Linear Systems

What puts the "linear" in "Linear Algebra?" Turns out that this adjective refers to properties of linear systems.

Definition 1. *Given a linear system, we can define special properties depending on the types of solutions it has.*

- *If a linear system has at least one solution, we say the system is* ***consistent****.*
- *If a linear system has exactly one solution, we say the solution is* ***unique****.*
- *If a linear system has no solutions, we say the system is* ***inconsistent****.*
- *If two linear systems have exactly the same solutions, we call them* ***equivalent****.*
- *If* $b_1 = b_2 = \cdots = b_m = 0$ *in a linear system with* m *equations, we say the system is* ***homogeneous****.*
- *If* $x_1 = x_2 = \cdots = x_n = 0$ *is a solution to a linear system in* n *unknowns, we say the system has the* ***trivial solution****.*

For each of the following, write down an example or explain why one can't exist:

Exercise 1 Question 2 *an inconsistent linear system in 2 unknowns;*

Exercise 3 Question 4 *an inconsistent homogeneous linear system in 2 unknowns;*

Question 5 *a unique solution to the linear system*

$$\begin{aligned} x + 4y &= 12 \\ 3x + 8y &= 4 \end{aligned}$$

Question 6 *a nonhomogeneous system in two unknowns with the trivial solution*

Question 7 *a linear system with three equations that is equivalent to the linear system*

$$\begin{aligned} x + 4y &= 12 \\ 3x + 8y &= 4 \end{aligned}$$

Question 8 *Jot down any conjectures you have about the way these various properties are related.*