HCMIU - Calculus I - Final Test Sample

Semester 1 - Year: 2024 $\backsim$ 2025 - Duration : 120 minutes

Date Modified : Wednesday, January 15$^{\text{th}}$, 2025

INSTRUCTIONS:

- Use of calculator is allowed. Each student is allowed one doubled-sized sheet of reference material (size A4 of similar). All other documents and electronic devices are forbidden
- You must explain your answers in detail; no points will be given for the answer alone.
- There are a total of 10 (ten) questions. Each one carries 10 points

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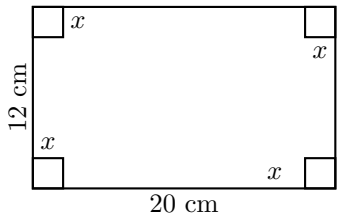
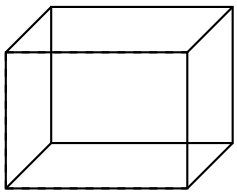
1. Find the limit
$$\lim_{x\to\infty} x \ln\left(1+\frac{2}{x}\right)$$

2. You are constructing a cardboard box with the dimension 12cm by 20cm. You then cut equal-size squares from each corner so you may fold the edges. What are the dimensions of the box with the largest volume? Refer to the figure below.





3. Use the Newton's method to find an approximate solution of the equation with the initial $x_0 = -3$, correct to 7 decimal places.
$$\frac{x^3}{3}+\frac{x^2}{2}+3=0$$

4. Let $F(x)=\int_x^{x^2} f(t)\,dt$. Assume that $F'(1)=3$. Find $f(1)$.

5. Use the L'Hospital rule to find the limit
$$\lim_{x\to 0}\frac{\int_0^{3x^2}\ln\left(t^2+2\right)dt}{x^2}.$$

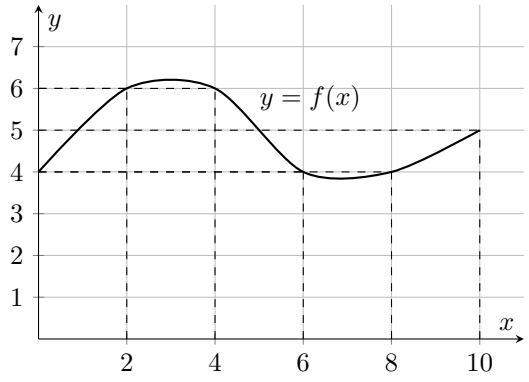
6. Evaluate the integral
$$\int_0^1 \frac{x^2}{(x^3+1)^4}\,dt$$

7. Evaluate the improper integral
$$\int_0^\infty x^2 e^{-2x}\,dx.$$

8. Find the volume of the solid generated by revolving the region under the curve $y=\frac{5}{x}$ about the x-axis, for x ranging from $x=3$ to $x=\infty$.

9. Let R be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Calculate the area of R and the volume obtained by rotating R about the y-axis.

10. Approximate the area under the curve $f(x)$ between $x = 0$ and $x = 10$ (as shown in the figure below), using the Trapezoidal Rule with n = 5 sub-intervals.



END OF TEST - BEST OF LUCK