HCMIU - Linear Algebra - Mid-term Test
Semester 3 - Year: 2023 $\backsim$ 2024 - Duration : 75 minutes
Date Modified : Saturday, July 26$^{\text{th}}$, 2025

INSTRUCTIONS: Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.

Question 1. *(20 marks)* Determine the matrices $\boldsymbol{A^T A}$ and $\boldsymbol{AA^T}$ if

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -5 \end{bmatrix}$$

Question 2. *(20 marks)* Solve the following system of equations using Gaussian elimination with back-substitution:

$$\begin{aligned} x_1 + 2x_2 - \ x_3 &= \ 3 \\ 3x_1 - \ x_2 + 2x_3 &= \ 1 \\ x_1 - \ x_2 + \ x_3 &= -1 \\ 4x_1 + \ x_2 + \ x_3 &= \ 4 \end{aligned}$$

Question 3. *(20 marks)* Use Cramer's Rule to find the solution of the system

$$\begin{aligned} -2x_1 + 3x_2 - x_3 &= \ 1 \\ x_1 + 2x_2 - x_3 &= \ 4 \\ -2x_1 - \ x_2 + x_3 &= -3 \end{aligned}$$

Question 4. *(20 marks)* Evaluate the determinant of

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

Question 5. *(20 marks)*

(a) Let **A** and **B** be n x n matrices. Show that if **AB** is non-singular, then **A** and **B** must be non-singular.

(b) A square matrix is **skew-symmetric** if $\boldsymbol{A^T = A}$. Show that if **B** is a square matrix of order n, then $\boldsymbol{A = \frac{1}{2}(B - B^T)}$ is skew-symmetric.

END