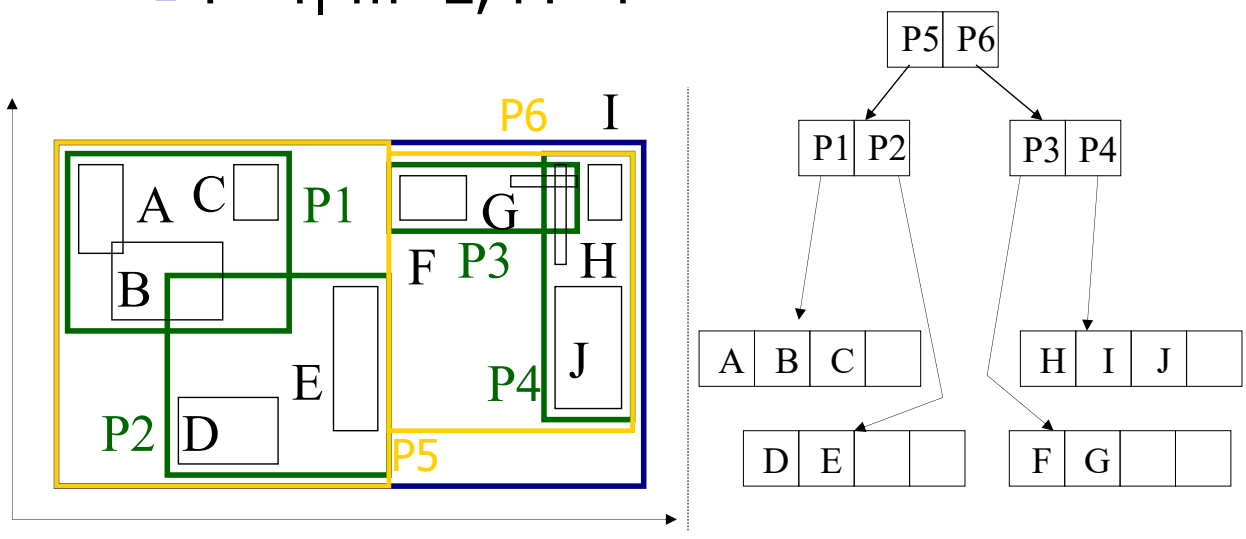
Example

- F=4



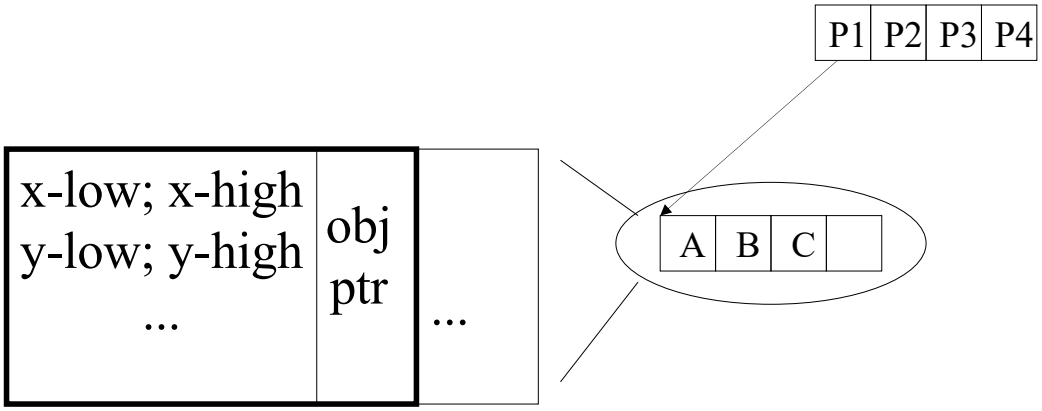
Example

- F=4| m=2, M=4



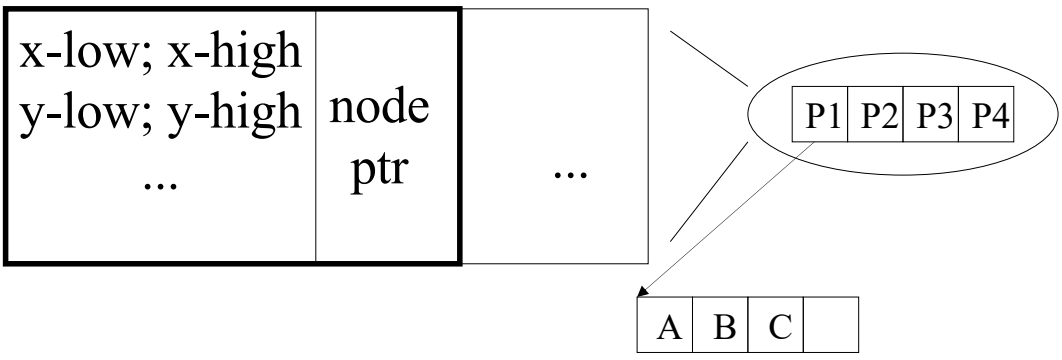
R-trees - format of nodes

- {(MBR; obj_ptr)} for leaf nodes

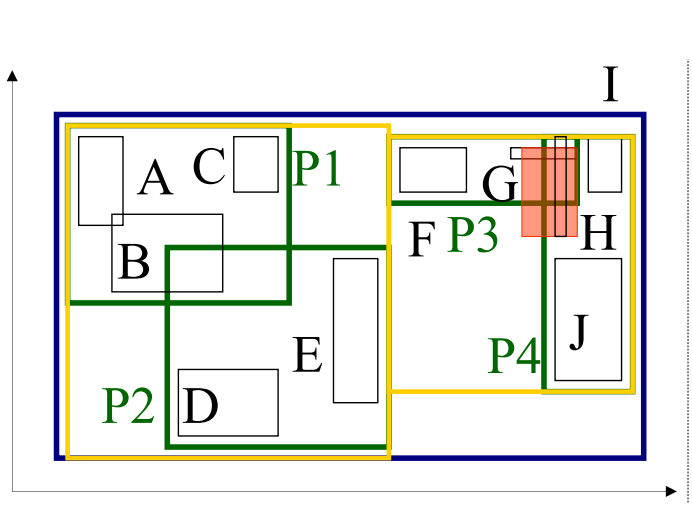


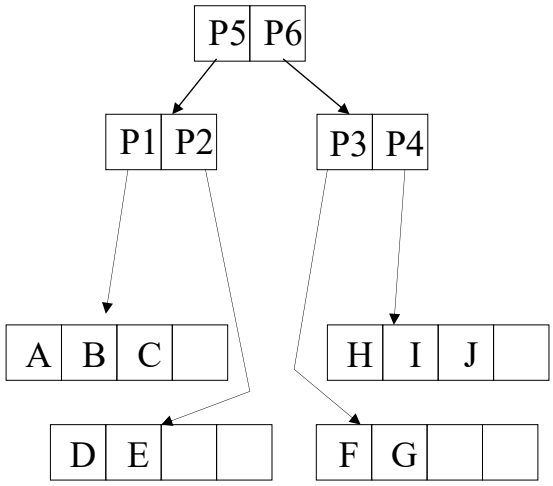
R-trees - format of nodes

- {(MBR; node_ptr)} for non-leaf nodes

