Measuring the Speed of Light*

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Special relativity postulates the speed of light is invariant across all inertial reference frames. Thus, determining the speed of light with precision is an invaluable measurement for the modern researcher. Using ultrafast timing measurements of photons generated from positron-electron annihilation, it is possible to measure the speed of light.

In this work, the speed of light was measured to be $0.97242 \cdot c \pm 0.03718 \cdot c$. Since the accepted value for the speed of light in a vacuum falls within our uncertainty margin, the measurement techniques outlined in this paper provide a valid methodology for measuring the speed of light.

I. INTRODUCTION & THEORY

There are very few real-world physics problems that have exact solutions. Most real-world problems require some degree of approximation, simplification, or numerical analysis. One of the few real-world physics problems with an elegant, simple solution is determining the speed of light in a vacuum. Albert Einstein predicted the exact solution in 1905 [2], and in this work, we will empirically measure the speed of light using ultrafast timing techniques.

A. Special Relativity

Einstein's theory of special relativity has only two postulates.

1. The laws of physics must hold in all inertial reference frames.
2. The speed of light in a vacuum c, is constant in all inertial reference frames.

These two simple rules, when applied, have drastic consequences. Because of the second postulate, spacetime must bend in order to accommodate the constant speed of light. Therefore, if there were a single metric to measure against, it would be c, since it is invariant between reference frames [3].

Since the speed of light in a vacuum is such an important, universal constant, it is of the utmost importance to modern research that it is measured with the highest possible. Here, we were able to calculate the speed of light in a vacuum using EQN. 1.

$$\frac{c}{2} = \frac{\Delta x}{ch.} \times \left(\frac{\Delta t}{ch.}\right)^{-1} \pm \sqrt{\sum_i \left(\frac{\partial}{\partial q_i}\left(\frac{\Delta x}{\Delta t}\right)\delta q_i\right)^2} \quad (1)$$

Since $\Delta x/ch.$ and $\Delta x/ch.$ can be measured using ultrafast timing techniques, outlined in SEC. II B, the speed of light can be easily calculated with this equation. Error propagation techniques used for EQN. 1 can be found in Taylor [4]. An explanation for the factor of $1/2$ in EQN. 1 can be found in SEC. IV.

B. Positron-Electron Annihilation

A source of light that radiates at least two photons simultaneously is necessary to measure the speed of light properly. The emitted light should also have enough energy to have a very low probability of interaction with the atmosphere. A source of light that does exactly this is a β^+ emitter. In this experiment, ^{22}Na was selected as our β^+ emitter.

^{22}Na is a radioactive isotope. Due to its shell structure, the most energetically favorable method of decay for this nucleus is to undergo positron emission to become ^{22}Ne [5]. The mechanics for positron emission can be found in EQN. 2.

$$^{22}\mathrm{Na} \Rightarrow ^{22}\mathrm{Ne}^* + W^+ \Rightarrow ^{22}\mathrm{Ne} + \gamma + e^+ + \nu_e \quad (2)$$

* from *Advanced Physics Laboratory Manual*, [1]
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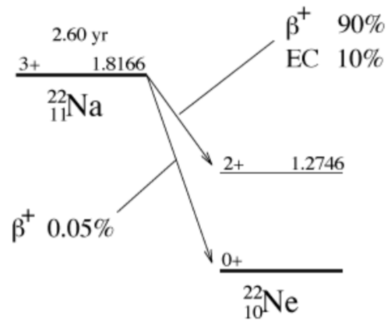


FIG. 1. The level scheme for ^{22}Na decaying into ^{22}Ne through β^+ emission

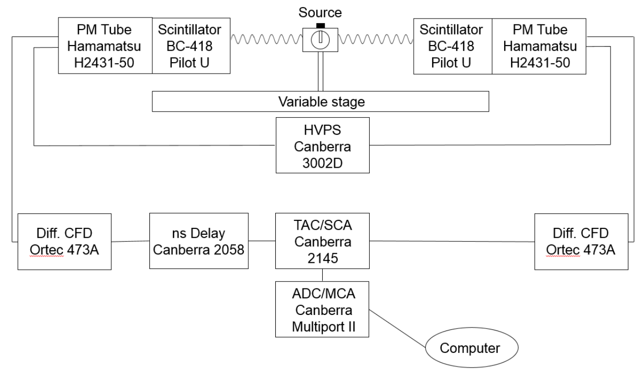


FIG. 2. This is the block diagram of our experimental setup. Our ^{22}Na souce was positioned on a movable table between our scintillation detectors. After positron-electron annihilation, two identical photons travel in opposite directions away from the ^{22}Na source. These photons are able to penetrate the aluminum shielding over the scintillation crystals, and induce scintillation. The light from the BC-418 plastic scintillation crystal is then measured by the PMT's and converted to an electronic signal. Both signals travel through Constant Fraction Discrimination modules, and one of the two is sent through a delay. Finally, if there is a valid event measured in the Time to Amplitude Converter, a signal is sent to the Multi-Channel Analyzer, and finally recorded on the Data Acquisition computer.

