Using Hidden Markov Random Fields to Estimate Functional Networks in fMRI Data
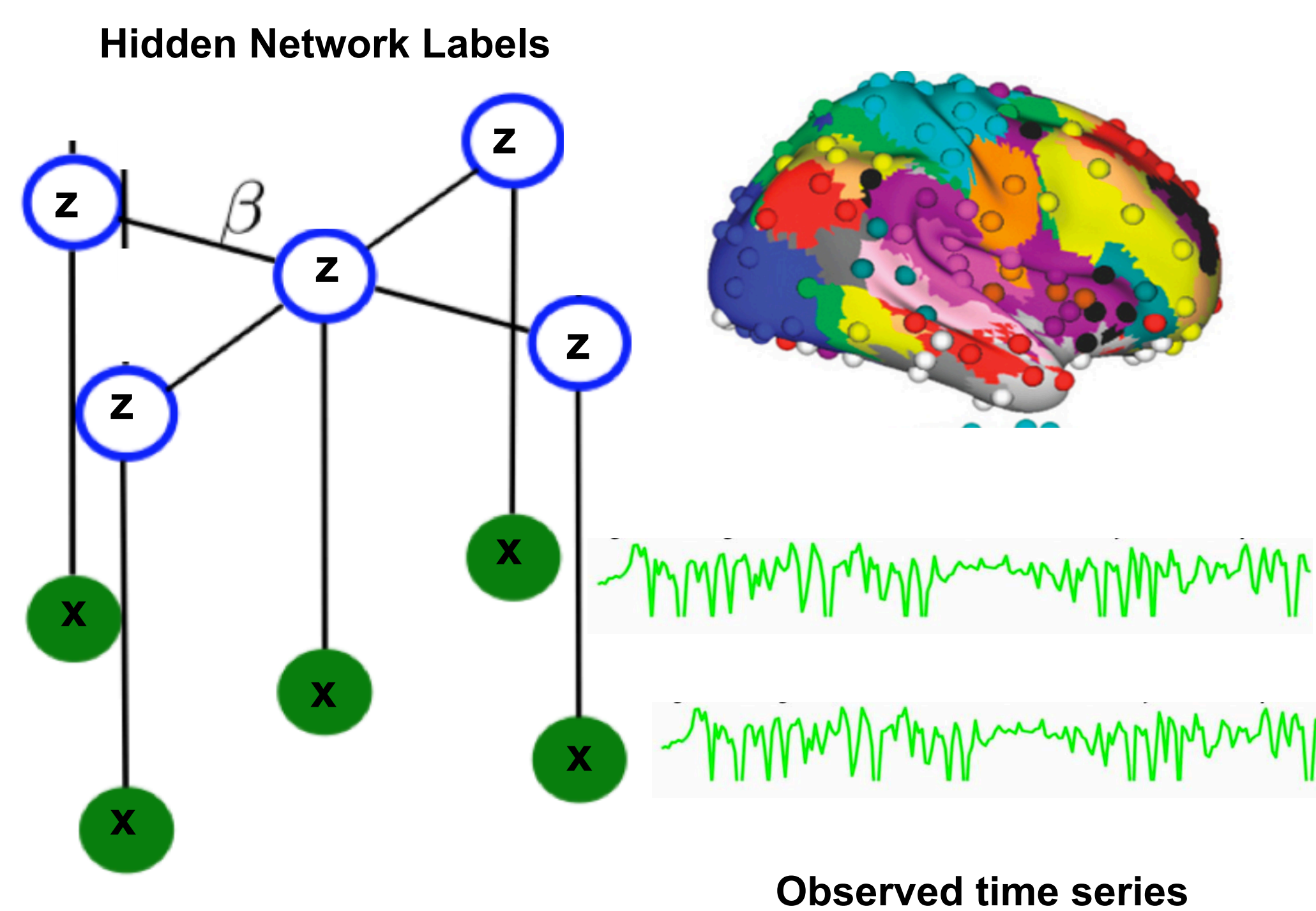
Chris Gagne

Background Problem

- Unsupervised estimation of functional connectivity networks in fMRI data based on similar time series
- Previous methods (ICA, K-means) do not exploit spatial information
- Liu 2012 proposed Hidden Markov Random Field (akin to use in image labeling)

