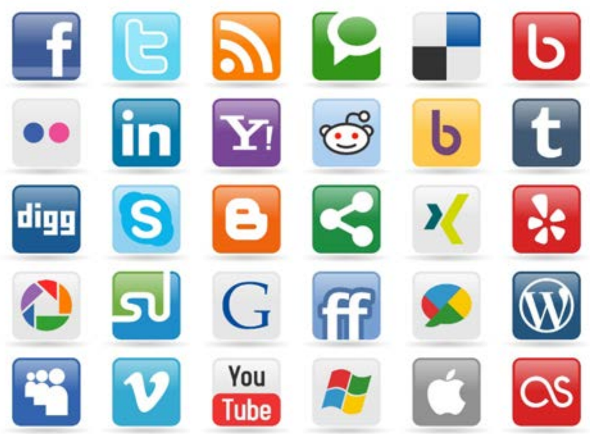
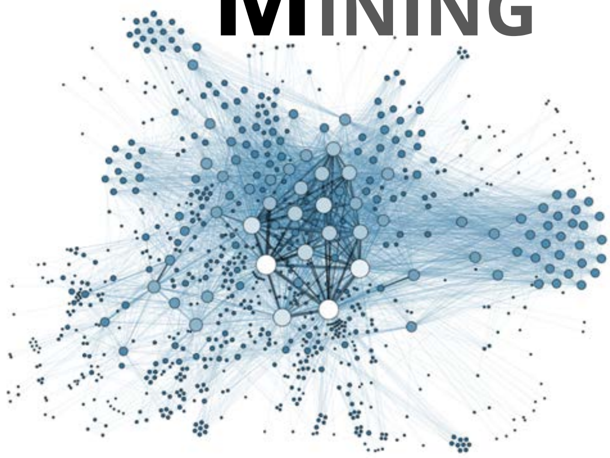


Social Media Mining



Information Diffusion

Definition

In February 2013, during the third quarter of Super Bowl XLVII, a power outage stopped the game for 34 minutes.

- Oreo, a sandwich cookie company, tweeted during the outage:
  - “Power out? No Problem. You can still dunk it in the dark”.
- The tweet caught on almost immediately, reaching
  - 15,000 retweets and 20,000 likes on Facebook in less than 2 days.
- Cheap advertisement reaching a large population of individuals.
  - companies spent as much as 4 million dollars to run a 30 second ad during the super bowl.

Example of **Information Diffusion**

Information Diffusion

- Information diffusion: process by which a piece of information (knowledge) is spread and reaches individuals through interactions.

- Studied in a plethora of sciences.

- We discuss methods from
  - Sociology, epidemiology, and ethnography
  - All are useful for social media mining.

- We focus on techniques that can model information diffusion.

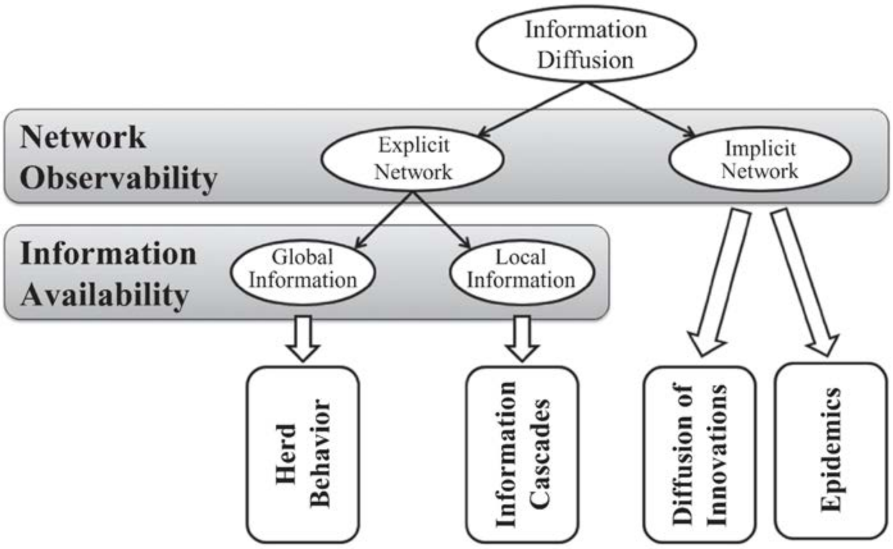
Information Diffusion

- **Sender(s).** A sender or a small set of senders that initiate the information diffusion process;

- **Receiver(s).** A receiver or a set of receivers that receive diffused information. Commonly, the set of receivers is much larger than the set of senders and can overlap with the set of senders; and

- **Medium.** This is the medium through which the diffusion takes place. For example, when a rumor is spreading, the medium can be the personal communication between individuals

Information Diffusion Types



We define the process of interfering with information diffusion by expediting, delaying, or even stopping diffusion as Intervention

Herd Behavior

- **Network is observable**
- **Only public information is available**

Herd Behavior Example

- Consider people participating in an online auction.
- In this case, individuals can observe the behavior of others by monitoring the bids that are being placed on different items.
- Individuals are connected via the auction's site where they can not only observe the bidding behaviors of others, but can also often view profiles of others to get a feel for their reputation and expertise.
- In these online auctions, it is common to observe individuals participating actively in auctions, where the item being sold might otherwise be considered unpopular.
- This is due to individuals trusting others and assuming that the high number of bids that the item has received is a strong signal of its value. In this case, Herd Behavior has taken place.

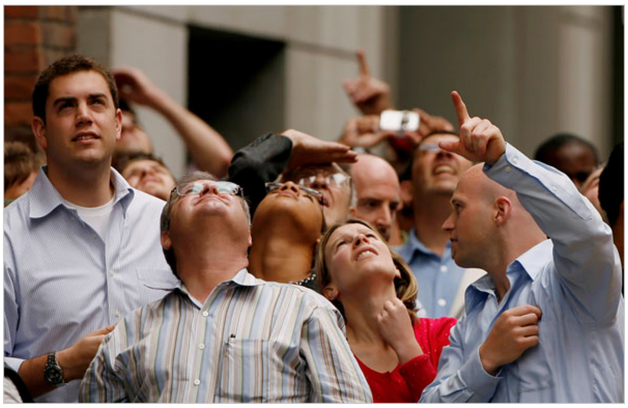
Herd Behavior: Popular Restaurant Example

- Assume you are on a trip in a metropolitan area that you are less familiar with.
- Planning for dinner, you find restaurant **A** with excellent reviews online and decide to go there.
- When arriving at **A**, you see **A** is almost empty and restaurant **B**, which is next door and serves the same cuisine, almost full.
- Deciding to go to **B**, based on the belief that other diners have also had the chance of going to **A**, is an example of herd behavior

Herd Behavior: Milgram's Experiment

Stanley Milgram asked one person to stand still on a busy street corner in New York City and stare straight up at the sky

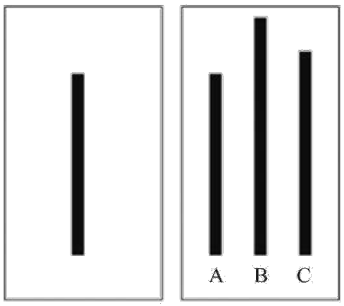
- About 4% of all passersby stopped to look up.



When 5 people stand on the sidewalk and look straight up at the sky, 20% of all passersby stopped to look up.

