```
> with(Student[LinearAlgebra])
```

[&x, `.`, AddRow, AddRows, Adjoint, ApplyLinearTransformPlot, BackwardSubstitute, BandMatrix, Basis, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, ColumnDimension, ColumnSpace, CompanionMatrix, ConstantMatrix, ConstantVector, CrossProduct, CrossProductPlot, Determinant, DeterminantSteps, Diagonal, DiagonalMatrix, Dimension, Dimensions, EigenPlot, EigenPlotTutor, Eigenvalues, EigenvaluesTutor, Eigenvectors, EigenvectorsTutor, Equal, GaussJordanEliminationTutor, GaussianElimination, GaussianEliminationTutor, GenerateEquations, GenerateMatrix, GramSchmidt, HermitianTranspose, HouseholderMatrix, Id, IdentityMatrix, IntersectionBasis, InverseTutor, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LUDecomposition, LeastSquares, LeastSquaresPlot, LinearSolve, LinearSolveTutor, LinearSystemPlot, LinearSystemPlotTutor, LinearTransformPlot, LinearTransformPlotTutor, MatrixBuilder, MatrixExponential, MatrixInverse, MinimalPolynomial, Minor, MultiplyRow, Norm, Normalize, NullSpace, Pivot, PlanePlot, ProjectionMatrix, ProjectionPlot, Pseudoinverse, QRDecomposition, RandomMatrix, RandomVector, Rank, ReducedRowEchelonForm, ReflectionMatrix, RotationMatrix, RowDimension, RowSpace, SetDefault, SetDefaults, SingularValues, SumBasis, SwapRow, SwapRows, Trace, Transpose, UnitVector, VectorAngle, VectorSumPlot, ZeroMatrix, ZeroVector] **(1)**

```
> with(LinearAlgebra)