Model

Hidden Network Labels
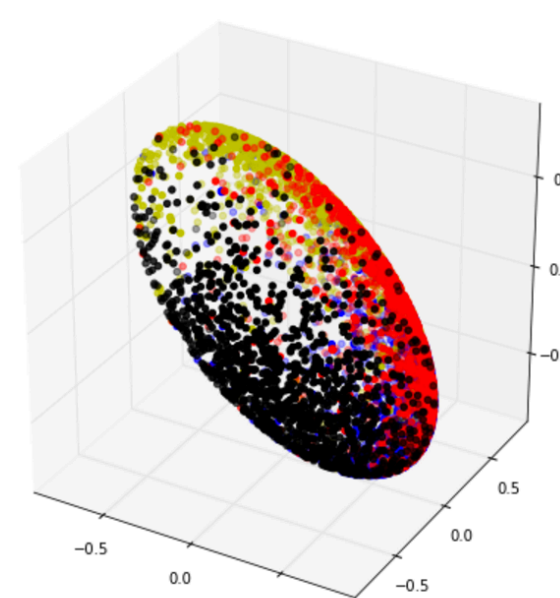

Observed time series

Prior on Network Labels

- Hidden Markov Random Field
- Joint probability of all states (Boltzmann)
- Energy function is Potts Model

$$P(Z) = \frac{1}{C} e^{-\beta \Sigma_s \Sigma_{j \in N_j} T(z_s \neq z_j)} \qquad P(z|z_{-s}) = \frac{e^{-\beta \Sigma_{j \in N_j} T(z_s \neq z_j)}}{\Sigma_l e^{-\beta \Sigma_{j \in N_j} T(l \neq z_j)}}$$

Liklihood: Von-Mises Mixture Model

- Normalized time-series lie on t-1 dimensional hyper-sphere.
- Clusters are modeled as direction and spread on sphere.

$$P(x|z, u, k) = C_p(k) e^{k u^T x}$$

Monte-Carlo EM

E-step: Gibbs Sampling

- Approximate posterior network labels P(Y|X) using local conditional distribution for sample.

$$P(z|z_{-s}, X) = \frac{e^{-\beta \Sigma_{j \in N_j} T(z_s \neq z_j) - k u^T x - log(C(k))}}{\Sigma_l e^{-\beta \Sigma_{j \in N_j} T(l \neq z_j) - k u^T x - log(C(k))}}$$

M-step: Parameter Estimation

- Uses MCMC samples to approximate expectation of complete log liklihood
- Use pseudo-liklihood instead.

$$\frac{1}{M} \sum_M log P(Z^M, X; \theta)$$

$$= \frac{1}{M} \sum_M log P(Z^M; \theta) + log P(X|Z^M; \theta)$$

- Calculate direction (u) by taking normalized mean of X from same cluster in the Gibbs samples.
- B, K require numerical solution.

