Programming Exercise of Model Predictive Control

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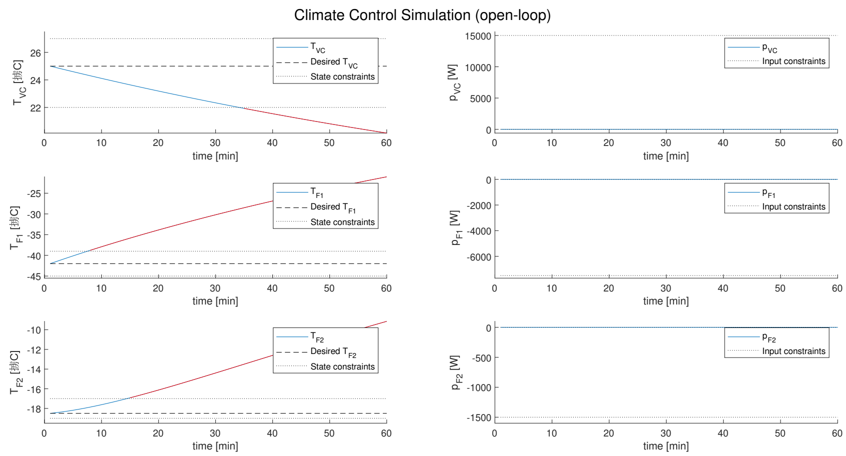

Figure 1: ex5

1 Ex.5

Plot and describe what you can see.

From the open-loop simulation result, it is indicated that the freezers do not work to respond to the temperature fluctuation. The powers of the temperature control system, namely the input variables, remain zero during the whole procedure. Therefore it is not surprising that the temperature being controlled exceeds the constraints and the aim to keep the system state variables in a normal range fails.

2 Ex.6

Implement the heuristic approach and visualize the corresponding result.Describe and interpret what you can see.

The numerical value obtained by the random simulations is as follows.The point filled in red represents the optimal choice of Q.

$$\begin{bmatrix} 4109749 & 0 & 0 \\ 0 & 2064982 & 0 \\ 0 & 0 & 2076632 \end{bmatrix} \tag{1}$$

Every simulation will bring q different Q due to the heuristic method.After some trials, it has slight influences for future tasks. And the optimal point

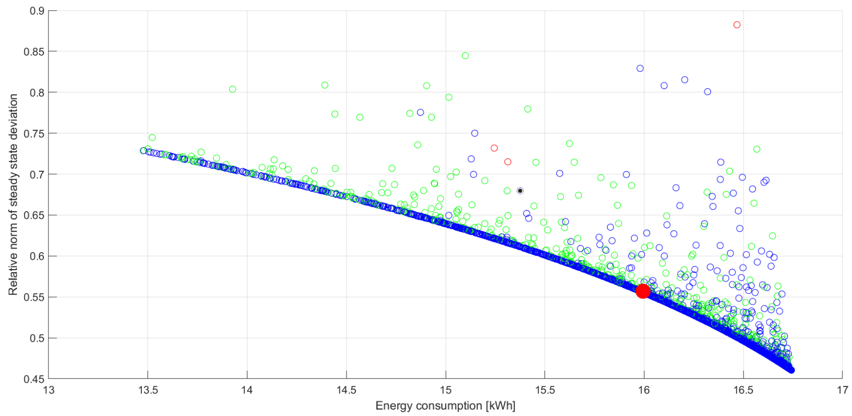

Figure 2: Ex6

position is similar for different runs.There is a clear lower bound in the scatter plot.From intuitive, there is a function to bind the relative deviation and the power consumed. For the LQR controller, when the input consumption is fixed, the system state value at specific moment must have a lower bound.

3 Ex.7

Provide the closed-loop simulation plot starting from T^1_{init}. What qualitative influences do changes of diagonal elements in Q and R have?

From observation and simple derivation, it can be deduced that when Q is fixed, as the diagonal element of R increases, the norm of the input vector will decrease by virtual of the cost function form. And when R is fixed, as the elements of Q increase, the norm of the state vector at every step will decline and it seems the settling time will be dwindled as well.

