Muon Lifetime Experiment - Determination of the Weak Coupling Constant*

Craig Reingold†
Department of Physics,
The University of Notre Dame

Patricia Huestis‡ and Shane Moylan§
Department of Physics,
The University of Notre Dame

(Experimental Methods in Physics, Spring 2016)
(Dated: March 21, 2016)

The primary method of determining the Fermi weak coupling constant experimentally is by measuring mean lifetime of the muon and calculating G_F using this value. Using a large scintillating detector and analog data acquisition system, the muon mean lifetime was measured to be $2.206 \pm 0.025\,\mu s$. The Fermi weak coupling constant was calculated to be $G_F = (1.161 \pm 0.007) \cdot 10^{-5}\,\text{GeV}^{-2}$. The accepted values for these measurements are $\tau = 2.197\,\mu s$ and $G_F = (1.166) \cdot 10^{-5}\,\text{GeV}^{-2}$. Both accepted values fall well within our margins of uncertainty.

I. INTRODUCTION & THEORY

A. Muon Decay

The muon is a lepton almost identical to the electron. Like the electron, the muon is a fundamental particle, with spin $\pm 1/2$ and charge $-e$. One of the differences between the two leptons is that $m_\mu \simeq 207 \cdot m_e$. The other is that while the electron is stable, the mean lifetime of a muon has been measured to be 2.197μs.

The decay of either a muon (μ^-) or an antimuon (μ^+) is mediated by the $W^\pm$ boson [2], as seen in EQN. 1.

$$\begin{aligned} \mu^- &\rightarrow W^- + \nu_\mu \rightarrow e^- + \bar{\nu}_e + \nu_\mu \\ \mu^+ &\rightarrow W^+ + \bar{\nu}_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \end{aligned} \tag{1}$$

The $\mu^\pm$ into a weak $W^\pm$ boson and muon neutrino/antineutrino to conserve muon lepton number. The $W^\pm$ boson then decays into an $e^\pm$ and an electron neutrino/antineutrino.

The muons detected for this experiment originated in Earth's atmosphere. Classically, the muon should decay before it reaches the atmosphere. According to special relativity, however, particles moving at an appreciable fraction of the speed of light experience time dilation.

The speed of light must remain constant in all reference frames [3]. Therefore, in the frame of the muon, the definition of time is adjusted such that the speed of light in that frame is still measured as the speed of light in a stationary reference frame. The stopped reference frame, for the purposes of this experiment, will be inside our detector (SEC. II A).

B. The Weak Coupling Constant

A relationship between the Fermi weak coupling constant G_F and the mean lifetime τ of a particle whose decay is governed by the weak force can be found in EQN. 2.

$$\frac{1}{\tau} = \frac{1}{\hbar} \frac{G_F^2}{(\hbar c)^6} \frac{\left(m_\mu c^2\right)^5}{192\pi^3} \tag{2}$$

The weak coupling constant predicts how likely a particle is to decay via a weak boson. Typically, this constant is measured by first measuring the mean lifetime of a particle whose decay is mediated by a weak boson, such as the muon.

To determine the mean lifetime of a particle, we begin with the differential equation found in EQN. 3.

$$\frac{dN}{dt} = -\lambda N \tag{3}$$

This differential equation mathematically models the number of unstable particles N present in a sample as a function of time. Therefore, it is appropriate to assume that the decay of a muon into its daughter particles (EQN. 1) is governed by this differential equation.

It is left as an exercise to the reader to validate that

$$N(t) = N_0 e^{-\lambda t}$$

is an appropriate solution to EQN. 3, where N_0 is the initial number of particles, λ is the decay constant. From here, it is straightforward to calculate the mean

---

* from *Advanced Physics Laboratory Manual*, [1]

† Primary Author,NSH 284, creingol@nd.edu
‡ NDRL 316, phuestis@nd.edu
§ NSH 284, smoylan1@nd.edu

lifetime $\langle t \rangle$ [4].

$$\langle t \rangle = \frac{\int_0^\infty t\,|dN/dt|\,dt}{\int_0^\infty |dN/dt|\,dt} = \frac{\int_0^\infty te^{-\lambda t}dt}{\int_0^\infty e^{-\lambda t}dt}$$

$$\langle t \rangle = \tau = \frac{1}{\lambda} \tag{4}$$

Therefore, measuring the decay constant for muon decay will allow for the indirect measurement of the Fermi weak coupling constant for muon decay.

