Iterated Belief Revision Games

Master Thesis

Cyril Reymond
Supervised by Professor Katsumi Inoue and Professor Samir Aknine

Université Claude Bernard Lyon 1, France
National Institute of Informatics, Japan

Résumé Iterated Belief Revision Games a pour objectif d'étudier l'évolution des croyances d'un groupe d'agents. Basé sur les principes de la révision des croyances et de la révision itérée, les croyances des agents sont représentés par des états épistemiques. Cette représentation des croyances est nécessaire pour conserver des informations importantes lors d'une succession d'étapes de révision. Ce jeu implique des agents échangeant leurs croyances avec leurs voisins et les agents mettent) jour leurs propres croyances à chaque étape du jeu selon des opérateurs de révision itérée. Dans ce papier est donnée une définition générale, ainsi que des résultats prometteurs.

Mots-clés: Revision des croyances, Révision itérée des croyances, Système Résilient, Belief Revision Games, Multi-Agent systèmes

Abstract. Iterated Belief Revision Games aims to study the evolution of beliefs of a group of agents. Based on belief revision and iterated revision principles, beliefs of agents are represented by epistemic states. It is necessary representation of beliefs to carry on important information during a succession of revision steps. This game involve agents exchanging beliefs with their acquaintances and revises their own beliefs at each steps of the game given some iterated revision operators. In this paper is given a general definition along with some promising results.

Keywords: Belief Revision, Iterated Belief Change, Resilient System, Belief Revision Games, Multi-Agent systems

Acknowledgements

I would like to pay my special regards to my supervisors, Professor Samir Aknine and Professor Katsumi Inoue, for providing guidance and feedback throughout this research. They made this internship come true and allowed me to live this wonderful experience.

I wish to express my deepest gratitude to Professor Emmanuel Planas for allowing me to stay in Japan during the virus outbreak and his continuous support.

I would also like to thank Nicolas Schwind for his help in my research and his advices. He was always willing giving me his time.

I wish to show my gratitude to Ms. Aya Gotoda, Ms. Rie Ayuzawa and Ms. Yuko Kataoka for their help to organize this internship and their true kindness.

In addition, I would like to thank my mother for her precious psychological support during those hard times. I finally thank all the people who supported me and help me correcting this thesis.

Table of Contents

Iterated Belief Revision Games ... 1
Cyril Reymond
Supervised by Professor Katsumi Inoue *and Professor Samir* Aknine
1 Introduction ... 4
2 State of the art ... 5
2.1 Logic Preliminaries ... 5
2.2 AGM Framework ... 6
2.3 Belief Revision Games ... 8
2.4 Limitation of Belief Revision ... 9
2.5 Iterated Belief Revision ... 10
2.6 Iterated Revision Operators ... 12
3 Proposed model ... 14
3.1 Iterated Belief Revision Games ... 15
3.2 Logical Properties ... 17
3.3 Motivating Example ... 18
3.4 Simulation Software ... 20
3.5 Results ... 21
4 Further work ... 22
5 Conclusion ... 22

1 Introduction

Multi-agent systems (MAS) are explored in artificial intelligence research for solving complex distributed problems. MAS involve autonomous entities, called agents, that can act cooperatively and/or competitively in an environment to achieve specific tasks. In many applications like games [20], robotics [14] and autonomous cars [13], MAS proved to be more efficient at achieving goals than individuals could do. The strength of MAS reside in the interaction of agents to reason, learn and act in a shared environment. In the past decades, the most standard way for agents to learn how to achieve a goal is with Reinforcement Learning (RL). Multi-agent learning research is directed towards RL as it gives agents a precise idea of which actions they should chose given the current state of the environment. As a reward-driven learning process, RL provides interesting results in closed environment but does not perform well in realistic continuous environment. In addition, we can argue on the independence of agents as they follow the optimal policy provided by RL algorithms and that communication among agents is barely used. In this paper, we focus on the interaction of agents in MAS and more specifically about belief change theory.

Belief change theory is a young research field that combines database researches and philosophy. In past decades, technologies and data have taken am important place in our lives, privately and professionally. The necessity of organising and storing huge amount of data forces computer scientist to use databases. However, back then the addition of new information into databases could cause conflicts with existing information. Artificial intelligence, through belief change theory, provided solutions to address this issue. This new research area showed high potential for MAS to design agents with beliefs. To support the initial works, research in philosophy about the interaction and rationality of human beliefs. Belief change theory provides various models and representation of beliefs to define precisely how new information should be added to a current knowledge base.

Over the years, belief change theory have been deeply studied and a multitude of models emerged. The original goal of this research field is to manage the addition of new data into an existing knowledge base. The major issue is when the new data causes conflicts with the previous information saved. Those conflicts have to be solved somehow. Each models follow the same goal but with different approaches. In the literature, three models seems the most promising: nonmonotonic logic [17], probabilistic reasoning [7,21,22] and belief revision from AGM framework [1]. Firstly, the nonmonotonic logic [17] introduces a model based on rules called *defaults*, which allow one to resonate following such pattern *"if nothing contradicts X, then assume X"*. Secondly, the model of probabilistic reasoning presents a way of applying some probability functions on a new evidence in order to incorporate it into a database [7,21,22]. Finally, the last model is belief revision from the original AGM framework [1], named after their authors. Belief changes are managed by rules called *postulates* and revision operators. This model uses propositional logic to describe beliefs. The AGM framework is designed to support a one step revision process which does show some limitations. Indeed, such model makes it difficult to follow the evolution of beliefs of agents as the previous iteration are not saved. The revision process can be considered as the creation of new agent.

Iterated belief revision aims at improving the AGM framework by updating the representation of beliefs and the postulates to define how beliefs should evolved given a sequence of formulae. Every previous iteration of revision play a role in the computation of next steps. Iterated change theory helps to precisely define beliefs of agents.

The work we propose in this paper is based on Belief Revision Games [18](BRG) that is a game interested in the dynamic of beliefs in a group of communicating agents. At each step, agents revise their beliefs depending on information they receive from neighbors and following the postulates provided by belief revision theory. Despite the model offers interesting study perspectives, it could still be upgraded in the same way as belief revision. As a consequence, we propose an extension of BRG, called Iterated Belief Revision Games (IBRG), to adapt the model with iterated change. In addition, we investigate some iterated operators that suits the new model and study existing logical properties to compare BRGs and IBRGs.

The rest of the paper is organised as follow. Section 2 is dedicated to the state of the art of belief revision. In Section 3 is detailed the proposed extension of the model along with theoretical aspects supported by an example and a simulation software. Finally, in Section 4 are discussed potential further work based on Iterated Belief Revision Games before the conclusion in Section 5.

2 State of the art

This section recall the fundamentals about belief theory change and iterated revision as well as introducing Belief Revision Games.

Based on belief revision, iterated revision have been designed as an general improvement. Instead of just one step, it is used to characterize the evolution of beliefs after a succession of revision operations. Iterated belief change allows the preservation of more information during the revision which leads to more rationality.

2.1 Logic Preliminaries

Before introducing the AGM framework, let us recall some logic definitions and notations that one can find in the literature.

Belief set Ψ represents the actual facts believed by an agent. Ψ is a finite set of propositional sentences build from a propositional language $\mathcal{L}$, and P is the set of propositional variables. μ is a sentence such as $\mu \in \mathcal{L}$. For example, the belief set $\Psi = \{a, a \Rightarrow b\}$ corresponds to the belief $a \wedge b$.

An interpretation is a total function from P to $\{0,1\}$. Let W be the set of all interpretations. The models of a sentence are defined by the set $Mod(\mu) = \{w \in W \mid w \models \mu\}$. Given $\mu = a$, the models of μ are $Mod(\mu) = \{10, 11\}$, as μ give no information on the proposition b.

A revision operation is managed by an operator $\circ$. $\Psi \circ \mu$ denotes the resulting belief set from the revision of the belief set Ψ by the sentence μ.

2.2 AGM Framework

This model describes properties and rules which belief revision operator have to respect in order to assure minimal belief changes.

First of all, there exist different kind of operations which can be executed on belief sets. In this framework, only three are considered.

Contraction of a belief base Ψ by a sentence μ results in a new subset of Ψ which does not imply μ and without deleting too many information.

Expansion consists in the addition of a new sentence which does not contradict any of the sentences of Ψ. It is the most basic and straightforward operation which guarantees success.