4 Ex.8

The initial option of Q is the one obtained in task 6, However, the heuristic result violates the temperature constraints slightly. After tuning several times, it is viable to find a proper solution. The corresponding Q is

$$\begin{bmatrix} 4056008 & 0 & 0 \\ 0 & 1917959 & 0 \\ 0 & 0 & 9803891 \end{bmatrix} \tag{2}$$

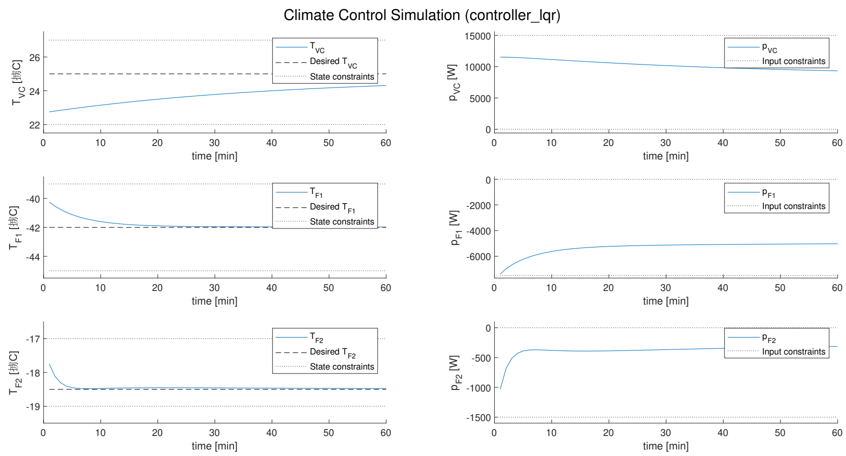

Figure 3: Ex.7 Closed-loop simulation result for $T_{init}^{(1)}$

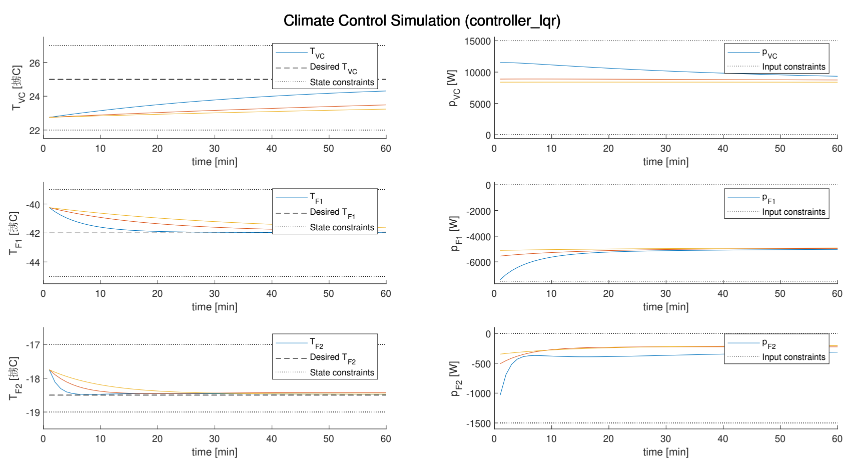

Figure 4: Ex.7 Result for R contrast

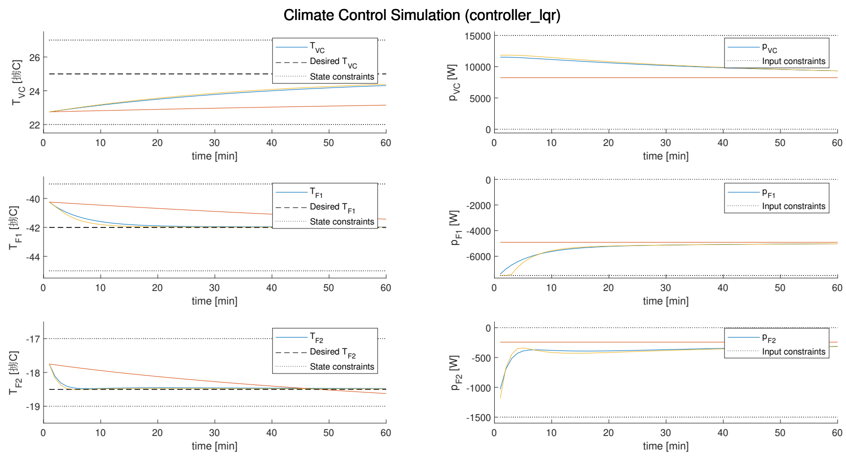

Figure 5: Ex.7:Result for Q contrast

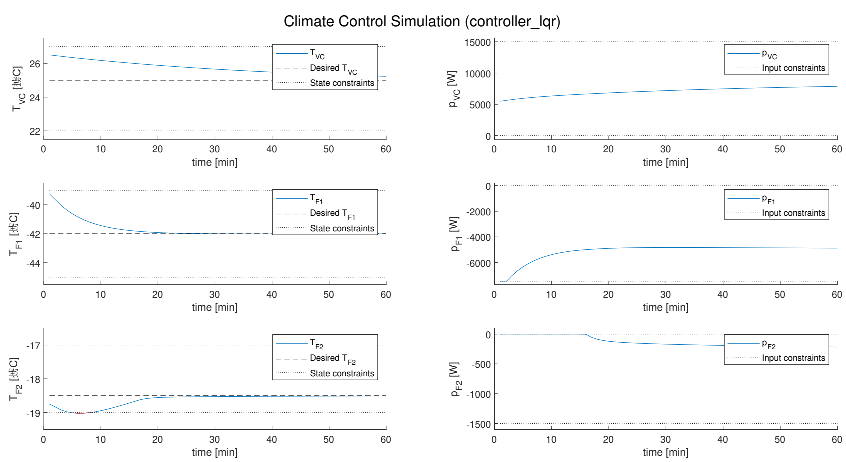

Figure 6: Ex.8 Closed-loop simulation result for $T_{init}^{(2)}$

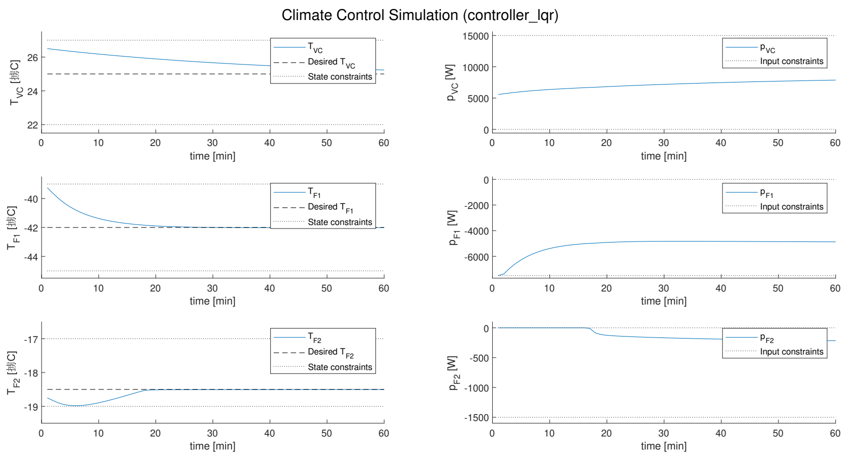

Figure 7: Ex.8: tuning Q

The effects of Q and R to the system is similar to task 7.

5 Ex.9

Plot the resulting Ployhedron together with the initial conditions of the previous steps.Make conclusions from the plot with regards to state and constraint satisfaction.