Finally, when a group of 18 people look up simultaneously, almost 50% of all passersby stopped to look up.

Herd Behavior: Solomon Asch's Experiment



- Groups of students participated in a vision test
- They were shown two cards, one with a single line segment and one with 3 lines
- The participants were required to match line segments with the same length
- Each participant was put into a group where all other group members were collaborators with Asch.
- These collaborators were introduced as participants to the subject.
- Asch found that in control groups with no pressure to conform, only 3% of the subjects provided an incorrect answer.
- However, when participants were surrounded by individuals providing an incorrect answer, up to 32% of the responses were incorrect.

Herding: Asch Elevator Experiment

https://www.youtube.com/watch?v=BgRoiTWkBHU

Herd Behavior

Herd behavior describes when a group of individuals performs actions that are highly correlated without any plans

Main Components of Herd Behavior

- Connections between individuals
- A method to transfer behavior among individuals or to observe their behavior

Examples of Herd Behavior

- Flocks, herds of animals, and humans during sporting events, demonstrations, and religious gatherings

Network Observability in Herb Behavior

In herd behavior, individuals make decisions by observing all other individuals' decisions

- In general, herd behavior's network is close to a complete graph where nodes can observe at least most other nodes and they can observe public information
  - For example, they can see the crowd

Designing a Herd Behavior Experiment

1. There needs to be a decision made.
   - In our example, it is going to a restaurant
2. Decisions need to be in sequential order
3. Decisions are not mindless and people have private information that helps them decide
4. No message passing is possible. Individuals don't know the private information of others, but can infer what others know from what is observed from their behavior.

Herding: Urn Experiment

- There is an urn in a large class with three marbles in it

Majority Blue
P[B,B,R]=50%

Majority Red
P[R,R,B]=50%

- During the experiment, each student comes to the urn, picks one marble, and checks its color in private.
- The student predicts **majority blue** or **majority red**, writes her prediction on the blackboard, and puts the marble back in the urn.
- Students cannot see the color of the marble taken out and can only see the predictions made by different students regarding the majority color on the board.

Urn Experiment: First and Second Student

- First Student:
  - *Board: -*
    - Observed: **B** → Guess: **B**

    -or-

    - Observed: **R** → Guess: **R**
- Second Student:
  - *Board:* ***B***
    - Observed: **B** → Guess: **B**

    -or-

    - Observed: **R** → Guess: **R**/**B** (flip a coin)

Urn Experiment: Third Student

- *If board: **B**, **R***
  - Observed: **B** → Guess: **B**, or
  - Observed: **R** → Guess: **R**

- *If board: **B**, **B***
  - Observed: **B** → Guess: **B**, or
  - **Observed: R → Guess: B (Herding Behavior)**

The forth student and onward
  - Board: **B**,**B**,**B**
  - **Observed: B/R → Guess: B**

Bayes's Rule in the Herding Experiment

Each student tries to estimate the conditional probability that the urn is **majority-blue** or **majority-red**, given what she has seen or heard

- She would guess **majority-blue** if:

  P[majority-blue|what she has seen or heard] > 1/2

- From the setup of the experiment we know:

  P[majority-blue] = P[majority-red]=1/2

  P[blue|majority-blue] = P[red|majority-red]=2/3

Bayes's Rule in the Herding Experiment

P[majority-blue|blue] = P[blue|majority-blue] x P[majority-blue] / Pr[blue]

P[blue] = P[blue|majority-blue] × P[majority-blue]
+ P[blue|majority-red] × P[majority-red]
= 2/3 × 1/2 + 1/3 × 1/2 = 1/2

P[majority-blue|blue] = (2/3 × ½)/(1/2)

- So the first student should guess **blue** when she sees **blue**
- The same calculation holds for the second student

Third Student

P[majority-blue|blue, blue, red] =
P[blue, blue, red|majority-blue] × P[majority-blue] / P[blue, blue, red]

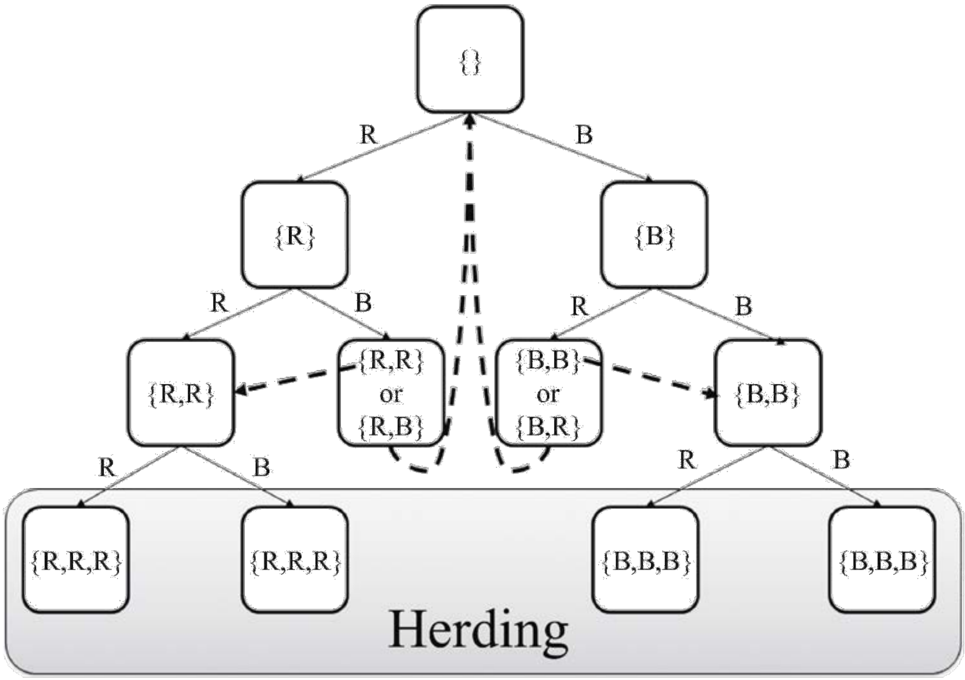
P[blue, blue, red|majority-blue] = 2/3 × 2/3 × 1/3 = 4/27

P[blue, blue, red] = P[blue, blue, red|majority-blue] × P[majority-blue]
+ P[blue, blue, red|majority-red] × P[majority-red]
= (2/3 × 2/3 × 1/3) × 1/2 + (1/3 × 1/3 × 2/3) × 1/2 = 1/9

P[majority-blue|blue, blue, red] = (4/27 × 1/2) / (1/9) = 2/3

- So the third student should guess **blue** even when she sees **red**
- All future students will have the same information as the third student

Urn Experiment

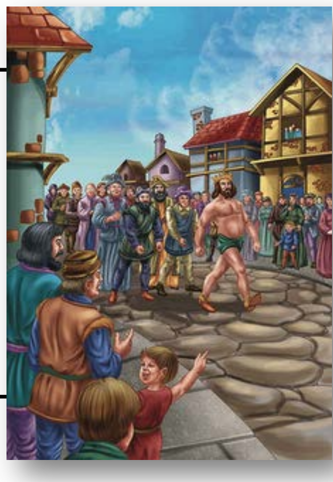


Herding Intervention

Herding: we only have access to public information

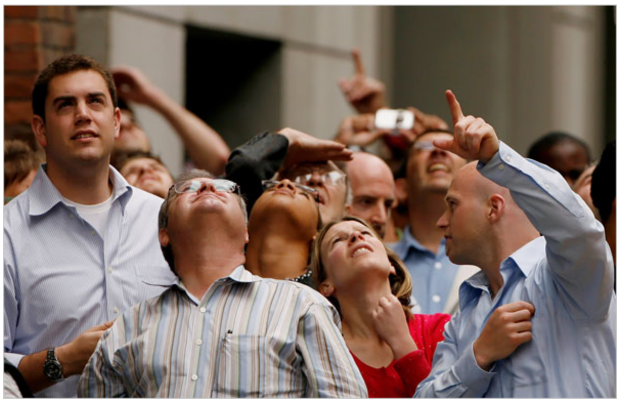
– Herding may be intervened by releasing private information which was not accessible before

The little boy in
"The Emperor's New Clothes"
story *intervenes* the herd by
shouting
"There is no clothe"



Herding Intervention

To intervene the herding effect, we need one person to tell the herd that there is nothing in the sky and the first person doing this to stop his nose bleeding



How Does Intervention Work?

