

Shape Evolution in Computational Fluid Dynamics

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1. Ω : Rectangular orthogonal simulation domain with center $\in (0,0)$ and $x \in [-L,L]$ and $y \in [-D,D]$.
2. Γ : PL simple closed curve $\in R^2$ (CCW with respect to Z) with center of gravity $\in (0,0)$.
3. F_L : Lift force, generated by the difference of pressures across a object and acting perpendicular to flow direction.
4. F_D : Drag force, generated by the relative movement between a object and a fluid and acting parallel to flow direction.

Outline

1. Introduction
2. Literature Review
3. Methodology and Results
4. Data Structure and Complexity
5. Conclusions

1. Introduction

In nature, constant perturbations make objects change their shape in order to maximize a biological function and evolve. Thanks to actual computational power this shape evolution can be applied in engineering to optimize machines performance by maximizing a mathematical function.

Aeronautics industry is one of the field where shape optimization is widely used in the design of optimal Airfoils for specific functions. This project, whit help of Computational Fluid Dynamics (CFD), studies the shape evolution of a wing submitted to air flow maximizing the lift force to identify which parameters have more influence in the Airfoil shape.

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2. Literature Review

1. Laminar flow occurs when a fluid flows in parallel layers with no disruption between them.
2. Steady flow occurs when velocity, pressure and density are not variable in time.
3. Bernaulli theorem dictates that the decrease of pressure of a moving fluid is due to its speed increase [CCO06].
4. Von Kármán effect is a repeating pattern of vortices caused by an unsteady separation of the flow [Wik].
5. The performance of an Airfoil can be characterized by three variables, lift, moment and drag. They represent the aerodynamic loads applied to a wing [Wau00].

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3.1. Problem Specification

Given:

1. A incompressible Newtonian fluid $\in R^2$.
2. Constant density and viscosity.
3. Steady laminar flow.
4. $v(x = -L) = (V_\infty, 0)$.
5. Γ_0 : Initial object, submerged in the fluid domain Ω .
6. A target F_L .

Goal:

1. To obtain a Γ_f by modifying its shape such that generates the target F_L and minimizes F_D .

3.2. Evolution Diagram

Figure 1 shows the evolution diagram for the object's shape

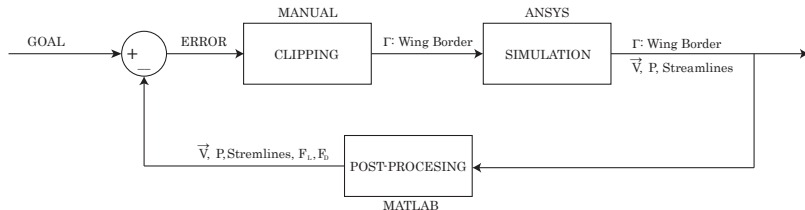


Figure 1: Shape evolution diagram. The output is the optimized object Γ

3.3. Simulation Parameters

| Geometry | |
|----------|-------|
| L | 35 m |
| D | 30 m |
| b | 3 m |
| h | 1.5 m |

Table 1: Geometry for Ω and Γ_0

| Mesh | |
|-------------------------|-------|
| Max element size | 0.3 m |
| Min element size | 0.1 m |

Table 2: Meshing parameters. Figure 3 shows the generated mesh.

| FLUENT (Solver) | |
|---------------------------------------|-----------------|
| Fluid | air |
| Density ρ | 1.225 kg/m^3 |
| Viscosity μ | 1.789 |
| Velocity V_∞ | 80 m/s |
| Flow model | Viscous Laminar |
| Constant density | |
| Constant viscosity | |