When the positron from this interaction comes into contact with an electron, the two particles annihilate each other. This happens since they are the anti-particles of one another. Since they are leptons, their interaction is governed by the electro-weak force, meaning their annihilation will result in a cascade of photons.

To conserve four-momentum, the photons must carry away the rest energy of the electron-positron pair, and travel anti-parallel to each other [6]. The photons generated from this interaction also have enough energy to be considered γ-rays, meaning the energy they lose to the atmosphere is negligible. For these reasons, ^{22}Na is a perfect light source for measuring the speed of light using ultrafast timing techniques.

II. EXPERIMENTAL SETUP

A. Apparatus

Ultrafast timing techniques were required to measure the speed of light. Our light source for this experiment was a ^{22}Na source. A full block diagram of our apparatus can be found in FIG. 2. Photons emitted from pair production were detected in scintillation detectors. These detectors are composed of BC-418 plastic scintillation crystals attached to Hamamatsu Photomultiplier Tubes (PMT's). BC-418 crystals are ideal for ultrafast timing experiments due to their 1.4 ns time constant [7]. This means, at the cost of energy resolution, these crystals provide precise timing information.

Signals from one detector were sent directly to the TAC as a start signal, while the other was sent through a 13 ns timing delay. The delayed information was used as a stop signal for the TAC. The delay was implemented in order to ensure only valid signals were recorded.

B. Procedure

Our measurement of the speed of light was actually a comparison of two calibrations. By comparing the linear correlation between MCA channel and ^{22}Na position, and the linear correlation between MCA channel and timing differences, we were able to extract the speed of light.

A correlation between channel and position was acquired by moving the ^{22}Na source 20 cm closer to the delayed scintillation detector every 24 hours. The position of the source was recorded, and later compared to the peak corresponding with that position in the MCA (See SEC. III B)

A correlation between channel and timing differences was obtained with a timing calibration module. The signal from the module was split, and sent through two cables of equal length. One of the cables was connected to the TAC start signal. The other was connected to the delay module, and then sent to the TAC stop signal. This method of using the timing calibration module as a signal generator allowed us to fit more than two points to a calibration.

III. DATA & ANALYSIS

A. Data

The data from this experiment is displayed for the reader in FIG. 3 and FIG. 4. The data in FIG. 3 was used for our position correlation, and FIG. 4 contains the data relevant to our timing correlation.

The curious reader will wonder why there is a noticeable difference in the amplitudes of all recorded peaks. The difference in counts between each of the peaks corresponds simply to the number of events recorded, and later binned, into a channel in the MCA. Therefore, events with larger peaks simply mean those events were recorded for a longer period of time. Since peak amplitude plays no role in our analysis (SEC. III B), these differences can be ignored.

It is also abundantly obvious that the timing peaks are have significantly lower standard deviations than the position peaks. This is because the position peaks are real timing measurements from a radioactive source, and the timing peaks were generated using a signal generator and a delay module. The accuracy of the data taken with the scintillation detectors is subject to the internal resolutions of the scintillation crystal and PMT. That is the primary reason for the noticeable difference in peak widths.

B. Analysis

The analysis of both data sets was identical. A Gaussian distribution was fit to each of the five peaks in FIG. 3, and the twenty-two peaks in FIG. 4. All fitting was performed in ROOT using a χ^2 minimization algorithm [8]. The fitting procedure allowed us to extract the centroid of each peak, and an associated standard deviation.

The centroids of the peaks detected by the scintillation detectors were then plotted against the known positions of the ^{22}Na source (FIG. 5, *left*). The timing difference Δt was then plotted as a function of corresponding peak centroid, as seen in FIG. 5, *right*.

The correlations extracted from these data were determined by fitting a first order polynomial to the data. The slope of this line was then found the to be the sought after correlation. A table of these slopes can be found in TABLE I.

Equation 1 from SEC. I, in combination with the data from TABLE I, has given us everything we need to calculate the speed of light. The measured speed of light can be found in TABLE II.

Date	Slope		Error
Position	$1.842 \cdot 10^{-1}$	$^{cm}/_{ch.}$	$7.013 \cdot 10^{-3}$
Timing	$1.263 \cdot 10^{-2}$	$^{ns}/_{ch.}$	$4.383 \cdot 10^{-5}$

TABLE I. Measured correlations, and their associated uncertainty. These correlations will be used to calculate the speed of light.

