

# Linear Systems

What puts the “linear” in “Linear Algebra?” Turns out that this adjective refers to properties of linear systems.

**Definition 1.** • If a linear system has at least one solution, we say the system is **consistent**.

- If a linear system has exactly one solution, we say the solution is **unique**.
- If a linear system has no solutions, we say the system is **inconsistent**.
- If two linear systems have exactly the same solutions, we call them **equivalent**.
- If  $b_1 = b_2 = \cdots = b_m = 0$  in a linear system with  $m$  equations, we say the system is **homogeneous**.
- If  $x_1 = x_2 = \cdots = x_n = 0$  is a solution to a linear system in  $n$  unknowns, we say the system has the **trivial solution**.

For each of the following, write down an example or explain why one can't exist:

**Question 1** an inconsistent linear system in 2 unknowns;

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**Question 2** an inconsistent homogeneous linear system in 2 unknowns;

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**Question 3** a unique solution to the linear system

$$\begin{aligned}x + 4y &= 12 \\ 3x + 8y &= 4;\end{aligned}$$

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**Question 4** a nonhomogeneous system with the trivial solution;

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**Question 5** *a linear system with three equations that is equivalent to the linear system*

$$\begin{aligned}x + 4y &= 12 \\ 3x + 8y &= 4.\end{aligned}$$

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**Question 6** *Jot down any conjectures you have about the way these various properties are related.*

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