Table 3: Solver model and boundary conditions

3.3. Geometry

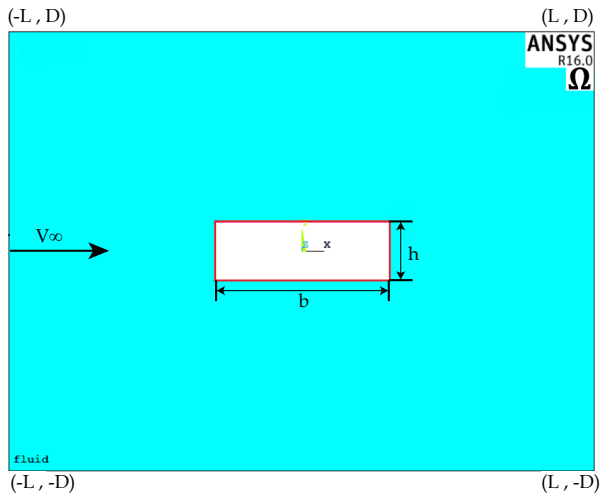


Figure 2: Geometry for initial conditions

3.3. Mesh

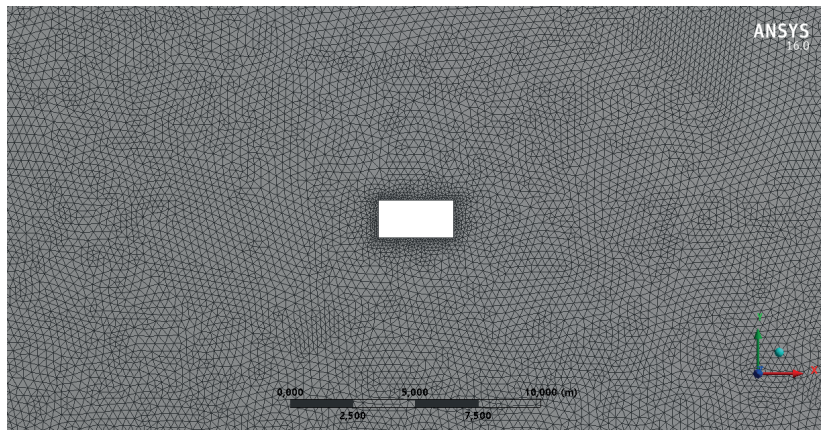


Figure 3: Mesh for initial conditions

3.4. Simulation Results

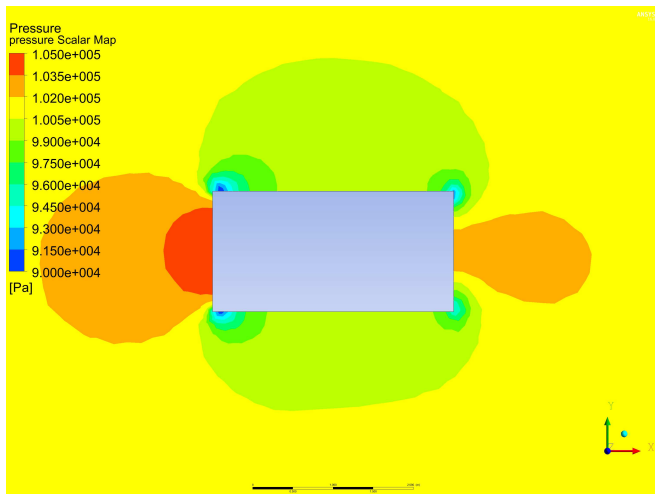


Figure 4: Pressure scalar map. Due to horizontal symmetry $F_L = 0$

3.4. Simulation Results

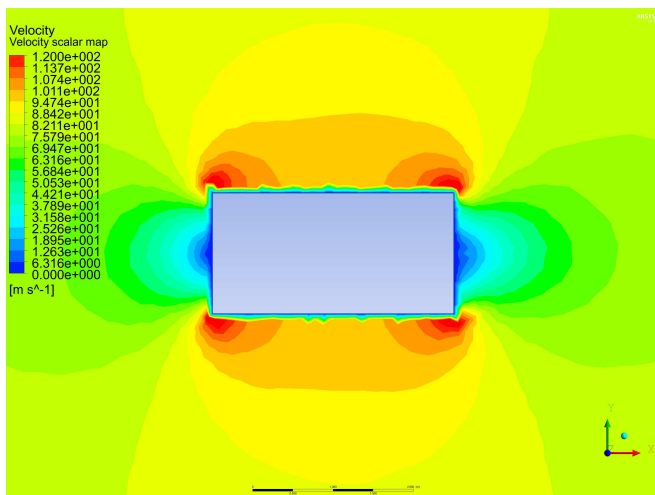


Figure 5: Velocity magnitude scalar map around Γ_0 . There is low pressure for high velocity magnitudes and high pressure for low velocity magnitude

