Wintersemester 2022/23

Automaten und Berechenbarkeit - Übung 02

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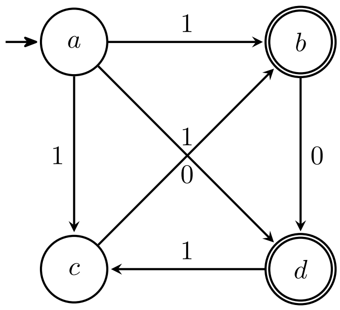
Aufgabe 1

NFA $M = (\{0,1\}, \{a,b,c,d\}, \delta, \{a,d\}, \{b,d\})$ δ:



Zustand	0	1
$\emptyset$	$\emptyset$	$\emptyset$
a	$\emptyset$	$\{b,c,d\}$
b	$\{d\}$	$\emptyset$
c	$\{b\}$	$\emptyset$
d	$\emptyset$	$\{c\}$

(a)

$$\begin{aligned}
\delta^*(\{a\}, 1001) &= \bigcup_{z \in \{a\}} \delta^*(\delta(\{a\}, 1), 001) \\
&= \delta^*(\{b,c,d\}, 001) \\
&= \bigcup_{z \in \{b,c,d\}} \delta^*(\delta(\{b,c,d\}, 0), 01) \\
&= \delta^*(\{d\}, 01) \cup \delta^*(\{b\}, 01) \cup \delta^*(\emptyset, 01) \\
&= \delta^*(\delta(\{d\}, 0), 1) \cup \delta^*(\delta(\{b\}, 0), 1) \\
&= \emptyset \cup \delta^*(\{d\}, 1) \\
&= \delta^*(\delta(\{d\}, 1), \lambda) \\
&= \delta^*(\{c\}, \lambda) \\
&= \{c\}
\end{aligned}$$

$$\begin{aligned}
\delta^*(\{d\}, 1000) &= \delta^*(\delta(\{d\}, 1), 000) \\
&= \delta^*(\{c\}, 000) \\
&= \delta^*(\delta(\{c\}, 0), 00) \\
&= \delta^*(\{b\}, 00) \\
&= \delta^*(\delta(\{b\}, 0), 0) \\
&= \delta^*(\{d\}, 0) \\
&= \delta^*(\delta(\{d\}, 0), \lambda) \\
&= \delta^*(\emptyset, \lambda) = \emptyset
\end{aligned}$$

(b)

Bestimmen Sie $\{w \in \{0,1\}^* \mid \delta^*(\{a\}, w) \cap \{d\} \neq \emptyset\}$!

$\delta^*(\{a\}, w)$... Menge der

(c)

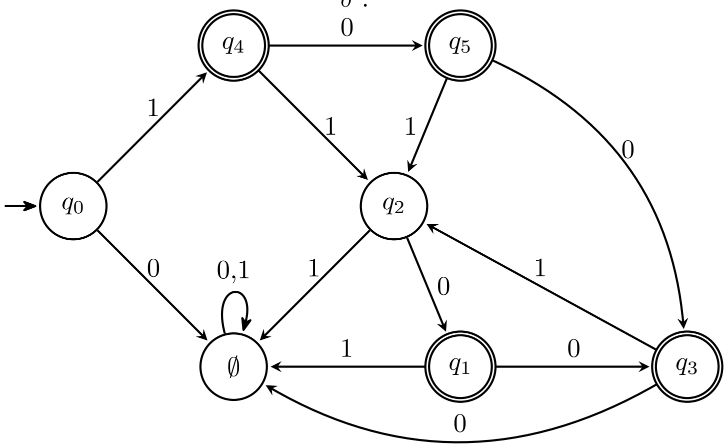
$$\delta^*(\{d,b\},0) =$$
$$\delta^*(\{d,b\},1) =$$
$$\delta^*(\{b,c,d\},0) =$$
$$\delta^*(\{b,c,d\},1) =$$

Zustand	0	1
$\emptyset$	$\emptyset$	$\emptyset$
a	$\emptyset$	$\{b,c,d\}$
b	$\{d\}$	$\emptyset$
c	$\{b\}$	$\emptyset$
d	$\emptyset$	$\{c\}$
$\{b,c,d\}$	$\{d,b\}$	$\{c\}$
$\{d,b\}$	$\{d\}$	$\{c\}$

Bzw.

Zustand	0	1
$\emptyset$	$\emptyset$	$\emptyset$
q_0	$\emptyset$	$\{q_4\}$
q_1	$\{q_3\}$	$\emptyset$
q_2	$\{q_1\}$	$\emptyset$
q_3	$\emptyset$	$\{q_2\}$
q_4	$\{q_5\}$	$\{q_2\}$
q_5	$\{q_3\}$	$\{q_2\}$

δ':