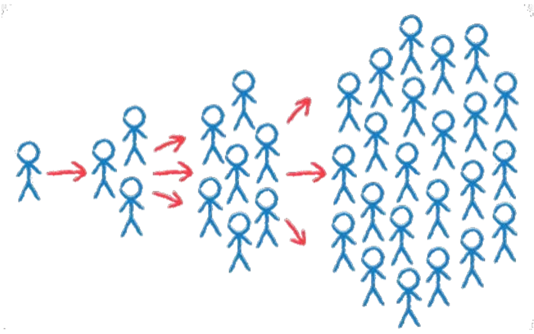
- When a new piece of private information releases,
  - The herd reevaluate their guesses and this may create completely new results
- The Emperor's New Clothes
  - When the boy gives his private observation, other people compare it with their observation and confirm it
  - This piece of information may change others guess and ends the herding effect
- In urn experiment, intervention is possible by
  1. A private message to individuals informing them that the urn is majority blue or
  2. Writing the observations next to predictions on the board.

Information Cascade

- **In the presence of a network**
- **Only local information is available**

Information Cascade

- Users often repost content posted by others in the network. T
  - Content is often received via immediate neighbors (friends).



Information propagates through friends

An information cascade: a piece of information/decision cascaded among some users, where

individuals are connected by a network and
individuals are only observing decisions of their immediate neighbors (friends).

Cascade users have less information available

- Herding users have almost all information about decisions

Notable example

- Between 1996/1997,
  - Hotmail was one of the first internet business's to become extremely successful utilizing viral marketing
  - By inserting the tagline "*Get your free e-mail at Hotmail*" at the bottom of every e-mail sent out by its users.
- Hotmail was able to sign up **12 million users** in 18 months.
- At the time, this was the fastest growth of any user-based company.
  - By the time Hotmail reached **66 million** users, the company was establishing **270,000** new accounts each day.

--
Get your free Email at Hotmail

Underlying Assumptions for Cascade Models

- The network is a directed graph.
  - Nodes are actors
  - Edges depict the communication channels between them.
- A node can only influence nodes that it is connected to
- Decisions are binary. nodes can
  - **Active**: the node has adopted the behavior/innovation/decision;
  - **Inactive**
- A activated node can activate its neighboring nodes; and
- Activation is a progressive process, where nodes change from inactive to active, but not vice versa

Independent Cascade Model (ICM)

- **Independent Cascade Model (ICM)**
  - Sender-centric model of cascade
  - Each node has **one chance** to activate its neighbors
- In ICM, nodes that are active are senders and nodes that are being activated as receivers
  - The *linear threshold model* concentrates on the receiver (to be discussed later in Chapter 8).

ICM Algorithm

- Node activated at time t, has one chance, at time step $t+1$, to activate its neighbors
- Assume v is activated at time t
  - For any neighbor w of v, there's a probability p_{vw} that node w gets activated at time $t+1$.
- Node v activated at time t has a single chance of activating its neighbors
  - The activation can only happen at $t+1$

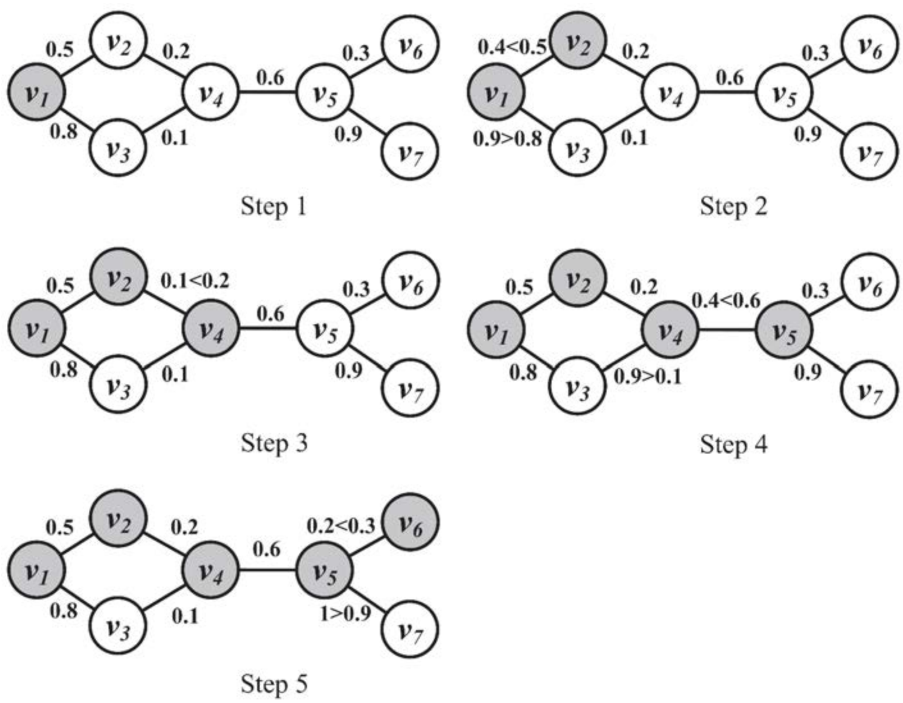
ICM Algorithm

Algorithm 1 Independent Cascade Model (ICM)

Require: Diffusion graph $G(V, E)$, set of initial activated nodes A_0, activation probabilities $p_{v,w}$

return Final set of activated nodes A_∞
i = 0;
while $A_i \neq \{\}$ **do**
  $i = i + 1;$
  $A_i = \{\};$
  for all $v \in A_{i-1}$ **do**
    for all w neighbor of $v, w \notin \cup_{j=0}^{i} A_j$ **do**
      rand = generate a random number in [0,1];
      if rand $< p_{v,w}$ **then**
        activate w;
        $A_i = A_i \cup \{w\};$
      end if
    end for
  end for
end while
$A_\infty = \cup_{j=0}^{i} A_j;$
Return A_∞;

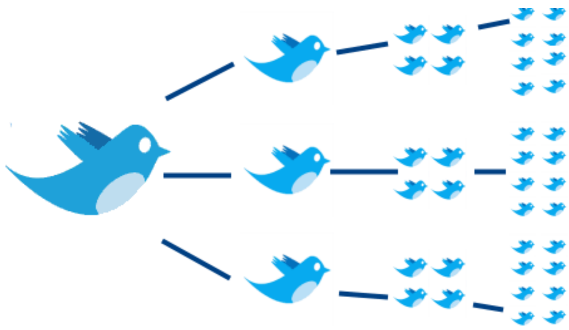
Independent Cascade Model: An Example



Maximizing the Spread of Cascades

Maximizing the spread of cascades

- **Maximizing the Spread of Cascades** is the problem of finding a small set of nodes in a social network such that their aggregated spread in the network is maximized
- Applications
  - Product marketing
  - Influence

