HCMIU - Probability - Mid-term Test

Semester 1 - Year: 2021 - 2022 - Duration : 90 minutes

Date Modified : Monday, Sep. 1st, 2025

INSTRUCTIONS:

- Each student is allowed one doubled-sized sheet of reference material (size A4 of similar). All other documents and electronic devices are forbidden, except scientific calculators.
- There is a total of 10 (ten) questions. Each one carries 10 points.

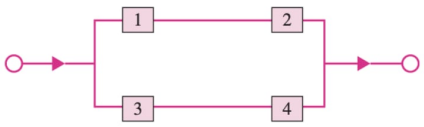
Question 1. (10 points) An insurance company classifies the arriving claims into four categories, depending on the size of these claims. From past data, it has been estimated that the first two categories, associated with the largest claims, arrive with the same probability, a claim from the third category is twice as much probable, and a claim from the fourth category is three times as much probable compared to a claim from the first or the second group. What is the probability that the next claim to arrive at the company will not be of the first category?

Question 2. (10 points) On the way driving back home from school, An has to pass through two sets of traffic lights. The probability that she has to stop at the first is 0.3, while the probability that she has to stop at the second one is 0.40. and the probability that she does not stop at a traffic light is 0.40. Calculate the probability that she has to stop at least at one set of traffic lights.

Question 3. (10 points) A telecommunications system transmits binary signals (0 or 1). The system includes a transmitter that emits the signals and a receiver which receives those signals. The probability that the receiver registers a signal 1 when the transmitter has sent a signal 1 is 99%, while the probability that the receiver registers a signal 0 when the transmitter has sent a signal 0 is 98%. Signals are transmitted every second. The probability that the signal 0 is transmitted is twice as the signal 1.

The last signal has been registered as 1. Find the probability that the original signal transmitted was also 1.

Question 4. (10 points) Consider an electrical system comprises four components as following



Each component functions independently of one another. The probabilities that the component 1, 2, 3, 4 work are, respectively, 0.9, 0.92, 0.85, and 0.95.

What is the probability that the system works?

Question 5. (20 points) A small internet company sells ebooks. The company has paid \$100 to obtain a new book electronically and sells each copy of the book at a price of \$50. The number of sales X for this book is a random variable with probability mass function

x	0	1	2	3	4	5
$P(X=x)$	$\frac{1}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{1}{15}$

(a) Find the probability mass function of the company's profit (or loss).

(b) What is the probability that the company will make a profit but that will be less than \$80?

(c) Compute the expected company's profit

Question 6. (20 points) The borrowing period (in days) for a particular book at a University library can be regarded as a continuous random variable X with probability density function

$$f(x) = \begin{cases} cx, & 0 \le x \le 5 \\ 0, & otherwise \end{cases}$$

(a) Determine c.

(b) What is the probability that a book is returned within 2 days?

(c) Four students borrow a book. Find the probability that at least two of them return the book within 2 days.

Question 7. (20 points) The daily demand for rice in a shop (in kilograms) has the normal distribution with mean $m = 800$ and standard deviation $s = 45$. The current stock of the shop is 950 kg

(a) Find the probability that all this stock will be sold within a day.

(b) What is the probability that after the end of the next working day, the shop will have at least 100 kg of rice still in stock?

END

Standard Distribution Table $Z \approx N(0,1) : \Theta(z) = \mathbb{P}(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-x^2}{2}}\, dx$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

SUGGESTED ANSWER

Question 1. Let A be the event that An stops at least one of the set of traffic lights, respectively S_1, S_2, S^C are first - second stop and non-stop. By Multiplication rule,

$$P(A) = P(S_1) \cdot P(S^C) + P(S_2) \cdot P(S^C) + P(S_1) \cdot P(S_2) = 0.3 \cdot 0.4 + 0.4 \cdot 0.4 + 0.4 \cdot 0.3 = 0.4$$

Question 2. Label $C_n,\ n = 1, 2, 3, 4$ as the likelihood of each claims to occur, derive the interpretation from the passage