Revision of Ψ is kind of a combination of both previous operations. If the incorporation of new sentence within the belief base. If it creates inconsistency, it is possible to remove older beliefs involved in the conflict.

Contraction and *revision* have to comply with postulates given by the framework to assure the efficiency of the belief change. As the AGM framework is a more general model, it did not suit propositional logic right away. The postulate below are the modified version from [10]:

— **(R1)** $\Psi \circ \mu \models \mu$
— **(R2)** if $\Psi \wedge \mu \not\models \bot$, then $\Psi \circ \mu \equiv \Psi \wedge \mu$
— **(R3)** if $\mu \not\models \bot$, then $\Psi \circ \mu \not\models \bot$
— **(R4)** if $\Psi_1 \equiv \Psi_2$ and $\mu_1 \equiv \mu_2$, then $\Psi_1 \circ \mu_1 \equiv \Psi_2 \circ \mu_2$
— **(R5)** $(\Psi \circ \mu) \wedge \Phi \models \Psi \circ (\mu \wedge \Phi)$
— **(R6)** if $(\Psi \circ \mu) \wedge \Phi \not\models \bot$, then $\Psi \circ (\mu \wedge \Phi) \models (\Psi \circ \mu) \wedge \Phi$

In this model, incoming information are considered more reliable than current beliefs which means that the new belief set must contain this evidence **(R1)**, it shows the success of the revision operation. In absence of conflicts, new evidence is simply added to the belief set like does the *expansion* operation **(R2)**. Achieving a revision operation on belief set should not create inconsistency within the set **(R3)**, otherwise the system is unstable. The syntax of the belief set and the evidence does not impact the result of the revision **(R4)**. **(R5)** and **(R6)** purpose is different as it concerns the minimal changes in the belief set after the revision. The minimal change of beliefs is characterized by some distance between the set of models of the belief set and an interpretation w. That is to say, the models of the revised belief set have to be the closest possible to the previous set of models. It does introduce the notion of ordering between interpretations.

Total Preorder A way of determining distance between set of models and an interpretation is to define an ordering between interpretations. Let $\leq$ be the preorder which is a binary relation, reflexive and transitive. Let $w_1, w_2 \in W$ be two interpretations, then the ordering $<$ is defined as $w_1 \leq w_2$ if and only if $w_1 \leq w_2$ and $w_2 \nleq w_1$. A preorder is total if $w_1 \leq w_2$ or $w_2 \leq w_1$ for all $w_1, w_2 \in W$. The notion preorder in belief revision gives more information about how much a belief is believed by an agent compared to others. By example, the expression $w_1 \leq w_2$ means that the sentence w_1 is more or equivalently believed than w_2. The ordering between interpretations allow one to characterize belief revision operators more precisely.

Belief Revision Operator The AGM postulates relate the general principles a revision must follow. Those rules are expressed through belief revision operator, noted $\circ$, if there exist a faithful assignment. Katsuno and Mendelzon [10] justified a similarity between the AGM postulates and a revision operation using total preorders which they called a faithful assignment:

Definition 1. *(faithful assignment) A function $\Psi \mapsto \leq_\Psi$ that maps each sentence Ψ in $\mathcal{L}$ to a total preorder over worlds $\leq_\psi$ is a faithful assignment for every $w_1, w_2 \in W$ if and only if:*
- *$w_1 \models \Psi$ and $w_2 \models \Psi$, then $w_1 =_\Psi w_2$, where $w_1 =_\Psi w_2$ is defined as $w_1 \leq_\Psi w_2$ and $w_2 \leq_\Psi w_1$;*
- *$w_1 \models \Psi$ and $w_2 \not\models \Psi$, then $w_1 <_\Psi w_2$.*

They also provided a representation for a revision operator as a faithful assignment considering the minimum worlds of the total preorder of Ψ:

Theorem 1. *(Katsuno and Mendelzon [10]) A revision operator $\circ$ satisfies postulates **(R1)-(R6)** if and only if there exists a faithful assignment that maps each belief set Ψ to a total preorder $\leq_\Psi$ such that $Mod(\Psi \circ \mu) = Min(Mod(\mu), \leq_\Psi)$.*

The above formula $Min(Mod(\mu), \leq_\Psi)$ represents the minimum worlds from the set $Mod(\mu)$ given the preorder $\leq_\Psi$. That is to say, the minimum worlds of $Mod(\mu)$ becomes the new beliefs believed in the revised set $\Psi \circ \mu$.

The previous definition and theorem show that only the minimum worlds, i.e, the first level, of the ordering are considered upon the revision process and constitute the actual beliefs of the sentence. The rest of the preorder is called *conditional beliefs*.

Conditional Beliefs The notion of total preorder introduces a separation between the objective beliefs of an agent, the facts that she is committed to believe, and the conditional beliefs which are the facts she might believe if a certain revision operation occurs. Given $\Psi, \Phi \in \mathcal{L}$ two sentences, the conditional belief $\Psi >> \Phi$ holds when an agent is committed to believe Φ whenever her belief set is revised by Ψ [3,6]. Even though the conditionals appear to carry valuable information for the revision process, they are not retained after a revision, while only the minimum worlds, and so the actual beliefs, is preserved. The lack of preservation of conditional beliefs is actually one of the limitations of the present model, and a distinction between belief set and belief state will be necessary to improve the model.

2.3 Belief Revision Games

Most of the researches made on belief revision are theoretical. However, to study the exactitude and performance of the AGM based revision, a framework which can simulate environment with agents could come handily. This is the aim of Belief Revision Games [18]. The framework summarize researches on belief revision into a game without human players. In this section, we recall BRG objectives and definitions.

Principles The framework considers a group of agents interacting with each other. An instance of a BRG can be seen as a graph where vertices represent agents and edges show connections between them. Each agent possesses a belief state representing her actual beliefs. At every step of the game, the agents exchange their beliefs with their acquaintances. By doing so, they also revise their beliefs according to their revision policy and the beliefs of their neighbors. The revision policies determine the way beliefs have to be merged. The game is finished when a specific behavior is reached.

Formalism

Definition 2. *(Belief Revision Game)[18] A Belief Revision Game (BRG) is a 5-tuple* $G = (V, A, \mathcal{L}_p, B, R)$ *where*
- $V = \{1, ..., n\}$ *is a finite set;*
- $A \subseteq V \times V$ *is an irreflexive binary relation on V;*
- $\mathcal{L}_p$ *is a finite propositional language;*
- B *is a mapping from* V *to* $\mathcal{L}_p$*;*
- $\mathcal{R} = \{R_1, ..., R_n\}$*, where each* R_i *is a mapping from* $\mathcal{L}_p \times \mathcal{L}_p^{in(i)}$ *to* $\mathcal{L}_p$ *with* $in(i) = \mid \{j \mid (j, i) \in A\} \mid$ *the in-degree of* i*.*

In the following, an in-depth explanation of each variables is provided.

The set V is the set of agents participating in the game G.

The set A holds the acquaintances between the agents. Given two agents $i, j \in V$, if $(i, j) \in A$, then agent i communicates her beliefs to agent j at every step of the game. If a graph is generated from an BRG instance, the set of acquaintances is represented by the oriented edges.

The variable B is a infinite set representing each belief of agents expressed by a formula from $\mathcal{L}_p$. For $\forall i \in V$, the formula B_i is called a *belief state* of agent i. Before the game starts, B_i represents the initial beliefs of agent i.

Before going further, another notion has to be introduced. As a consequence of the set of acquaintances, each agent may receive new information from multiple sources. This idea is captured by the concept of *context*, noted C_i for the *context* of agent i. It designates all the incoming beliefs. The *context* of i is defined as the sequence $C_i = B_{i_1}, ..., B_{i_{in(1)}}$ where $i_1 < ... < i_{in(i)}$ and $\{i_1, ..., i_{in(i)}\} = \{i_j \mid (i_j, i) \in A\}$. Then, each $R_i \in \mathcal{R}$ is called the *revision policy* of i. It corresponds to the revision of the belief set of i by her current context which produces a new belief state characterized by the formula $R_i(B_i, C_i)$.