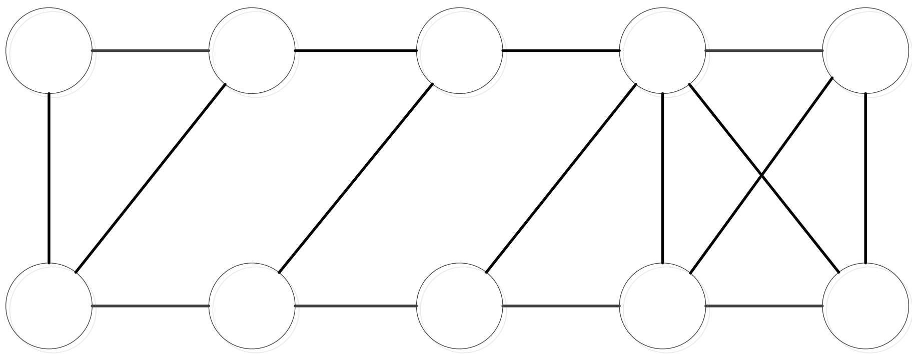


Problem Setting

- **Given**
  - A limited budget **B** for initial advertising
    - Example: give away free samples of product
  - Estimating spread between individuals
- **Goal**
  - To trigger a large spread
    - i.e., further adoptions of a product
- **Question**
  - Which set of individuals should be targeted at the very beginning?

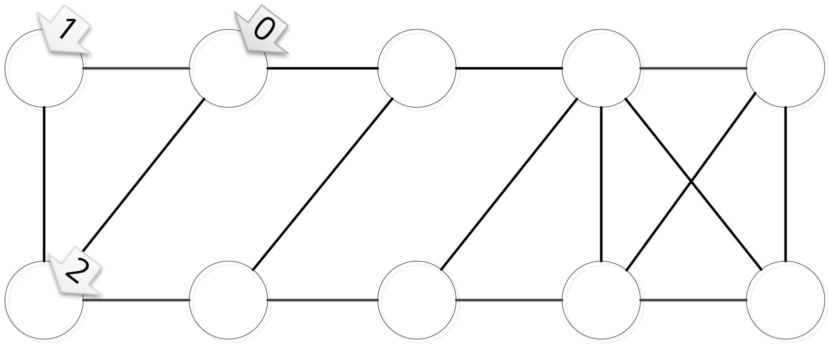
Maximizing the Spread of Cascade: Example

- We need to pick k nodes such that maximum number of nodes are activated

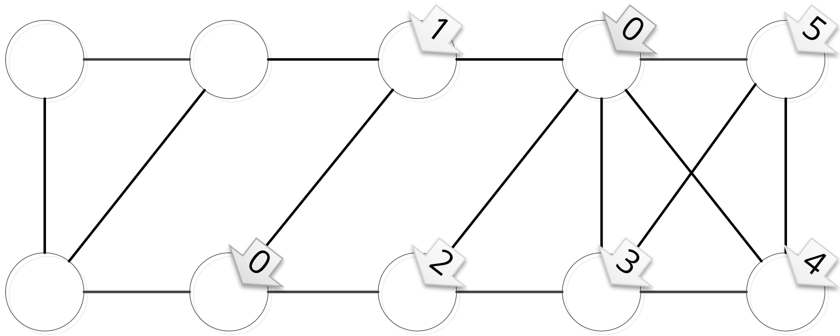


Maximizing the Spread of Cascade

Select one seed



Select two seeds



Problem Statement

- Spread of node set S: $f(S)$
  - An expected <u>number</u> of activated nodes at the end of the cascade, if set S is the initial active set
- Problem:
  - Given a parameter k (budget), find a k-node set S to maximize $f(S)$
  - A constrained optimization problem with $f(S)$ as the objective function

f(S): Properties

1. **Non-negative** (obviously)

2. **Monotone**

$$f(S+v) \geq f(S)$$

3. **Submodular**
   - Let N be a finite set
   - A set function is submodular if and only if

$$f : 2^N \mapsto \Re$$
$$\forall S \subset T \subset N, \forall v \in N \setminus T,$$
$$f(S+v) - f(S) \geq f(T+v) - f(T)$$

Some Facts Regarding this Problem

- **Bad News**
  - Consider a non-negative, monotone, submodular function f
  - Finding a k-element set S for which $f(S)$ is maximized is **NP-hard**
    - It is NP-hard to determine the optimum initial set for cascade maximization
- **Good News**
  - We can use a greedy algorithm
    - Start with an empty set S
    - For k iterations:
      Add node v to S that maximizes $f(S \cup \{v\}) - f(S)$.
  - How good (or bad) it is? (Kempe et al., before that Nemhauser et al.)
    - **Theorem**: the greedy algorithm provides a $(1 - 1/e)$ approximation.
    - The resulting set S activates **at least** $(1 - 1/e) \approx 63\%$ of the number of nodes that any size-k set S could activate.

Greedy Algorithm

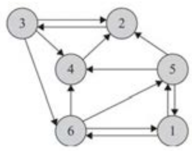
Algorithm 1 Maximizing the spread of cascades – Greedy algorithm

Require: Diffusion graph $G(V, E)$, budget k

```
1: return  Seed set S (set of initially activated nodes)
2: i = 0;
3: S = {};
4: while i ≠ k do
5:     v = argmax_{v∈V\S} f(S ∪ {v});
       or equivalently argmax_{v∈V\S} f(S ∪ {v}) − f(S)
6:     S = S ∪ {v};
7:     i = i + 1;
8: end while
9: Return S;
```


Example of Maximizing the Spreads of Cascades

Example 7.3. *For the following graph, assume that node i activates node j when* $|i - j| \equiv 2 \ (mod\ 3)$. *Solve cascade maximization for* $k = 2$.



To find the first node v, we compute $f(\{v\})$ *for all v. We start with node 1. At time 0, node 1 can only activate node 6, because*

$$|1 - 6| \equiv 2 \ (mod\ 3), \quad (7.11)$$

$$|1 - 5| \not\equiv 2 \ (mod\ 3). \quad (7.12)$$

At time 1, node 1 can no longer activate others, but node 6 is active and can activate others. Node 6 has outgoing edges to nodes 4 and 5. From 4 and 5, node 6 can only activate 4:

$$|6 - 4| \equiv 2 \ (mod\ 3) \quad (7.13)$$

$$|6 - 5| \not\equiv 2 \ (mod\ 3). \quad (7.14)$$

At time 2, node 4 is activated. It has a single out-link to node 2 and since $|4 - 2| \equiv 2 \ (mod\ 3)$, *2 is activated. Node 2 cannot activate other nodes; therefore,* $f(\{1\}) = 4$. *Similarly, we find that* $f(\{2\}) = 1$, $f(\{3\}) = 1$, $f(\{4\}) = 2$, $f(\{5\}) = 1$, *and* $f(\{6\}) = 4$. *So, 1 or 6 can be chosen for our first node. Let us choose 6. If 6 is initially activated, nodes 1, 2, 4, and 6 will become activated at the end. Now, from the set* $\{1,2,3,4,5,6\} \setminus \{1,2,4,6\} = \{3,5\}$, *we need to select one more node. This is because in the setting for this example,* $f(\{6,1\}) = f(\{6,2\}) = f(\{6,4\}) = f(\{6\}) = 4$. *In general, one needs to compute* $f(S \cup \{v\})$ *for all* $v \in V \setminus S$ *(see Algorithm 7.2, line 5). We have* $f(\{6,3\}) = f(\{6,5\}) = 5$, *so we can select one node randomly. We choose 3. So,* $S = \{6,3\}$ *and* $f(S) = 5$.