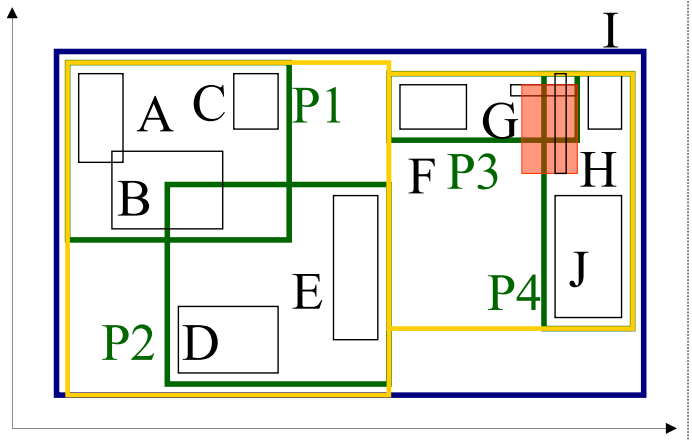


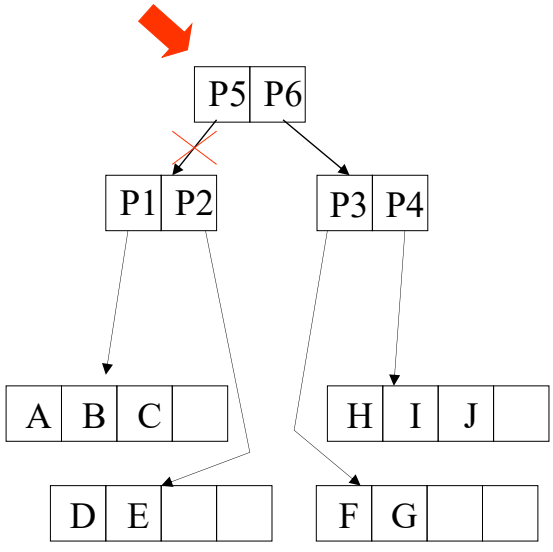
R-trees:Search





R-trees:Search



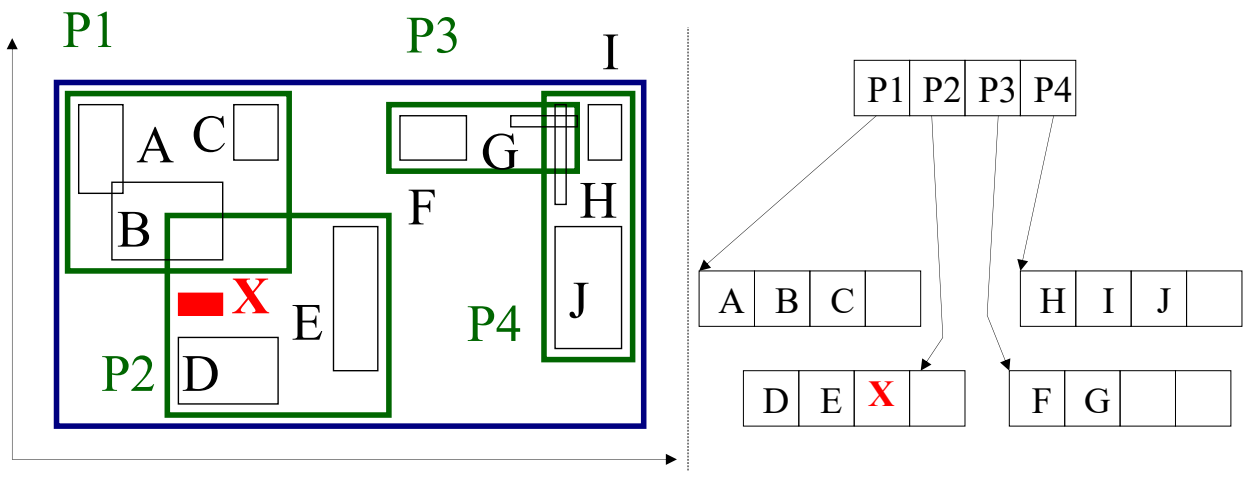


R-trees:Search

- Main points:
  - every parent node completely covers its 'children'
  - a child MBR may be covered by more than one parent - it is stored under ONLY ONE of them. (ie., no need for dup. elim.)
  - a point query may follow multiple branches.