3.4. Simulation Results

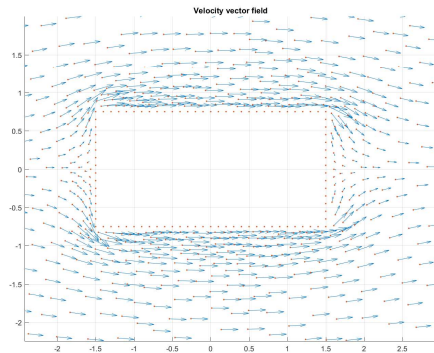


Figure 6: Velocity vector field exported from ANSYS and plotted with Matlab

3.4. Simulation Results

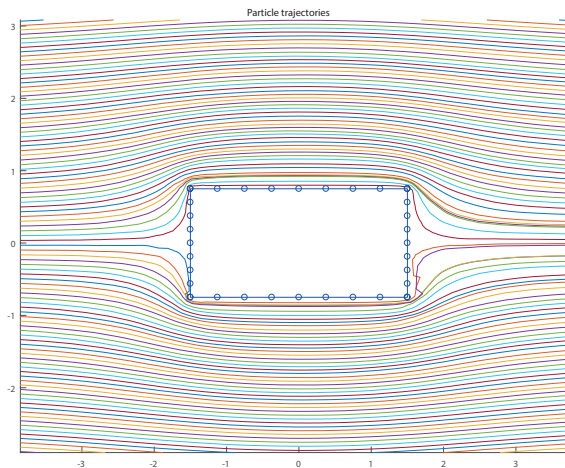
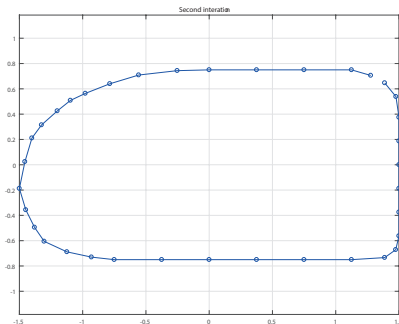


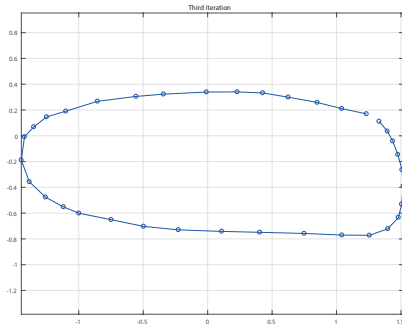
Figure 7: Particle trajectory exported from ANSYS and plotted with Matlab. Changes in paths curvature produces change in velocity magnitude.

3.5. Wing Border Clipping

1. **Goal:** Satisfy a minimal Lift force F_L .
2. Tuning variable: Γ
3. **Criteria:** Produce a high pressure underneath the wing by reducing there the stream velocity. Avoid zero velocities around the wing

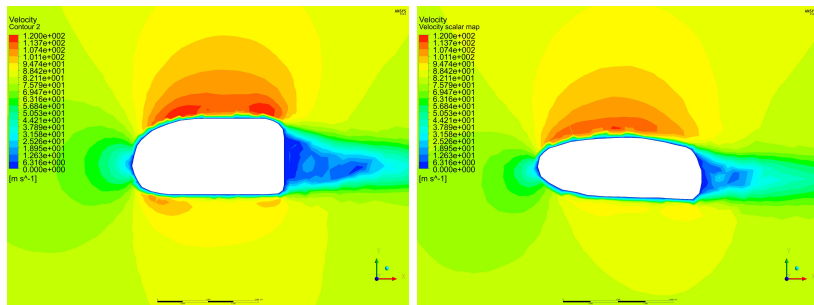


(a) Second state



(b) Third state

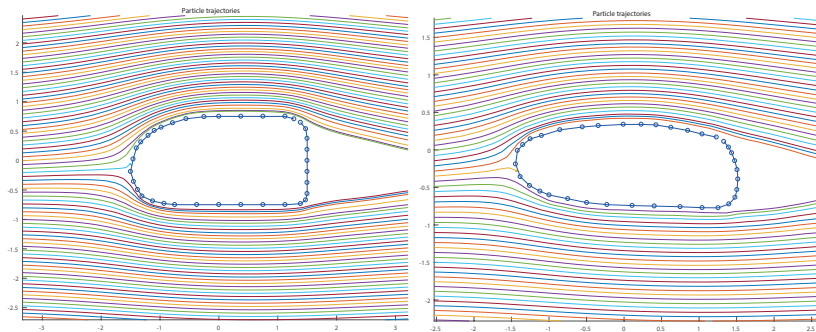
3.5.1. Velocity Results



(a) Low velocities after Γ generate vacuum
(b) Γ tends to have a tail on order to reduce the low velocity area

