

Linear Systems

What puts the “linear” in “Linear Algebra?” Turns out that this adjective refers to properties of linear systems.

Definition 1. Given a linear system, we can define special properties depending on the types of solutions it has.

- If a linear system has at least one solution, we say the system is **consistent**.
- If a linear system has exactly one solution, we say the solution is **unique**.
- If a linear system has no solutions, we say the system is **inconsistent**.
- If two linear systems have exactly the same solutions, we call them **equivalent**.
- If $b_1 = b_2 = \cdots = b_m = 0$ in a linear system with m equations, we say the system is **homogeneous**.
- If $x_1 = x_2 = \cdots = x_n = 0$ is a solution to a linear system in n unknowns, we say the system has the **trivial solution**.

For each of the following, write down an example or explain why one can't exist:

Question 1 an inconsistent linear system in 2 unknowns;

Question 2 an inconsistent homogeneous linear system in 2 unknowns;

Question 3 a unique solution to the linear system

$$\begin{aligned}x + 4y &= 12 \\ 3x + 8y &= 4\end{aligned}$$

Question 4 a nonhomogeneous system with the trivial solution

Question 5 *a linear system with three equations that is equivalent to the linear system*

$$x + 4y = 12$$

$$3x + 8y = 4$$

Question 6 *Jot down any conjectures you have about the way these various properties are related.*