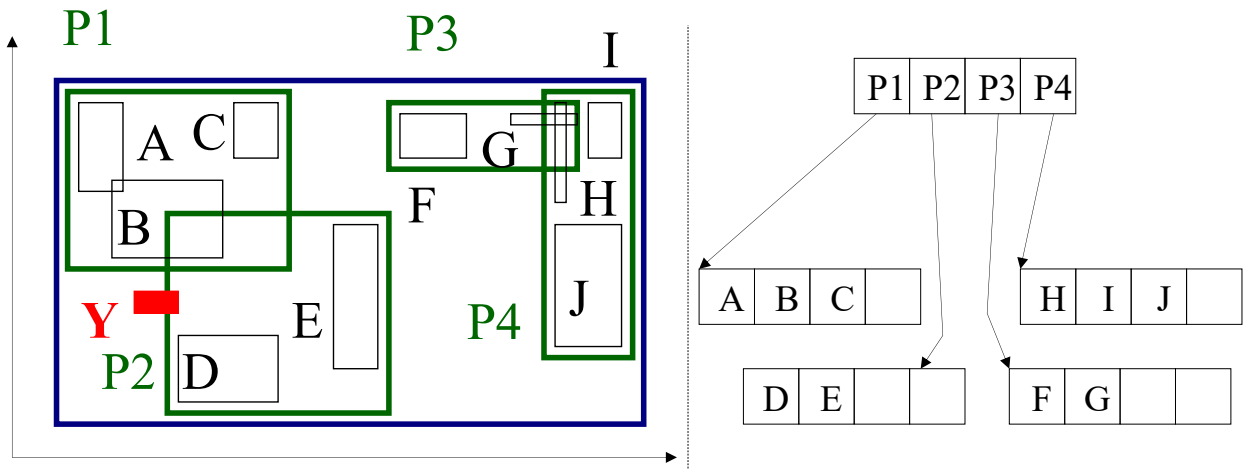
R-trees:Insertion

Insert X



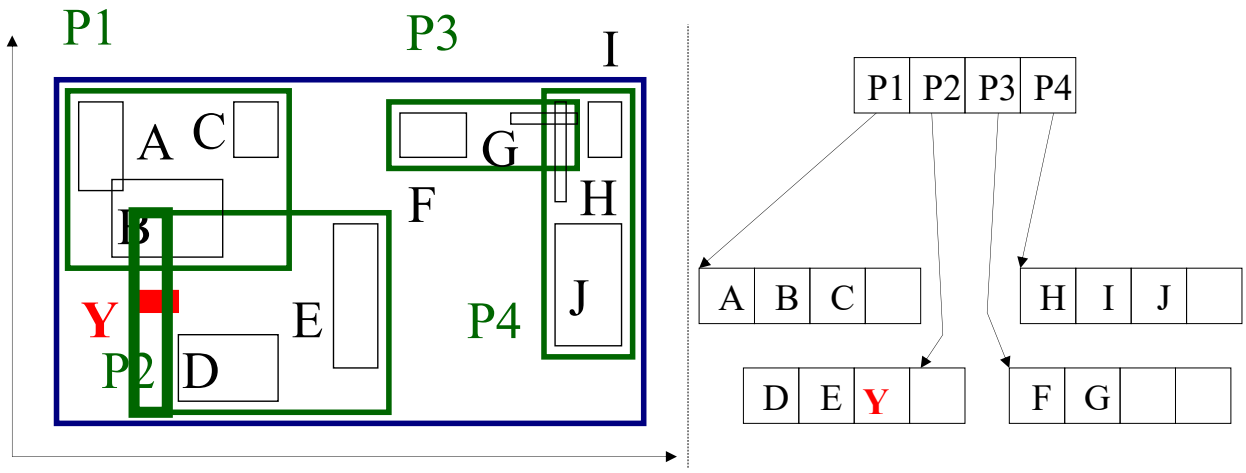
R-trees:Insertion

Insert Y



R-trees:Insertion

- Extend the parent MBR

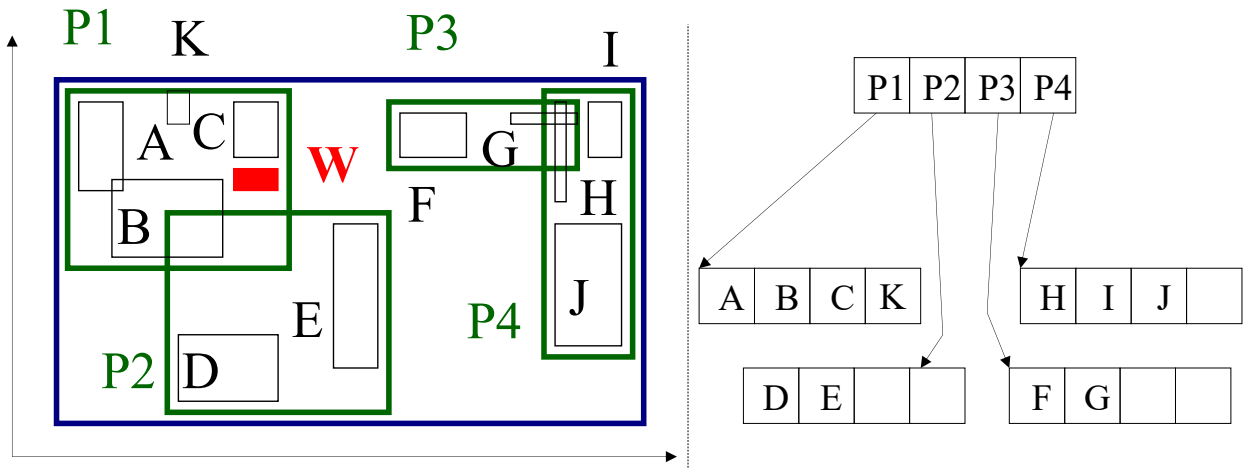


