# Linear Systems

*What puts the "linear" in "Linear Algebra?" Turns out that this adjective refers to properties of linear systems.*

**Definition 1.**

- *If a linear system has at least one solution, we say the system is **consistent**.*
- *If a linear system has exactly one solution, we say the solution is **unique**.*
- *If a linear system has no solutions, we say the system is **inconsistent**.*
- *If two linear systems have exactly the same solutions, we call them **equivalent**.*
- *If $b_1 = b_2 = \cdots = b_m = 0$ in a linear system with $m$ equations, we say the system is **homogeneous**.*
- *If $x_1 = x_2 = \cdots = x_n = 0$ is a solution to a linear system in $n$ unknowns, we say the system has the **trivial solution**.*

For each of the following, write down an example or explain why one can't exist:

**Question 1** *an inconsistent linear system in 2 unknowns;*

**Question 2** *an inconsistent homogeneous linear system in 2 unknowns;*

**Question 3** *a unique solution to the linear system*

$$\begin{aligned} x + 4y &= 12 \\ 3x + 8y &= 4; \end{aligned}$$

**Question 4** *a nonhomogeneous system with the trivial solution;*

**Question 5** *a linear system with three equations that is equivalent to the linear system*

$$\begin{aligned} x + 4y &= 12 \\ 3x + 8y &= 4. \end{aligned}$$

**Question 6** *Jot down any conjectures you have about the way these various properties are related.*