TPK4171 - Advanced Industrial Robotics Exercise 1, 2024

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Problem 1

The camera model is given by

$$\tilde{\boldsymbol{s}} = \frac{\mathbf{1}}{\boldsymbol{z}}\boldsymbol{r} \tag{1}$$

$$\tilde{\boldsymbol{p}} = \boldsymbol{K}\tilde{\boldsymbol{s}} \tag{2}$$

The camera parameter matrix is

$$\boldsymbol{K} = \begin{bmatrix} k & 0 & u_0 \\ 0 & k & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The inverse of the camera parameter matrix is

$$\boldsymbol{K^{-1}} = \begin{bmatrix} 1/k & 0 & -u_0/k \\ 0 & 1/k & -v_0/k \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

a) The camera parameters given by the problem are $k = 1500$, $u_0 = 640$ and $v_0 = 512$, and the 3D points are given by

$$\boldsymbol{r_1} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.5 \end{bmatrix}, \boldsymbol{r_2} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \boldsymbol{r_3} = \begin{bmatrix} 0.1 \\ 0.2 \\ 1 \end{bmatrix}$$

Find $\tilde{\boldsymbol{s}}_{\mathbf{1}}$, $\tilde{\boldsymbol{s}}_{\mathbf{2}}$ and $\tilde{\boldsymbol{s}}_{\mathbf{3}}$ and $\tilde{\boldsymbol{p}}_{\mathbf{1}}$, $\tilde{\boldsymbol{p}}_{\mathbf{2}}$ and $\tilde{\boldsymbol{p}}_{\mathbf{3}}$.

From Equation (1) we can compute $\tilde{\boldsymbol{s}}_{\mathbf{1}}$, $\tilde{\boldsymbol{s}}_{\mathbf{2}}$ and $\tilde{\boldsymbol{s}}_{\mathbf{3}}$, but first we need to know the value of $\boldsymbol{z}$. This value can be found just taking the third value of the 3D points $\boldsymbol{r_1}$, $\boldsymbol{r_2}$ and $\boldsymbol{r_3}$.

So knowing this, we can compute the values for $\tilde{\boldsymbol{s}}_{\mathbf{1}}$, $\tilde{\boldsymbol{s}}_{\mathbf{2}}$ and $\tilde{\boldsymbol{s}}_{\mathbf{3}}$ from Equation (1).

$$\tilde{s_1} = \frac{1}{z_1} r_1 = \frac{1}{0.5} \begin{bmatrix} 0.1 \\ 0.2 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.4 \\ 1 \end{bmatrix}$$

$$\tilde{s_2} = \frac{1}{z_2} r_2 = \frac{1}{5} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.4 \\ 1 \end{bmatrix}$$

$$\tilde{s_3} = \frac{1}{z_3} r_3 = \frac{1}{1} \begin{bmatrix} 0.1 \\ 0.2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ 1 \end{bmatrix}$$

Now that we have computed the values for the normalized image coordinates we can compute the values for the pixel coordinates because we already have the values of $\tilde{\boldsymbol{s}}_{\boldsymbol{1}}$, $\tilde{\boldsymbol{s}}_{\boldsymbol{2}}$ and $\tilde{\boldsymbol{s}}_{\boldsymbol{3}}$. Using (2) and the values of the camera parameters stated at the beggining we get

$$\tilde{p_1} = K\tilde{s_1} = \begin{bmatrix} 1500 & 0 & 640 \\ 0 & 1500 & 512 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.4 \\ 1 \end{bmatrix} = \begin{bmatrix} 940 \\ 1112 \\ 1 \end{bmatrix}$$

$$\tilde{p_2} = K\tilde{s_2} = \begin{bmatrix} 1500 & 0 & 640 \\ 0 & 1500 & 512 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.4 \\ 1 \end{bmatrix} = \begin{bmatrix} 940 \\ 1112 \\ 1 \end{bmatrix}$$

$$\tilde{p_3} = K\tilde{s_3} = \begin{bmatrix} 1500 & 0 & 640 \\ 0 & 1500 & 512 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2 \\ 1 \end{bmatrix} = \begin{bmatrix} 790 \\ 812 \\ 1 \end{bmatrix}$$

b) With pixel points

$$\tilde{\boldsymbol{p}}_{\boldsymbol{a}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tilde{\boldsymbol{p}}_{\boldsymbol{b}} = \begin{bmatrix} 740 \\ 612 \\ 1 \end{bmatrix}, \tilde{\boldsymbol{p}}_{\boldsymbol{c}} = \begin{bmatrix} 1280 \\ 1024 \\ 1 \end{bmatrix}$$

