Method Results

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September 2025

1. Incremental search method

Incremental search Method Pseudocode:

```
BEGIN
 READ x0, deltaX, f, maximum_iterations
 INTEGER counter
 counter = 0


 PRINT |Iteration| xi | f(xi) |
 PRINT | 0 | x0 | f(x0) |

 WHILE TRUE DO:
  counter = counter + 1
  x1 = x0 + deltaX
  PRINT |counter| x1 | f(x1) |
  IF f(x0)*f(x1) < 0 THEN
   PRINT In [x0,x1] there is at least one root
   BREAK WHILE
  ELSE THEN
   x0=x1
  END IF
  IF counter == maximum_iterations THEN
   PRINT Limit reached
   BREAK WHILE
  END IF
 END WHILE
END BEGIN
```

Incremental search Method Results:

i	x_i	$f(x_i)$
0	-2.450000000	-6.7e-03
1	-1.950000000	3.7e-02

In the interval [-2.450000000,-1.950000000 there is at least one root

Table 1: Iterations of the Newton method for $f(x) = x^2 - x + 1 - e^x + \frac{99}{200}$

2. Bisection method

Bisection Pseudocode:

```
Begin

 READ a, b, f, tolerance, maximum_iterations

 INTEGER counter, cbefore

 counter = 0

 cbefore = (a+b)/c

 PRINT |Iteration| xi | f(xi) | E |
 PRINT | 0 |cbefore|f(cbefore)|  |

 WHILE True:
  counter = counter + 1
  IF f(a)*f(cbefore)<0 THEN
   b = c
  ELSE THEN
   a = c
  c = (a+b)/2
  abs_error = abs(c-cbefore)
  PRINT | counter | cbefore | f(cbefore) | abs_error
  IF abs_error < tolerance THEN
   PRINT Root = c
            BREAK WHILE
  ELSE THEN
   cbefore = c
  IF counter == maximum_iterations THEN
   PRINT "Limited reached"
            BREAK WHILE
END BEGIN
```

Bisection Method Results:

i	x_i	$f(x_i)$	Error
0	0.250000000	2.3e-02	-
1	0.250000000	2.3e-02	3.8e-01
2	0.625000000	-6.1e-01	1.9e-01
3	0.437500000	-3.0e-01	9.4e-02
4	0.343750000	-1.4e-01	4.7e-02
5	0.296875000	-5.9e-02	2.3e-02
6	0.273437500	-1.8e-02	1.2e-02
7	0.261718750	2.6e-03	5.9e-03
8	0.267578125	-7.8e-03	2.9e-03
9	0.264648438	-2.6e-03	1.5e-03
10	0.263183594	1.6e-05	7.3e-04
11	0.263916016	-1.3e-03	3.7e-04
12	0.263549805	-6.3e-04	1.8e-04
13	0.263366699	-3.1e-04	9.2e-05
14	0.263275146	-1.5e-04	4.6e-05
15	0.263229370	-6.5e-05	2.3e-05
16	0.263206482	-2.4e-05	1.1e-05
17	0.263195038	-3.9e-06	5.7e-06
18	0.263189316	6.3e-06	2.9e-06
19	0.263192177	1.2e-06	1.4e-06
20	0.263193607	-1.3e-06	7.2e-07

An approximation of the root was found at $x \approx 0.263193607$

Table 2: Iterations of the Newton method for $f(x) = x^2 - x + 1 - e^x + \frac{99}{200}$

3. False position method

False position Method Pseudocode:

```
Begin

 READ a, b, f, tolerance, maximum_iterations

 INTEGER counter, cbefore

 counter = 0

 cbefore = b - ((f(b)*(b-a))/(f(b)-f(a))

 PRINT |Iteration| xi | f(xi) | E |
 PRINT | 0 |cbefore|f(cbefore)|  |

 WHILE True DO
  counter = counter + 1
  c = cbefore
  IF f(a)*f(cbefore)<0 THEN
   b = c
  ELSE THEN
```

```
    a = c
   END IF
   c = b - ((f(b)*(b-a))/(f(b)-f(a))
   abs_error = abs(c-cbefore)
   PRINT | counter | cbefore | f(cbefore) | abs_error
   IF abs_error < tolerance THEN
    PRINT Root = c
   ELSE THEN
    cbefore = c
   END IF
   IF counter == maximum_iterations THEN
    PRINT "Limited reached"
   END IF
  END WHILE
END BEGIN
```