In conclusion, the problem can be regarded as a convex optimization due to the constraint area. The initial conditions starts in this area will not violate the input and state constraints for the problem.One initial state does not be located in the invariable set, which can explain the temperature violation for starting from $T^{(}_{init}2)$

6 Ex.10

Compute the infinite horizon cost as a function of $x(0)$.

$$J(x(0)) = x(0)^T P_0 x(0) \tag{3}$$

$$P_i = A^T P_{i+1} A + Q - A^T P_{i+1} B (B^T P_{i+1} B + R)^{-1} B^T P_{i+1} A \tag{4}$$

The matrix P can be solved by matlab function dlqr() or the DARE equation.

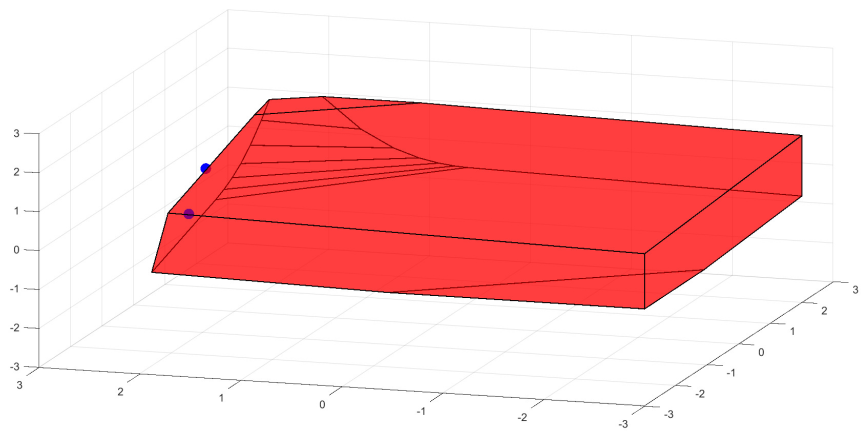

Figure 8: Ex.9 constraints area

7 Ex.11

Provide closed-loop simulation plots for two initial conditions and discuss the difference with respect to the results from Task 7 and 8.

As for $T0_init^1$ because of the proper initial condition, there is no significant difference. When it comes to $T0_init^2$, the MPC controller can adjust the whole system not violating the temperature constraints and the deviation from the desired value will be decreased.