R-trees:Insertion

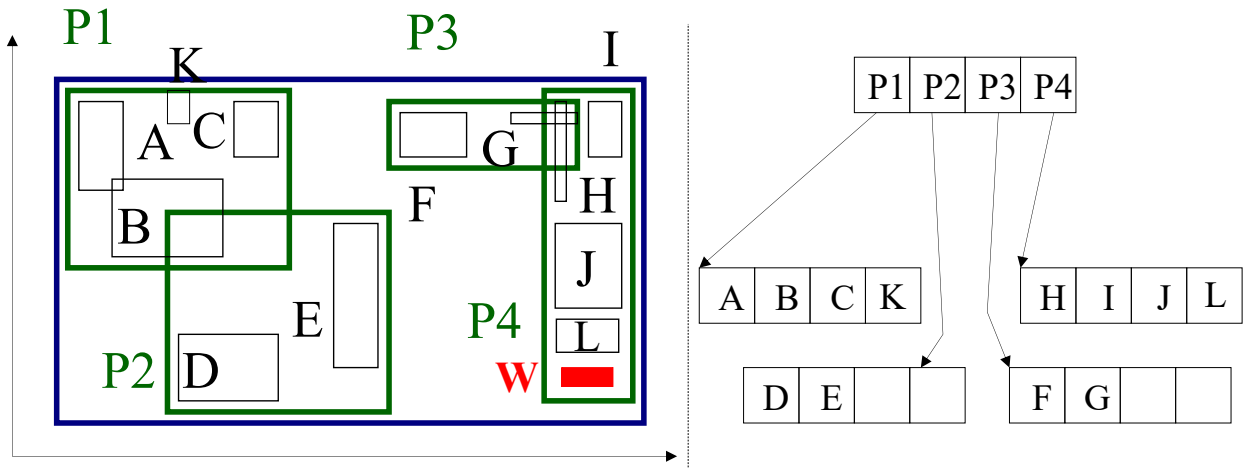
- How to find the next node to insert the new object?
  - Using ChooseLeaf: Find the entry that needs the least enlargement to include Y. Resolve ties using the area (smallest)