Units	c	δc
$^{m}/_{s}$	$291,525,034$	$11,145,664$
% c	97.242 %	3.718 %

TABLE II. The measured speed of light in this experiment. The speed of light is given here in both meters per second, and as a fraction of the known speed of light, for simplicity.

IV. DISCUSSION

Our final measurement for the speed of light was $0.97242 \cdot c \pm 0.03718 \cdot c$. The accepted value for the speed of light falls within our calculated uncertainty, thus validating our method for measuring c. While this has been proven to be an effective method for measuring c, there is still room for improvement for future experiments.

EQN. 1 contains a factor of one half that, up until this point, has not been addressed. The factor of two arises from the correlation factor generated from the position data. When the ^{22}Na source is moved 20 cm closer to one of the scintillation detectors, it is simultaneously moved 20 cm farther from the other detector. Therefore, the change in distance is not 20 cm, it is actually 40. This is why EQN. 1 only calculates half the speed of light, and why we must multiply by a factor of two for our final answer.

Upon closer examination of FIG. 4, there appears to be a source of systematic error arising from the delay module. The spacing between the peaks is not uniform, as expected. Instead, it appears as if the peaks are clustered in groups of four. While there is currently no good explanation for this pattern, there is a strong justification for using the calibration regardless.

With twenty-two data points, small deviations from the fit contribute in a very minor way to the overall uncertainty in the calculated slope, as seen in TABLE I. The data also appear to fall symmetrically on either side of the fit, meaning we are not systematically over- or under-estimating the correlation. Finally, the analysis was conducted using only one of the clusters of four points, as well as the way the manual originally called for, with only two points. Both of

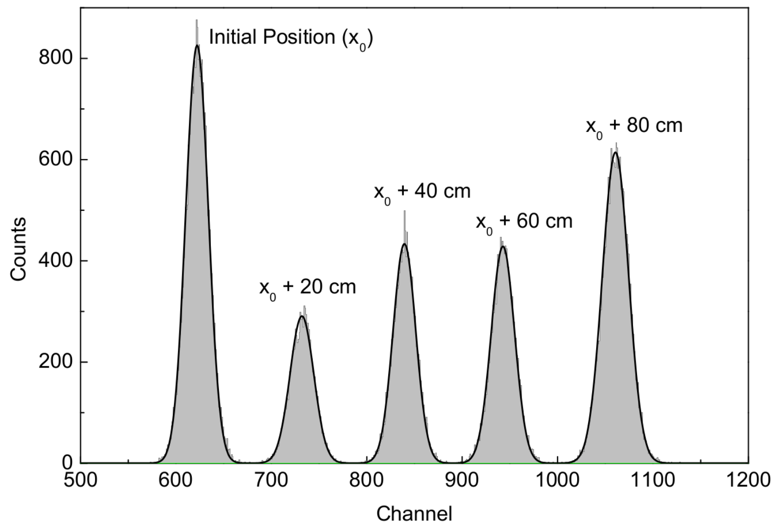


FIG. 3. This figure contains the raw data from our PMT's. Each of the peaks seen above correspond to a different position of the ^{22}Na source. The difference in amplitude of each of these peaks corresponds only to how long the source was left at that position. Since normalization does not play a factor in our analysis, (SEC. III B), the differences in peak amplitudes will not effect our measurement.