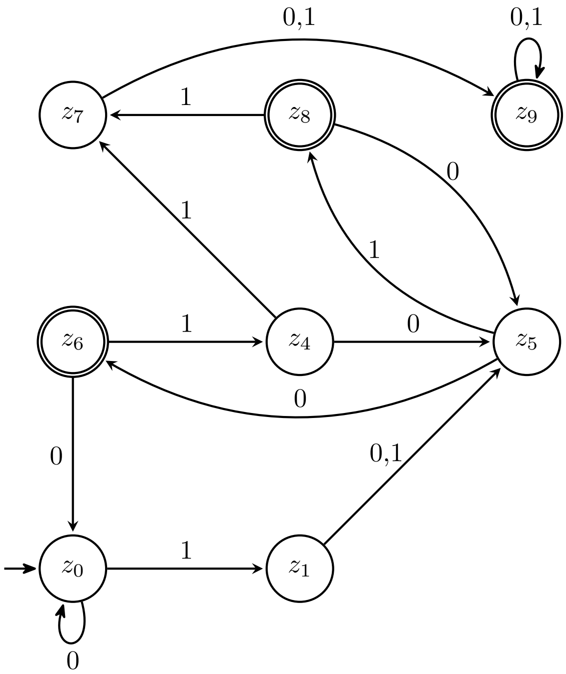
$M' = (\{0,1\}, Z', \delta', S', Z_E')$

Aufgabe 2

(a)

Der DFA ist mittel Potenzmengenkonstruktion aus dem NFA in (b) entstanden. Die Zustände z_2 und z_3 wurden entfernt, da Sie nicht erreichbar sind.

$M' = (\{0,1\}, Z', \delta', S', Z_E')$ δ:



Zustand	0	1
z_0	$\{z_0\}$	$\{z_1\}$
z_1	$\{z_0, z_2\}$	$\{z_0, z_2\}$
z_2	$\{z_0, z_3\}$	$\{z_0, z_3\}$
z_3	$\{z_0\}$	$\{z_0\}$
$\{z_0, z_1\}$	$\{z_0, z_2\}$	$\{z_0, z_1, z_2\}$
$\{z_0, z_2\}$	$\{z_0, z_3\}$	$\{z_0, z_1, z_3\}$
$\{z_0, z_3\}$	$\{z_0\}$	$\{z_0, z_1\}$
$\{z_0, z_1, z_2\}$	$\{z_0, z_1, z_2, z_3\}$	$\{z_0, z_1, z_2, z_3\}$
$\{z_0, z_1, z_3\}$	$\{z_0, z_2\}$	$\{z_0, z_1, z_2\}$
$\{z_0, z_1, z_2, z_3\}$	$\{z_0, z_1, z_2, z_3\}$	$\{z_0, z_1, z_2, z_3\}$

Bzw.

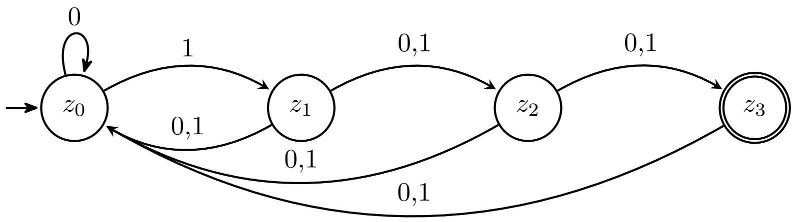
Zustand	0	1
z_0	$\{z_0\}$	$\{z_1\}$
z_1	$\{z_5\}$	$\{z_5\}$
z_2	$\{z_6\}$	$\{z_6\}$
z_3	$\{z_0\}$	$\{z_0\}$
z_4	$\{z_5\}$	$\{z_7\}$
z_5	$\{z_6\}$	$\{z_8\}$
z_6	$\{z_0\}$	$\{z_4\}$
z_7	$\{z_9\}$	$\{z_9\}$
z_8	$\{z_5\}$	$\{z_7\}$
z_9	$\{z_9\}$	$\{z_9\}$

Beweis. " $\subseteq$ " □

Beweis. " $\supseteq$ " □

(b)

$M = (\{0,1\}, Z, \delta, S, Z_E)$ δ:

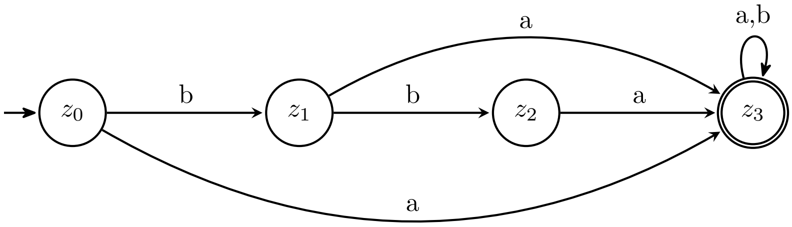


Zustand	0	1
z_0	$\{z_0\}$	$\{z_1\}$
z_1	$\{\mathrm{z}_0, z_2\}$	$\{\mathrm{z}_0, z_2\}$
z_2	$\{\mathrm{z}_0, z_3\}$	$\{\mathrm{z}_0, z_3\}$
z_3	$\{z_0\}$	$\{z_0\}$

Beweis. " $\subseteq$ " $\square$

Beweis. " $\supseteq$ " $\square$

Aufgabe 3



Zustand	a	b
z_0	z_3	z_1
z_1	z_3	z_2
z_2	z_3	$\emptyset$
z_3	z_3	z_3

Beweis. " $\subseteq$ " $\square$

Beweis. " $\supseteq$ " $\square$

Aufgabe 4

$$G_1 = (\{a,b\}, \{S, A, B, E1, E2\}, S, R) \text{ mit } R : \begin{cases} S & \rightarrow aA \mid Bb \\ A & \rightarrow E_1 b \mid b \\ B & \rightarrow aE_2 \mid a \\ E_1 & \rightarrow aA \\ E_2 & \rightarrow Bb \end{cases}$$

Beweis. " $\subseteq$ " $\square$

Beweis. " $\supseteq$ " $\square$