HCMIU - Calculus I - Final Test Sample
Semester 1 - Year: Unknown - Duration : 90 minutes
Date Modified : Saturday, December 14^{th}, 2024

Full Name: ______________________________

Student's ID: ______________________________

INSTRUCTIONS:

- Use of calculator is allowed. Each student is allowed one double-sized sheet of reference material (size A4 or similar). All other documents and electronic devices are forbidden.
- You must explain your answers in detail; no point will be given for the answer alone.
- There is a total of 10 (ten) questions. Each one carries 10 points.

1. Evaluate the limit
$$\lim_{x\to 0} \frac{\int_0^{2x} [e^t + \sin t]\, dt}{\ln(1+10x)}$$

2. A spherical snowball is melting in such a way that its radius is decreasing at a rate of 1 cm/min. At what rate is the volume of the snowball decreasing when the radius is 9 cm?

3. Find the linear approximation of the function $f(x) = \sqrt[3]{1+x}$ at $x_0 = 0$. Use this approximation to approximate the number $\sqrt[3]{0.97}$ and $\sqrt[3]{1.1}$.

4. (a) *[5 points]* Show that the equation $x^3 + x + 4 = 0$ has a *unique* real root on $\mathbb{R}$.
   (b) *[5 points]* Use Newton's method to approximate the root of $x^3 + x + 4 = 0$ correct to seven decimal places. Let x_0 = -1.5 be the initial approximation.

5. Find the derivative of the function
$$G(x) = \int_x^{3x^2} \ln(t^2+1)\, dt.$$

6. Evaluate the improper integral
$$\int_3^{\infty} \frac{1}{(x+6)\sqrt{x+6}}\, dx.$$

7. An oil storage tank cracks at time t = 0 and oil then leaks from the tank at a rate of $r(t) = t \cdot e^{-0.01 \cdot t}$ liters per minute.
   (a) *[5 points]* Find the time at which the rate has its maximum value.
   (b) *[5 points]* How much oil leaks out during the first ten minutes?

8. Sketch the region enclosed by the curves $y = 2$, $y = \sqrt{x}$ and $x + y = 2$. Then find the area of the region.

9. An object is moving along a straight line path with an initial velocity of $v(0) = 5\,\text{m/s}$. The table below presents the dependence of the acceleration of this object on time t (in m/s^2). Use the Trapezoidal Rule to approximate the velocity of the object at $t = 2$ seconds.

Time t (in seconds)	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Acceleration (in m/s^2)	1	1.5	1.8	2	2.5	3	3.5	4	4.5	4.2	4

10. The region bounded by $y = sin(2x)$ with $0 \leqslant x \leqslant \frac{\pi}{2}$, and $y = 0$ is rotated about the y-axis. Find the volume of the solid of revolution.