Algorithm 7.2 Maximizing the spread of cascades – Greedy algorithm

Require: Diffusion graph $G(V, E)$, budget k

```
1: return  Seed set S (set of initially activated nodes)
2: i = 0;
3: S = {};
4: while i ≠ k do
5:     v = arg max_{v∈V\S} f(S ∪ {v});
       or equivalently arg max_{v∈V\S} f(S ∪ {v}) − f(s)
6:     S = S ∪ {v};
7:     i = i + 1;
8: end while
9: Return S;
```

Intervention

- By limiting the number of out-links
  – Disconnected nodes don't get to activate anyone
- By limiting the number of in-links
  – Reducing the chance of getting activated by others
- By decreasing the activation probability p_{vw}
  – Reducing the chance of activating others.

Diffusion of Innovations

- **The network is <u>not observable</u>**
- **Only public information is observable**

Diffusion of Innovation

- An innovation is "*an idea, practice, or object that is perceived as new by an individual or other unit of adoption*"
- The theory of diffusion of innovations aims to answer **why** and **how** innovations spread.
- It also describes the **reasons** behind the diffusion process, individuals involved, as well as the rate at which ideas spread.

Innovation Characteristics

For an innovation to be adopted, the individuals adopting it (adopters) and the innovation must have certain qualities

Innovations must:

- **Be Observable**,
  - The degree to which the results of an innovation are visible to potential adopters
- **Have Relative Advantage**
  - The degree to which the innovation is perceived to be superior to current practice
- **Be Compatible**
  - The degree to which the innovation is perceived to be consistent with socio-cultural values, previous ideas, and/or perceived needs
- **Have Trialability**
  - The degree to which the innovation can be experienced on a limited basis
- **Not be Complex**
  - The degree to which an innovation is difficult to use or understand.

Diffusion of Innovations Models

- **First model was introduced by Gabriel Tarde in the early 20th century**

I. The Iowa Study of Hybrid Corn Seed

- Ryan and Gross studied the adoption of hybrid seed corn by farmers in Iowa
  - The hybrid corn was highly resistant to diseases and other catastrophes such as droughts
- Despite the fact that the use of new seed could lead to an increase in quality and production, the adoption by Iowa farmers was slow
  - Farmers did not adopt it due to its high price and its inability to reproduce
    - i.e., new seeds have to be purchased from the seed provider

I. The Iowa Study of Hybrid Corn Seed

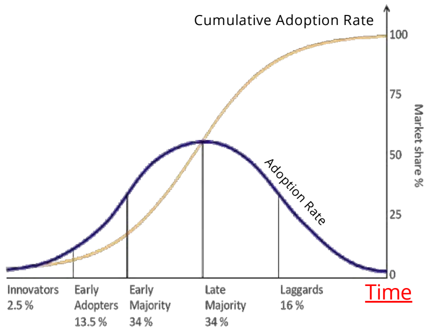
Farmers received information through two main channels

- **Mass communications** from companies selling the seeds (i.e., information)
- **Interpersonal communications** with other farmers. (i.e., influence)

- Adoption depended on a combination of information and influence.

- The study showed that the adoption rate follows an S-shaped curve and that there are 5 different types of adopters based on the order that they adopt the innovations, namely:

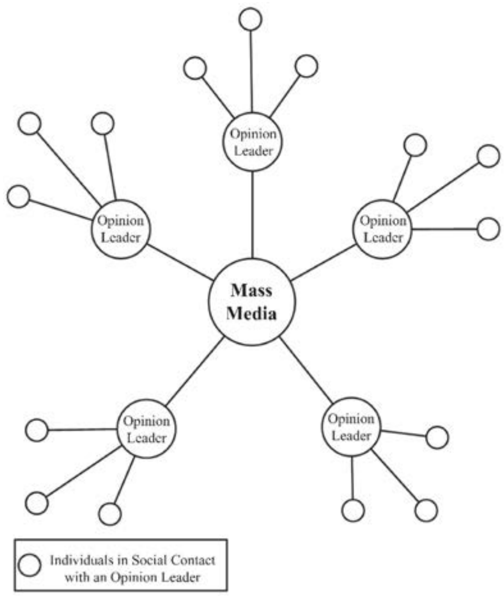
1) **Innovators** (top **2.5%**)
2) **Early Adopters** (next **13.5%**)
3) **Early Majority** (next **34%**)
4) **Late Majority** (next **34%**)
5) **Laggards** (last **16%**)



II. Katz Two-Step Flow Model

- **Two-step Flow Model.** most information comes from mass media, which is then directed toward influential figures called *opinion leaders*.

- These leaders then convey the information (or form opinions) and act as hub for other members of the society



III. Rogers: Diffusion of Innovations: The Process

Adoption process:

1. **Awareness**
   – The individual becomes aware of the innovation, but her information regarding the product is limited
2. **Interest**
   – The individual shows interest in the product and seeks more information
3. **Evaluation**
   – The individual tries the product in his mind and decides whether or not to adopt it
4. **Trial**
   – The individual performs a trial use of the product
5. **Adoption**
   – The individual decides to continue the trial and adopts the product for full use

Modeling Diffusion of Innovations

The diffusion of innovations models can be concretely described as

$$\frac{dA(t)}{dt} = i(t)[P - A(t)]$$

- $A(t)$ is the total population that adopted the innovation
- $i(t)$ denotes the coefficient of diffusion corresponding to the innovativeness of the product being adopted
- P is the total number of potential adopters (till time t)

- The rate depends on how innovative the product is
- The rate affects the potential adopters that have not yet adopted the product.

Modeling Diffusion of Innovations

We can rewrite $A(t)$ as $$A(t) = \int_{t_0}^{t} a(t)dt$$ Let $A_0 = A(0)$

The adopters at time t

We can define the diffusion coefficient $i(t)$ as a function of number of adopters $A(t)$

$$i(t) = \alpha + \alpha_0 A_0 + \ldots + \alpha_t A(t) = \alpha + \sum_{i=t_0}^{t} \alpha_i A(i)$$

We can simplify this linear combination

Diffusion Models

Three models of diffusion:
i.e., each having different ways to compute i(t):

$$\frac{dA(t)}{dt} = i(t)[P - A(t)]$$

$i(t) = \alpha,$	External-Influence Model
$i(t) = \beta A(t),$	Internal-Influence Model
$i(t) = \alpha + \beta A(t).$	Mixed-Influence Model