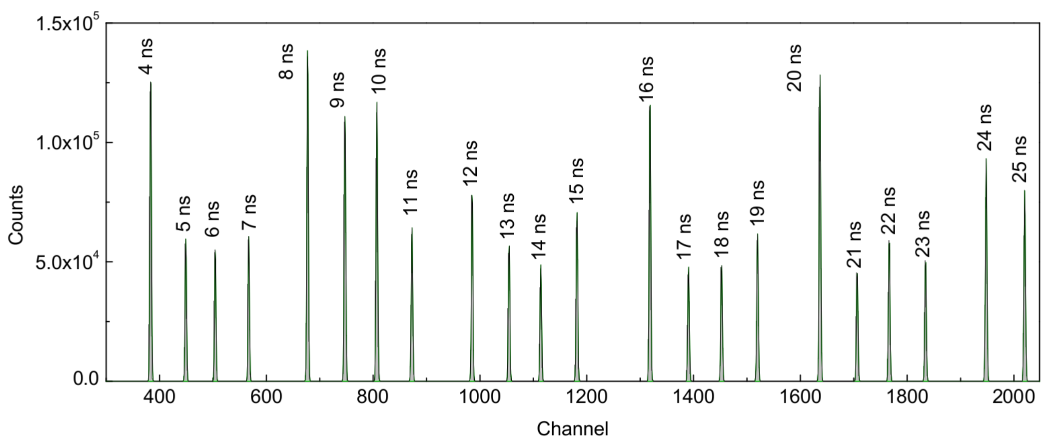
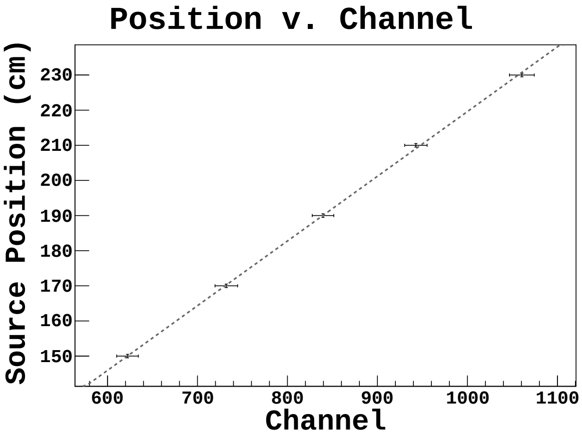


FIG. 4. This figure contains the raw data from our timing calibration. The timing difference, Δt, corresponding to each peak can be found above the peaks.

these analyses failed miserably[1]. Not only were we unable to come within $0.2 \cdot c$ of the accepted value, but also because propagating the error associated with only fitting two or four points to a straight line was ridiculous.

Nonetheless, the first step toward a more accurate measurement would involve using a delay module that did not include the same grouping of data. Another improvement to this technique could be to use scintillation crystals and PMT's with faster response times. While the data acquisition system used in this experiment was acceptable, modern digitizing electronics could have been used instead of the analog CFD and TAC modules to improve our ultrafast timing measurements. These improvements, however, would only improve the accuracy of this measurement. We were able to accurately measure the speed of light using ultrafast timing measurements.

[1] If the reader is interested, the results of the four-point calibration were saved to the same ROOT file as the published results. The speed of light from that analysis was $c = 0.797 \cdot c \pm 0.048 \cdot c$. Analysis macro, and ROOT file containing the data can be found at `https://www.github.com/creingold/ExpMeth2016/SpeedOfLight/`



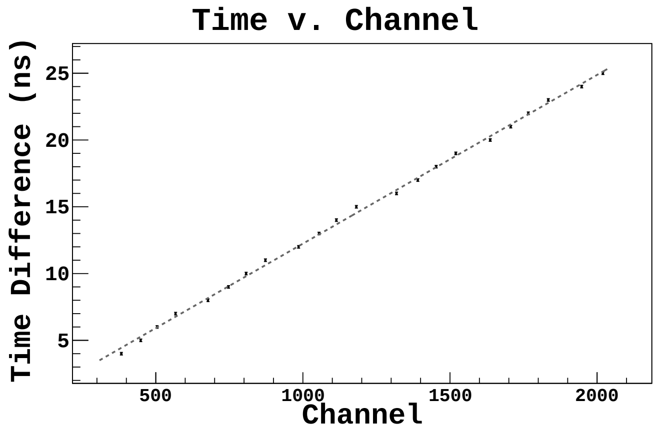


FIG. 5. *Left:* Measured position of the ^{22}Na source from the scintillation detector feeding signal into the start of the TAC against the corresponding channel centroid. *Right:* Quoted time difference Δt from the delay module against the corresponding channel centroid. Straight lines were fit to the data, and have been superimposed on the plots. Equations of the lines are as follows: *Left:* $y = 35.399 + 0.184 \cdot x$ *Right:* $y = -0.389 + 0.013 \cdot x$

[1] J. Hammer, *Advanced Physics Laboratory Manual* (Department of Physics, University of Notre Dame, 2008).

[2] A. Einstein, Annalen der Physik **17**.

[3] *Le Systeme International d'Unites (SI) = The International System of Units (SI)* (Bureau International des Poids et Mesures, Sevres, 2006).

[4] J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements* (U Science, Sausalito, CA, 1997).

[5] NNDC, *Nuclear Science References* (Brookhaven National Laboratory, www.nndc.bnl.gov, 2016).

[6] K. S. Krane and D. Halliday, *Introductory Nuclear Physics* (Wiley, New York, 1988).

[7] Saint-Gobain, *Organic Scintillation Materials* (Saint-Gobain Crystals, Hiram, OH, 2015) Chap. Plastic Scintillators.

[8] R. Brun and F. Rademakers, Nucl. Inst. & Meth. in Phys. Res. A **389**.