It is posible to compute the normalized image coordinates $\tilde{\boldsymbol{s}}_{\boldsymbol{a}}$, $\tilde{\boldsymbol{s}}_{\boldsymbol{b}}$ and $\tilde{\boldsymbol{s}}_{\boldsymbol{c}}$ by using Equation (2) and solving for $\tilde{\boldsymbol{s}}$

$$\tilde{\boldsymbol{s}} = \boldsymbol{K}^{-\boldsymbol{1}} \tilde{\boldsymbol{p}}$$

Using Equation (4) we can compute for the normalized image coordinates

$$\tilde{s}_a = K^{-1}\tilde{p}_a = \begin{bmatrix} 1/1500 & 0 & -640/1500 \\ 0 & 1/1500 & -512/1500 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.427 \\ -0.341 \\ 1 \end{bmatrix}$$

$$\tilde{s}_b = K^{-1}\tilde{p}_b = \begin{bmatrix} 1/1500 & 0 & -640/1500 \\ 0 & 1/1500 & -512/1500 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 740 \\ 612 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.067 \\ 0.067 \\ 1 \end{bmatrix}$$

$$\tilde{s}_c = K^{-1}\tilde{p}_c = \begin{bmatrix} 1/1500 & 0 & -640/1500 \\ 0 & 1/1500 & -512/1500 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1280 \\ 1024 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.427 \\ 0.341 \\ 1 \end{bmatrix}$$

c) The displacement of the oject frame o with respect to the camera frame c is given by

$$\boldsymbol{T_o^c} = \begin{bmatrix} \boldsymbol{R_o^c} & \boldsymbol{t_c^c o} \\ \boldsymbol{0^T} & 1 \end{bmatrix} \tag{5}$$

Where

$$\boldsymbol{t_{co}^c} = [0, 0, 4]^T$$

and

$$\boldsymbol{R_o^c} = \boldsymbol{R_x}(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi/2) & -\sin(\pi/2) \\ 0 & \sin(\pi/2) & \cos(\pi/2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

A point P is given by its position $\boldsymbol{r_{op}^o} = [1, 1, 0]^T$ in the object frame. With this information find the matrix $\boldsymbol{C}$ that satisfies the following expression.

$$z\boldsymbol{\tilde{s}} = \boldsymbol{C}\boldsymbol{\tilde{r}_{op}^o} \tag{6}$$

This $\boldsymbol{C}$ matrix can be found by looking at Equation (25) from the Vision.pdf provided by Professor Olav Egeland

$$\boldsymbol{\tilde{s}} = \frac{1}{z}\boldsymbol{\Pi}\boldsymbol{T_o^c}\boldsymbol{\tilde{r}_{op}^o} \tag{7}$$

This equation can be written in another way by multiplying all the equation by $\boldsymbol{z}$. This will result in the following equation

$$z\boldsymbol{\tilde{s}} = \boldsymbol{\Pi}\boldsymbol{T_o^c}\boldsymbol{\tilde{r}_{op}^o} \tag{8}$$

So now we can compare equations (6) and (8) and observe that

$$\boldsymbol{C} = \boldsymbol{\Pi}\boldsymbol{T_o^c}$$

And from the Vision.pdf notes provided by Professor Olav Egeland we can know that

$$\boldsymbol{\Pi}\boldsymbol{T_o^c} = \begin{bmatrix} \boldsymbol{R_o^c} & \boldsymbol{t_{co}^c} \end{bmatrix} \in \mathbb{R}^{3\times4}$$

Hence

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{R_o^c} & \boldsymbol{t_{co}^c} \end{bmatrix} \in \mathbb{R}^{3\times4}$$

The values of $\boldsymbol{R_o^c}$ and $\boldsymbol{t_{co}^c}$ are given so C can be computed

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

Find the normalized image coordinates $\boldsymbol{\tilde{s}}$ and the pixel coordinate $\boldsymbol{\tilde{p}}$ of point P. To compute this z needs to be obtained first. This value is found by taking the third value of the vector $\boldsymbol{\tilde{r}_{cp}^c}$ which is defined by

$$\boldsymbol{\tilde{r}_{cp}^c} = \boldsymbol{T_o^c}\boldsymbol{\tilde{r}_{op}^o}$$