II. EXPERIMENTAL SETUP

A block diagram [5] the experimental setup is displayed in FIG. 1.

A. Apparatus

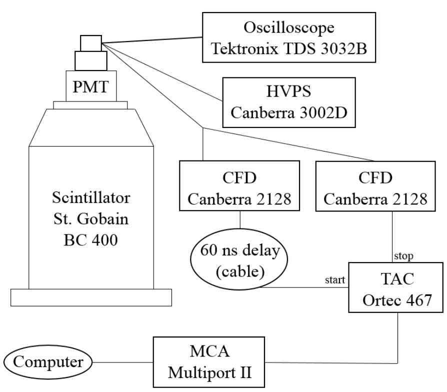

FIG. 1. A block diagram of our experimental setup. The oscilloscope was used to set potential thresholds for our constant fraction discriminators (CFD), not for actual data acquisition.

The cosmic ray muons interact with the large scintillating material in our detector two times; when they first deposit energy into the detector, and when the muon undergoes decay. The initial interaction produces a larger signal in the detector. The 60 ns delay prevented pileup in the DAQ.

B. Procedure

The signal from the PMT is split and sent to two CFDs. One CFD was set with a higher threshold potential than the other, and had a 60 ns delay immediately after output. The higher threshold, delayed CFD signal was used as the start for our TAC, and lower threshold signal was sent as the stop.

This allowed us to begin measurements when a muon initially interacted with our detector, and stop measuring when that muon decayed. Since the decays will be governed by EQN. 3, we expect our data to follow an exponential decay, with some background (SEC. III B). Our DAQ was run for two weeks, due to the low probability of muon interaction with our detector.

III. DATA & ANALYSIS

A. Data

The data from our timing calibration can be found in SEC. V. Calibrated data from our experiment can be found in FIG. 2, and plotted in log scale in FIG. 3.

B. Analysis

The analysis began with a timing calibration. The calibration was determined by finding the centroids of the peaks in FIG. 4. A first order polynomial was fit to a scatter plot of actual time differences versus centroid position, producing a linear function relating channel to time.

After the linear calibration was applied to the raw data, the calibrated data were fit to an exponential plus a background [6]. The calibrated data with the superimposed fit can be found in FIG. 2. The decay

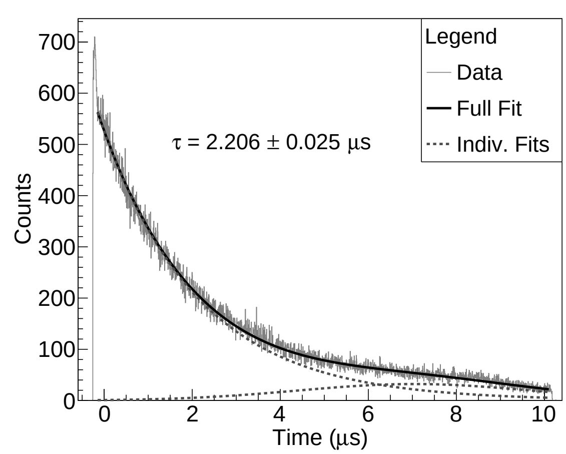

FIG. 2. Above is the muon decay spectrum, with fits superimposed over the data. The individual fits are an exponential function, and a Gaussian probability distribution.

constant λ was extracted from the fit, and was used to calculate the mean lifetime τ using EQN. 4. Finally, using EQN. 2, we were able to calculate our measured $G_F = (1.161 \pm 0.007) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$.

IV. DISCUSSION

The measured value for the mean lifetime of the muon is $\tau = 2.206 \pm 0.025\,\mu s$. The accepted value $\tau = 2.197\,\mu s$ is well within the range of the measured value. Since our measured value for the mean lifetime was incredibly close to the actual value, it should follow that our measured G_F is also accurate. The actual value of $G_F = (1.166) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$, with falls within our margin of uncertainty.

