

## HCMIU - Calculus I - Mid-term Test

Semester 3 - Year: 2021 ~ 2022 - Duration : 90 minutes

Date Modified : Thursday, June 26<sup>th</sup>, 2025**INSTRUCTIONS:**

- Use of calculator is allowed. Each student is allowed one doubled-sized sheet of reference material (size A4 of similar). All other documents and electronic devices are forbidden
- You must explain your answers in detail; no points will be given for the answer alone.
- There are a total of 5 (five) questions. Each one carries 20 points

**Question 1.** Find the limit of the following sequences:

$$(a) a_n = \frac{\ln(2n+1)}{n} \quad (b) a_n = \sin\left(\frac{n\pi}{6n+1}\right)$$

**Question 2.** Determine whether the given series is convergent or divergent:

$$(a) \sum_{n=1}^{\infty} \frac{n\sqrt{n}}{n^3+1} \quad (b) \sum_{n=2}^{\infty} \frac{1}{n \ln(n^2)}$$

**Question 3.** Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (2x-4)^n}{n6^n}$$

**Question 4.** Do the following requests:

- Find a nonzero vector orthogonal to the plane through the points  $P(1, -1, 0)$ ,  $Q(x_1, y_1, z_1)$ ,  $R(x_2, y_2, z_2)$ , and find the area of triangle PQR.
- Find an equation of the plane that passes through the line of intersection of the planes  $y - z = 1$  and  $x + 2y = 2$ , and is perpendicular to the plane  $2x + 3y - z = 4$ .

**Question 5.** Do the following requests:

- Find parametric equations for the tangent line to the curve

$$r(t) = \langle t^2, 2t, e^{t-1} \rangle, \quad 0 \leq t \leq 2$$

at the point  $(1, 2, 1)$ .

- Find the length of the space curve

$$r(t) = \left\langle t, t^2, \frac{4t^{3/2}}{3} \right\rangle, \quad 0 \leq t \leq 1$$

*END OF TEST - BEST OF LUCK*