Where

$$\tilde{\boldsymbol{r}}^{\boldsymbol{o}}_{\boldsymbol{op}} = \begin{bmatrix} \boldsymbol{r}^{\boldsymbol{o}}_{\boldsymbol{op}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Hence

$$\tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{cp}} = \boldsymbol{T}^{\boldsymbol{c}}_{\boldsymbol{o}} \tilde{\boldsymbol{r}}^{\boldsymbol{o}}_{\boldsymbol{op}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

From this result $\tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{cp}}$ is obtained and $\tilde{\boldsymbol{s}}$ can be computed were $z = \tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{cp}}(3) = 5$

$$\tilde{\boldsymbol{s}} = \frac{1}{z} \boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{cp}} = \frac{1}{5} \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0 \\ 1 \end{bmatrix}$$

And $\tilde{\boldsymbol{p}}$ can be computed with

$$\tilde{\boldsymbol{p}} = \boldsymbol{K} \tilde{\boldsymbol{s}} = \begin{bmatrix} 1500 & 0 & 640 \\ 0 & 1500 & 512 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 940 \\ 512 \\ 1 \end{bmatrix}$$

Problem 2

A camera is used to find points in a horizontal plane. The displacement from the camera frame to the object frame is given by

$$\boldsymbol{T}^{\boldsymbol{c}}_{\boldsymbol{o}} = \begin{bmatrix} \boldsymbol{R_x}(120°) & \boldsymbol{t} \\ \boldsymbol{0^T} & 1 \end{bmatrix}$$

$$\boldsymbol{R_x}(120°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(120°) & -\sin(120°) \\ 0 & \sin(120°) & \cos(120°) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

Where $\boldsymbol{t} = [0, 0, 2]^T$ and 4 points are given in the object frame as the corners of a quadratic rectangle with coordinates

$$\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{o1}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{o2}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{o3}} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{o4}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

a) Find the coordinates $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c1}}$, $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c2}}$, $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c3}}$, $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c4}}$ of the points of the camera frame.

$$\tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{c1}} = \boldsymbol{T}^{\boldsymbol{c}}_{\boldsymbol{o}} \tilde{\boldsymbol{r}}^{\boldsymbol{o}}_{\boldsymbol{o1}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{c2}} = \boldsymbol{T}^{\boldsymbol{c}}_{\boldsymbol{o}}\tilde{\boldsymbol{r}}^{\boldsymbol{o}}_{\boldsymbol{o2}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{c3}} = \boldsymbol{T}^{\boldsymbol{c}}_{\boldsymbol{o}}\tilde{\boldsymbol{r}}^{\boldsymbol{o}}_{\boldsymbol{o3}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ 2.866 \\ 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{r}}^{\boldsymbol{c}}_{\boldsymbol{c4}} = \boldsymbol{T}^{\boldsymbol{c}}_{\boldsymbol{o}}\tilde{\boldsymbol{r}}^{\boldsymbol{o}}_{\boldsymbol{o4}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \\ 2.866 \\ 1 \end{bmatrix}$$

b) Find the normalized image coordinates of the points

$$\tilde{\boldsymbol{s}}_{\boldsymbol{1}} = \frac{1}{z}\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c1}} = \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{s}}_{\boldsymbol{2}} = \frac{1}{z}\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c2}} = \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{s}}_{\boldsymbol{3}} = \frac{1}{z}\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c3}} = \frac{1}{2.866}\begin{bmatrix} 1 \\ -0.5 \\ 2.866 \end{bmatrix} = \begin{bmatrix} 0.349 \\ -0.174 \\ 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{s}}_{\boldsymbol{4}} = \frac{1}{z}\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c4}} = \frac{1}{2.866}\begin{bmatrix} 0 \\ -0.5 \\ 2.866 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.174 \\ 1 \end{bmatrix}$$

c) The rectangle is not quadratic in normalized image coordinates because there is an element of distortion due to the angle in which the picture is been taken. In the coordinates we can see that on the third value of the vectors $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c1}}$, $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c2}}$, $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c3}}$, $\boldsymbol{r}^{\boldsymbol{c}}_{\boldsymbol{c4}}$ that would be the value related to the depth, the points 3 and 4 are deeper than the points 1 and 2, hence this will mean that in a plane, points 1 and 2 will be farther away from each other compared to points 3 and 4, due to the fact the factor of depth in the picture. If the camera would have been directly on top of the points, meaning that the depth would be the same for all 4 points, then the normalized coordinates will draw a perfect square.