- α **- Innovativeness factor of the product**
- β **- Imitation factor**

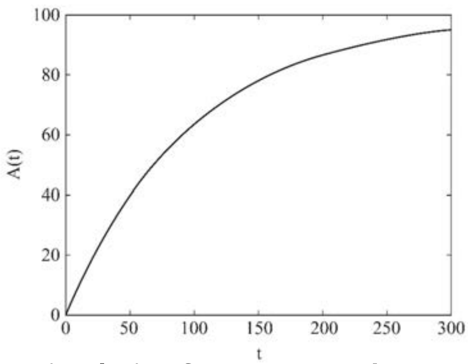
1. External-Influence Model

The adoption rate is a function that depends on external entities, $i(t) = \alpha$

– Assuming $A(t = t_0 = 0) = 0$

$$\frac{dA(t)}{dt} = \alpha[P - A(t)] \Rightarrow A(t) = P(1 - e^{-\alpha t})$$

The number of adopters increases exponentially and then saturates near P



Simulation for $P = 100$ and $\alpha = 0.01$

2. Internal-Influence Model (Pure imitation Model)

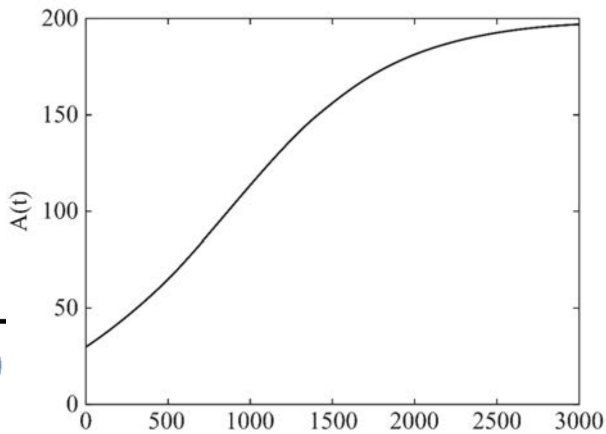
The adoption rate is a function that depends only on the number of already activated individuals

– $i(t) = \beta A(t)$

$$\frac{dA(t)}{dt} = \beta A(t)[P - A(t)]$$

$$\Downarrow$$

$$A(t) = \frac{P}{1 + \frac{P - A_0}{A_0} e^{-\beta P(t - t_0)}}$$



Simulation for $A_0 = 30$, $P = 200$ and $\beta = 0.00001$

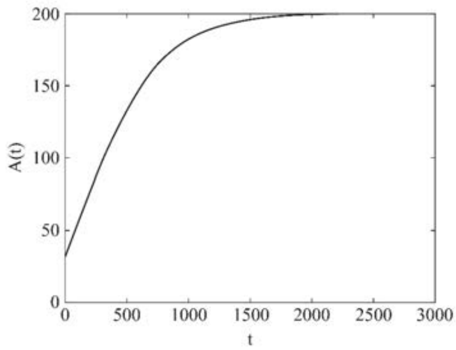
3. Mixed-Influence Model

The adoption rate is a function that depends on both the number of already activated individuals and external forces,

– $i(t) = \alpha + \beta A(t)$

$$\frac{dA(t)}{dt} = \alpha + \beta A(t)A(t)[P - A(t)]$$

$$A(t) = \frac{P - \frac{\alpha(P-A_0)}{\alpha+\beta A_0} e^{-(\alpha+\beta P)(t-t_0)}}{1 + \frac{\beta(P-A_0)}{\alpha+\beta A_0} e^{-(\alpha+\beta P)(t-t_0)}}$$



Simulation for
$P = 200$
$A_0 = 30$,
$\beta = 0.00001$ and $\alpha = 0.001$

Diffusion of Innovation: Intervention

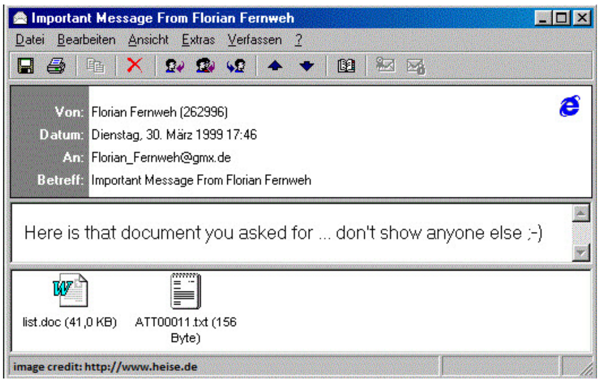
1. **Limiting the distribution of the product or the audience that can adopt the product.**

2. **Reducing interest in the product being sold.**
   – A company can inform adopters of the faulty status of the product.

3. **Reducing interactions within the population.**
   – Reduced interactions result in less imitations on product adoptions and a general decrease in the trend of adoptions.

Epidemics

Epidemics: Melissa computer worm

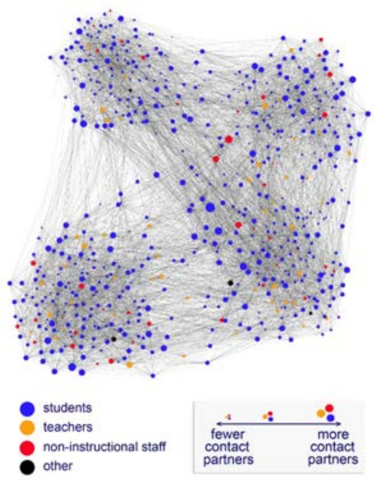


- Started on March 1999
- Infected MS Outlook users
- The user
  - Receives email with a word document that has a virus
  - Once opened, the virus sends itself to the first 50 users in the outlook address book
- First detected on Friday, March 26
  - On Monday, March 29, the virus had infected more than 100K computers

Epidemics

Epidemics describes the process by which diseases spread. This process consists of

- A pathogen
  - The disease being spread
  - Tweet being retweeted
- A population of hosts
  - Humans, animals, plants, etc.
- A spreading mechanism
  - Breathing, drinking, sexual activity, etc.



Comparing Epidemics and Cascades

- Epidemic models assume an **<u>implicit network</u>** and unknown connections between users.
  - Unlike information cascades and herding
  - Similar to diffusion of innovations models,
- Epidemic models are more suitable when we are interested in global patterns
  - Trends
  - Ratios of people getting infected
  - Not suitable for who infects whom

How to Analyze Epidemics?

I. Using Contact Network

- look at how hosts contact each other and devise methods that describe how epidemics happen in networks.
- **Contact network**: a graph where nodes represent the hosts and edges represent the interactions between these hosts.
  - E.g., In influenza contact network, hosts (nodes) that breathe the same air are connected

II. Fully-mixed Method

- Analyze only the rates at which hosts get infected, recover, etc. and avoid considering network information

The models discussed here will assume:
- No contact network information is available
- The process by which hosts get infected is unknown

Basic Epidemic Models

- SI
- SIR
- SIS
- SIRS

SI Model: Definition

SI model:

- The *susceptible* individuals get infected
- Once *infected,* they will never get cured

Two Types of Users:

- **Susceptible**
  – When an individual is in the susceptible state, he or she can potentially get infected by the disease.
- **Infected**
  – An infected individual has the chance of infecting susceptible parties

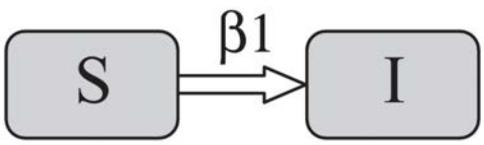
Notations

- N: size of the crowd
- $S(t)$: number of susceptible individuals at time t
  – $s(t) = S(t)/N$
- $I(t)$: number of infected individuals at time t
  – $i(t) = I(t)/N$
- β: Contact probability
  – if $\beta = 1$ everyone comes to contact with everyone else
  – if $\beta = 0$ no one meets another individual