Revision Process In the BRG framework, the revision process is more detailed. It is a combination of belief merging and revision policies. A merging operator $\Delta^{d,f}$ is used to combine a formula μ and a vector of sentences $\mathcal{K}$ into a belief state $\Delta_{\mu}^{d,f}(\mathcal{K})$. The formula μ is acting an *integrity constraint* orienting the direction of the revision. The variable d represents the distance used between interpretations. The variable f is the aggregation function used by merging operators. The revision of each agent belief state is directed by a set of six revision policies using merging operators. Each of those policies gives agents a different view on the opinion of their neighbors. It allows one to study distinct behaviors of beliefs exchange between agents. See [18] for more precision.

Game Behavior and Properties As the goal is to study the evolution of belief of agents, the game shows some interesting behaviors and properties. There are two main behaviors that can occurs during the game.

Firstly, at some point when no more agents change their beliefs after receiving information from their neighbors, the game stabilize. The game is in a state of *convergence*. Once this state is reached, the game is finished. On the point of view of agents, *convergence* expresses that agents agreed on facts that should be believed true and so will not change any further their opinion.

Secondly, there is a notion of *belief cycle* which concerns more each agent individually than the general state of the game. At every step of the game, each agent revises her beliefs. From the variable B, one can track the evolution of each agent beliefs. If an agent i is said to be in *belief cycle* state, it does mean that she enters a loop where her next belief states have already been assumed previously. When it happens, the agents can not agree on the same opinion and no more progress can be made. *Convergence* and *belief cycle* are quite similar, as in fact *convergence* is obtained when each agent have a *belief cycle* of size 1.

Lastly, there are more logical properties which can emerge from a game. The authors conducted several comparison on each each of those properties, such as *convergence*, *monotonicity* and *responsiveness* [18], on which integrity constraint they respect given revision policies used. The results presented in the paper shows that logical properties may not be obtained in general cases. Revision policies appear to play a decisive role in beliefs evolution and emergence of logical properties from a BRG instance.

2.4 Limitation of Belief Revision

Belief revision throughout AGM framework give direct solutions on how to manage the addition of one information into a database, especially when it causes inconsistency within the base. The revision process considers only one step revision. It does not offer a response to Iterated Revision which concerned with a succession of belief revisions. In fact, AGM framework does not specify any constraints on the preservation of conditional beliefs [3,4,8,15]. In order for an agent to face any kind of situation, it could be necessary to carry over previous information on beliefs.

2.5 Iterated Belief Revision

The revision as describe in AGM framework is not sufficient to preserve conditional beliefs after a succession of revision steps. Darwiche and Pearl [4] related this issue about the original postulates and proposed a refinement of them as well as introduced new postulates. Before detailing each of those postulates, it is necessary to make the distinction between belief set and belief state, also called *epistemic state*.

Epistemic State A belief set represents the actual belief of an agent. It is insufficient on its own to perform iterated revision. The iteration process requires keeping track of the way revision policies are modified after each steps. This information is captured by conditional beliefs and so it is important to determine which conditional beliefs have to be preserved. An epistemic state handles more information than a simple belief set. From now on, the letter Ψ will denote an epistemic state. Ψ is composed of two elements. On one hand, it contains a belief set which represents objective belief of an agent, noted $Bel(\Psi)$. On the other hand, it supports the set of conditional beliefs, beliefs that an agent is ready to adopt in a further revision.

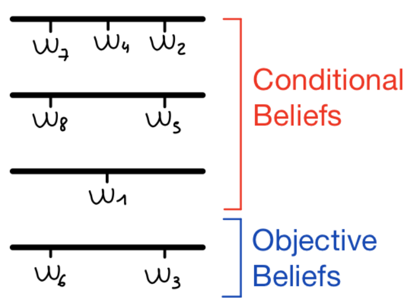

Fig. 1: Representation of an epistemic state as an ordering between worlds.

Figure 1 shows a way to represent the ordering $\leq_\Psi$, i.e, the total preorder over worlds of epistemic state Ψ. It can be compared to stairs: the first stair corresponds to the belief set, $Bel(\Psi)$, of the agent. In this case, $Bel(\Psi) = \{w_6, w_3\}$. The other stairs represent the relative entrenchment, and so the set of conditional beliefs. The ordering gives more importance of worlds close to the first level. It means that the world w_1 is more believed plausible than world w_7 for an agent with this epistemic state. The relation between those two worlds is $w_1 <_{\Psi} w_7$.

Iterated Revision Postulates The original AGM postulates are defined for belief sets but not epistemic states. An improvement is required, especially to minimize modification of conditional beliefs [4]:

— **(R*1)** $Bel(\Psi \circ \mu) \models \mu$
— **(R*2)** if $Bel(\Psi) \wedge \mu \not\models \perp$, then $Bel(\Psi \circ \mu) \equiv Bel(\Psi) \wedge \mu$
— **(R*3)** if $\mu \not\models \perp$, then $Bel(\Psi \circ \mu) \not\models \perp$
— **(R*4)** if $\Psi_1 = \Psi_2$ and $\mu_1 \equiv \mu_2$, then $Bel(\Psi_1 \circ \mu_1) \equiv Bel(\Psi_2 \circ \mu_2)$
— **(R*5)** $Bel(\Psi \circ \mu_1) \wedge \mu_2 \models Bel(\Psi \circ (\mu_1 \wedge \mu_2))$
— **(R*6)** if $Bel(\Psi \circ \mu_1) \wedge \mu_2 \not\models \perp$, then $Bel(\Psi \circ (\mu_1 \wedge \mu_2)) \models Bel(\Psi \circ \mu_1) \wedge \mu_2$

The above postulates are very similar to the ones in the original model proposed in [1]. In fact, most of the changes concern usage of epistemic state instead of only belief set. However, few comments can still be made. First of all, to capture the needs of iteration revision, belief change is conducted on epistemic state level, and not at belief set level. The expression $\Psi \circ \mu$ represents the revision operation of the epistemic state Ψ by the sentence μ. The new sentence is still considered more reliable than the current beliefs even when the conditional beliefs are preserved **(R*1)**. The most notable improvement concerns **(R*4)**. The revision by μ of two epistemic states produces the same result when the two epistemic states are strictly equal, meaning that their corresponding ordering are equals. In **(R4)**, the belief sets only had to be equivalent, as different set of worlds can result in the same belief. This new restriction is necessary as the revision operation occurs at epistemic state level.

In addition to the modification of original postulates, Darwiche and Pearl introduced four new postulates **(CR1) - (CR4)** to assure that the revision operation preserves as much as possible conditional beliefs [4]:

— **(CR1)** if $w_1 \models \mu$ and $w_2 \models \mu$, then $w_1 \leq_{\Psi} w_2$ iff $w_1 \leq_{\Psi \circ \mu} w_2$
— **(CR2)** if $w_1 \models \neg\mu$ and $w_2 \models \neg\mu$, then $w_1 \leq_{\Psi} w_2$ iff $w_1 \leq_{\Psi \circ \mu} w_2$
— **(CR3)** if $w_1 \models \mu$ and $w_2 \models \neg\mu$, then $w_1 <_{\Psi} w_2$ only if $w_1 <_{\Psi \circ \mu} w_2$
— **(CR4)** if $w_1 \models \mu$ and $w_2 \models \neg\mu$, then $w_1 \leq_{\Psi} w_2$ only if $w_1 \leq_{\Psi \circ \mu} w_2$

The below example gives a representation of a revision directed by **(R*1)-(R*6)** and **(CR1)-(CR4)** postulates:

In figure 2, the total preorder $\leq_{\Psi}$ is composed by eight worlds. Initially, the belief set of Ψ is $Bel(\Psi) = \{w_2, w_6\}$. The epistemic state is revised by the sentence $\mu = \{w_1, w_4, w_8\}$, resulting in $\leq_{\Psi \circ \mu}$. Given the representation as total preorders, during the revision process the minimum worlds of μ become the new belief set, i.e. $Bel(\Psi \circ \mu) = \{w_1, w_4\}$. As required by iteration revision, the ordering of conditional beliefs must be preserved as much as possible. Some changes

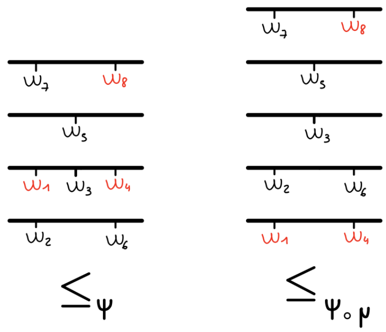

Fig. 2: Revision of epistemic state Ψ by the evidence μ with $Mod(\mu) = \{w_1, w_4, w_8\}$.

can be made to reach this goal. In the above example, worlds $w_2, w_6, w_3 \models \neg\mu$ as they are not part of μ worlds. Then, postulate **(CR2)** forces the addition of another level of ordering for the world w_3 to preserve their relative entrenchment. Furthermore, although $w_8 \models \mu$, its position within the ordering does not change and so it is assumed that some part of μ are not important whereas the evidence is supposed to be more reliable than old beliefs. Another postulate is required to give more importance to all μ worlds in the total preorder considered [2,9]:

(PR) if $w_1 \models \mu$ and $w_2 \not\models \mu$, then $w_1 \leq_\Psi w_2$ only if $w_1 <_{\Psi \circ \mu} w_2$

Applying **(PR)** to example in figure 2 results in an increase of plausibility of w_8 over w_7 and so another stair is created as follow $w_8 <_{\Psi \circ \mu} w_7$.

