Course Notes

The following notes are intended to remind you what we covered each week. They are not a substitute for attending class and not a substitute for reading the textbook.

Week 1

This week we covered the following topics:

Week 2: Matrix Operations and Theorems

Matrix Addition

Definition 1 Matrix Addition. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be matrices in $M_{m,n}$. The *sum* of A and B, denoted $A + B$, is the matrix $C = [c_{ij}]$ where $c_{ij} = a_{ij} + b_{ij}$ for all $1 \leq i \leq m$ and $1 \leq j \leq n$. $\Diamond$

Proposition 2 Additive Identity. *Let $A \in M_{m,n}$, and let Z be the $m \times n$ zero matrix, meaning each entry of Z is zero. Then $A + Z = Z + A = A$.*

Proof. For all i and j, $[A + Z]_{ij} = a_{ij} + 0 = a_{ij}$. By definition of matrix equality, $A + Z = A$. The argument for $Z + A$ is identical. $\blacksquare$

Remark 3 These properties should feel familiar: matrix addition behaves exactly like addition of real numbers.

One guiding principle of linear algebra is that we want matrices to behave like numbers whenever possible. When they fail to do so, that failure usually signals something important.

Remark 4 Every matrix has an additive inverse, but this is not true for multiplicative inverses. We're going to get into that more, and more, and more...

Scalar Multiplication and Linear Combinations

Definition 5 Scalar Multiplication. Let $A = [a_{ij}] \in M_{m,n}$ and let $r \in \mathbb{R}$. The *scalar multiple* of A by r, denoted rA, is the matrix $C = [c_{ij}]$ where $c_{ij} = ra_{ij}$. $\Diamond$

Example 6 Compute $5A + B$, where$A = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.$ $\square$

Definition 7 Linear Combination. Given matrices $A_1, \ldots, A_k \in M_{m,n}$ and scalars $c_1, \ldots, c_k \in \mathbb{R}$, the matrix $\sum_{i=1}^{k} c_i A_i$ is called a *linear combination* of the matrices $A_1, \ldots, A_k$. $\Diamond$

Remark 8 Linear combinations are one of the most important ideas in this course. We will return to them again and again, in many different disguises.

Whenever you see an expression like $c_1 A_1 + \cdots + c_k A_k$, mentally translate it as "a weighted sum."