R-trees:Insertion

- If node is full then Split : ex. Insert w

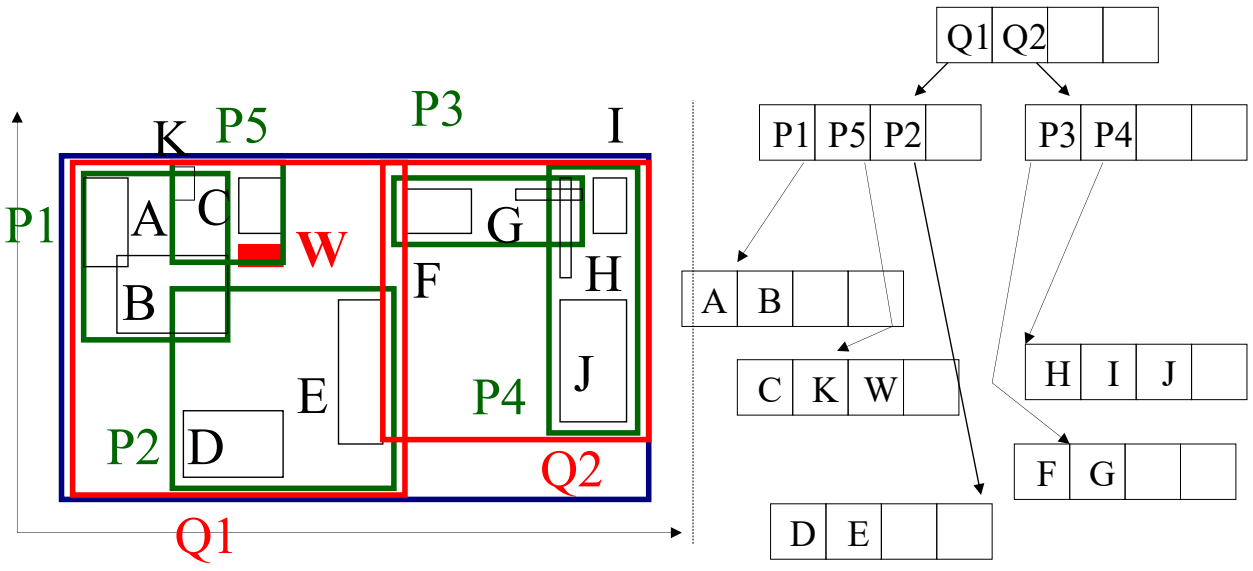


R-trees:Insertion



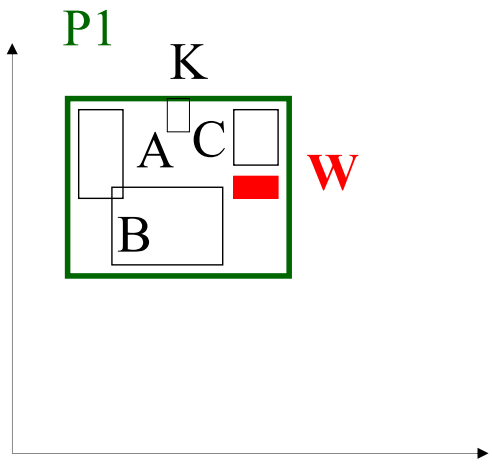
R-trees:Insertion

- If node is full then Split : ex. Insert w



R-trees:Split

- Split node P1: partition the MBRs into two groups.



- (A1: plane sweep, until 50% of rectangles)
- A2: 'linear' split
- A3: quadratic split
- A4: exponential split: 2^{M-1} choices

R-trees: Split

- Two requirements
  - Minimize the total area of the two nodes
  - Minimize the overlapping the two nodes

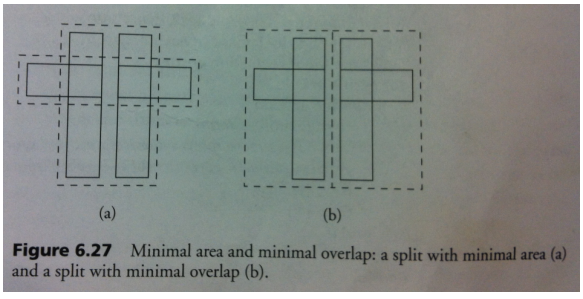
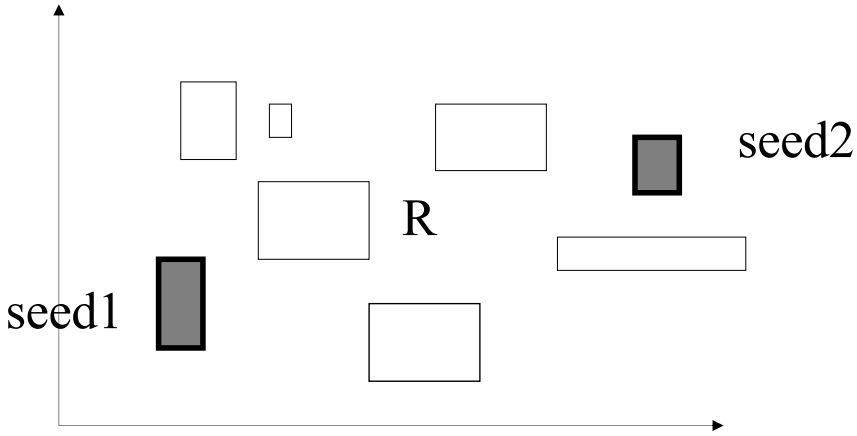


Figure 6.27 Minimal area and minimal overlap: a split with minimal area (a) and a split with minimal overlap (b).

- Two requirements are not always compatible
  - R-tree Splitting tries to minimize the total area first.