2.6 Iterated Revision Operators

In addition to the postulates, there are plenty of iterated operators which bring specific behaviors to the revision process. In this section, four operators from the literature are presented [2,3,12,16]. Those operators are widely used and studied [19]. Revision operator is represented by an additional postulate serving as another constraint on the preservation of conditional beliefs and the reliability of the new evidence within the epistemic state, whereas they might not satisfy original constraints given by **(CR1)-(CR4)** and **(PR)**.

Lexicographic Revision Nayak's operator, represented by $\circ_N$, might be the most straightforward one as all worlds of new evidence μ are considered more believable than any other beliefs from the epistemic state. It is expressed by postulate **(R)** [16], for all $w_1, w_2 \in W$:

— **(R)** if $w_1 \models \mu$ and $w_2 \not\models \mu$, then $w_1 <_{\Psi \circ_N \mu} w_2$

Figure 3 captures the lexicographic operator behavior which it similar to postulate **(PR)** but stricter. The worlds of μ are given more relative importance

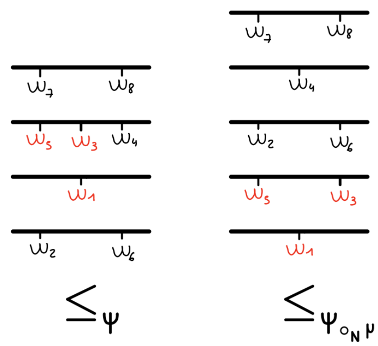

Fig. 3: Lexicographic Revision of Ψ by μ with $Mod(\mu) = \{w_1, w_5, w_3\}$.

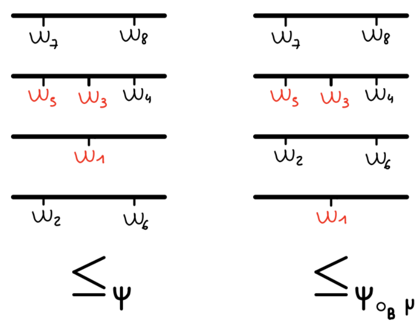

Fig. 4: Natural Revision of Ψ by μ with $Mod(\mu) = \{w_1, w_5, w_3\}$.

within $\leq_{\Psi \circ_N \mu}$. As a consequence, worlds w_5 and w_3 are more likely to be believed true.

Natural Revision Boutilier's operator, represented by $\circ_B$, aims at conserving as much as possible the ordering intact. On that matters, natural revision focuses on highlighting the minimum worlds of μ and keeping conditional beliefs unchanged [3], for all $w_1, w_2 \in W$:

— **(CBR)** if $w_1, w_2 \not\models Bel(\Psi \circ_B \mu)$, then $w_1 \leq_\Psi w_2$ iff $w_1 \leq_{\Psi \circ_B \mu} w_2$

As represented in figure 4, any worlds which are not contained in the new belief set within $\leq_{\Psi \circ_B \mu}$ keep their relative ordering. In fact, natural operator contradicts postulate **(PR)**. In the above example, $w_5, w_3 \models \mu$ and $w_4 \not\models \mu$ but there is no modification of plausibility between those worlds. It does indeed preserves conditional beliefs ordering by introducing few changes.

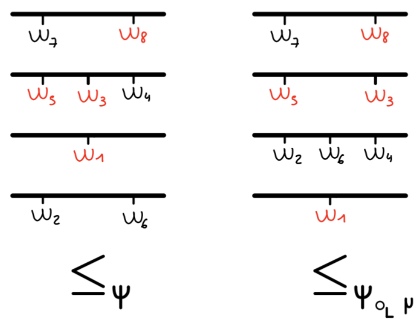

Fig. 5: Ranked Revision of Ψ by μ with $Mod(\mu) = \{w_1, w_5, w_3, w_8\}$.

Ranked Revision Lehmann's operator [12], represented by $\circ_L$, tends to increase the plausibility of worlds which are not part of $Mod(\mu)$. In some way, those worlds will be regrouped on the minimum level possible after the objective beliefs if there exists some worlds of μ above them. Postulates **(L1)-(L2)** have been rephrased to correspond with total preorder over worlds [19], for all $w_1, w_2, w_3 \in W$:

— **(L1)** if $w_1, w_2 \not\models \mu$ and $\forall w_3 \models \mu$, $w_1 \leq_\Psi w_3$, then $w_1 \leq_{\Psi \circ_L \mu} w_2$

— **(L2)** if $\exists w_3 \models \mu$ s.t. $w_3 <_\Psi w_2$, then $w_1 \leq_\Psi w_2$ iff $w_1 \leq_{\Psi \circ_L \mu} w_2$

Figure 5 presents the particular behavior of ranked operator. This operator gives more weight to old beliefs compared to ones carried by μ as long as they are not contained in $Bel(\Psi \circ_L \mu)$. In the example, rule **(L1)** implies that w_4 becomes more reliable and reaches the same level as w_2 and w_6.

Restrained Revision Booth and Meyer's operator, represented by $\circ_{BM}$, is a combination of increasing plausibility of μ worlds and keeping the previous ordering intact as possible. An agent using such operator will not give up her beliefs easily while still considering the new evidence as more reliable in most cases. Restrained operator is defined as follow, for all $w_1, w_2 \in W$ [2]:

— **(DR)** if $w_1 \models \mu$, $w_1 \not\models Bel(\Psi \circ_{BM} \mu)$ and $w_2 \not\models \mu$, then $w_2 <_\Psi w_1$ only if $w_2 <_{\Psi \circ_{BM} \mu} w_1$

In figure 6, revision process managed by the restrained operator appears to be at the intersection of natural and lexicographic revisions. On one hand, most of the ordering is preserved, respecting the minimal change of conditional beliefs principle. On the other hand, worlds w_5 and w_3 become more trustworthy than w_4.

3 Proposed model

Belief revision have been widely studied and improved over the years. Coming from AGM framework to iterated revision, it is in constant evolution. It is natural

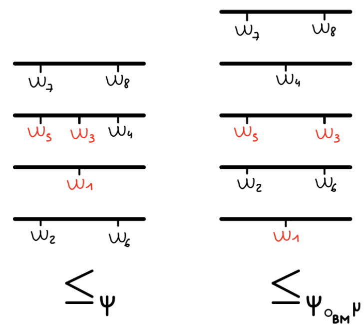

Fig. 6: Restrained Revision of Ψ by μ with $Mod(\mu) = \{w_1, w_5, w_3\}$.

that artificial intelligence systems implementing such principles have to evolve following the same tendency. In this section we propose the general definition of Iterated Belief Revision Games along with comparison of differences between the two models. Throughout an example, one can understand the motivation behind this necessary upgrade to avoid undesired behavior from the original model. Supporting this work, a software have been developed to simulate instances of Iterated Belief Revision Games and details are given about algorithms used. This section will conclude by general results of the new model.

3.1 Iterated Belief Revision Games

IBRGs is based on iterated change theory and so represent beliefs of agents as epistemic state. For this work, we assume that any epistemic state E is associated a total preorder over interpretations $\leq_E$ that is used for iterated revision operators. The definition is similar as the original framework but also include the updated belief representation.