8 Ex.12

Why is the origin an asymptotically stable equilibrium point for the resulting closed-loop system.

When there exists feasible solution, the origin will become the stable equilibrium point for the result. According to the Lyapunov stability theory, if a system admits a Lyapunov function $V(x)$, then $x = 0$ is asymptotically stable in the invariable set. In this task the Lyapunov function is just the cost function which satisfies the definition of Lyapunov function

$$V(0) = 0, V(x) > 0, V(g(x)) - V(x) \leq -\alpha(x) \tag{5}$$

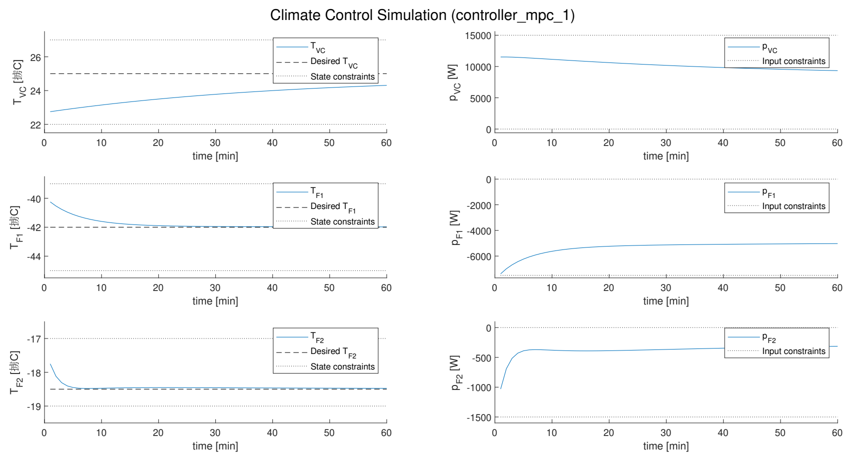

Figure 9: Ex.11 result for $T_init^(1)$

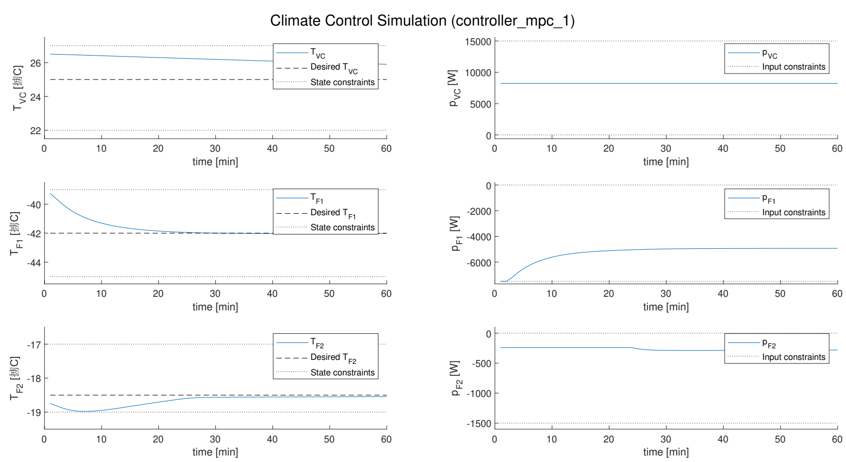

Figure 10: Ex.11:result for $T_init^(2)$

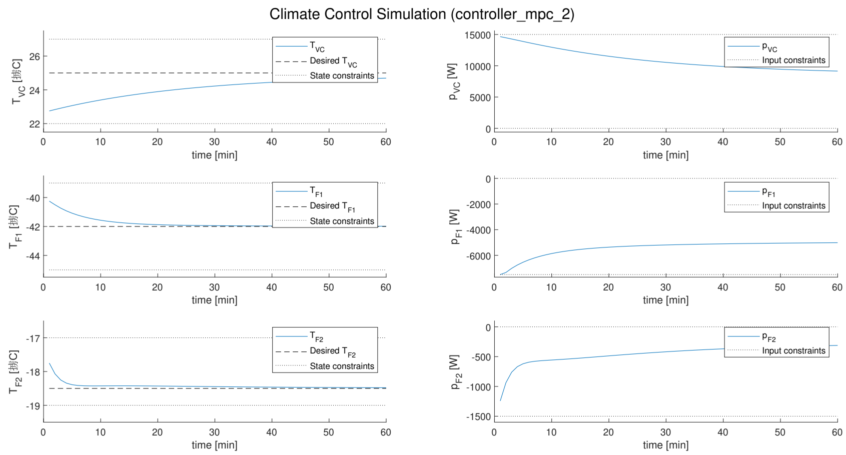

Figure 11: Ex.13 result for $T^{(}_{init}1)$

9 Ex.13

Provide closed-loop simulation plots for two initial conditions.

