A Variety of Ways to Solve a Problem

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Example Problem

1			
			4
	3	2	
2			

Rules: Fill each cell with the numbers 1, 2, 3, 4 so they appear exactly once in each row, column, and box.

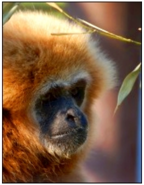
How many solutions does this puzzle have if you omit the clue 4?

♫ (switching media) ♫

http://habanero.math.cornell.edu:3690/

Application

- **Evolutionary Biology**
  These Gibbons share a common ancestor:



`www.ippl.org/gibbon/courtney/`

`people.hamilton.edu/cgibbons`

When did this ancestor roam the world?

Application

- **Evolutionary Biology**
  These Gibbons share a common ancestor:

`www.ippl.org/gibbon/courtney/` `people.hamilton.edu/cgibbons`

When did this ancestor roam the world?
(about 19 million years ago)

To learn more

To play with the code I just presented, email me!
`crgibbon@hamilton.edu`
(*you'll need Macaulay2 to run it*)

To learn more about:

- Algorithms that make these computations possible:
  Ideals, Varieties, and Algorithms
  by Cox, Little, and O'Shea
- Using algebra and geometry to solve problems from biology:
  Algebraic Statistics for Computational Biology
  by Pachter and Sturmfels

Homework (email solutions to crgibbon@hamilton.edu)

Problem 1: Show that each polynomial in B is necessary. That is, by omitting one of each type of rule independently, find a pathological solution.

$$\begin{array}{|cc|cc|} \hline a & b & c & d \\ c & d & a & b \\ \hline b & c & d & a \\ d & a & b & c \\ \hline \end{array} = \underline{a} \in \mathbb{C}^{16}$$

To get started, here's a "solved" board. What if you omit the polynomial

$$(x_{1,1} - 1)\,(x_{1,2} - 2)\,(x_{1,3} - 3)\,(x_{1,4} - 4)?$$

Problem 2: Ideals and varieties are a match made in heaven. For example, when $I \subseteq J$ as ideals, $V(J) \subseteq V(I)$ as varieties.
Assume I and J are ideals in the polynomial ring $P = F[x_1, \ldots, x_n]$.

(a) Prove that $I \subseteq J$ implies $V(J) \subseteq V(I)$.

(b) Prove that $V(I \cup J) = V(I) \cap V(J)$.

Thanks!