Definition 3. ***(Iterated Belief Revision Game)*** *An Iterated Belief Revision Game is a 5-tuple $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ where*

- *$V = 1, \ldots, n$ is a finite set;*
- *$A \subseteq V \times V$ is an irreflexive binary relation on V;*
- *$\mathcal{L}_p$ is a finite propositional language;*
- *E is a mapping from V to a total preorder over interpretations;*
- *$\mathcal{R} = \{R_1, \ldots, R_n\}$, where each R_i is a mapping from $E \times \mathcal{L}_p^{in(i)}$ to E.*

As one can see, definition 2 and 3 are almost identical. The variable E does now represent each epistemic state of agents from V. We assume that for each agent $i \in V$, to every E_i is associated a total preorder over interpretations $\leq_i$. This modification also imply to define new revision policies as stated by variable R. Then, the definition of belief sequence is updated accordingly :

Definition 4. ***(Epistemic State Sequence)*** *Given a IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ and an agent $i \in V$, the epistemic state sequence of i, denoted $(E_i^s)_{s\in\mathbb{N}}$, states how the beliefs of agent i evolve during the game. $(E_i^s)_{s\in\mathbb{N}}$ is inductively defined, for every $E_i \in (E_i^s)$ represented as $\leq_{E_i}$, as follows:*

- *— $E_i^0 = E_i$*
- *— $E_i^{s+1} = R(E_i^s, C_i^s)$ for every $s \in \mathbb{N}$, where C_i^s is the context of i at step s.*

E_i^{s+1} denotes the epistemic state of agent i after s moves. The characteristic of belief cycle can be retained for epistemic state sequence as the number of interpretations is finite and so only many epistemic states can be reached.

Definition 5. ***(Iterated Belief Cycle)*** *A sequence $(K^s)_{s\in\mathbb{N}}$ of epistemic states, where each belief set $B(K^s) \in \mathcal{L}_p$, is cyclic if there exists a finite subsequence $K^b, \dots, K^e$ such that every $j > e$, we have $K^j = K^{b+((j-b)mod(e-b+1))}$. In this case, the iterated belief cycle of $(K^s)_{s\in\mathbb{N}}$ is defined by the subsequence $K^b, \dots, K^e$ for which b and e are minimal.*

In addition of considering epistemic states over only belief sets, definition 5 requires the strict equality of two epistemic states, i.e, $\leq_{E_1}=\leq_{E_2}$, in order to constraint effectively the equality on objective and conditional beliefs. Given the representation of epistemic states as total preorders over interpretations, we can prove that:

Proposition 1. *For every IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ and every agent $i \in V$, the epistemic state sequence of i is cyclic.*

We then denote $Cyc(E_i)$ the iterated belief cycle associated to each agent i.

We define a new set of revision policies R based on iterated change theory, called **DM-AGM revision policies**. Before introducing the new revision policies, we have to address an issue about the revision of the epistemic state of an agent while she has more than one acquaintances. Indeed, iterated revision involve an epistemic state and one formula. As in BRG, we use a merging operator Δ along with the revision policy $R^2_\Delta(B_i^s, C_i^s) = \Delta_{\Delta(\langle C_i^s\rangle)}(\langle B_i^s\rangle)$, where the context of i is replaced by the tautology, as it merges beliefs of neighbors into a single formula which is as close as possible to all those beliefs.

Definition 6. ***(DP-AGM Revision Policies)***

- *— $R^1_{\circ,\Delta}(E_i^s, C_i^s) = E_i^s \circ_N \Delta_\top(\langle C_i^s\rangle)$;*
- *— $R^2_{\circ,\Delta}(E_i^s, C_i^s) = E_i^s \circ_B \Delta_\top(\langle C_i^s\rangle)$;*
- *— $R^3_{\circ,\Delta}(E_i^s, C_i^s) = E_i^s \circ_L \Delta_\top(\langle C_i^s\rangle)$;*
- *— $R^4_{\circ,\Delta}(E_i^s, C_i^s) = E_i^s \circ_{BM} \Delta_\top(\langle C_i^s\rangle)$.*

The particular behavior of each of these policies depends on the DP-AGM operator involved. For $R^1_{\circ,\Delta}$, the aggregated opinion of the neighbors is considered more reliable than any of the old beliefs of an agent. For $R^2_{\circ,\Delta}$, the agent is ready to adopt the aggregated objective beliefs of her neighbors. For $R^3_{\circ,\Delta}$, the agent adopt the aggregated opinion of her neighbors while giving more weight to her own beliefs within the conditional beliefs. For $R^4_{\circ,\Delta}$, an agent considers equally reliable the aggregated beliefs of her neighbors and her own beliefs.

3.2 Logical Properties

We study now some logical properties of BRG that can be adapted for IBRG. We focus on IBRGs which are instantiated with one of the four revision policies previously defined, while all agents sharing the same revision policy. Given a revision policy $R^k_{\circ,\Delta}$, $\mathcal{G}(R^k_{\circ,\Delta})$ is the set of all IBRGs $(V, A, \mathcal{L}_p, E, \mathcal{R})$ where for each $R_i \in \mathcal{R}, R_i = R^k_{\circ,\Delta}$. For any of the below logical property, We assume that the merging operator Δ satisfies one or more *integrity constraints* [11] and the disjunction postulate **(Disj)** [5] when it is required as detailed in [18]. Proof of proposition are available in appendix.

First of all, we have updated preservation properties that assure the resiliency of epistemic states after interaction between agents.

Definition 7. ***(Consistency Preservation (CP))*** *A IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ satisfies **(CP)** if for each $Bel(E_i) \in E$, if $Bel(E_i)$ is consistent then all $Bel(E^s_i)$ from $(E^s_i)_{s\in\mathcal{N}}$ are consistent.*

(CP) expect that initial objective beliefs of agents remain consistent and not conflicting in the epistemic state sequence.

Proposition 2. *For every $k \in \{1, \ldots, 4\}$, $R^k_{\circ,\Delta}$ satisfies **(CP)**.*

Definition 8. ***(Agreement preservation)*** *A IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ satisfies **(AP)** if given any consistent formula $\varphi \in \mathcal{L}_p$, if for each $Bel(E_i) \in E$, $\varphi \models Bel(E_i)$, then for each $Bel(E_i) \in E$ and at every step $s \geq 0$, $\varphi \models Bel(E_i)$.*

(AP) states that if all agents share the same opinion, i.e, equivalent objective beliefs, then their beliefs will not evolve in opposition of their original opinion.

Proposition 3. *For every $k \in \{1, \ldots, 4\}$, $R^k_{\circ,\Delta}$ satisfies **(AP)**.*

Definition 9. ***(Unanimity Preservation (UP))*** *A IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ satisfies **(UP)** if given any formula $\varphi \in \mathcal{L}_p$, if for each $E_i \in E$, $Bel(E_i) \models \varphi$, then for each $E_i \in E$ and at every step $s \geq 0$, $Bel(E^s_i) \models \varphi$.*

(UP) is an interesting property that allow us the effectively study the evolution of beliefs by choosing a formula that is a logical consequence of initial objective beliefs of agents and will be logical consequence of every beliefs in their epistemic state sequence. The formula is then unanimously accepted by agents involve in the IBRG.

Proposition 4. *For every $k \in \{1, \ldots, 4\}$, $R^k_{\circ,\Delta}$ satisfies **(UP)**.*

The next property concerns the stability of beliefs of each agents and so the stability of the game. As expected, we cannot conclude in the convergence of IBRGs in the general case.

Definition 10. *(Convergence (Conv)) A IBRG satisfies **(Conv)** if it is stable.*

Proposition 5. *For every $k \in \{1, \ldots, 4\}$, $R_{\circ,\Delta}$ does not satisfies **(Conv)**.*

Directed acyclic graph (DAG) assure that the IBRG graph does not contain any cycle implying that the **(Conv)** property is obtained in any cases.

Proposition 6. *For $k \in \{1, \ldots, 4\}$, all directed acyclic IBRGs from $\mathcal{G}(R^k_{\circ,\Delta})$ satisfy **(Conv)**.*

Before closing this section, we further investigate the notion of global cycle to conclude on the equivalency of both models. A global cycle in BRGs and IBRGs is a sequence of vector of each agent beliefs.

Definition 11. *(Global belief sequence) Given a BRG $G = (V, A, \mathcal{L}_p, B, \mathcal{R})$, the global belief sequence of G, denoted $(T^s_G)_{s\in\mathbb{N}}$, states how the beliefs of every agent $i \in V$ evolve during the game. $(T^s_G)_{s\in\mathbb{N}}$ is inductively defined as follows:*
— $T^0_G = \left\langle B^0_1, \ldots, B^0_n \right\rangle$*, where n is the number of agents from V;*
— $T^{s+1}_G = \left\langle B^{s+1}_1, \ldots, B^{s+1}_n \right\rangle$

Global belief sequence definition can be right away applied to IBRGs by considering only the objective beliefs of the epistemic states.