10 Ex.14

Provide closed-loop simulation plots for two initial conditions.

11 Ex.15

Compare the closed-loop trajectories as well as the optimization costs J_{MPC} for the previous three controllers for two initial conditions.

The result for the T^1_{init}

- $J_1 = 8.5632 \times 10^9$
- $J_2 = 9.4865 \times 10^9$
- $J_3 = 8.5632 \times 10^9$

There is no significant difference of the control trajectory between controller1 and controller3. As for controller 2, the energy consumption is greater than others and the responding rate for the system is more rapid.

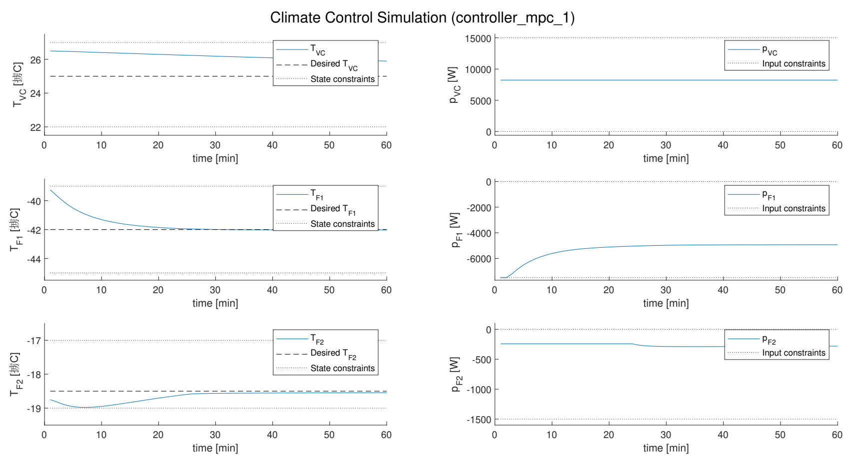

Figure 12: Ex.13 result for $T^{(}_{init}2)$

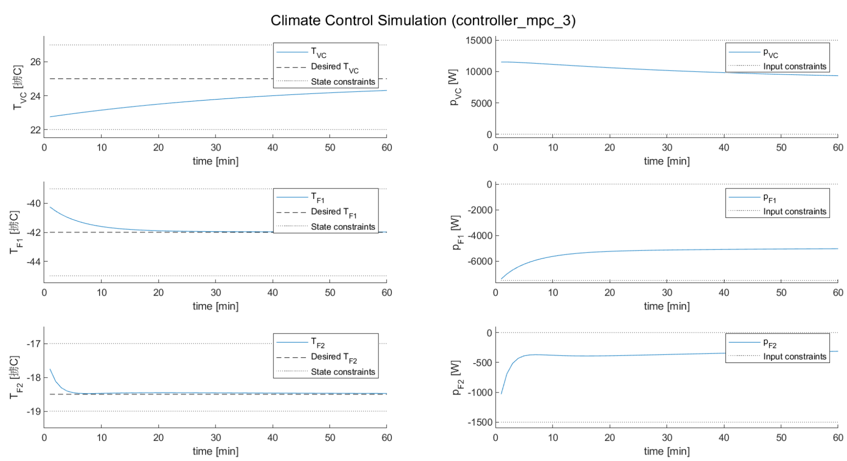

Figure 13: Ex.14 result for $T^{(}_{init}1)$

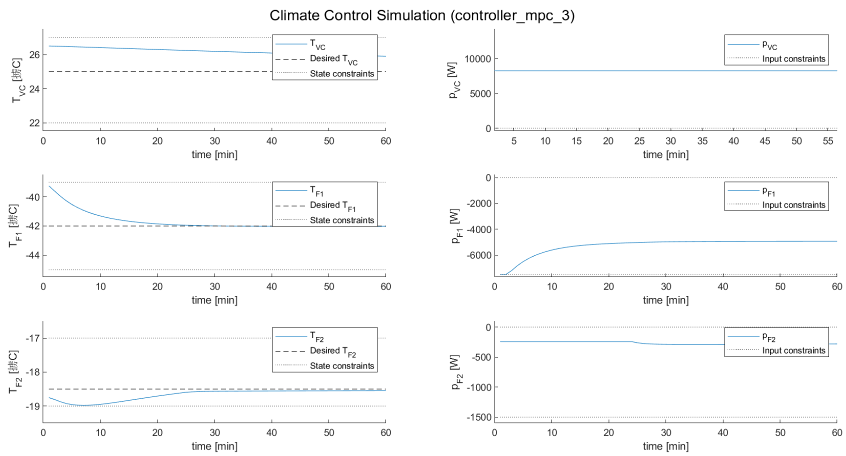

Figure 14: Ex.14 result for $T^{(}_{init}2)$

The result for the T^{2}_{init}

- $J_1 = 4.8176 \times 10^9$
- $J_2 = 4.5999 \times 10^9$
- $J_3 = 4.8176 \times 10^9$

The discrepancy between controller1 and controller3 is still not obvious.The performance of the controller 2 is greater than others.The responding speed is faster and the energy consumption is less than others.