A major factor that had to be dealt with in order to properly extract the mean lifetime of the muon was our background. As seen in FIG. 3, our data did not follow EQN. 3 after about $3\,\mu s$. If it did, we should expect this plot to be completely linear. The most appropriate

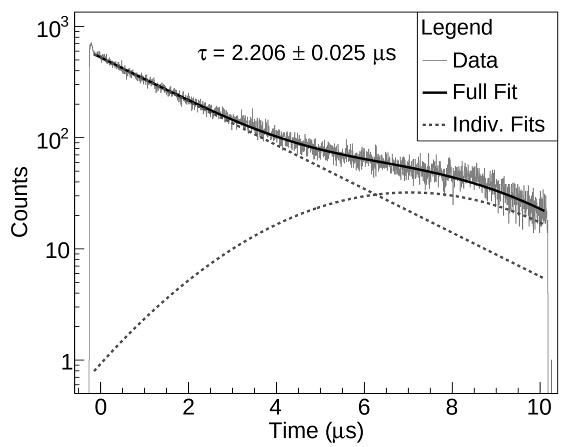

FIG. 3. Above is the muon decay spectrum plotted in logarithmic scale, with fits superimposed over the data. The individual fits are an exponential function, and a Gaussian probability distribution. It is abundantly clear from this plot that the background effects every portion of our data. This was the motivation to actually model the background mathematically, instead of cut off our fit for times less than 3 μs.

mathematical model for our background was a Gaussian probability distribution. FIG. 2 and FIG. 3 both verify that an exponential function added to a very broad Gaussian distribution describe the behavior of our data very well.

A Gaussian distribution was chosen not only because it appears to fit the data, but also because physical intuition validated this choice. Physically, background in the detector arises from unwanted radiation interacting with the detector. Under normal circumstances, the background can be modeled using a first or second order polynomial.

One explanation for the Gaussian background is that a quadratic background was detected that could also be described as a Gaussian. To first order approximation,

$$e^{-x^2} \approx 1 - x^2 + \cdots$$

Therefore, it would not be unreasonable to guess that we used a Gaussian distribution to model a quadratic background.

Another explanation for this background choice is that there was an extraneous source of radiation within the range of our detector. After running for two weeks continuously with a very large, unshielded detector, it would not be unrealistic for a typically inert source of background radiation to be detected. Since our experiment was designed for timing measurements, any background we detect should have very poor energy resolution.

What we appear to see in our detector is some background source being detected with incredibly poor energy resolution. That source could be ^{40}K in the concrete, a cosmic ray byproduct, or a radioactive source from another experiment. Since there is no way to determine what the source was, it felt safest to model the background mathematically with a Gaussian probability distribution.

[1] J. Hammer, *Advanced Physics Laboratory Manual* (Department of Physics, University of Notre Dame, 2008).
[2] A. Das and T. Ferbel, *Introduction to Nuclear and Particle Physics* (World Scientific Publishing, Hackensack, NJ, 2006).
[3] A. Einstein, Annalen der Physik **17**.
[4] K. S. Krane and D. Halliday, *Introductory Nuclear Physics* (Wiley, New York, 1988).
[5] P. Huestis, (2016).
[6] R. Brun and F. Rademakers, Nucl. Inst. & Meth. in Phys. Res. A **389**.

V. APPENDIX

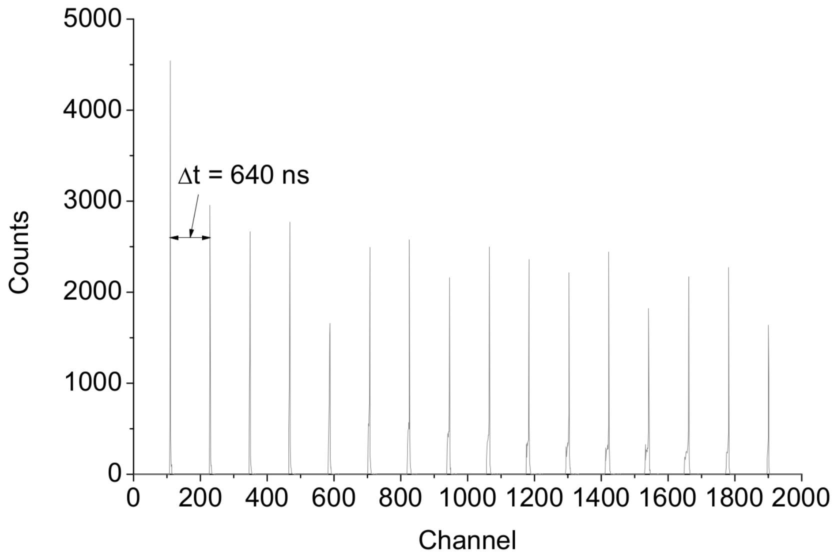

FIG. 4. Raw data used for our timing calibration