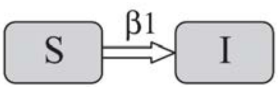
$$N = S(t) + I(t)$$

SI Model

- At each time stamp, an infected individual will meet βN people on average and will infect βS of them
- Since I are infected, βIS will be infected in the next time step



SI Model: Equations

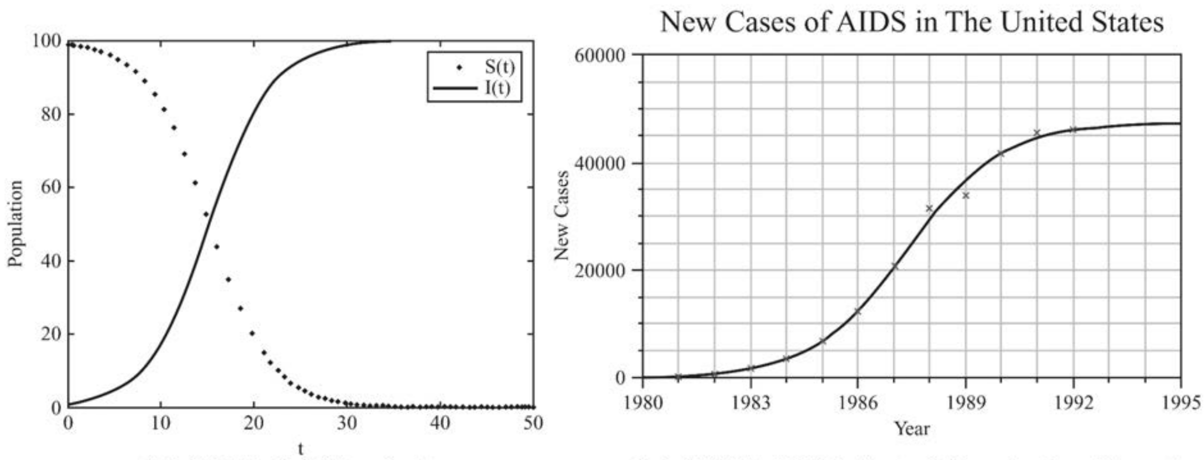


$$\frac{dS}{dt} = -\beta IS, \qquad \frac{dI}{dt} = \beta IS.$$

$$(S + I = N) \Rightarrow \frac{dI}{dt} = \beta I(N - I) \Rightarrow I(t) = \frac{N I_0 e^{\beta t N}}{N + I_0(e^{\beta t N} - 1)}$$

I_0 is the number of individuals infected at time 0

SI Model: Example



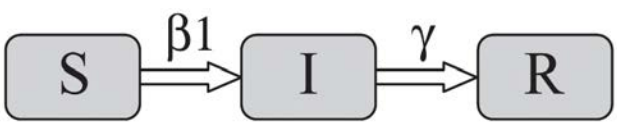
(a) SI Model Simulation (b) HIV/AIDS Infected Population Growth

Logistic growth function compared to the HIV/AIDS growth in the United States

SIR Model

SIR model:

- In addition to the **I** and **S** states, a <u>recovery state</u> **R** is present
- Individuals get infected and some recover
- Once hosts recover (or are removed) they can no longer get infected and are not susceptible



SIR Model, Equations

$$I + S + R = N$$

$$\begin{aligned}\frac{dS}{dt} &= -\beta IS,\\ \frac{dI}{dt} &= \beta IS - \gamma I,\\ \frac{dR}{dt} &= \gamma I.\end{aligned}$$

γ defines the recovering probability of an infected individual at a time stamp

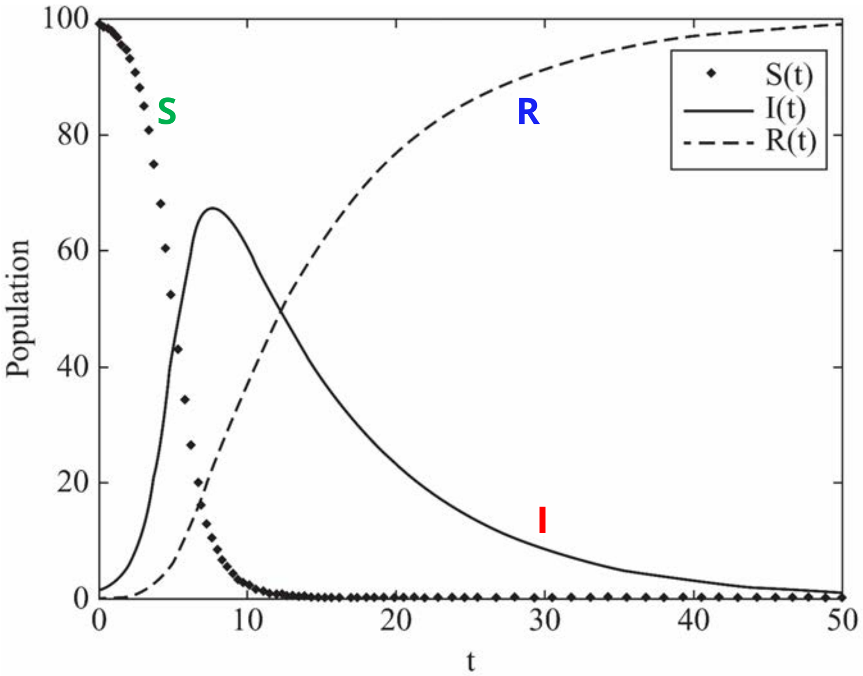
SIR Model, Equations, Cont.

$$\begin{aligned}\frac{dS}{dt} &= -\beta IS,\\ \frac{dI}{dt} &= \beta IS - \gamma I,\\ \frac{dR}{dt} &= \gamma I.\end{aligned} \Rightarrow \frac{dS}{dR} = -\frac{\beta}{\gamma}S \Rightarrow \log\frac{S_0}{S} = \frac{\beta}{\gamma}R \quad (R_0 = 0)$$

$$\begin{aligned}\frac{dR}{dt} &= \gamma(N - S - R)\\ \frac{dR}{dt} &= \gamma(N - S_0 e^{-\frac{\beta}{\gamma}R} - R)\end{aligned} \Rightarrow t = \frac{1}{\gamma}\int_0^R \frac{dx}{N - S_0 e^{-\frac{\beta}{\gamma}x} - x}$$

There is no closed form solution for this integration and only numerical approximation is possible.

