Discrete Mathematics (2024 Program) - HCMUS - January 8th, 2025

Semester 1 : Year 2024 - 2025 - Time Duration: 90 minutes

Question 1 (1.5 points)

Given $a_0 = 4, a_1 = 24$ and $a_{n+2} = 6a_{n+1} - 9a_n - (4n - 17)2^n$. Calculate a_n in accordance to n ($n \geq 0$).

Question 2 (3.25 points = 1.25pts + 1.25pts + 0.75pts)

Given $m = 43615880, n = -22198176, a = 36567$ and $b = 6886$.

a) Analyze elements m and n so as to find $d = (m, n), e = [m, n]$ and a minimalist form of $\frac{m}{n}$

b) Use Euclidean Algorithm to find $r, s, u, v \in \mathbb{Z}$ satisfying $\mathbf{l} = ra + sb$ and $\frac{l}{ab} = \frac{u}{a} + \frac{v}{b}$

c) Describe the integer divisors of m and calculate the possible number of those in m?

Question 3 (3.25 points = 1.25pts + 1pt + 1pt)

a) Given a binary relation $\mathcal{R}$ on $S = \{1, 2, 3\}$ defined by $\forall x, y \in S,\ x\,\hat{\mathcal{R}}\,y \iff (x - y)^2 \leq 1$. List all sets $H = \{(x, y) \in S^2 |\ x\,\hat{\mathcal{R}}\,y\}$. Evaluate the following properties: Reflexive, Symmetric, Anti-symmetric and Transitive of relation $\mathcal{R}$.

b) Upon $T = \{1, 2, 4, 5, 7, 10, 12, 24, 30\}$, giving ordinal relation Ω identified by $\forall x, y \in T,\ x\,\hat{\Omega}\,y \iff y = x$ or containing **even integer k** such that y = **kx** (k depends on x and y). Draw Hasse diagram of (T, Ω) and find smallest - largest - minimum and maximum (if possible) of (T, Ω).

c) Apply b) part of **Question 2** in solving the equation $\overline{6886} \cdot \overline{y} = \overline{238}$ in $\mathbb{Z}_{36567}$. Derive the solution of the equation $\overline{6886 \times 6} \cdot \overline{x} = \overline{238 \times 6}$ in $\mathbb{Z}_n$ with **n** = 36567 x 6

Question 4 (2 points = 1pt + 1pt) Given boolean function f following by boolean variables x, y, z, t identified by

$$f(x, y, z, t) = x\,\overline{y}\,z\,t \vee \overline{x}\,\overline{z}\,t \vee x\,y\,z\,\overline{t} \vee \overline{x}\,y\,\overline{t} \vee x\,\overline{y}\,z\,\overline{t} \vee \overline{x}\,z\,t$$

a) Draw the Karnaugh map for f and identify its largest implicants (prime implicants).

b) Find the minimal expressions for f (i.e, the minimized Boolean expressions).

END