Figure 9: Velocity magnitude for second (a) and third (b) states

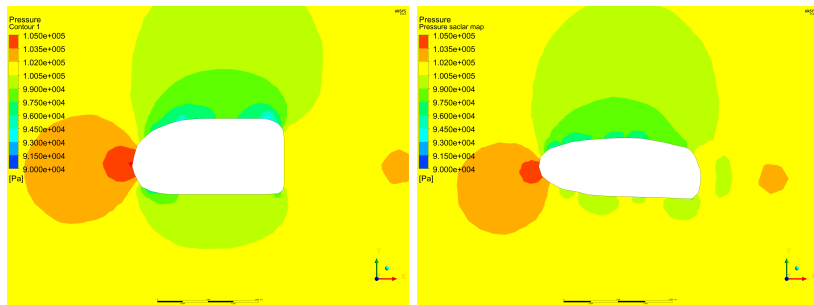
3.5.2. Particle trajectories



- (a) Trajectories present high curvature values they meet with Γ
- (b) Curvatures above the object are reduced in order to have a higher velocity than below it

Figure 10: Particle trajectories for second (a) and third (b) states

3.5.3. Pressure scalar map



(a) Pressure in front of Γ is higher than second state generating more drag (b) Pressure difference is augmented by decreasing of velocity under Γ

Figure 11: Pressure scalar maps for second (a) and third (b) states

3.5.4. Final state

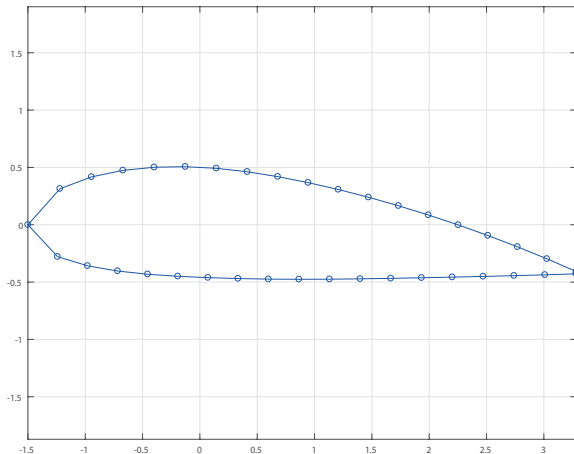
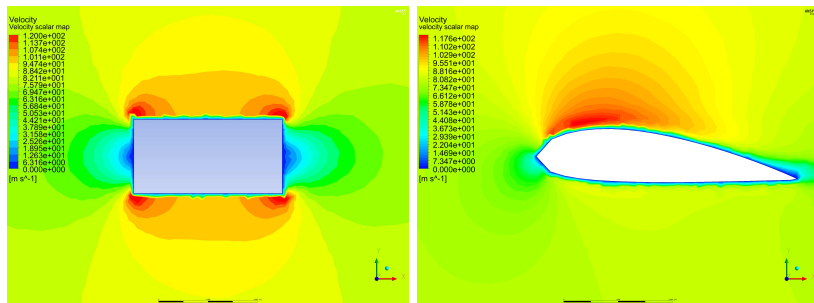


Figure 12: Final state of the Pruning

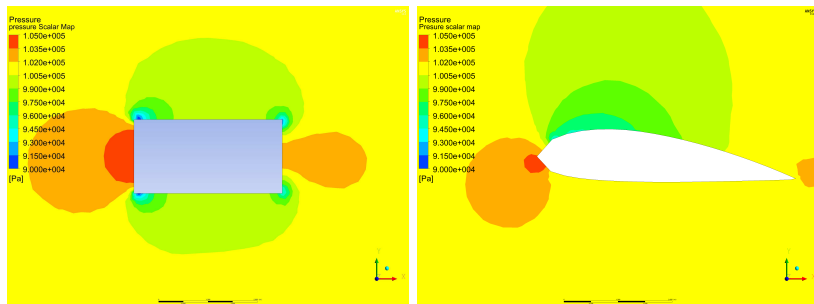
3.5.5. Initial and final states



- (a) Velocity presents horizontal symmetry
- (b) There is no horizontal difference of pressure in Γ is not 0