12 Ex.16

The set $X_f \cap X_{LQR}$ is still an invariant set under such condition. And this terminal set can yield an asymptotically stable MPC controller as long as the set X_f is still convex thereby the solution properties will not change.

13 Ex.17

What do you notice in the closed-loop simulation plot?

From the plot, the MPC controller designed previously fails to regulate the temperature in the desired range.It is not capable enough to resist the disturbance under the specific scenario.

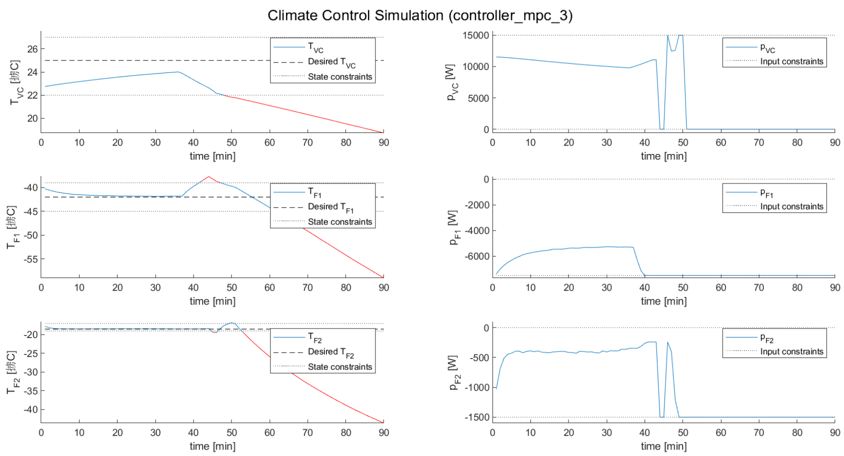

Figure 15: Ex.17 result for $T^{(}_{init}1)$

14 Ex.18

Provide the closed-loop simulation plot.

15 Ex.19

Provide a closed-loop simulation plot showing that the behavior using controller3 and controller4 is unchanged from initial condition $T^{(1)}_{init}$ under scen1.

16 Ex.20

Incorporate the expected future disturbances explicitly into the prediction dynamics of the controller.

The violation of the constrained can be metigated by the knowledge of the future disturbance.After some tuning the result can be improved.

