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*A free, open-source text*

# Open Calculus

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# 1 Introduction

## 1.1 About this text

**Welcome!** Before we start getting into the calculus, I thought it would be best to introduce both myself and this text. My name is Dixon Crews and I'm a student at North Carolina State University in Raleigh, N.C. majoring in Computer Science. I'm also minoring in Music Performance with a concentration in piano. I was first introduced to calculus in my senior year of high school in an AP Calculus BC course. I have found the study and applications of calculus to be utterly fascinating, and I have also seen first hand the need for a free and open-source calculus text to aid those currently enrolled in a college-level math course.

This text is not meant to replace a conventional calculus textbook. It is merely meant to serve as a second resource for students who may be struggling to grasp a difficult concept or want to see something expressed a different way. It should be apparent that I am not an authority on all things mathematical, and therefore this book should not be seen as the be all end all of calculus texts. Instead, we will approach the topics of college-level calculus courses with a reasonable but not exhaustive rigor that anyone may understand. At this time, I am only planning to cover what most colleges call Calculus I and II, mainly consisting of single variable differentiation and integration.

This is an *ongoing, ever-changing* text. I welcome and encourage all suggestions, comments, and questions in the form of an email to `dixon@opencalculus.org`.



## 1.2 What is Calculus?

**Calculus** is a branch of mathematics focused on limits, functions, derivatives, integrals, and infinite series. It has two major branches, differential calculus and integral calculus, which are related to the fundamental theorem of calculus. Calculus has widespread applications in science, economics, and engineering and can solve many problems for which algebra alone is insufficient. [1]

# 2 Limits

## 2.1 Introduction

**Definition:** The limit of $f(x)$ as $x$ approaches $a$ equals $L$

$$\lim_{x \to a} f(x) = L$$

if we can make the values of $f(x)$ as close to $L$ as we like by taking $x$ to be close to $a$ but not equal to $a$. It is important to note that we do not consider the case where $x = a$ when evaluating limits. $f(x)$ may not be defined at $x = a$, so we are just concerned with the behavior of $f(x)$ as it approaches $x = a$.

### 2.1.1 Numerical Limit Evaluation

We can numerically evaluate limits by creating a table of values and letting the independent variable (usually $x$ or $t$) get very close to our $a$ value. Let's look at an example.

**Evaluate** $\lim_{x \to 2} \sqrt{x}$ given

| $x$ | $\sqrt{x}$ |
|---|---|
| 1.98 | 1.4071 |
| 1.99 | 1.4107 |
| 2.00 | ? |
| 2.01 | 1.4177 |
| 2.02 | 1.4213 |

Let's choose 1.414 as our limit. $\sqrt{2}$ to four decimal places is actually 1.4142, so this is quite a good approximation!

[1] `http://en.wikipedia.org/wiki/Calculus`

### 2.2 One-Sided Limits

We can also look at limits from both directions. If we're evaluating a limit and see that we approach a different value from the left than we do when approaching from the right, the limit **does not exist (DNE)**.

A left-sided limit is denoted by changing $a$ to $a^-$: $\lim_{x\to a^-} f(x) = L$

Similarly, a right-sided limit is denoted by changing $a$ to $a^+$: $\lim_{x\to a^+} f(x) = L$

$$\boxed{\lim_{x\to a} f(x) = L \textbf{ iff } \lim_{x\to a^-} f(x) = L \textbf{ and } \lim_{x\to a^+} f(x) = L}$$

### 2.3 Properties of Limits

### 2.4 Continuity

### 2.5 Infinite Limits

## 3 Differentiation

## 4 Integration

## 5 Sequences and Series