Problem 3

Use the Hough transform $\rho = x\cos\theta + y\sin\theta$ to find the lines which corresponds to the data-set in Table 1. Sketch the curves, and find the line with the highest number of points. From

Point	1	2	3	4	5
x	0	1	2	3	0
y	0	0	0	0	1

Table 1: Data-set for Hough transform

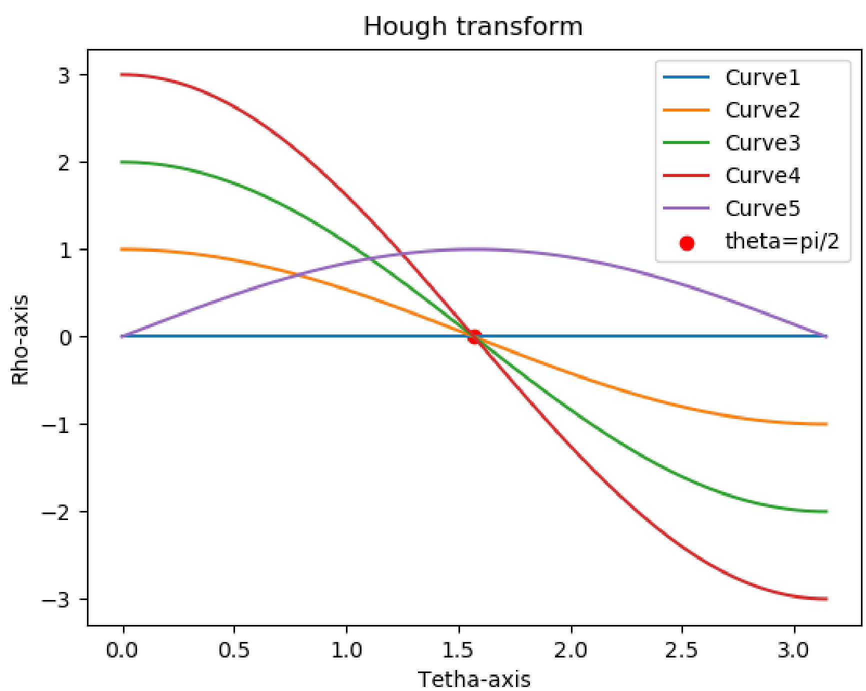

Figure 1: Plot of the sketched curves for each set of points.

this plot it can be seen that the point in which most curves intersect is $\boldsymbol{P = (\rho, \theta)} = (0, \pi/2)$, this means that the line will be a line through the origin orthogonal to $\pi/2$. Meaning it is an horizontal line $y = 0$.

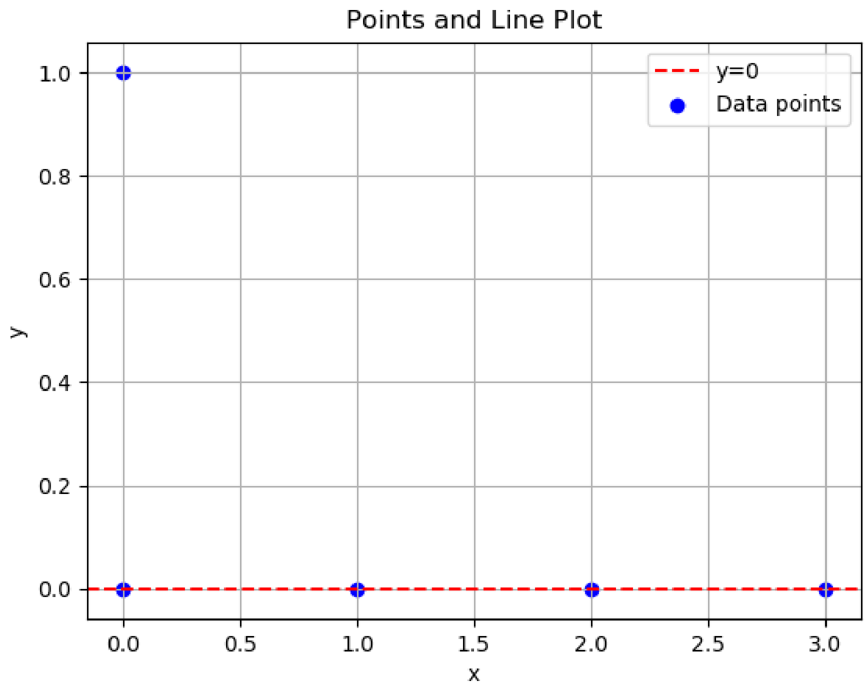

Figure 2: Plot of the sketched resulting line.