



Liberal Safety for Answer Set Programs with External Sources

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1. Motivation

- ▶ HEX-programs extend ASP by **external sources**
- ▶ Rule bodies may contain **external atoms** of the form
 $\&p[q_1, \dots, q_k](t_1, \dots, t_l),$
 p ... external predicate name
 q_i ... predicate names or constants: $\tau(\&p, i) \in \{\text{pred}, \text{const}\}$
 t_j ... terms
 Semantics:
 $1 + k + l$ -ary Boolean **oracle function** $f_{\&p}$:
 $\&p[q_1, \dots, q_k](t_1, \dots, t_l)$ is true under assignment A
 iff $f_{\&p}(A, q_1, \dots, q_k, t_1, \dots, t_l) = 1$.
- ▶ Traditional safety **not** sufficient due to **value invention**
- ▶ Current notion of **strong safety** is **unnecessarily restrictive**

Example

$$\Pi = \left\{ \begin{array}{ll} r_1: t(a). & r_3: s(Y) \leftarrow t(X), \&cat[X, a](Y). \\ r_2: \text{dom}(aa). & r_4: t(X) \leftarrow s(X), \text{dom}(X). \end{array} \right\}$$

Goal:

- ▶ New **more liberal safety criteria**
- ▶ Still guarantee **finite groundability**
- ▶ **Future extensibility**

Main result:

- ▶ **Flexible framework** which **subsumes other notions** and allows for **combination of syntactic and semantic safety criteria**

2. Grounding Operator

We use the following **canonical grounding operator**:

$$G_{\Pi}(\Pi') = \bigcup_{r \in \Pi} \{r_{\theta} \mid A \subseteq \mathcal{A}(\Pi'), A \not\models \perp, A \models B^+(r_{\theta})\},$$

where $\mathcal{A}(\Pi') = \{Ta, Fa \mid a \in A(\Pi')\} \setminus \{Fa \mid a \leftarrow \cdot \in \Pi\}$
 and r_{θ} is the instance of r under variable substitution $\theta: \mathcal{V} \rightarrow \mathcal{C}$.

For **liberally domain-expansion safe programs**, the **fixpoint** of this operator is reached after **finitely many steps** and preserves all answer sets.

Example

Program Π :

$$\begin{array}{lll} r_1: s(a). & r_2: \text{dom}(ax). & r_3: \text{dom}(axx). \\ r_4: s(Y) \leftarrow s(X), \&cat[X, x](Y), \text{dom}(Y). \end{array}$$

Least fixpoint of G_{Π} :

$$\begin{array}{lll} r'_1: s(a). & r'_2: \text{dom}(ax). & r'_3: \text{dom}(axx). \\ r'_4: s(ax) \leftarrow s(a), \&cat[a, x](ax), \text{dom}(ax). \\ r'_5: s(axx) \leftarrow s(ax), \&cat[ax, x](axx), \text{dom}(axx). \end{array}$$

3. Two Key Concepts

- ▶ A **term** is **bounded** if $G_{\Pi}(\Pi')$ contains only finitely many substitutions for this term
- ▶ An **attribute** (=argument position of a predicate or external predicate) is **(liberally) de-safe** if $G_{\Pi}(\Pi')$ contains only finitely many values at this attribute position

A program is de-safe iff all its attributes are de-safe

4. How to Check Safety

1. Start with empty set of **bounded terms** B_0 and **de-safe attributes** S_0
2. For all $n \geq 0$ until B_n and S_n did not change
 - a Declare additional terms as bounded $\Rightarrow B_{n+1}$
 (assuming that B_n are bounded and S_n are de-safe)
 - b Identify additional de-safe attributes $\Rightarrow S_{n+1}$
 (assuming that B_{n+1} are bounded and S_n are de-safe)

Identification of bounded terms in 2a by **term bounding functions (TBFs)**

Concrete safety criteria can be plugged in by specific TBF $b(\Pi, r, S, B)$

\Rightarrow TBFs are **easily exchangeable** but must fulfill certain **preconditions**

5. Example: Syntactic Term Bounding Function

$t \in b_{\text{syn}}(\Pi, r, S, B)$ iff

- (i) t is a constant in r ; or
- (ii) there is an ordinary atom $q(s_1, \dots, s_{ar(q)}) \in B^+(r)$ s.t. $t = s_j$, for some $1 \leq j \leq ar(q)$ and $q \setminus j \in S$; or
- (iii) for some external atom $\&g[\vec{X}](\vec{Y}) \in B^+(r)$, we have that $t = Y_i$ for some $Y_i \in \vec{Y}$, and for each $X_i \in \vec{X}$,

$$\begin{cases} X_i \in B, & \text{if } \tau(\&g, i) = \text{const}, \\ X_i \setminus 1, \dots, X_i \setminus ar(X_i) \in S, & \text{if } \tau(\&g, i) = \text{pred}. \end{cases}$$

6. Key Propositions

Proposition 1:

If $b_i(\Pi, r, S, B)$, $1 \leq i \leq \ell$, are TBFs, then $\bigcup_{1 \leq i \leq \ell} b_i(\Pi, r, S, B)$ is a TBF.
 \Rightarrow **Easy combination of multiple TBFs**

Proposition 2:

If Π is a de-safe program, then $G_{\Pi}^{\infty}(\emptyset)$ is finite.

Proposition 3:

A de-safe program Π is finitely restrictable and $G_{\Pi}^{\infty}(\emptyset) \equiv^{pos} \Pi$.
 \Rightarrow Operator G is a **witness for finite groundability**

Note: The results hold for **any** TBF!

7. Advantages

- ▶ **Strictly more liberal** than **strongly safe** [Eiter et al., 2006], **VI-** [Calimeri et al., 2007] and **ω -restricted** [Syrjänen, 2001] programs.
- ▶ **Modularity** of the approach
- ▶ **Extensible** due to easy **combination** of multiple TBFs

8. References

- ▶ Calimeri, F., Cozza, S., and Ianni, G. (2007). External Sources of Knowledge and Value Invention in Logic Programming. Ann. Math. Artif. Intell. 50(3–4):333–361.
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- ▶ Syrjänen, T. (2001). Omega-restricted logic programs, LPNMR'01, volume 2173 of LNCS, pages 267–279.
- ▶ Zantema, H. (1994). Termination of term rewriting: Interpretation and type elimination, J. Symb. Comp., 17(1):23–50.