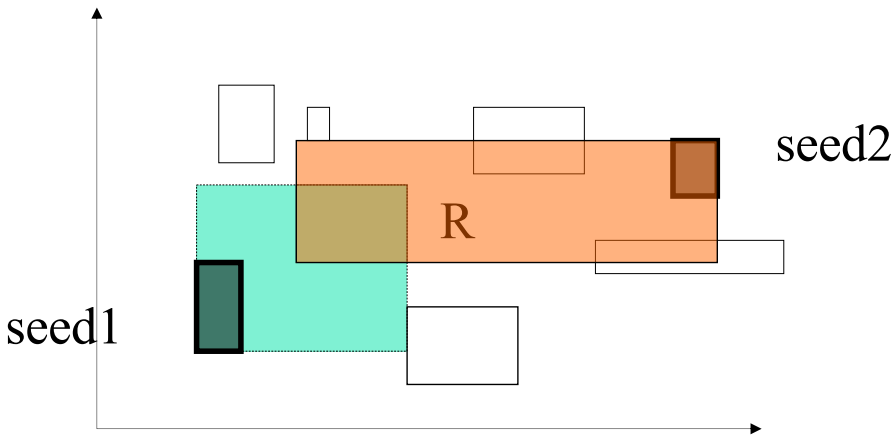
R-trees:Split

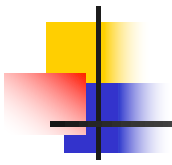
- pick two rectangles as ‘seeds’;
- assign each rectangle ‘R’ to the ‘closest’ ‘seed’



R-trees:Split

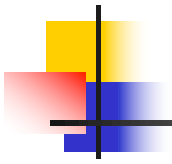
- pick two rectangles as ‘seeds’;
- assign each rectangle ‘R’ to the ‘closest’ ‘seed’:
- ‘closest’: the smallest increase in area





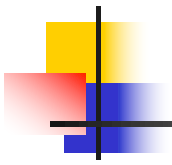
R-trees:Split

- How to pick Seeds:
  - Linear:Find the highest and lowest side in each dimension, normalize the separations, choose the pair with the greatest normalized separation
  - Quadratic: For each pair E1 and E2, calculate the rectangle J=MBR(E1, E2) and d= J-E1-E2. Choose the pair with the largest d



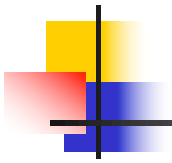
R-trees:Insertion

- Use the **ChooseLeaf** to find the leaf node to insert an entry E
- If leaf node is full, then **Split,** otherwise insert there
  - Propagate the split upwards, if necessary
- Adjust parent nodes



R-Trees:Deletion

- Find the leaf node that contains the entry E
- Remove E from this node
- If underflow:
  - Eliminate the node by removing the node entries and the parent entry
  - Reinsert the orphaned (other entries) into the tree using **Insert**

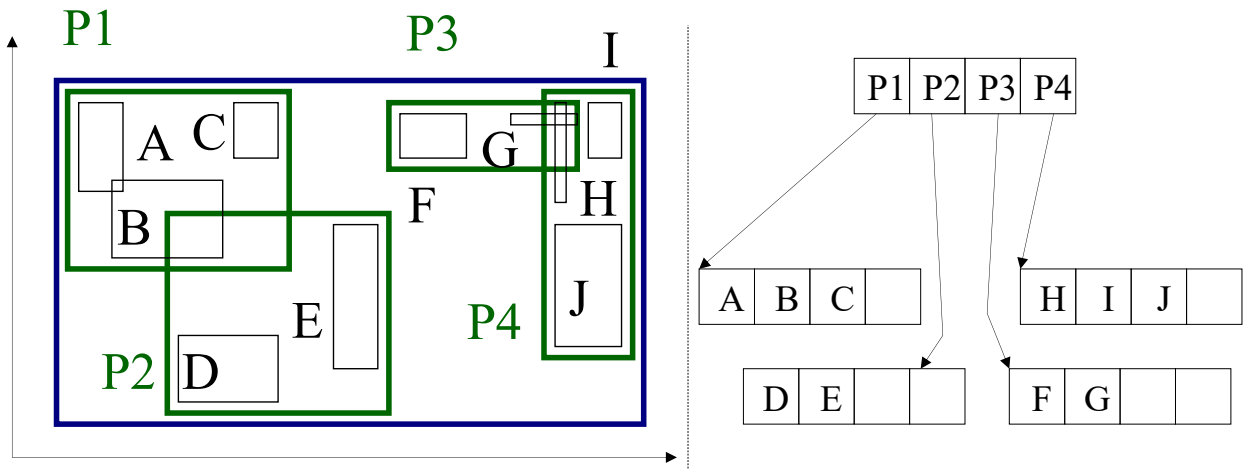


R-trees: Variations

- R+-tree: DO not allow overlapping, so split the objects (similar to z-values)
- R*-tree: change the insertion, deletion algorithms (minimize not only area but also perimeter, forced re-insertion)
- Hilbert R-tree: use the Hilbert values to insert objects into the tree

R-tree

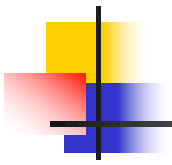
Multi-way external memory structure, indexes MBRs
Dynamic structure



R-tree

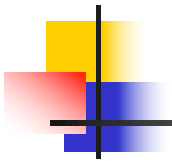
- The original R-tree tries to minimize the area of each enclosing rectangle in the index nodes.
- Is there any other property that can be optimized?

R*-tree → Yes!