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, **(2)**

SmithForm, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

> *with*(*linalg*)

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] **(3)**

> $A := Matrix([[0,-2,0],[1,-2,0],[0,0,-2]])$

$$A := \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad \textbf{(4)}$$

> *Determinant*(*A*)

$$-4 \quad \textbf{(5)}$$

> *inverse*(*A*)

$$\begin{bmatrix} -1 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad \textbf{(6)}$$

> *Eigenvalues*(*A*)

$$\begin{bmatrix} -2 \\ -1-I \\ -1+I \end{bmatrix} \quad \textbf{(7)}$$

> *Eigenvectors*(*A*)

$$\begin{bmatrix} -2 \\ -1+I \\ -1-I \end{bmatrix}, \begin{bmatrix} 0 & 1+I & 1-I \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \textbf{(8)}$$

> $CharacteristicPolynomial(A, r)$

$$r^3 + 4\,r^2 + 6\,r + 4 \tag{9}$$

> $u1 := \langle 0, 0, 1 \rangle$

$$u1 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{10}$$

> $A\_u1 := A \cdot u1$

$$A\_u1 := \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{11}$$

> $Lam\_u1 := (-2) \cdot u1$

$$Lam\_u1 := \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{12}$$

> $A\_u1 - Lam\_u1$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

> $u2 := \langle 1 + I, 1, 0 \rangle$

$$u2 := \begin{bmatrix} 1 + \mathrm{I} \\ 1 \\ 0 \end{bmatrix} \tag{14}$$

> $A\_u2 := A \cdot u2$

$$A\_u2 := \begin{bmatrix} -2 \\ -1 + \mathrm{I} \\ 0 \end{bmatrix} \tag{15}$$