Figure 13: Velocity magnitude for first (a) and final states

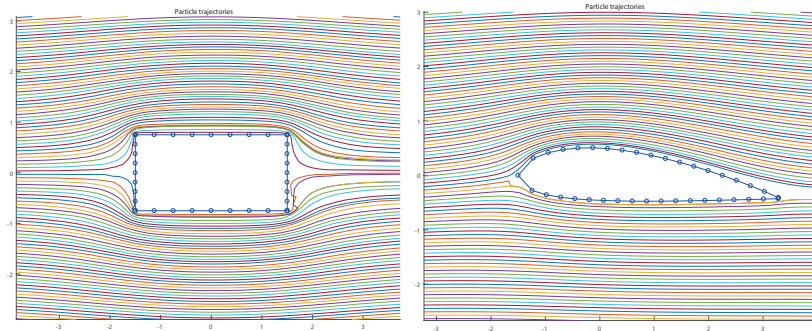
3.5.5. Initial and final states



(a) As shown in velocity scalar maps, Symmetry produces no lift
(b) In final state Γ presents higher pressure under it that produces lift

Figure 14: Pressure scalar maps for first (a) and final (b) states

3.5.5. Initial and final states



(a) Trajectories present high curvature (b) Curvatures are significantly reduced values they meet with Γ

Figure 15: Particle trajectories for first (a) and final (b) states

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4.1. Matlab Data Structure

- Structure array of Matlab is a data type that groups related data containers called fields. This structure is used to storage the the obtained informations by the simulation.

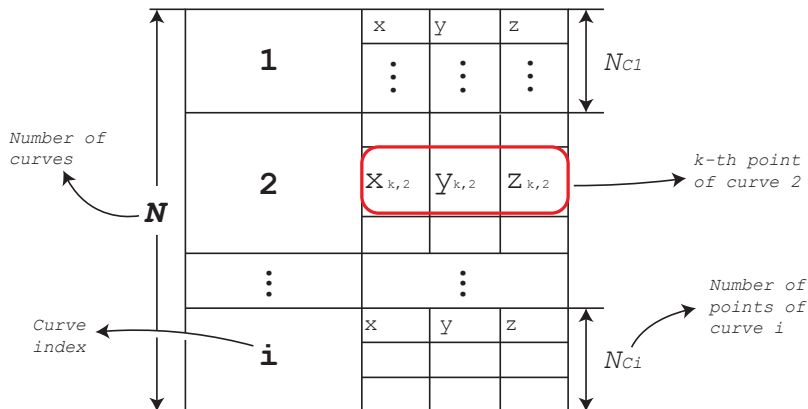


Figure 16: Matlab Struct Array for particle pathlines

4.2. Use of the structure

Being A the structure array that contains the streamlines information.

1. $A(i).curve$ = number of the curve i of the structure A.
2. $A(i).streamline(k,:)$ = k-th point of i-th streamline of structure A

4.3. Complexity calculation

1. To calculate the complexity of the algorithms it is necessary to traduce the measure variable to an input variable of the problem. In this case that variable is N_e the number of the mesh elements .

Assumptions

1. The number of streamlines is $O(\sqrt{N_e})$.
2. The number of points of the streamlines is $O(\sqrt{N_e})$.
3. n is the number of points of Γ
4. The number of elements is greater than the number of points of the polyline Γ .
5. Basic operations refers to Matlab functions and operations that are $O(1)$. (e.g sums, multiplications, normalizations, extract a value from an array, etc).

4.3. Complexity calculation

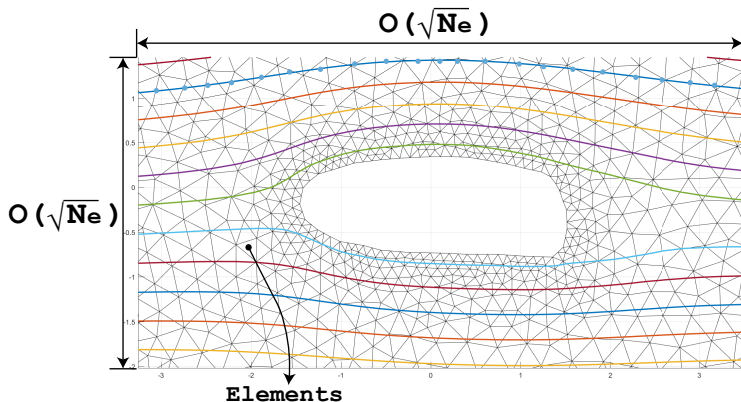


Figure 17: Relation between the number of elements, streamlines and points of the streamlines