False position method Results:

i	x_i	$f(x_i)$	**Error**
0	1.333333333	-9.6e-01	-
1	1.333333333	-9.6e-01	3.1e-01
2	1.642857143	7.9e-01	1.4e-01
3	1.503250975	-1.1e-01	1.7e-02
4	1.519780718	-9.5e-03	1.5e-03
5	1.521239851	-8.3e-04	1.3e-04
6	1.521367483	-7.3e-05	1.1e-05
7	1.521378639	-6.3e-06	9.7e-07
8	1.521379613	-5.5e-07	8.5e-08

An approximation of the root was found at $x \approx 1.521379613$

Table 3: Iterations of the Newton method for $f(x) = x^3 - x - 2$

4. Roots multiplicity method

Roots multiplicity Pseudocode:

```
BEGIN
 READ x0, tolerance, maximum_iterations, f(x), f'(x), f''(x)
 INTEGER counter
 counter = 0

 PRINT |Iteration| xi | f(xi) | E |
 PRINT | 0 |x0|f(x0)|  |

 WHILE TRUE DO:
  counter = counter + 1
  x1 = x0-((f(x0)*f'(x0)/(f'(x0)^2-(f(x0)*f''(x0)))))

  abs_error = abs(x1-x0)
  PRINT | counter |x1|f(x1)| abs_error |
  IF abs(x1-x0) < tolerance THEN
```

```
   PRINT Root: x1
  ELSE IF THEN
   x0 = x1
  END IF
  IF counter == maximun_iterations THEN
   PRINT Limit reached
  END IF
 END WHILE
END BEGIN
```

Roots multiplicity Results:

i	x_i	$f(x_i)$	**Error**
0	-2.450000000	-6.7e-03	-
1	-2.450000000	-6.7e-03	5.4e-02
2	-2.503601660	-8.3e-03	1.2e-01
3	-2.624312939	-1.9e-02	6.2e+00
4	3.535792042	6.2e+00	2.0e-01
5	3.331527089	6.1e+00	3.3e-01
6	3.004132288	5.8e+00	5.8e-01
7	2.426699035	4.8e+00	1.8e+00
8	0.614528262	3.1e+00	8.6e-02
9	0.528389228	3.1e+00	1.3e-01
10	0.397688963	3.1e+00	2.0e-01
11	0.197275711	3.0e+00	3.1e-01
12	-0.114851802	2.7e+00	4.9e-01
13	-0.605695172	1.9e+00	7.3e-01
14	-1.334755925	5.7e-01	7.5e-01
15	-2.087747208	6.5e-03	7.0e-02
16	-2.157597192	-1.0e-03	1.2e-02
17	-2.146068699	-6.4e-05	7.3e-04
18	-2.145340742	-2.2e-07	2.5e-06
19	-2.145338243	-2.6e-12	2.9e-11

An approximation of the root was found at $x \approx -2.1453382432$

Table 4: Iterations of the Newton method for $f(x) = \ 1 - sin^2(x) + x + 1.85$

5. Newton Method

Newton Method Pseudocode:

```
Function NewtonMethod(f, df, x0, tol, N)
    x = x0
    For i = 1 To N Do
        fx = f(x)
        If Abs(fx) < tol Then
            Return x
        EndIf
```

```
        x = x - fx / df(x)
    EndFor
    Return x
EndFunction
```

Newton Method Results:

i	x_i	$f(x_i)$	**Error**
0	0.500000000	-2.9e-01	-
1	0.928391990	-4.7e-03	4.3e-01
2	0.936366741	-2.2e-05	8.0e-03
3	0.936404580	-5.0e-10	3.8e-05