Definition 12. *(Global belief cycle) A sequence $(T^s_G)_{s\in\mathbb{N}}$ of vector of beliefs is cyclic if there exists a finite subsequence $T^b_G, \ldots, T^e_G$ such that every $j > e$, we have $T^j_G = T^{b+((j-b) mod (e-b+1))}_G$. In this case, the global belief cycle of $(T^s_G)_{s\in\mathbb{N}}$ is defined by the subsequence $T^b_G, \ldots, T^e_G$ for which b and e are minimal.*

Proposition 7. *For every IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$, if $T^s_G = \left\langle Bel(E^s_1), \ldots, Bel(E^s_n) \right\rangle$ is cyclic, then $T'^s_G = \left\langle E^s_1, \ldots, E^s_n \right\rangle$ is also cyclic.*

There are no direct connections between belief cycle and epistemic state cycle due to the different belief representation. However, from Proposition 7 we conclude that for a similar setting an instance of IBRG is a generalization of a BRG. The extension proposed in this paper do not impact performances of the model and opens the way for further studies of beliefs dynamics in a network of agents.

3.3 Motivating Example

The necessity of the extension of the model presented in this paper can be supported by a motivating example showing the differences between belief revision and iterated revision. Classical belief revision presents in some cases limits that iterated change theory aims at resolving.

Context Two residents of Tokyo, Akane and Yumi, are trying to figure out what is happening during the virus outbreak. Akane believes that *airports are closed*, while Yumi thinks that if the city is in *lockdown*, then *airports are closed*. Later on, they both learn from the news the city is indeed in *lockdown* state. At that point, they agree on the fast that the city is in *lockdown* and *airports are closed*. Finally, the government declare that *airports are open*. We then ask each resident: do you think the city is in *lockdown* ?

Propositional Logic In this example, beliefs are build from a propositional language $\mathcal{L}$ based on the set of propositional variables $P = \{s, t\}$ where s stands for *lockdown* and f for *airports are closed*.

Belief Revision In the case of classical belief revision, this example can be described as two agents using the same revision policies and having as initial belief sets $\Psi^0_{Akane} = \{t\}$ and $\Psi^0_{Yumi} = \{s \Rightarrow t\}$. Conditional beliefs are not represented as they do not play a role in the revision process. The table 1 presents the evolution of their belief sets given the step i and the evidence μ.

Step i	μ	Ψ^i_{Akane}	Ψ^i_{Yumi}
0		t	$s \Rightarrow t$
1	s	$s \wedge t$	$s \wedge t$
2	$\neg t$	$s \wedge \neg t$	$s \wedge \neg t$

Table 1: Evolution of agent's beliefs using classical belief revision.

Both agents shares the same opinion, $\Psi^1_{Akane} = \Psi^1_{Yumi} = \{s \wedge t\}$, after the first revision and so their beliefs will evolve in the same way from that moment. It is expected that they continue to agree on the same beliefs and both believe that the city is in *lockdown* and the *airports are open*. The result seems natural, given the context, but may be incorrect if one considers their previous beliefs.

Iterated Revision In the context of iterated revision, it is necessary to make use of conditional beliefs. In order to compare effectively the two models, iterated revision uses natural revision operator [3] as it conducts minimum changes of conditional beliefs. Also, Ψ_i does now represents the epistemic state of agent i and $Bel(\Psi_i)$ her belief set.

In figure 7, after the first revision step, results are equivalent and both agents share again the same opinion. However, at the end their belief sets diverge. Agent Akane believes that *airports are open*, while agent Yumi believes that the city is *not in lockdown* and *airports are open*, i.e. $Bel(\Psi)^2_{Akane} = \{\neg t\}$ and $Bel(\Psi)^2_{Yumi} = \{\neg s \wedge \neg t\}$. As a consequence, agent Akane cannot answer the question and agent Yumi replies negatively. Despite results seem unrealistic, they reflect the true evolution of the state of mind of both agents.

Analysis This example raises an issue about equivalency of two belief sets. Even though they represent the same beliefs, they may not have been acquired in the same way. In that matter, next step beliefs could be misinterpreted. Taking into consideration conditional beliefs of an agent gives a more natural idea of what truly thinks this agent. In iterated revision every steps matters to determine the future beliefs and epistemic states. This example brings enough attention

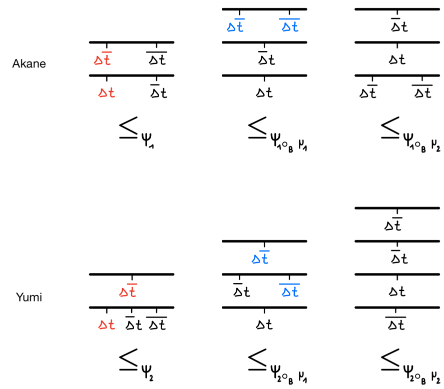

Fig. 7: Evolution of agent's beliefs using iterated revision with natural revision operator and evidences $\mu_1 = \{s\}$ and $\mu_2 = \{\neg t\}$ successively.

towards iterated revision that it motivates the upgrade of Belief Revision Games [18] to Iterated Belief Revision Games.

3.4 Simulation Software

To support this theoretical work, a software have been developed. It allows one to simulate instances of IBRG. In this section, details are given about the implementation, functionalities and algorithms used. The general interface of the application can be seen in Appendix 1.

Code The language used is Java. The simulation software requires to have a clear and easily configurable graphical user interface which this language permits. As mentioned in a previous section, an instance of this game can be represented as a graph where agents are the vertices and acquaintances between them are directed edges. The graph representation of the simulation is managed by the framework Jung.

Software Functionalities From the menu, users can generate a graph with specific layout such as random graph and circular graph. Automatic generated graphs randomize initial beliefs of each agent. Also, it is possible to create manually a graph by adding nodes and edges in the middle of the window. Before running a simulation, belief set of an agent can be modified, and a new epistemic state will be generated accordingly, as well as the iterated operator can be chosen. To simplify matters, each agent uses the same revision operator. More details are available in the help menu.

Iterated Revision Operator The most important part of the code is the revision process and the implementation of revision operators. Each operators can be used with two fusion operator based on distances : Hamming sum and Drastic sum. Each agent exchange only their belief sets and so when one have multiple neighbors, their beliefs must be combined by these fusion operators before merging it into the epistemic state. Every epistemic state are built from a total of sixteen worlds based on a propositional language of four variables. As every iterated operators are different, below is given the algorithm used to implement each of them. Every algorithms works in the same fashion: fusion of neighbors beliefs to produce the evidence μ, find the minimum worlds of μ given the considered epistemic state, then rearrange the total preorder depending on which operator chosen. Epistemic states are represented as total preorders and each worlds are given a position. Worlds from the first level, i.e, from $Bel(\Psi)$, are at the position 0. As an example, algorithm 1 presents the way ranked operated is implemented. Briefly, the main point of this algorithm is to lower the position of worlds within $\leq_\Psi$ as presented in figure 5.

Data: epistemic state e, list of neighbor's beliefs n
Result: updated e
$\mu \leftarrow$ Fusion(n);
allMu $\leftarrow$ FindMuWorlds(μ, e);
minMu, otherMu $\leftarrow$ Separate(μ);
nonMu $\leftarrow$ FindNonMuWorlds(μ, e) ;
minPos $\leftarrow$ LowestPosition(nonMu) ;
foreach $w \in nonMu$ **do**
    if $\exists w_1 \in otherMu$ *such as* $w <_e w_1$ **then**
        SetPosition(w, minPos);
    end
end
tmp $\leftarrow$ Combine(otherMu, nonMu) ;
$e \leftarrow$ UpdatePreOrder(minMu, tmp) ;

Algorithm 1: Ranked revision operator's algorithm.

3.5 Results

First of all, the most important result is about the global belief cycle. A global belief cycle defines the behavior of the whole network considering each beliefs of agents at each steps of the game. Global belief cycle can be considered on two levels: belief sets and total preorders over interpretations. From Proposition 7 we proved that both cycles are interconnected increasing the connection between belief sets and epistemic states. The addition of conditional beliefs does not limits the network performances.