> $Lam\_u2 := (-1 + I) \cdot u2$

$$Lam\_u2 := \begin{bmatrix} -2 \\ -1 + \mathrm{I} \\ 0 \end{bmatrix} \tag{16}$$

> $A\_u2 - Lam\_u2$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

> $u3 := \langle 1 - I, 1, 0 \rangle$

$$u3 := \begin{bmatrix} 1 - I \\ 1 \\ 0 \end{bmatrix} \quad (18)$$

> $A\_u3 := A \cdot u3$

$$A\_u3 := \begin{bmatrix} -2 \\ -1 - I \\ 0 \end{bmatrix} \quad (19)$$

> $Lam\_u3 := (-1 - I) \cdot u3$

$$Lam\_u3 := \begin{bmatrix} -2 \\ -1 - I \\ 0 \end{bmatrix} \quad (20)$$

> $A\_u3 - Lam\_u3$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

> $P := Matrix(\langle u1|u2|u3\rangle)$

$$P := \begin{bmatrix} 0 & 1 + I & 1 - I \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (22)$$

> $J := DiagonalMatrix([-2, -1 + I, -1 - I])$

$$J := \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 + I & 0 \\ 0 & 0 & -1 - I \end{bmatrix} \quad (23)$$

> $PJinvP := P \cdot J \cdot inverse(P)$

$$PJinvP := \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (24)$$

> $A - PJinvP$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (25)$$

> $MatrixExponential(t \cdot J)$

(26)

$$\begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-t}\cos(t) + I e^{-t}\sin(t) & 0 \\ 0 & 0 & e^{-t}\cos(t) - I e^{-t}\sin(t) \end{bmatrix} \quad (26)$$

> *MatrixExponential*$(t \cdot A)$

$$\begin{bmatrix} e^{-t}\cos(t) + e^{-t}\sin(t) & -2\,e^{-t}\sin(t) & 0 \\ e^{-t}\sin(t) & e^{-t}\cos(t) - e^{-t}\sin(t) & 0 \\ 0 & 0 & e^{-2t} \end{bmatrix} \quad (27)$$