17 Ex.21

$$A_{aug} = \begin{bmatrix} A & I \\ \mathbf{0} & I \end{bmatrix} \tag{6}$$

$$B_{aug} = \begin{bmatrix} B \\ \mathbf{0} \end{bmatrix} \tag{7}$$

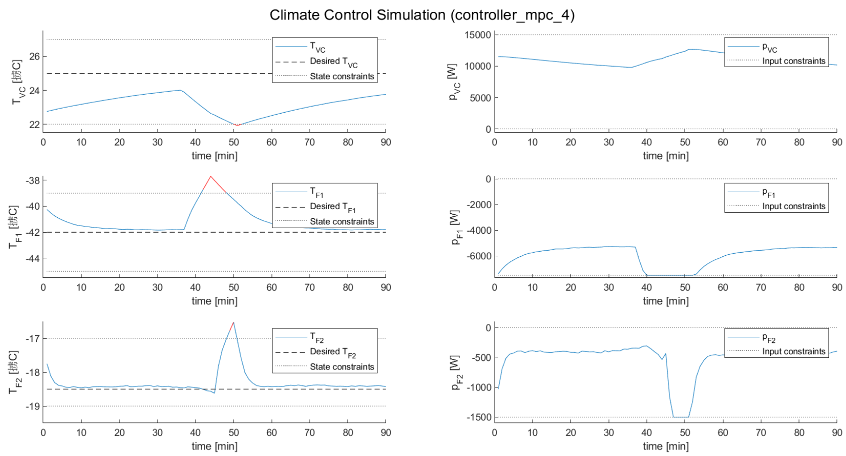

Figure 16: Ex.18:result for $T_{init}^{(}1)$

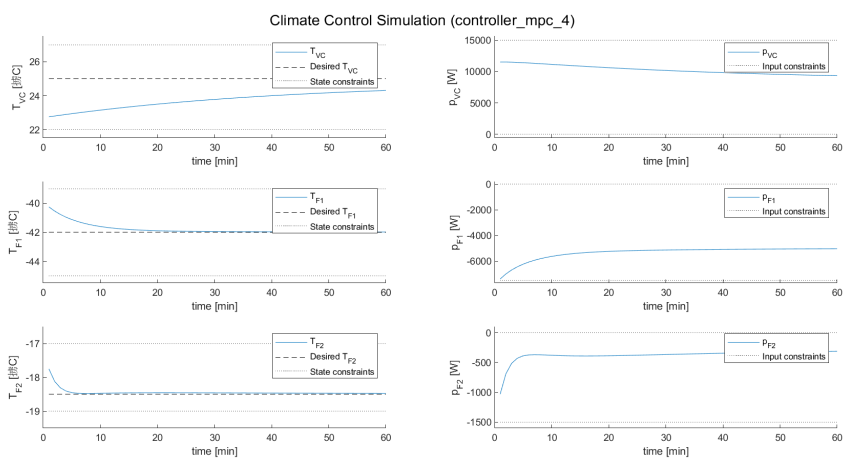

Figure 17: Ex.19:MPC-4 result for $T_{init}^{(1)}$

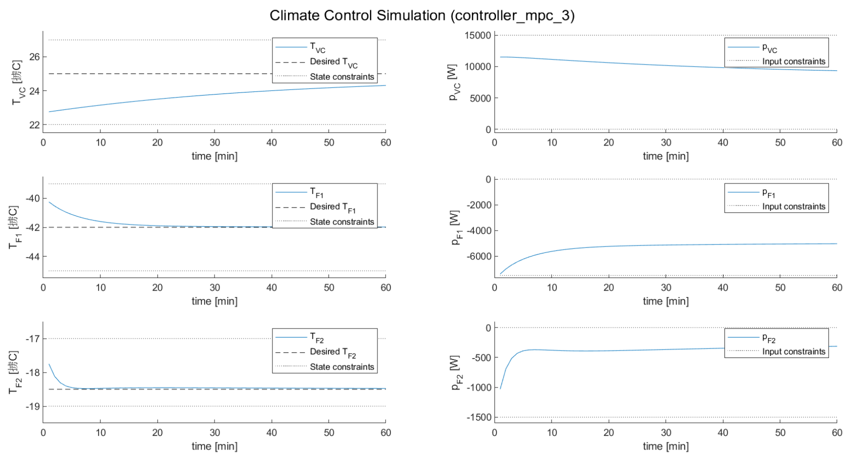

Figure 18: Ex.19:MPC-3 result for $T_{init}^{(1)}$

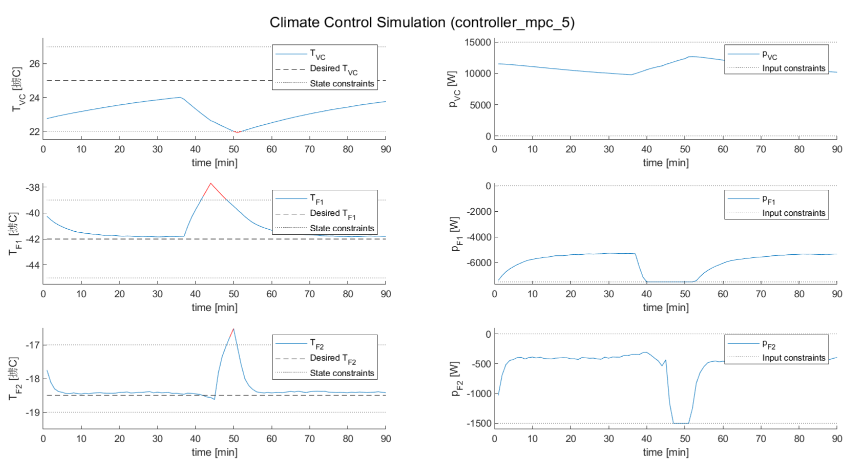

Figure 19: Ex.20 result for expected future disturbance

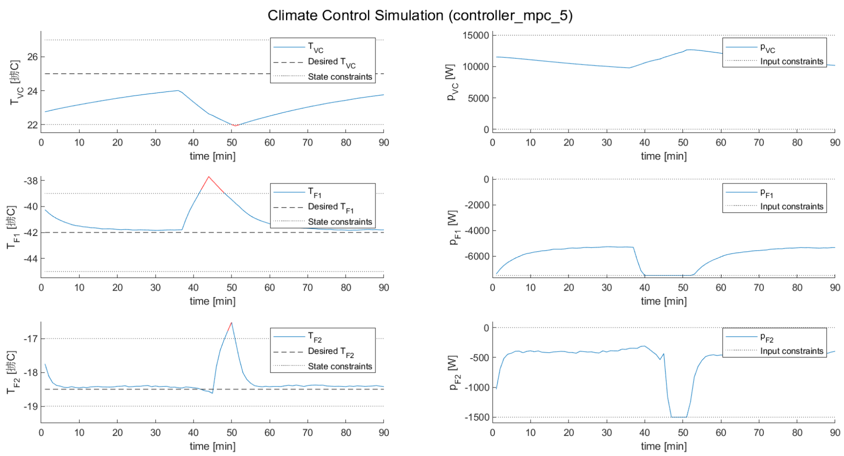

Figure 20: Ex.20 result for tuning disturbance

$$C_{aug} = \begin{bmatrix} I \\ \mathbf{0} \end{bmatrix} \tag{8}$$

$$D_{aug} = \begin{bmatrix} \mathbf{0} \end{bmatrix} \tag{9}$$

18 Ex.22

