HCMIU - Linear Algebra - Mid-term Test
Semester X - Year: Unknown - Duration : 120 minutes
Date Modified : Saturday, July 26$^{\text{th}}$, 2025

INSTRUCTIONS: Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.

Part A: True / False Questions. *Each question carries 3 points - Fill your answer in the answer sheet below.*

ANSWER SHEET FOR PART A

1. (T) (F)	4. (T) (F)	7. (T) (F)	10. (T) (F)	13. (T) (F)
2. (T) (F)	5. (T) (F)	8. (T) (F)	11. (T) (F)	14. (T) (F)
3. (T) (F)	6. (T) (F)	9. (T) (F)	12. (T) (F)	15. (T) (F)

1. If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a one-to-one linear transformation, then T is also onto.
2. Every matrix transformation is a linear transformation.
3. If A and B are symmetric, then AB^2A is symmetric.
4. $X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & -1 \\ 3 & -1 & 2 \end{bmatrix}$ corresponding to eigenvalue $\lambda = 2$
5. If A is a matrix for a linear transformation, then A is invertible.
6. If V is a set that contains the 0-vector, and such that whenever u and v are in V, then $u + v$ is in V, then V is a vector space.
7. If A is orthogonal, then A^T is orthogonal.
8. Any two vector spaces of dimension six are isomorphic.
9. Suppose Q is orthonormal and A is an arbitrary matrix of the same size. Then QAQ^T is similar to A.
10. There is a 5×5 matrix of rank 4 and determinant 1.
11. If A and B are symmetric invertible matrices, then ABA^{-1} is also symmetric and invertible.
12. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^n = \begin{bmatrix} a^n & b^n \\ c^n & d^n \end{bmatrix}$.
13. The matrix $\begin{bmatrix} 0 & 1 & -b \\ 0 & b & 1 \\ 1 & 0 & 0 \end{bmatrix}$ has rank 3 for every real value of b.
14. Suppose that $\vec{v}_1$ and $\vec{v}_2$ form a basis for V, a subspace of $\mathbb{R}^4$; and $\vec{w}_1$, $\vec{w}_2$ form a basis for W, a subspace of $\mathbb{R}^5$. If a mapping $T : V \to W$ is defined $T(\vec{v}_1 + \vec{v}_2) = a\vec{w}_1 + \vec{w}_2$, then T is an isomorphism.
15. If A is 5×5 matrix with two eigenvalues, one eigenspace is 2 dimensional, and the other is 3 dimensional, then A is diagonalizable.

Part B: Show your work in details and indicate answers clearly.

1. *(10 points)* Find the matrix for $T(\vec{w}) = proj_{\tilde{\mathbf{v}}}(\tilde{\mathbf{w}})$ where $\vec{v} = \begin{bmatrix} 1 & -2 & 3 \end{bmatrix}^T$.
2. *(20 points)* Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be defined as

$$T\vec{x} = \begin{bmatrix} 3 & 2 & 1 & 8 \\ 2 & 2 & -2 & 6 \\ 1 & 1 & -1 & 3 \end{bmatrix} \vec{x}$$

   Find the dimension and a basis for *Im(T)*, as well as the dimension and a basis for *Ker(T)*.

3. *(30 points)* Let $A = \begin{bmatrix} 13 & 1 & 4 \\ 1 & 13 & 4 \\ 4 & 4 & 10 \end{bmatrix}$.
   (a) Find eigenvalues and eigenvectors of A
   (b) Write A as $A = PDP^T$ where D is a diagonal matrix and P is an orthogonal matrix and find A^{2023}
   (c) Find the solution of the differential equation

$$\begin{cases} x'(t) = 13x(t) + y(t) + 4z(t) \\ y'(t) = x(t) + 13y(t) + 4z(t) \\ z'(t) = 4x(t) + 4y(t) + 10z(t) \end{cases}$$

   with the initial condition $x(0) = 1, y(0) = 0, z(0) = -1$.

END