4.3. Complexity calculation

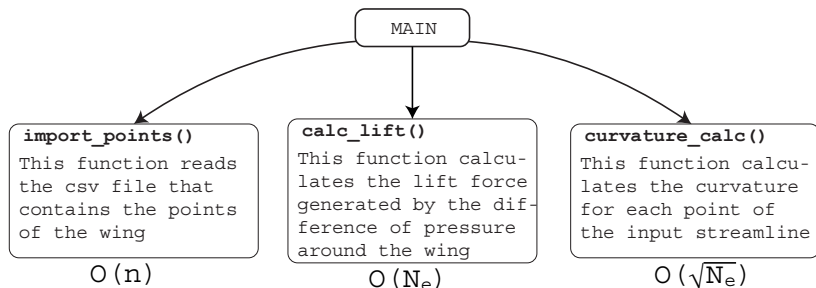


Figure 18: Functions called by the Main program

1. Matlab functions `plot()`, `quiver()`, `load()` and `csvread()` have $O(m)$ where m is the size of the input variable.

4.3.1. Curvature calculation of Streamlines

```
1  function curvature_calc(streamline)
2      for i = 2 to number of points - 1     $O(\sqrt{N_e})$ 
3          basic operations                   $O(1) * O(\sqrt{N_e})$ 
3      end for
4      return curvature                       $O(1)$ 
```

- Therefore the complexity of the function is

$$O(\sqrt{N_e} + \sqrt{N_e} + 1) = O(\sqrt{N_e}) \quad (1)$$

4.3.2. Lift Force Calculation

```
1  function calc_lift(presure,gamma)
2      for i = 2 to number of points - 1     $O(n)$ 
3          basic operations                   $O(1) * O(n)$ 
4              for j = 1 number of elements  $O(N_e) * O(n)$ 
5                  basic operations           $O(N_e) * O(n)$ 
6              end for
7          end for
8      return lift                            $O(1)$ 
```

- Therefore the complexity of the function is

$$\begin{aligned} O(n + N_e + N_e * n) &= O(N_e, n) \\ N_e \gg n \therefore \text{complexity is } &O(N_e) \end{aligned} \tag{2}$$

4.3.3. Main algorithm

| | | |
|----|------------------------------------|---------------------------------|
| 1 | import wing points | $O(n)$ |
| 2 | import velocity vector field | $O(N_e)$ |
| 3 | import pressure information | $O(N_e)$ |
| 4 | calc_lift() | $O(N_e)$ |
| 5 | plot velocity vectorfield | $O(N_e)$ |
| 6 | plot wing border | $O(n)$ |
| 7 | import streamlines | $O(N_e)$ |
| 8 | find separations of streamlines | $O(N_e)$ |
| 9 | for i = 1 to number of streamlines | $O(\sqrt{N_e})$ |
| 10 | plot i-th streamline | $O(\sqrt{N_e}) * O(\sqrt{N_e})$ |
| 11 | curvature_calc() | $O\sqrt{N_e}) * O(\sqrt{N_e})$ |
| 13 | end for | |

4.3.3. Main algorithm

- Therefore, the complexity of the function is

$$\begin{aligned} O(7N_e + n) &= O(N_e, n) \\ N_e \gg n \therefore \text{complexity is } O(N_e) \end{aligned} \tag{3}$$

4.4. Execution times

| N_e | calc_lift() | curvature_calc() |
|----------------|--------------------|-------------------------|
| 97600 | 3.33 | 0.15 |
| 400000 | 23.65 | 0.78 |
| 1000000 | 41.3 | 1.13 |

Table 4: Execution time for three different number of elements in seconds

| n | import_points() |
|--------------|------------------------|
| 35 | 1.13 |
| 3500 | 1.63 |
| 35000 | 3.2 |

Table 5: Execution time for number of wing points in seconds




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5. Conclusions

1. To reduce Drag force is necessary to allow smooth flow as possible in front of the wing. This can be made by modifying its frontal curvature.
2. The angle of attack is a very sensible parameter in the lift generation.
3. The wing tends to have a tail in order to avoid zero velocities after the it.
4. The calculation of lift force is the function that takes more execution time, followed by the functions that import the data.

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