R*-tree

- A variation of the R-tree that provides several improvements to the insertion algorithm



R*-tree

- Optimization Criteria:
  - (O1) Area covered by an index MBR
  - (O2) Overlap between directory MBRs
  - (O3) perimeter of a node's directory rectangle
  - (O4) Storage utilization
- Sometimes it is impossible to optimize all the above criteria at the same time!

R*-tree

- ChooseSubtree for a node insertion:
  - If next node is a leaf node, choose the node using the following criteria:
    - Least overlap enlargement
    - Least area enlargement
    - Smaller area
  - Else
    - Least area enlargement
    - Smaller area

R*-tree

- SplitNode
  - Choose the axis to split
  - Choose the two groups along the chosen axis
- ChooseSplitAxis
  - Along each axis, sort rectangles and break them into two groups (M-2m+2 possible ways where one group contains at least m rectangles). Compute the sum S of all margin-values (perimeters) of each pair of groups. Choose the one that minimizes S
    - Split M+1 entries into 2 groups with at least m entries
    - M-2m+2 possible ways to split M+1 entries into two groups
- ChooseSplitIndex
  - Along the chosen axis, choose the grouping that gives the minimum overlap-value

R*-tree

- Splitting Strategies by finding the best axis

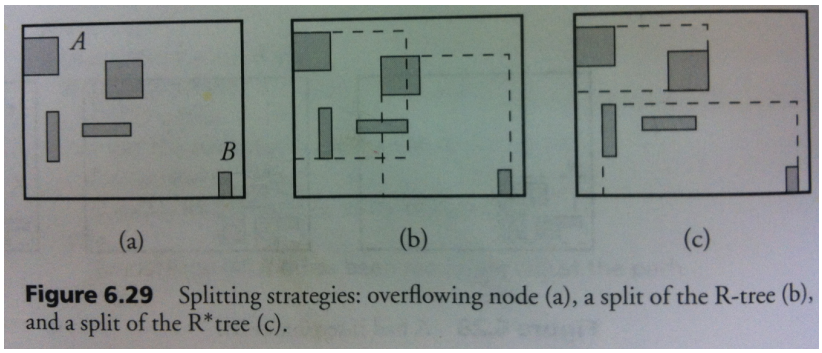


Figure 6.29 Splitting strategies: overflowing node (a), a split of the R-tree (b), and a split of the R*tree (c).

R*-tree

- Forced Reinsert:
  - defer splits, by forced-reinsert, i.e.: instead of splitting, temporarily delete some entries, shrink overflowing MBR, and re-insert those entries

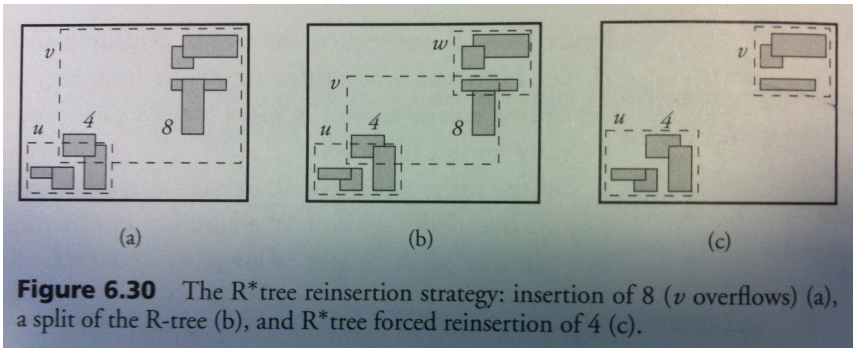
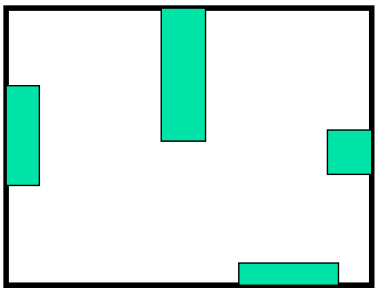


Figure 6.30 The R*tree reinsertion strategy: insertion of 8 (v overflows) (a), a split of the R-tree (b), and R*tree forced reinsertion of 4 (c).

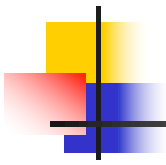
R*-tree

- Forced Reinsert:
  - Which ones to re-insert?
  - How many? A: 30%



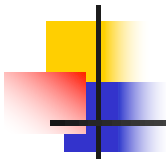
R tree Packing

- R and R* trees support incremental dynamic updates
  - The evolution of the structure with time may lead to a performance degradation (i.e., space utilization) as the # of insertions and deletions grows.
- When the collection of rectangles is stable with time, pre-processing the data before creating R-tree leads to the optimized R-tree structure
  - R tree packing : STR(Sort-Tile-Recursive) algorithm



STR R-tree Packing Algorithm

- N: the size of data set
- M: the node capacity
- P = $\lceil N/M \rceil$: the minimal # of leaf MBR entries of R tree
- Organize leaf MBRs to have vertical and horizontal slices
  - Requires that the # of vertical slices be the same as # of horizontal slices and equal to $\lceil \sqrt{P} \rceil$