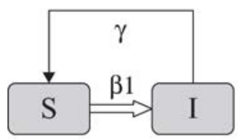
SIR Model: Example



SIR model simulated with $S_0 = 99$, $I_0 = 1$, $R_0 = 0$, $\beta = 0.01$, and $\gamma = 0.1$

SIS Model

- The **SIS** model is the same as the **SI** model with the addition of infected nodes recovering and becoming susceptible again



$$\frac{dS}{dt} = \gamma I - \beta I S, \quad \frac{dI}{dt} = \beta I S - \gamma I$$

$$\frac{dI}{dt} = \beta I(N - I) - \gamma I = I(\beta N - \gamma) - \beta I^2$$

SIS Model

$$\frac{dI}{dt} = \beta I(N - I) - \gamma I = I(\beta N - \gamma) - \beta I^2$$

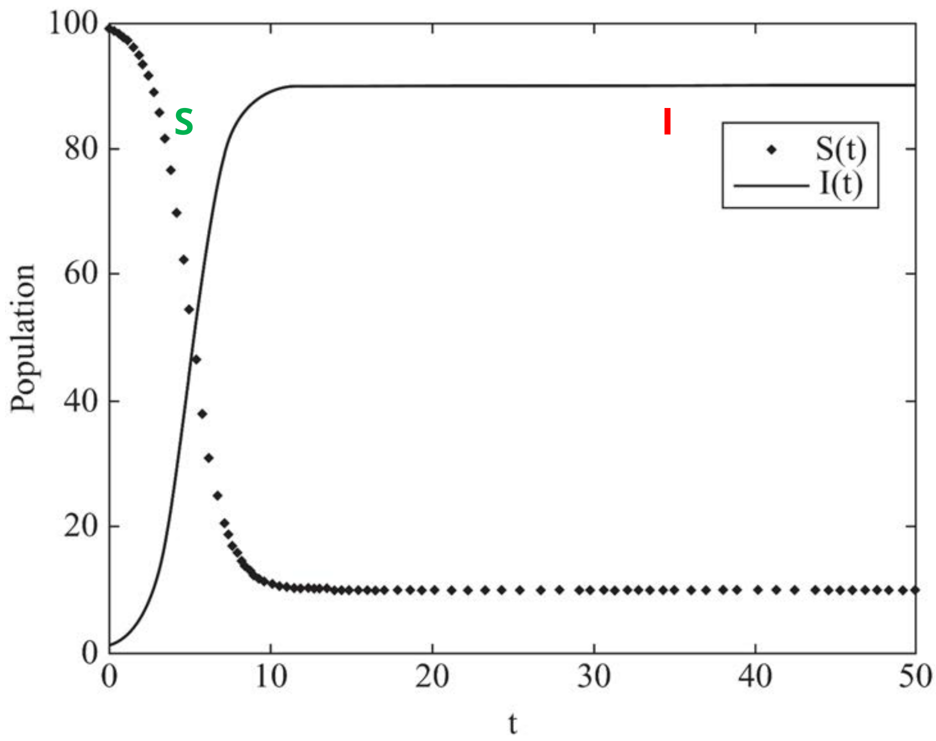
Case 1: When $\beta N \leq \gamma$ (or when $N \leq \frac{\gamma}{\beta}$):

- The first term will be at most zero or negative
- The whole term becomes negative
- In the limit, I(t) will decrease exponentially to zero

Case 2: When $\beta N > \gamma$ (or when $N > \frac{\gamma}{\beta}$):

- We will have a logistic growth function like the **SI** model

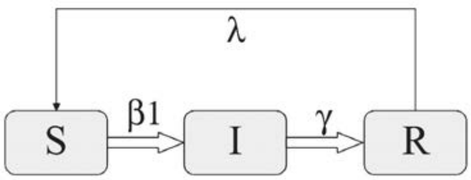
SIS Model



SIS model simulated with $S_0 = 99$, $I_0 = 1$, $\beta = 0.01$, and $\gamma = 0.1$

SIRS Model

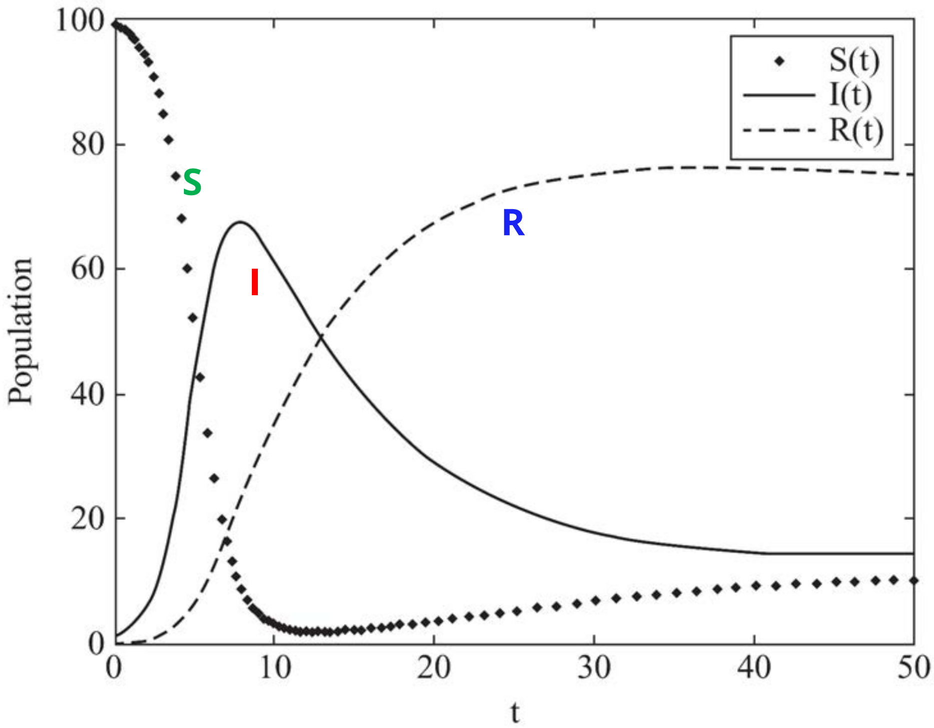
The individuals who have recovered will lose immunity after a certain period of time and will become susceptible again



$$\frac{dS}{dt} = \lambda R - \beta IS,$$
$$\frac{dI}{dt} = \beta IS - \gamma I,$$
$$\frac{dR}{dt} = \gamma I - \lambda R.$$

Like the SIR, model this model has no closed form solution, so numerical integration can be used

SIRS Model



SIRS model simulated with $S_0 = 99$, $I_0 = 1$, $R_0 = 1$, $\beta = 0.01$, $\lambda = 0.02$, and $\gamma = 0.1$

Epidemic Intervention

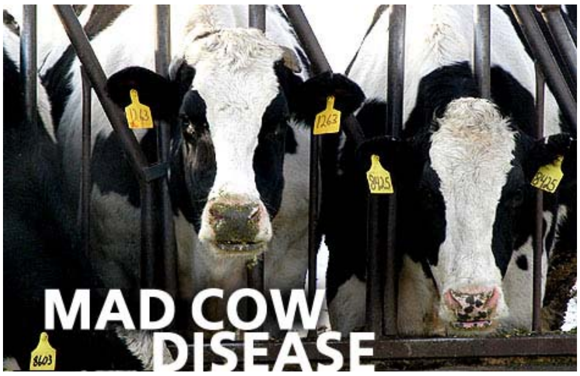
Epidemic Intervention

- Suppose that we have a susceptible society and want to prevent more spread by vaccinating the most vulnerable individuals
- How to find the most vulnerable individuals?

Randomly pick some nodes and ask them who is the most vulnerable from their point of view, then vaccinate those individuals!

Epidemic Intervention: *Mad Cow Disease*

- Jan. 2001
  - First case observed in UK
- Feb. 2001
  - 43 farms infected
- Sep. 2001
  - 9000 farms infected



In the mad cow disease case, we have weak ties

- Animals being bought and sold
- Soil from tourists, etc.

How to stop the disease:

- Banned movement (make contagion harder)
- Killed millions of animals (remove weak ties)