- $P(C_1) = P(C_2) = p, \quad P(C_3) = 2p, \quad P(C_4) = 3p$
- Sum Of Probabilities: $\sum_{n=1}^{4} P(C_n) = 1 \Longrightarrow p = \frac{1}{7}$

Hence, the probability that the next claim arriving at the company will not belong to 1st category is $P(S_1^C) = 1 - \frac{1}{7} = \frac{6}{7}$

Question 3. Assign each possible events

- T_1: transmit signal 1
- T_0: transmit signal 0
- R_0: receive signal 1
- R_1: receive signal 0
- $P(R_1|T_1) = 0.99, \quad P(R_0|T_1) = 0.01$
- $P(R_0|T_0) = 0.98, \quad P(R_1|T_0) = 0.02$

The probability that the signal 1 is transmitted given that the previous signal registered was 1 (Bayes' Theorem):

$$P(T_1|R_1) = \frac{P(T_1 \cap R_1)}{P(R_1)} = \frac{P(T_1)P(R_1|T_1)}{P(R_1|T_1)P(T_1) + P(R_1|T_0)P(T_0)} \approx 0.961$$

Question 4. Let A and B be respectively the events of components 1 - 2 work and components 3 - 4 work. According to the graph, the system requires only one of two branches to shed the electricity from one to another endpoint, thus the event that both branches turning on at the same time will produce redundant cases. By law of total probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$$

$$= 0.9 \cdot 0.92 + 0.85 \cdot 0.95 - 0.9 \cdot 0.92 \cdot 0.85 \cdot 0.95 \approx 0.9669$$

Question 5. Let x be the number of days for a particular book to be borrowed in library. The random variable indicates the assignment of each period to the sample space (values).

1. Since $\sum_{k=0}^{5} p_x(k) = 1$ (Sum Of Probabilities)

$$P(0 \le X \le 5) = \int_0^5 f(x)\,dx = \int_0^5 cx\,dx = \frac{25}{2}c = 1 \Longrightarrow c = \frac{2}{25}$$

2. The book is returned in 2 days

$$P(X \le 2) = \int_0^2 f(x)\,dx + \int_2^\infty f(x)\,dx = \int_0^2 \frac{2}{25}x\,dx = \frac{4}{25} = 0.16$$

3. The probability that atleast two students (assigned by Y) borrow the book in library can be interpreted as geometric distribution probability (Successes might be repetitive among persons)

$$P(Y \geq 2) = \sum_{k=2}^{4} p^k (1-p)^{4-k} \binom{4}{k} \approx 0.034$$

Question 6. Number of sale books converted to profit (revenue) as indicated by the random variable function $Y = 50X - 100$ (\$) with the initial price of buying books is 100\$

(a) Set up the table of P.M.F (Probability Mass Function):

X	0	1	2	3	4	5
$Y = 50X - 100$	-100	-50	0	50	100	150

(b) The profit of total books must be less than 80\$ (notice that it's not cumulative)

$$P(Y < 80) = P(X < 4) = P(X = 2) + P(X = 3) = \frac{4}{15}$$

(c) The expected company's profit is the average mass function of each book selling milestone:

$$E[X] = -100 \cdot 0 - 50 \cdot 1 + 0 \cdot 2 + 50 \cdot 3 + 100 \cdot 4 + 150 \cdot 5 = 8600(\$)$$

Question 7. The probability that all of this stock will be sold is calculated by Gaussian distribution formula and normalized following by the standard distribution table (please refer to second page).

(a)

$$P(X = 950) \approx P\left(\mathbb{Z} = \frac{X - \mu}{\sigma}\right) \approx P\left(\mathbb{Z} = \frac{950 - 800}{45}\right) P(\mathbb{Z} = 3.33) \approx 0.9996$$

(b) After the end of the next working day, the consumption of the stock is estimated to be no more than $X' = 950 - 150 = 850(kg)$.

$$P(X \leq 850) = P\left(\mathbb{Z} = \frac{X' - \mu}{\sigma}\right) = P\left(\mathbb{Z} = \frac{850 - 800}{45}\right) \approx P(\mathbb{Z} = 1.11) \approx 0.8665$$

END