STR R-tree Packing Algorithm

- Packing Method:
  - Sort the source MBRs on the x coordinate of their centroids and partition the sorted list into $\lceil \sqrt{P} \rceil$ groups (i.e., vertical slices)
  - For each loaded vertical slices sorted by y coordinate of their centroid, they are grouped into runs of size M
    - Each group of size M defines the leaf entries of the packed R-tree
  - Construct recursively the upper levels of R-tree

STR R-tree Packing Algorithm

- All leaves (except those of the last vertical slice) are completely filled thus maximizing space utilization

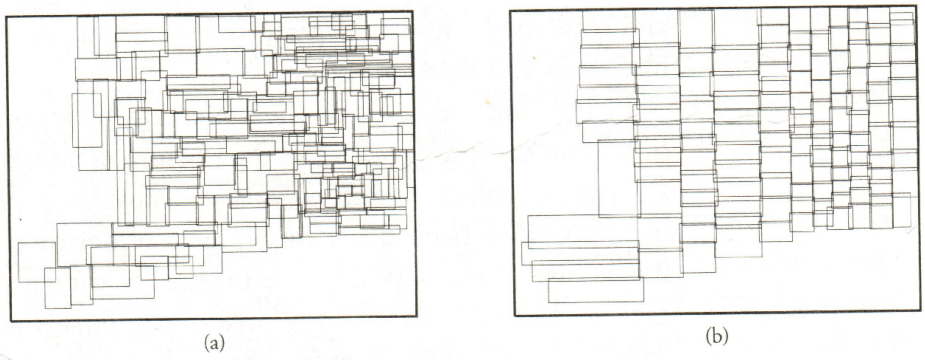
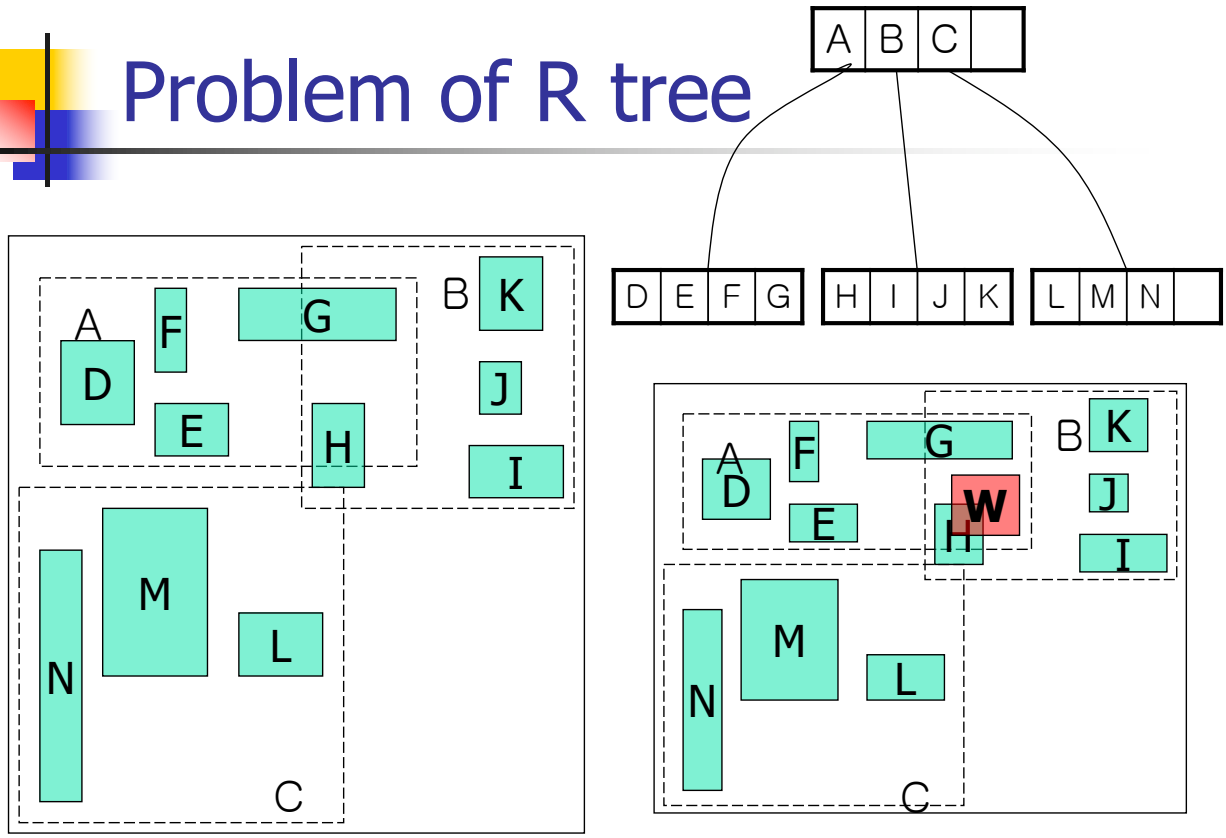


Figure 6.32 R-tree (a) and STR packed R-tree (b).

R+ tree

- Avoids multiple paths during searching.
- Objects may be stored in multiple nodes
- MBRs of nodes at same tree level do not overlap
- On insertion/deletion the tree may change downward or upward in order to maintain the structure

Problem of R tree



R+ tree