TPK4171 - Advanced Industrial Robotics
Exercise 1, 2024

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Deadline: 2024-01-24

The problems should be solved in Python or MATLAB, and the code is to be included. Python code can be based on the scripts found in the lecture notes.

Problem 1

The camera model is given by

$$\tilde{\boldsymbol{s}} = \frac{1}{z}\boldsymbol{r} \tag{1}$$

$$\tilde{\boldsymbol{p}} = \boldsymbol{K}\tilde{\boldsymbol{s}} \tag{2}$$

where $\boldsymbol{r} = \boldsymbol{r}_{cp}^{c} = [x, y, z]^{\mathrm{T}}$ is the position of a 3D point in the scene in the camera frame, and the camera parameter matrix is

$$\boldsymbol{K} = \begin{bmatrix} k & 0 & u_0 \\ 0 & k & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The inverse of the camera parameter matrix is

$$\boldsymbol{K} = \begin{bmatrix} 1/k & 0 & -u_0/k \\ 0 & 1/k & -v_0/k \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

a) Let the camera parameters be $k = 1500$, $u_0 = 640$ and $v_0 = 512$. The 3D points

$$\boldsymbol{r}_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.5 \end{bmatrix}, \quad \boldsymbol{r}_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \quad \boldsymbol{r}_3 = \begin{bmatrix} 0.1 \\ 0.2 \\ 1 \end{bmatrix}$$

in the scene are given in the camera frame. Find the corresponding normalized image coordinates $\tilde{\boldsymbol{s}}_1$, $\tilde{\boldsymbol{s}}_2$ and $\tilde{\boldsymbol{s}}_3$ and the corresponding pixel coordinates $\tilde{\boldsymbol{p}}_1$, $\tilde{\boldsymbol{p}}_2$ and $\tilde{\boldsymbol{p}}_3$.

b) The pixel points

$$\tilde{\boldsymbol{p}}_a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \tilde{\boldsymbol{p}}_b = \begin{bmatrix} 740 \\ 612 \\ 1 \end{bmatrix}, \quad \tilde{\boldsymbol{p}}_c = \begin{bmatrix} 1280 \\ 1024 \\ 1 \end{bmatrix}$$

are given. Find the corresponding normalized image coordinates $\tilde{\boldsymbol{s}}_a$, $\tilde{\boldsymbol{s}}_b$ and $\tilde{\boldsymbol{s}}_c$

c) The displacement of the object frame o with respect to the camera frame c is given by

$$\boldsymbol{T}_o^c = \begin{bmatrix} \boldsymbol{R}_o^c & \boldsymbol{t}_{co}^c \\ \boldsymbol{0}^{\mathrm{T}} & 1 \end{bmatrix}$$

where $\boldsymbol{t}_{co}^c = [0, 0, 4]^{\mathrm{T}}$ and

$$\boldsymbol{R}_o^c = \boldsymbol{R}_x(\pi/2)$$

is the rotation matrix of a rotation by an angle $\pi/2$ about the x axis. A point P is given by its position $\boldsymbol{r}_{op}^o = [1, 1, 0]^{\mathrm{T}}$ in the object frame. Find the camera matrix $\boldsymbol{C}$ which satisfies

$$z\tilde{\boldsymbol{s}} = \boldsymbol{C}\tilde{\boldsymbol{r}}_{op}^o$$

where $\tilde{\boldsymbol{r}}_{op}^o = [(\boldsymbol{r}_{op}^o)^{\mathrm{T}}, 1]^{\mathrm{T}}$ and z is the z coordinate of

$$\tilde{\boldsymbol{r}}_{cp}^c = \boldsymbol{T}_0^c \tilde{\boldsymbol{r}}_{op}^o$$

Find the normalized image coordinates $\tilde{\boldsymbol{s}}$ and the pixel coordinates $\tilde{\boldsymbol{p}}$ of the point.

Problem 2

a) A camera is used to find points in a horizontal plane. The object frame has a vertical z_o axis. The displacement from the camera frame to the object frame is given by

$$\boldsymbol{T}_o^c = \begin{bmatrix} \boldsymbol{R}_x(120^\circ) & \boldsymbol{t} \\ \boldsymbol{0}^{\mathrm{T}} & 1 \end{bmatrix} \tag{5}$$

where $\boldsymbol{t} = [0, 0, 2]^{\mathrm{T}}$. 4 points are given in the object frame as the corners of a quadratic rectangle with coordinates $\boldsymbol{r}_{o1}^o = [0, 0, 0]^{\mathrm{T}}$, $\boldsymbol{r}_{o2}^o = [1, 0, 0]^{\mathrm{T}}$, $\boldsymbol{r}_{o3}^o = [1, 1, 0]^{\mathrm{T}}$ and $\boldsymbol{r}_{o4}^o = [0, 1, 0]^{\mathrm{T}}$.

a) Find the coordinates $\boldsymbol{r}_{c1}^c$, $\boldsymbol{r}_{c2}^c$, $\boldsymbol{r}_{c3}^c$ and $\boldsymbol{r}_{c4}^c$ of the points in the camera frame.

b) Find the normalized image coordinates of the points.

c) Explain why the rectangle is not quadratic in normalized image coordinates.

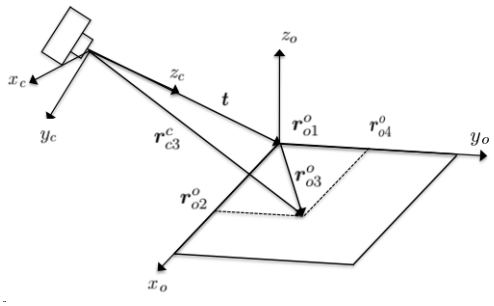

Figure 1: Camera and object frame.

Problem 3

Point	1	2	3	4	5
x	0	1	2	3	0
y	0	0	0	0	1

Table 1: Data-set for Hough transform

Use the Hough transform $\rho = x\cos\theta + y\sin\theta$ to find the lines which corresponds to the data-set in Table 1. Sketch the curves, and find the line with the highest number of points.