Secondly, logical properties are interesting to express necessary beliefs integrity preservation during the game and global behaviors. All of them are based on the notion of belief cycle. It was so required to investigate them when the representation of beliefs change to epistemic states. All of the logical properties from BRGs can be retrieved in IBRGs when using the merging-based revision policy R^2_Δ to deal with beliefs of neighbors. The fact that both models behave similarly logically wise provides one more argument in favor of iterated change.

Finally, by the above arguments we can state on the efficiency of the model extension proposed in this paper. Through the simulation software we can indeed confirm the theoretical studies on equivalency of both models. However, IBRGs propose more rationality about beliefs of agents and their importance in the revision process. The evolution of belief states of agents is more human like and gives opportunities to study other aspects in network dynamics.

4 Further work

The main idea about Iterated Belief Revision Games is to study the dynamics of beliefs of agents in a network environment but more research area can be concerned with this model. As the base of the game is the interaction between agents. However, communication is basic and do not serve much purposes. In such game, it can be interesting to increase the communication potential of agents by allowing them to exchange other information than beliefs. It can be used for solving team formation challenges. In games, usually the learning process of agents is based on reward-driven algorithms like reinforcement learning but shows limits in realistic complex environment where data are continuous. IBRGs may be the base of further studies about learning mechanisms using communication as agents have rational beliefs allowing them more expressiveness and perhaps giving sense to their actions.

IBRGs can also serve for network reasoning concerning the propagation of beliefs in a social network and study particular behaviors like manipulation. Moreover, the iterated change theory is based on the postulate that the incoming information is more reliable than the current beliefs. It can still remain so to simplify matter but an interesting point would be to give the possibility of agents to reason about the reliability of the new evidence. By doing so, agents can define by themselves an objective to achieve and act accordingly.

5 Conclusion

Multi-agent systems provide interesting tools to model complex realistic environment. It is necessary that agents behaviors are well defined to someday increase their interaction with humans. Designing rational agents with precise beliefs on their environment is the key for their complete autonomy. Also, most of information can be transmitted through communication which make an interesting field of research.

While multiple models exist to represent beliefs dynamics, in this paper we focused on the promising belief revision and iterated revision. The main idea is to revise a belief state with a new evidence while conducting the minimum changes and preserving the consistency of the base. Iterated revision permits to follow effectively the evolution of beliefs thanks to epistemic states. Epistemic states carries out more valuable information at every revision step which increases completeness of beliefs.

We proposed in this paper an extension of Belief Revision Games, called Iterated Belief Revision Games, that opens the way for further interesting researches about belief revision and network reasoning. The new model is roughly similar but the few changes make it more efficient and increase the rationality of agents. The updated logical properties and simulations with the software proved that IBRGs are a generalisation of BRGs and offers the same performances. IBRGs can be used as a foundation for more complete resilient system.

References

1. Alchourrón, C., Gärdenfors, P., Makinson, D.: On the logic of theory change: Partial meet contraction and revision functions. J. Symb. Log. 50, 510–530 (06 1985)
2. Booth, R., Meyer, T.: Admissible and restrained revision. Journal of Artificial Intelligence Research 26, 127–151 (2006)
3. Boutilier, C.: Revision Sequences and Nested Conditionals. IJCAI pp. 519–525 (1993)
4. Darwiche, A., Pearl, J.: On the logic of iterated belief revision. Artificial Intelligence 89(1-2), 1–29 (jan 1997)
5. Everaere, P., Konieczny, S., Marquis, P.: Disjunctive Merging: Quota and Gmin Merging Operators. Artificial Intelligence pp. – (2010), `https://hal.archives-ouvertes.fr/hal-00731928`
6. Gärdenfors, P.: Knowledge in flux. modeling the dynamics of epistemic states. Studia Logica 49(3), 421–424 (1988)
7. Goldszmidt, M., Pearl, J.: Qualitative probabilities for default reasoning, belief revision, and causal modeling. Artificial Intelligence 84(1), 57 – 112 (1996)
8. Hansson, S.O.: In defense of the ramsey test. The Journal of Philosophy 89(10), 522–540 (1992)
9. Jin, Y., Thielscher, M.: Iterated belief revision, revised. In: 7th International Symposium on Logical Formalizations of Commonsense Reasoning, Commonsense 2005 (2005)
10. Katsuno, H., Mendelzon, A.O.: Propositional knowledge base revision and minimal change. Artificial Intelligence 52, 263–294 (1991)
11. Konieczny, S., Pérez, R.: Merging information under constraints: A logical framework. J. Log. Comput. 12, 773–808 (2002)
12. Lehmann, D.: Belief revision, revised. In: Proceedings of the 14th International Joint Conference on Artificial Intelligence - Volume 2. p. 1534–1540. IJCAI'95, Morgan Kaufmann Publishers Inc., San Francisco, CA, USA (1995)
13. Li, C., Czarnecki, K.: Urban driving with multi-objective deep reinforcement learning. In: Elkind, E., Veloso, M., Agmon, N., Taylor, M.E. (eds.) Proceedings of

the 18th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS '19, Montreal, QC, Canada, May 13-17, 2019. pp. 359–367. International Foundation for Autonomous Agents and Multiagent Systems (2019), http://dl.acm.org/citation.cfm?id=3331714

14. Liu, M., Ma, H., Li, J., Koenig, S.: Task and path planning for multi-agent pickup and delivery. p. 1152–1160. AAMAS '19, International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC (2019)
15. Morreau, M.: Epistemic semantics for counterfactuals. Journal of Philosophical Logic 21(1), 33–62 (1992)
16. Nayak, C., Pagnucco, M., Peppas, P.: Dynamic belief revision operators. Artificial Intelligence 146, 193–228 (06 2003)
17. Reiter, R.: A logic for default reasoning. Artificial Intelligence 13(1), 81 – 132 (1980)
18. Schwind, N., Inoue, K., Bourgne, G., Konieczny, S., Marquis, P.: Belief revision games. Proceedings of the National Conference on Artificial Intelligence 2, 1590–1596 (2015)
19. Schwind, N., Konieczny, S., Lagniez, J.M., Marquis, P.: On computational aspects of iterated belief change. In: Bessiere, C. (ed.) Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence, IJCAI-20. pp. 1770–1776. International Joint Conferences on Artificial Intelligence Organization (7 2020), https://doi.org/10.24963/ijcai.2020/245
20. Silver, D., Huang, A., Maddison, C.J., Guez, A., Sifre, L., van den Driessche, G., Schrittwieser, J., Antonoglou, I., Panneershelvam, V., Lanctot, M., Dieleman, S., Grewe, D., Nham, J., Kalchbrenner, N., Sutskever, I., Lillicrap, T., Leach, M., Kavukcuoglu, K., Graepel, T., Hassabis, D.: Mastering the game of Go with deep neural networks and tree search. Nature 529(7587), 484–489 (2016), https://doi.org/10.1038/nature16961
21. Spohn, W.: Ordinal Conditional Functions: A Dynamic Theory of Epistemic States, pp. 105–134. Springer Netherlands, Dordrecht (1988)
22. Spohn, W.: A general non-probabilistic theory of inductive reasoning. Machine Intelligence and Pattern Recognition, vol. 9, pp. 149 – 158. North-Holland (1990)

Appendix 1: Software

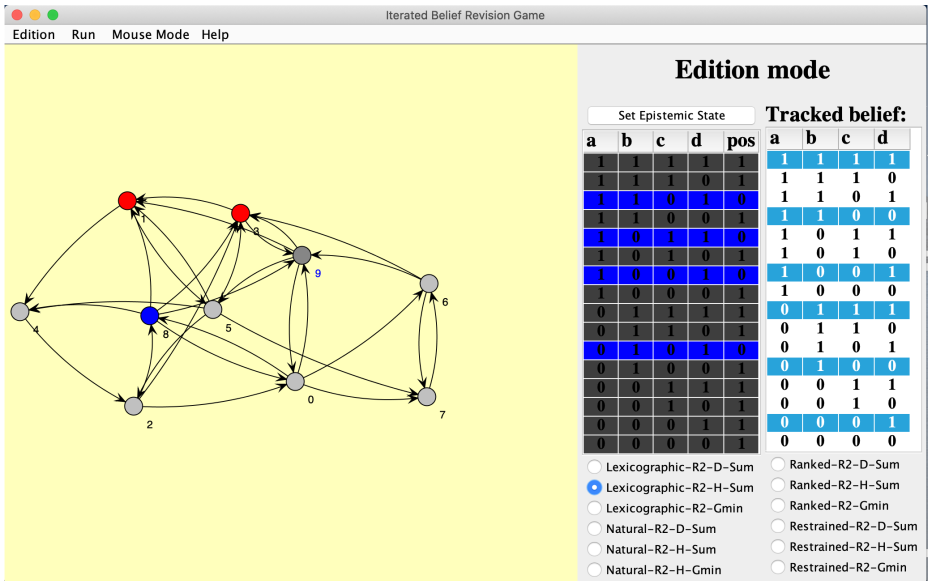

Fig. 8: Screenshot of the application interface.