An approximation of the root was found at $x \approx 0.9364045800189902$

Table 5: Iterations of the Newton method for $f(x) = \ln(\sin^2(x) + 1) - \frac{1}{2}$

6. Fixed Point Method Results

Fixed Point Pseudocode:

```
Function FixedPointMethod(g, x0, tol, N)
    x = x0
    For i = 1 To N Do
        x_new = g(x)
        If Abs(x_new - x) < tol Then
            Return x_new
        EndIf
        x = x_new
    EndFor
    Return x
EndFunction
```

Fixed Point Results:

i	x_i	$g(x_i)$	$f(x_i)$	E
0	-0.50000000	-0.29310873	2.07e-01	-
1	-0.29310873	-0.41982154	-1.27e-01	1.27e-01
2	-0.41982154	-0.34630452	7.35e-02	7.35e-02
3	-0.34630452	-0.39095846	-4.47e-02	4.47e-02
4	-0.39095846	-0.36440503	2.66e-02	2.66e-02
5	-0.36440503	-0.38042630	-1.60e-02	1.60e-02
6	-0.38042630	-0.37083680	9.59e-03	9.59e-03
7	-0.37083680	-0.37660565	-5.77e-03	5.77e-03
8	-0.37660565	-0.37314542	3.46e-03	3.46e-03
9	-0.37314542	-0.37522464	-2.08e-03	2.08e-03
10	-0.37522464	-0.37397659	1.25e-03	1.25e-03
11	-0.37397659	-0.37472622	-7.50e-04	7.50e-04
12	-0.37472622	-0.37427613	4.50e-04	4.50e-04
13	-0.37427613	-0.37454643	-2.70e-04	2.70e-04
14	-0.37454643	-0.37438413	1.62e-04	1.62e-04
15	-0.37438413	-0.37448159	-9.75e-05	9.75e-05
16	-0.37448159	-0.37442307	5.85e-05	5.85e-05
17	-0.37442307	-0.37445821	-3.51e-05	3.51e-05
18	-0.37445821	-0.37443711	2.11e-05	2.11e-05
19	-0.37443711	-0.37444978	-1.27e-05	1.27e-05
20	-0.37444978	-0.37444217	7.61e-06	7.61e-06
21	-0.37444217	-0.37444674	-4.57e-06	4.57e-06
22	-0.37444674	-0.37444399	2.74e-06	2.74e-06
23	-0.37444399	-0.37444564	-1.65e-06	1.65e-06
24	-0.37444564	-0.37444465	9.90e-07	9.90e-07
25	-0.37444465	-0.37444525	-5.94e-07	5.94e-07
26	-0.37444525	-0.37444489	3.57e-07	3.57e-07
27	-0.37444489	-0.37444510	-2.14e-07	2.14e-07
28	-0.37444510	-0.37444498	1.29e-07	1.29e-07
29	-0.37444498	-0.37444505	-7.73e-08	7.73e-08

An approximation of the root was found at $x \approx -0.3744449757$

Table 6: Iterations of the fixed-point method for $f(x) = \ln(\sin^2(x) + 1) - \frac{1}{2} - x$

7. Secant Results

Secant Method pseudocode:

```
Function SecantMethod(f, x0, x1, tol, N)
    For i = 1 To N Do
        If Abs(f(x1)) < tol Then
            Return x1
        EndIf
        x_new = x1 - f(x1)*(x1 - x0)/(f(x1) - f(x0))
        x0 = x1
        x1 = x_new
```

```
    EndFor
    Return x1
EndFunction
```

Secant Method results:

Iteration	x_i	$f(x_i)$	Error
0	0.500000000	-2.9e-01	-
1	1.000000000	3.5e-02	-
2	0.946166222	5.6e-03	5.4e-02
3	0.935996581	-2.4e-04	1.0e-02
4	0.936407002	1.4e-06	4.1e-04
5	0.936404581	3.4e-10	2.4e-06
6	0.936404581	-5.0e-16	5.9e-10