> *Map*$(limit, MatrixExponential(t \cdot A), t = \text{infinity})$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (28)$$

>

>

> $dsolve(\{diff(x(t), t) = 1 - (x(t))^2, x(0) = -1\}, x(t))$

$$x(t) = -1 \quad (29)$$

> $dsolve(\{diff(x(t), t) = 1 - (x(t))^2, x(0) = 1\}, x(t))$

$$x(t) = 1 \quad (30)$$

> $p1 := rhs(dsolve(\{diff(x(t), t) = 1 - (x(t))^2, x(0) = -2\}, x(t)))$

$$p1 := \coth\left(-\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \quad (31)$$

> $convert(convert(p1, \exp), \exp)$

$$\frac{e^{2t} + 3}{e^{2t} - 3} \quad (32)$$

> $p2 := rhs(dsolve(\{diff(x(t), t) = 1 - (x(t))^2, x(0) = 0\}, x(t)))$

$$p2 := \tanh(t) \quad (33)$$

> $convert(p2, \exp)$

$$\frac{e^{t} - e^{-t}}{e^{t} + e^{-t}} \quad (34)$$

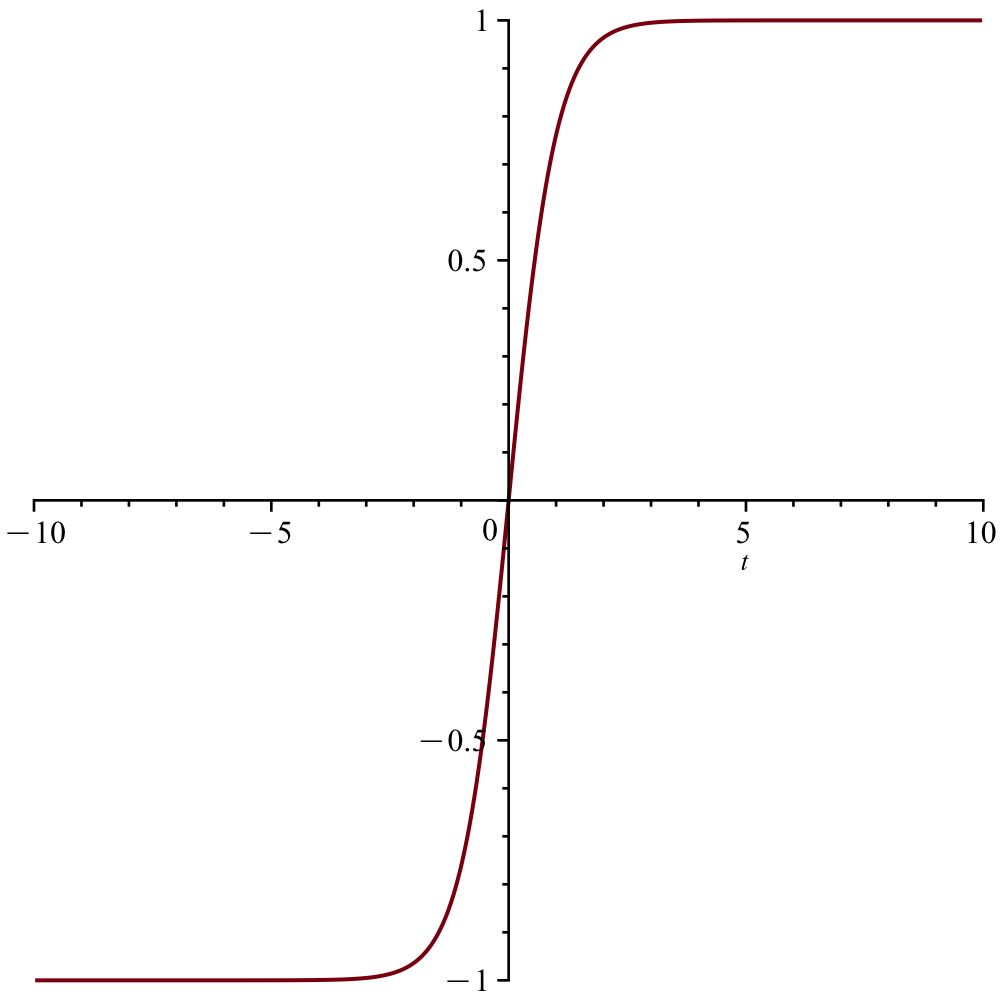
> $p3 := rhs(dsolve(\{diff(x(t), t) = 1 - (x(t))^2, x(0) = 2\}, x(t)))$

$$p3 := \coth\left(\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \quad (35)$$

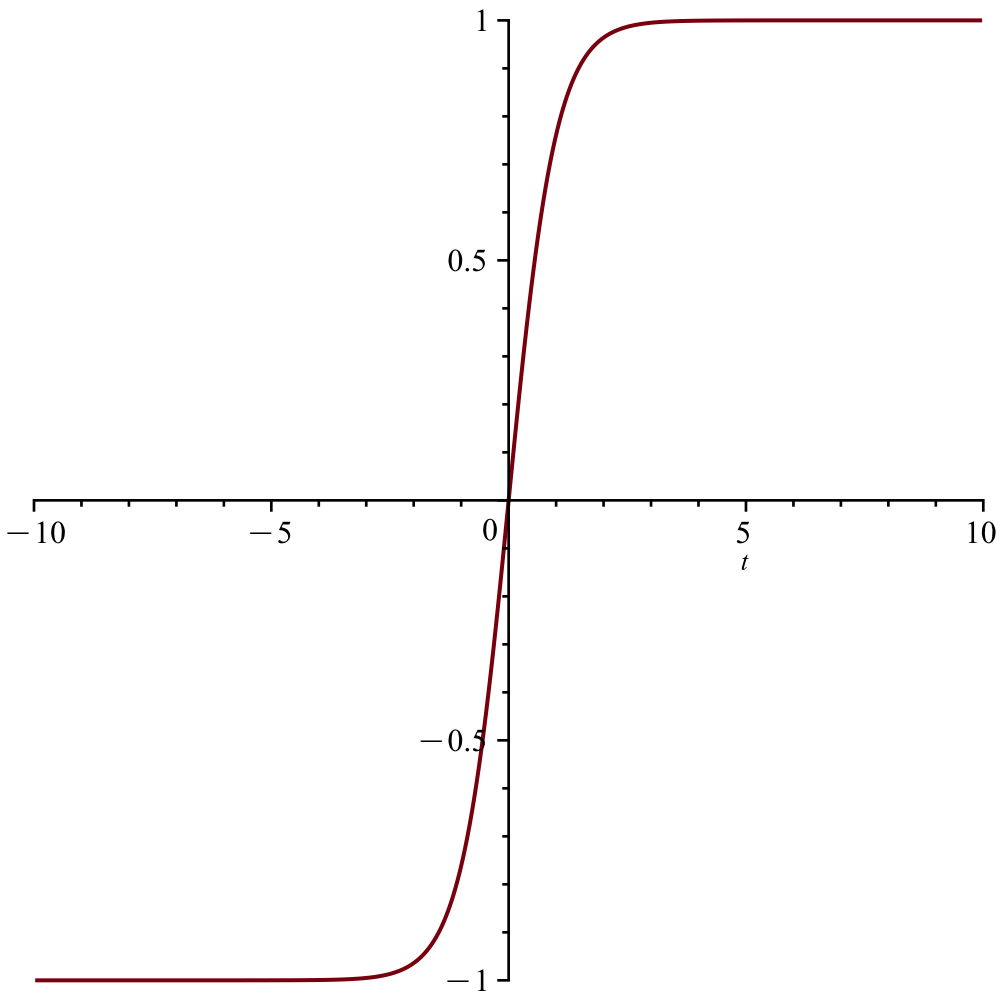
> $convert(convert(p3, \exp), \exp)$

$$\frac{3\,e^{2t} + 1}{3\,e^{2t} - 1} \quad (36)$$

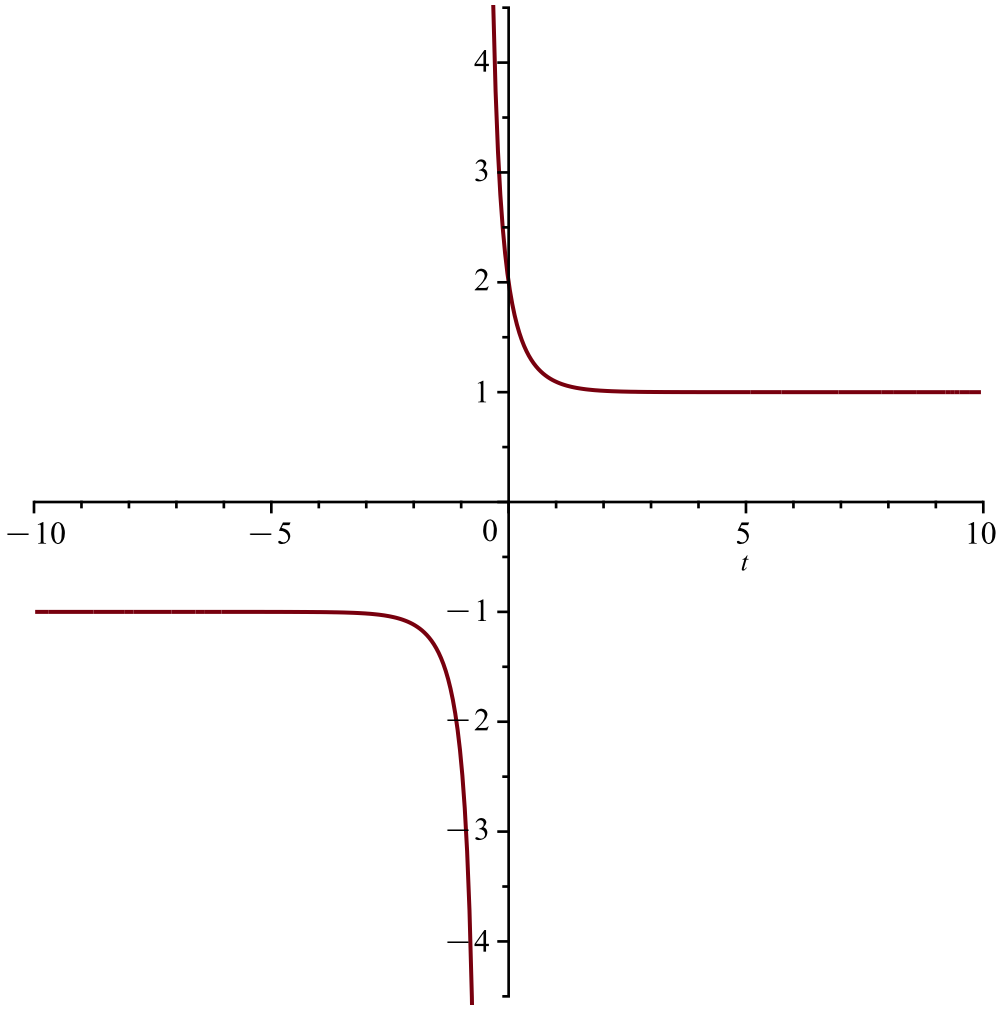
> $plot(p1)$




```
> plot(p2)
```




```
> plot(p3)
```



> $limit(p1, t = -\infty)$

$$-1 \quad \textbf{(37)}$$

> $limit(p2, t = -\infty)$

$$-1 \quad \textbf{(38)}$$

> $limit(p2, t = -\infty)$

$$-1 \quad \textbf{(39)}$$

> $limit(p3, t = \infty)$

$$1 \quad \textbf{(40)}$$

(41)