Simulation

1. Simulate network labels by sampling from prior (gibbs or MH)
2. Generate time series data for each cluster
3. Scramble the labels
4. Apply K-means to get closer to solution
5. Run Monte-Carlo 8EM
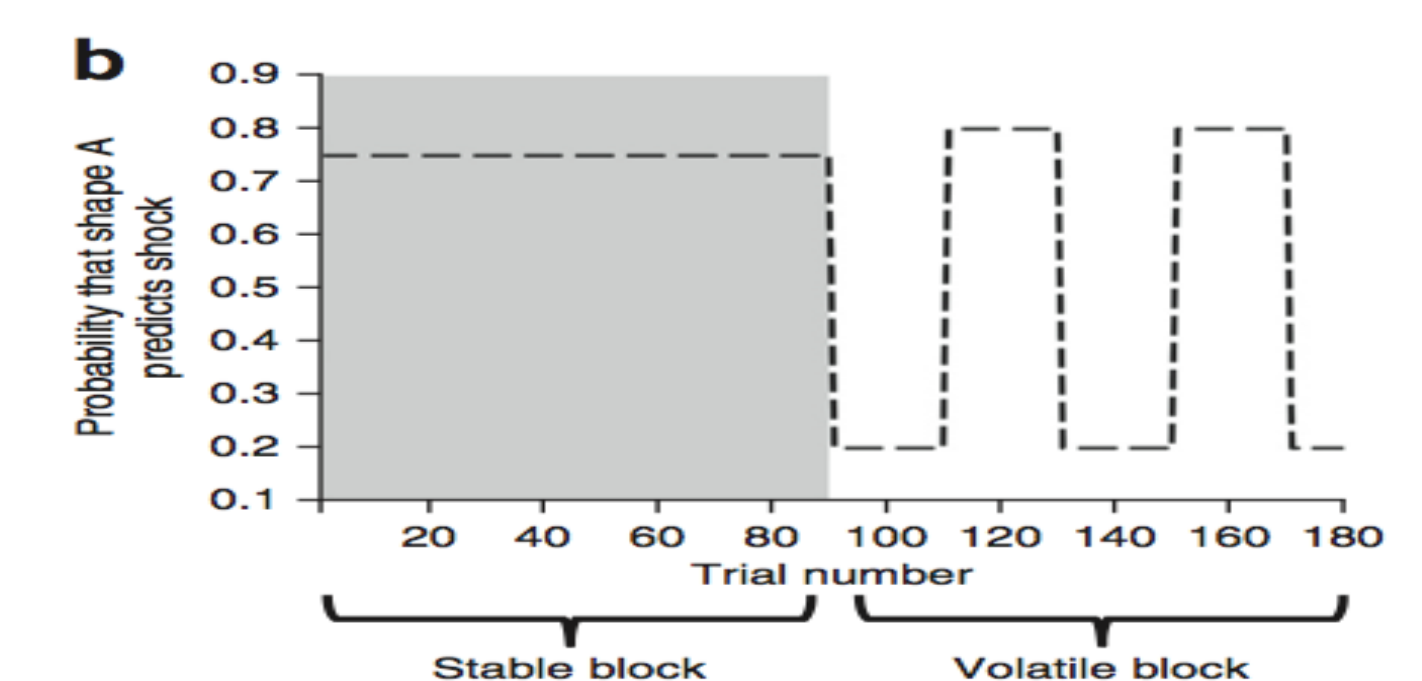6. Apply iterated conditional means to last Gibbs Sample.
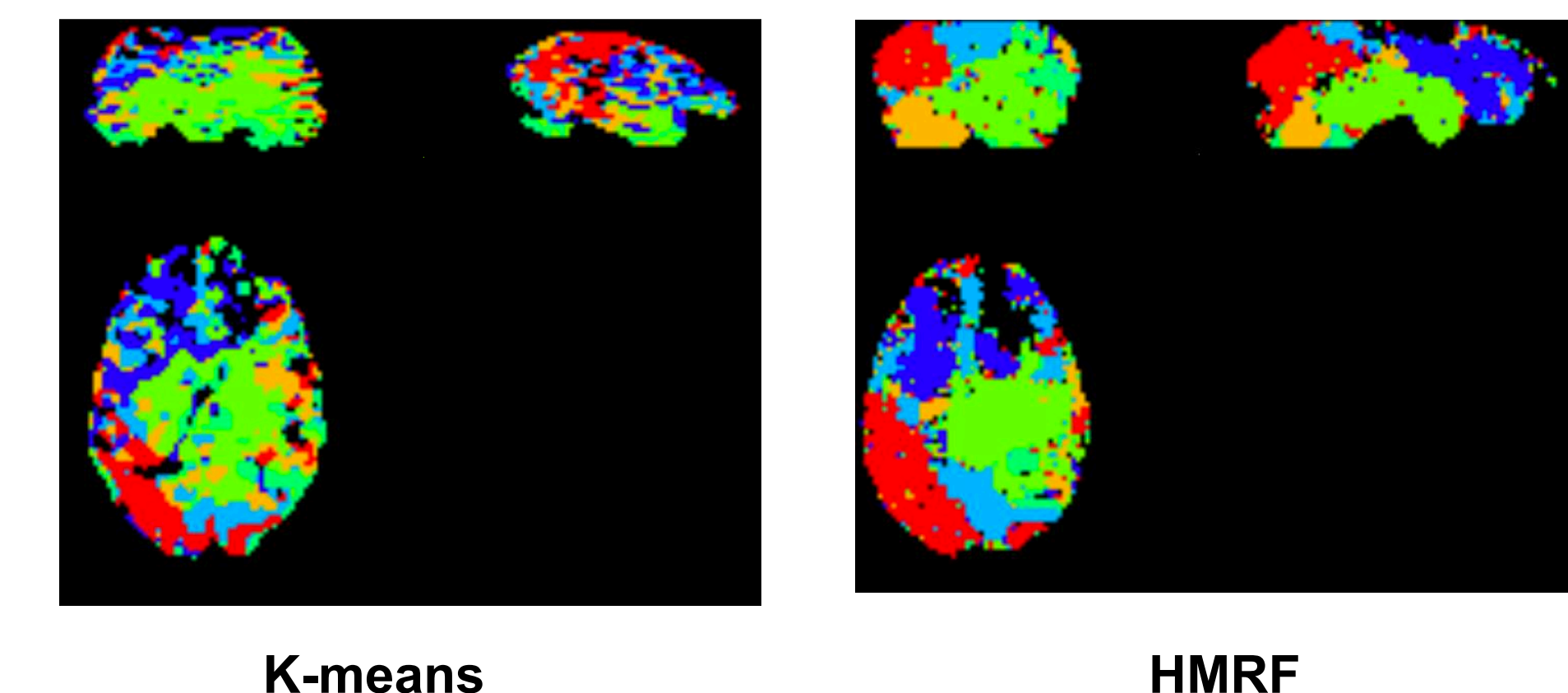
Prior acts as a spatial regularizer