Appendix 2: Proofs of Propositions

Proposition 2 For every $k \in \{1, \ldots, 4\}$, $R^k_{\circ,\Delta}$ satisfies **(CP)**.

Proof. Let $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ be a IBRG from any of the classes $\mathcal{G}(R^k_{\circ,\Delta})$ where $k \in \{1, \ldots, 4\}$. We must prove that G satisfies **(CP)**. Assume that for each $Bel(E_i) \in E$, $Bel(E_i)$ is consistent. We prove that for each step $s \geq 0$, $Bel(E_i^s)$ is consistent by recursion on s. This is trivial for each $k \in \{1, \ldots, 4\}$ when $s = 0$. Now, let $s \geq 0$ and assume that for each $E_i^s \in G^s$, $Bel(E_i^s)$ is consistent. Then, as every $R^k_{\circ,\Delta}$, $k \in \{1, \ldots, 4\}$, satisfy DP-AGM postulates **(R*1-R*6)**, by postulate **(R*3)** we have that at every revision step s, $E \circ \mu$ is consistent if μ is consistent. In G, μ is the formula obtained after merging beliefs of agent's neighbors, which are all consistent by definition. Hence, every $Bel(E_i^s)$ from $(E_i^s)_{s \in \mathcal{N}}$ are consistent.

Proposition 3 For every $k \in \{1, \ldots, 4\}$, $R^k_{\circ,\Delta}$ satisfies **(AP)**.
The proof uses the following lemma:

Lemma 1 Let $\circ$ be a DP-AGM operator which satisfies **(R*2)**. Then for every consistent propositional formula φ, for every formula μ and every epistemic state E, if $\varphi \models \mu$ and $\varphi \models Bel(E)$ for every E, then $\varphi \models E \circ \mu$.

Proof. Let $\circ$ be a DP-AGM operator which satisfies **(R*2)**, φ be a consistent formula, μ be a formula and E be an epistemic state such that $\varphi \models \mu$ and $\varphi \models Bel(E)$. Since φ is consistent, $Bel(E) \wedge \mu$ is consistent and $\varphi \models Bel(E) \wedge \mu$. Then by **(R*2)**, $E \circ \mu \equiv Bel(E) \wedge \mu$. Therefore, $\varphi \models E \circ \mu$.

Then, we can prove proposition 3:

Proof. Let $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ be a IBRG from any of the classes $\mathcal{G}(\mathcal{R}^{\|}_{\circ,\cdot})$ where $k \in \{1, \ldots, 4\}$ and where $\circ$ satisfies **(R*2)**. We must prove that G satisfies **(AP)**. Let φ be a consistent propositional formula from $\mathcal{L}_p$ and that for each $E_i \in E$, $\varphi \models Bel(E_i)$. We prove that for each $E_i \in E$ and at every step $s \geq 0$, $\varphi \models Bel(E_i^s)$ by recursion on s. This is trivial for each $k \in \{1, \ldots, 4\}$ when $s = 0$. Now, let $s \geq 0$ and assume that for each $E_i^s \in G^s$, $\varphi \models Bel(E_i^s)$. Then using lemma 1, for each $k \in \{1, \ldots, 4\}$ since $\circ$ satisfies **(R*2)**, it can easily be checked that for each agent $i \in V$, $Bel(E_i^{s+1})$ is consistent.

Proposition 4 For every $k \in \{1, \ldots, 4\}$, $R^k_{\circ,\Delta}$ satisfies **(UP)**.

Proof. Let $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ be a IBRG from any of the classes $\mathcal{G}(R^k_{\circ,\Delta})$ where $k \in \{1, \ldots, 4\}$. We must prove that G satisfies **(UP)**. Let φ be a propositional formula from $\mathcal{L}_p$ and assume that for each $E_i \in E$, $Bel(E_i) \models \varphi$. We prove that for each $E_i \in E$ and at every step $s \geq 0$, $Bel(E_i^s) \models \varphi$ by recursion on s. This is trivial for each $k \in \{1, \ldots, 4\}$ when $s = 0$. Now, let $s \geq 0$ and assume that for each $E_i^s \in G^s$, $Bel(E_i^s) \models \varphi$. We assume that Δ satisfies **(IC0)** and **(Disj)**, then we can infer from the proof in (CITATION) that $\Delta_\top(\langle C_i^s \rangle) \models \varphi$. So by **(R*1)**, we get that $Bel(E_i^{s+1}) \models \Delta_\top(\langle C_i^s \rangle)$. Hence, $Bel(E_i^{s+1}) \models \varphi$.

Proposition 5 For every $k\imath n\{1, \ldots, 4\}$, $R_{\circ,\Delta}$ does not satisfies **(Conv)**.

Proof. For every $k \in \{1, \ldots, 4\}$, we must prove that $R^k_{\circ,\Delta}$ does not satisfy **(Conv)**. That is to say, for each revision policy we must show that there exists a IBRG from $\mathcal{G}(R^k_{\circ,\Delta})$ which is not stable. To simplify the proof, we consider here only the belief cycle as we proved in **??** that it is equivalent to iterated belief cycle. Then let $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$ be a IBRG from $\mathcal{G}(R^k_{\circ,\Delta})$ defined as $V = \{1, 2, 3\}$, $A = \{(1, 2), (2, 3), (3, 1)\}$, $\mathcal{L}_p$ is the propositional language defined from $P = \{a, b\}$, $Bel(E_1) = Bel(E_1^0) = \neg a$, $Bel(E_2) = Bel(E_2^0 = a \wedge b)$ and $Bel(E_3) = Bel(E_3^0 = a \wedge \neg b)$. Then for every $k \in \{1, \ldots, 4\}$, we can verify that at step 1, $Bel(E_1^1) \equiv Bel(E_3^0) \equiv a \wedge \neg b$, $Bel(E_2^1) \equiv Bel(E_1^0) \equiv \neg a$ and $Bel(E_3^1) \equiv Bel(E_2^0) \equiv a \wedge b$. From then, for every step $s \neq 1$, we have $Bel(E_1^{s+1}) \equiv Bel(E_3^s)$, $Bel(E_2^{s+1}) \equiv Bel(E_1^s)$ and $Bel(E_3^{s+1}) \equiv Bel(E_2^s)$. This means that for each agent $i \in V$, $\mid Cyc(E_i) \mid = 3$. Hence, G is not stable.

Proposition 7 For every IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$, if $T^s_G = \left\langle Bel(E^s_1), \ldots, Bel(E^s_n) \right\rangle$ is cyclic, then $T'^s_G = \left\langle E^s_1, \ldots, E^s_n \right\rangle$ is also cyclic.

Proof. Given a IBRG $G = (V, A, \mathcal{L}_p, E, \mathcal{R})$, we have to prove that if there is a global cycle based on belief sets, then there is a global cycle based on total preorders over interpretations as well in G. Let us assume that if we have $T^s_G = \left\langle Bel(E^s_1), \ldots, Bel(E^s_n) \right\rangle$ cyclic then there is no global cycle such that $T'^s_G = \left\langle E^s_1, \ldots, E^s_n \right\rangle$. If so, it means that within the global sequence of G, at least two total preorders are different whereas they should be equal. As both total preorders are different, their beliefs set must also be different. In this setup, there could not exist a global cycle based on belief sets in the first place which contradicts the initial postulate. We can then conclude that if $T^s_G = \left\langle Bel(E^s_1), \ldots, Bel(E^s_n) \right\rangle$ is cyclic, then $T'^s_G = \left\langle E^s_1, \ldots, E^s_n \right\rangle$ is also cyclic.