$$\begin{bmatrix} x(k+1) - \hat{x}(k+1) \\ d(k+1) - \hat{d}(k+1) \end{bmatrix} = \left(\begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} + \begin{bmatrix} L_x \\ L_d \end{bmatrix} \begin{bmatrix} C & C_d \end{bmatrix} \right) \begin{bmatrix} x(k) - \hat{x}(k) \\ d(k) - \hat{d}(k) \end{bmatrix} \tag{10}$$

The function *place()* in matlab control toolbox can be utilized to obtain a proper L matrix so that the error dynamics are stable and converge to zero.

19 Ex.23

Both controllers can achieve the goal to regulate the system variables, but the controller6 consumes less energy than controller3.

20 Ex.24

The running time invoking ordinary optimizer is $t_sim = 5.0725$

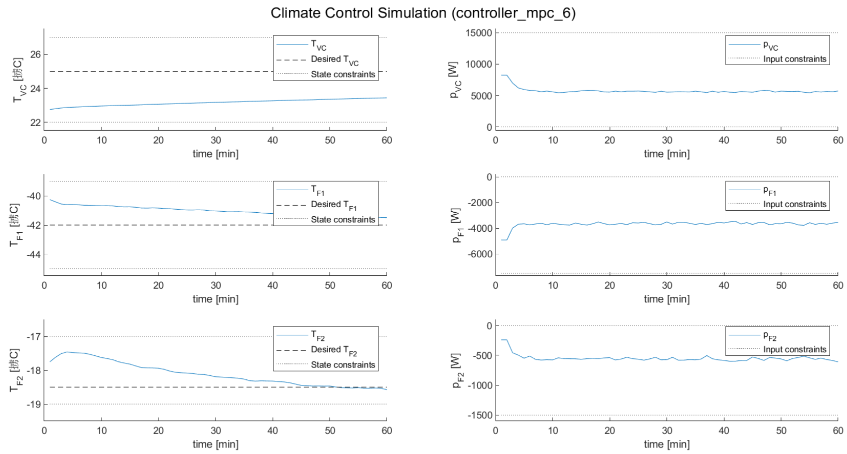

Figure 21: Ex.23: result for $T_{init}^{(1)}$ by controller6

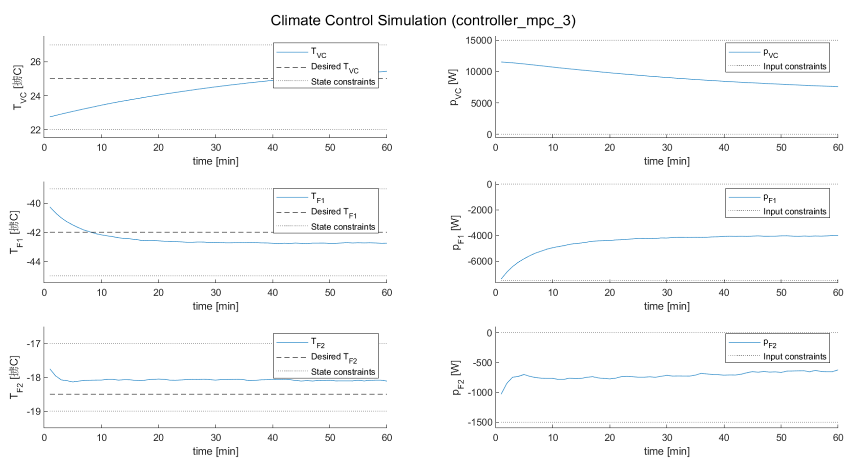

Figure 22: Ex.23: result for $T_{init}^{(1)}$ by controller3