Truth

Observed

K-means

HMRF
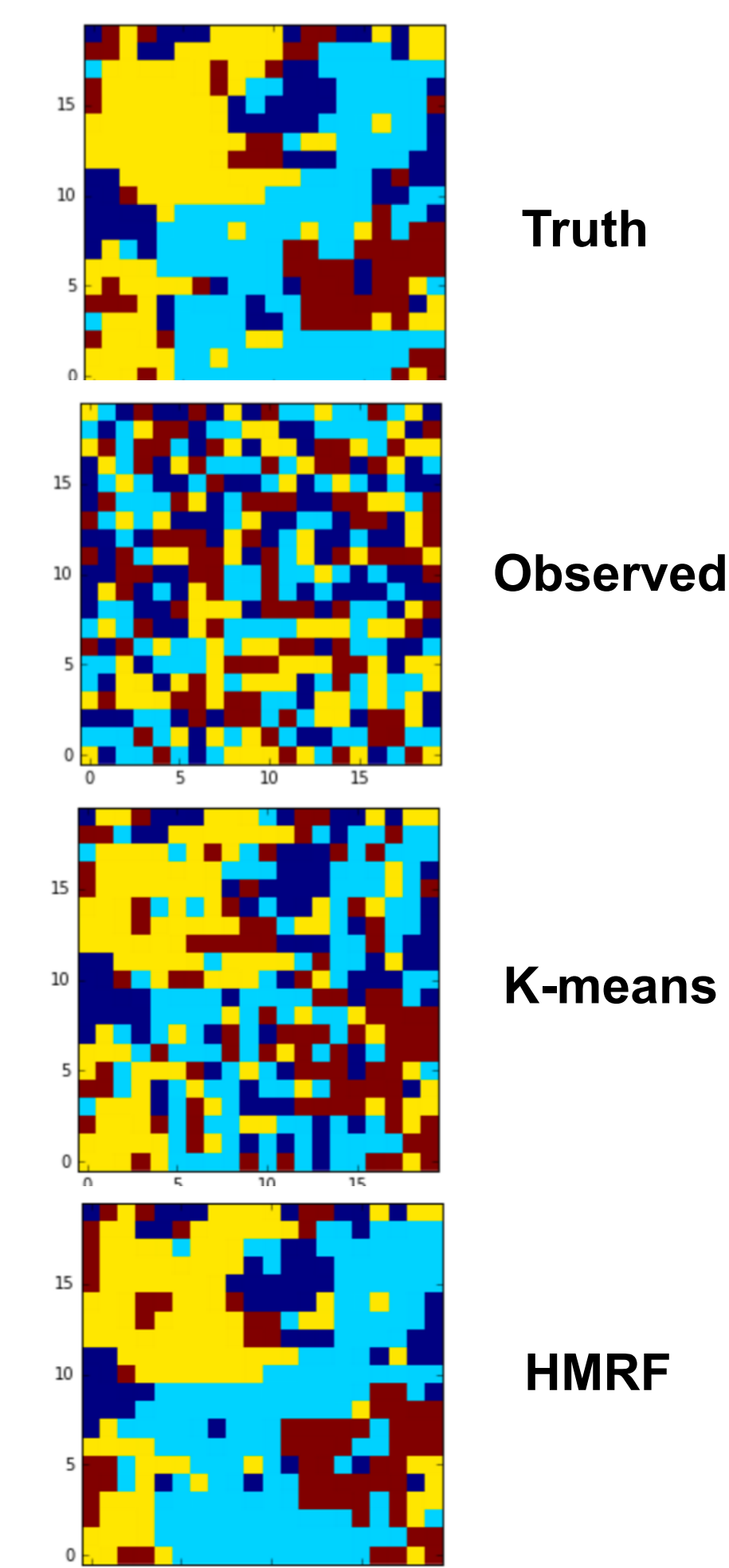
fMRI Data

Do we have different functional connectivity when we are uncertain?

Task

Results

K-means   **HMRF**

Issues

- Estimating B,
- Fixed number of clusters
- MCMC high computational cost
- Remove event related responses from non-resting state data.

Extensions

- Group estimation Liu 2014