An approximation of the root was found at $x \approx 0.9364045808795615$

Table 7: Iterations of the Secant method for $f(x) = \ln(\sin^2(x) + 1) - \frac{1}{2}$

8. Gaussian Elimination Results

Gaussian Elimination pseudocode:

```
Function GaussianElimination(A[1..n,1..n+1])
    # Forward elimination
    For k = 1 To n-1 Do
        For i = k+1 To n Do
            factor = A[i,k] / A[k,k]
            For j = k To n+1 Do
                A[i,j] = A[i,j] - factor * A[k,j]
            EndFor
        EndFor
    EndFor

    # Back substitution
    x[n] = A[n,n+1] / A[n,n]
    For i = n-1 DownTo 1 Do
        sum = 0
        For j = i+1 To n Do
            sum = sum + A[i,j]*x[j]
        EndFor
        x[i] = (A[i,n+1] - sum)/A[i,i]
    EndFor

    Return x
EndFunction
```

Gaussian Elimination results:
We solve the system using Gaussian elimination on the augmented matrix:

$$[A|b] = \begin{bmatrix} 2 & -1 & 0 & 3 & 1 \\ 1 & 0.5 & 3 & 8 & 1 \\ 0 & 13 & -2 & 11 & 1 \\ 14 & 5 & -2 & 3 & 1 \end{bmatrix}$$

Step 0: Initial augmented matrix

$$\begin{bmatrix} 2 & -1 & 0 & 3 & 1 \\ 1 & 0.5 & 3 & 8 & 1 \\ 0 & 13 & -2 & 11 & 1 \\ 14 & 5 & -2 & 3 & 1 \end{bmatrix}$$

Step 1: Zeroing first column below pivot

$$\begin{bmatrix} 2 & -1 & 0 & 3 & 1 \\ 0 & 1 & 3 & 6.5 & 0.5 \\ 0 & 13 & -2 & 11 & 1 \\ 0 & 12 & -2 & -18 & -6 \end{bmatrix}$$

Step 2: Zeroing second column below pivot

$$\begin{bmatrix} 2 & -1 & 0 & 3 & 1 \\ 0 & 1 & 3 & 6.5 & 0.5 \\ 0 & 0 & -41 & -73.5 & -5.5 \\ 0 & 0 & -38 & -96 & -12 \end{bmatrix}$$

Step 3: Zeroing third column below pivot

$$\begin{bmatrix} 2 & -1 & 0 & 3 & 1 \\ 0 & 1 & 3 & 6.5 & 0.5 \\ 0 & 0 & -41 & -73.5 & -5.5 \\ 0 & 0 & 0 & -27.878 & -6.9024 \end{bmatrix}$$

Solutions (back substitution)

$$x_4 \approx 0.2476$$
$$x_3 \approx -0.3097$$
$$x_2 \approx -0.1802$$
$$x_1 \approx 0.0385$$

9. Partial pivoting method

Partial pivoting pseudocode:

...

Partial pivoting results:
We solve the system using partial pivoting on the augmented matrix:

$$[A|b] = \begin{bmatrix} 2 & 3 & 1 & 5 \\ 4 & 7 & 2 & 1 \\ 1 & 3 & 3 & 0 \end{bmatrix}$$

Step 0

$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ 4 & 7 & 2 & 1 \\ 1 & 3 & 3 & 0 \end{bmatrix}$$

Step 1

$$\begin{bmatrix} 4 & 7 & 2 & 1 \\ 0 & -0.5 & 0 & 4.5 \\ 0 & 1.25 & 2.5 & -0.25 \end{bmatrix}$$

Step 2

$$\begin{bmatrix} 4 & 7 & 2 & 1 \\ 0 & 1.25 & 2.5 & -0.25 \\ 0 & 0 & 1 & 4.4 \end{bmatrix}$$