TPK4171 - Advanced Industrial Robotics
Exercise 2, 2024

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Problem 1

The lines $\boldsymbol{\ell}_1 = [1, -1, 0]^{\mathrm{T}}$ and $\boldsymbol{\ell}_2 = [0, 1, -1]^{\mathrm{T}}$ are given.

a) Write the lines $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$ in the form $y = Ax + B$, find the point that is closest to the origin for both lines. Sketch the lines.

b) Find the intersection point $\boldsymbol{x}$ of the lines $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$. Sketch the lines and the intersection point.

c) Find the distance from $\boldsymbol{\ell}_1$ to the point $\boldsymbol{x}_1 = [3, 1, 1]^{\mathrm{T}}$.

d) Find the line $\boldsymbol{\ell}_d$ through the points $\boldsymbol{x}_2 = [1, 0, 0]^{\mathrm{T}}$ and $\boldsymbol{x}_3 = [0, 1, 1]^{\mathrm{T}}$. Sketch the line.

e) Find the line $\boldsymbol{\ell}_e$ through the points $\boldsymbol{x}_4 = [1, 0, 0]^{\mathrm{T}}$ and $\boldsymbol{x}_5 = [0, 1, 0]^{\mathrm{T}}$. What type of line is $\boldsymbol{\ell}_e$? Is it possible to write the line in the form $y = Ax+B$ and make a sketch of the line and the point?

Problem 2

A camera is used to find points in a horizontal plane. The object frame has a vertical z_o axis. The displacement from the camera frame to the object frame is given by

$$\boldsymbol{T}_o^c = \begin{bmatrix} \boldsymbol{R}_x(120^\circ)\boldsymbol{R}_z(45^\circ) & \boldsymbol{t} \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix} \tag{1}$$

where $\boldsymbol{t} = [0, 0, 2]^{\mathrm{T}}$.

4 points are given in the xy plane of the object frame as the corners of a quadratic rectangle with coordinates $\boldsymbol{r}_{o1}^o = [0, 0, 0]^{\mathrm{T}}$, $\boldsymbol{r}_{o2}^o = [1, 0, 0]^{\mathrm{T}}$, $\boldsymbol{r}_{o3}^o = [1, 1, 0]^{\mathrm{T}}$ and $\boldsymbol{r}_{o4}^o = [0, 1, 0]^{\mathrm{T}}$. In the xy plane of the object frame these points can be written as the homogeneous points $\boldsymbol{x}_1 = [0, 0, 1]^{\mathrm{T}}$, $\boldsymbol{x}_2 = [1, 0, 1]^{\mathrm{T}}$, $\boldsymbol{x}_3 = [1, 1, 1]^{\mathrm{T}}$ and $\boldsymbol{x}_4 = [0, 1, 1]^{\mathrm{T}}$ where the the first two coordinates are the x and y coordinates in the xy plane of the object frame, and the third coordinate is the homogeneous coordinate.

a) Find the homogeneous representation in the xy plane of the object frame for the line $\boldsymbol{\ell}_{12}$ defined by the points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, the line $\boldsymbol{\ell}_{23}$ defined by the points $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$, the line $\boldsymbol{\ell}_{34}$ defined by the points $\boldsymbol{x}_3$ and $\boldsymbol{x}_4$, and the line $\boldsymbol{\ell}_{41}$ defined by the points $\boldsymbol{x}_4$ and $\boldsymbol{x}_1$. Find the homogeneous intersection point $\boldsymbol{y}_1$ of the lines $\boldsymbol{\ell}_{12}$ and $\boldsymbol{\ell}_{34}$, and the homogeneous intersection point $\boldsymbol{y}_2$ of the lines $\boldsymbol{\ell}_{23}$ and $\boldsymbol{\ell}_{41}$. What type of points are $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$?

b) Find the homogeneous normalized image coordinate vectors $\tilde{\boldsymbol{s}}_1, \ldots, \tilde{\boldsymbol{s}}_4$ corresponding to the points $\boldsymbol{r}_{o1}^o, \ldots, \boldsymbol{r}_{o4}^o$.

c) Find the homogeneous lines $\boldsymbol{\lambda}_{12} = \tilde{\boldsymbol{s}}_1 \times \tilde{\boldsymbol{s}}_2$, $\boldsymbol{\lambda}_{23} = \tilde{\boldsymbol{s}}_2 \times \tilde{\boldsymbol{s}}_3$, $\boldsymbol{\lambda}_{34} = \tilde{\boldsymbol{s}}_3 \times \tilde{\boldsymbol{s}}_4$ and $\boldsymbol{\lambda}_{41} = \tilde{\boldsymbol{s}}_4 \times \tilde{\boldsymbol{s}}_1$ in the normalized image plane. This lines will be the image of the lines $\boldsymbol{\ell}_{12}$, $\boldsymbol{\ell}_{23}$, $\boldsymbol{\ell}_{34}$ and $\boldsymbol{\ell}_{41}$.

d) Find the homogeneous intersection point $\boldsymbol{z}_1$ of the lines $\boldsymbol{\lambda}_{12}$ and $\boldsymbol{\lambda}_{34}$, and the intersection point $\boldsymbol{z}_2$ of the lines $\boldsymbol{\lambda}_{23}$ and $\boldsymbol{\lambda}_{41}$ and the corresponding points in the normalized image plane. Which points in the xy plane of the object frame will correspond to the points $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$?

e) Find the line $\boldsymbol{\lambda}_z = \boldsymbol{z}_1 \times \boldsymbol{z}_2$. What is the interpretation of this line?

Problem 3

Point	1	2	3	4	5	6
x	0.0	1.0	2.0	3.0	5.0	5.5
y	1.0	2.0	3.0	4.2	5.3	6.25

Table 1: Data-set for RANSAC

a) Use SVD to find the line $\boldsymbol{\ell} = [a, b, c]^{\mathrm{T}}$ that is the total least-squares fit to all the data-points in Table 1.

b) In a RANSAC setting, let the candidate solution for the line be determined by points 1 and 2. Find the inliers and the outliers for this solution when the requirement for a point to be inlier is that the magnitude of the distance from the point to the line is less than $\delta = 0.1$. It is given that the distance from a line $\boldsymbol{\ell} = [a, b, c]^{\mathrm{T}}$ to a point $\boldsymbol{x}$ is $\delta = \boldsymbol{\ell}^{\mathrm{T}}\boldsymbol{x}/|\boldsymbol{n}|$, where $\boldsymbol{n} = [a, b]^{\mathrm{T}}$ is the normal vector of the line.

c) Find the total least-squares solution for the line when only the inliers from b) are used to determine the line.