Notes

Verifying that F^n is a vector space over F

- closure on additin

$$(u_1, u_2, \dots u_n) + (v_1, v_2, \dots v_n) \in F^n$$

$$F + F \in F$$

$$(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \in F^n \checkmark$$

bla bla for rest of axioms too.

$$F^\infty = \{(x_1, x_2, \dots) : x_j \in \mathbb{F} \text{ for } j = 1, 2, \dots\}$$

- ex, verifying associativity of scalar multiplication in $\mathbb{F}^\infty$

$$(ab)v = a(bv) \qquad a, b \in F \quad v \in F^\infty$$

$$v = (x_1, x_2, \dots) ; x_j \in \mathbb{F}$$

$$(abx_1, abx_2, \dots) = a(bx_1, bx_2, \dots) \checkmark$$

$\mathbb{F}^S$ set of functions from S to $\mathbb{F}$

so an element in $\mathbb{F}^S$ has a domain S and a range $\mathbb{F}$

Additive identity: $\exists (0 \in V \mid \forall v \in V \; v + 0 = v)$

$$\exists 0 \in F^S \mid \forall f \in F^S \; f + 0 = f$$

$$\to (f + g)(x) = f(x) + g(x) \quad \forall x \in S$$

$\mathcal{O}(x) = 0$, $0 \in \mathbb{F}$

b/c $f(x) \in \mathbb{F}$, $g(x) = 0$

an elem in $\mathbb{F}$ + 0 = an elem in $\mathbb{F}$

$\therefore \exists\, 0 \in \mathbb{F}^S \mid \forall f \in \mathbb{F}^S\ f + 0 = f \rightarrow \mathcal{O}(x) = 0$

! $\mathbb{F}^n$, $\mathbb{F}^\infty$ are special cases of $\mathbb{F}^S$

$\mathbb{F}^n$ is the set of all lists of $\mathbb{F}$ that are n long

A list n long is a function that goes from $\{1, 2, \ldots n\}$ (the index) to $\mathbb{F}$

eg $S = \{1, 2, \ldots n\}$

→ the additive identity must be unique

(my own attempted proof)

suppose 0 and 0' are valid under the additive identity and V is a vector space

$\forall v \in V$, $v + 0 = v$, $v + 0' = v$, $0 \neq 0'$

$v = v$

$v + 0 = v$

$v + 0 = v + 0'$

$0 = 0'$ ← contradiction, $0 \neq 0'$

$\therefore$ must be unique

Exercises

① $\forall v \in V,\ -(-v) = v$

defn $\to$ $v - v = 0$

$\underbrace{-(-v)} - v = 0$

has to equal v $\therefore$ maybe?

③ $\exists x \in V,\ v + 3x = w$

suppose $v + 3x = w$, $v + 3x' = w$, $x \neq x'$

$3x' = w - v$

$x = \frac{1}{3}(w - v)$ $\quad x' = \frac{1}{3}(w - v)$

$\frac{1}{3}(w - v) = \frac{1}{3}(w - v)$ contradicts

QED

④ $\{\}$ does not adhere to the additive identity because $\nexists\, 0 \in \{\}$

⑤ $0v = 0\ \forall v \in V$

number additive identity of V

want to show isomorphic with

$\forall v \in V,\ \exists w \in V \to v + w = 0$

??? How does this make sense