In-Class Activities

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These activities are designed to be used in class to help students understand the material. They may or may not be part of the graded work for the class. Please ask your professor if you need help!

If you are reviewing these materials outside of class, you can post to the course Zulip for help, too.

Reading Mathematics

Reading mathematics, and especially mathematics textbooks, is not like reading a novel for pleasure. Often, one sentence of a book or paper contains a paragraph's worth of information. This worksheet will help you develop some habits to read math effectively.

1. **Advice from the Authors.** On page xviii, our textbook authors give advice to a student reader. Summarize the advice here.
2. **Using the End of Section Materials.** Head to the back of Section 1.1 and jot down of the key terms. Leave space to write notes about each term. Do this before you start reading the material in the chapter.

   How many terms are in your list?
3. **Reading for Meaning.** The first term in the Key Terms list is **linear equation**. Start reading the chapter and keep an eye out for this term.

   We find the first appearance of the term on page 1. It is defined as an equation of the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ where the variables x_i $(1 \leq i \leq n)$ are unknowns and $a_1, \ldots, a_n$ and b are real (or complex) numbers.

   We can see that it is closely related to the next terms in the Key Terms list, **solution** and **linear system**. Write those definitions in your list of terms, too. It's helpful to include a page numnber where it appears in the book in case you want to find it again.
4. **Reading Comprehension: Examples and Non-Examples.** When you encounter a defintion, it's a good idea to jot down an example and a non-example.

   With your team, come up with an example and non-example of a linear system based on page 1 of our textbook. Do not reuse the (non-)examples in the book! Make note of how many equations and how many unknowns are in your examples and non-examples.

   Why is your non-example *not* a linear system? Are the other ways to make a non-example?

   Continue reading until Example 1 on page 2. Make sure to fill out the definitions of the Key Terms you encounter.

   Before you read Example 1, see how the new terms relate to your example of a linear system. Does your linear system have a **solution**? If not, what term should we use to describe it? If so, what would you check next?
5. **Examples in the Textbook.** The authors include an example (Example 1) in the textbook on page 2.

   What are the authors trying to demonstrate with the example? That is, what's the point of the example and why is it being shown to the reader now?

   Write a note for yourself that "Example 1, page 2, is showing..."
6. **Stamina Time! Keep reading, practicing the habits above..** There is one more key term on page 2; don't miss it

   Pages 3-5 contain Examples 2-5. Repeat the previous exercise for each of these examples.

   After Example 5, the authors summarize some material and present figures with geometric interpretations. If you have questions about these figures, jot them down so you can ask about them in class (or on Zulip before class).

   Finish reading the chapter.
7. **Reading Comprehension: Trying Exercises.** Your professor has assigned some homework problems, but she did not assign number 1. Try that with individually and check with your teammates when you all finish.

   Your professor assigned problems 8, 12, and 16. Decide which group member will be the lead recorder for each problem (this role should rotate), and talk through the problems together.

   If you are stuck as a team, flag down your professor for help!

Key Terms

linear equation page 1
an equation like $ax+by=c$ where a,b,c are real (or complex) and x,y are variables. This can have any number of variables and coefficients (not just two).

solution of a linear equation page

linear system page

unknowns page

inconsistent system page

consistent system page

homogeneous system page

trivial solution page
Note: only applies to homogeneous systems

nontrivial solution page
Note: only applies to homogeneous systems

equivalent systems page

unique solution page

no solution page

infinitely many solutions page

manipulations on linear systerms page

method of elimination page
See also box on page 6!

Examples and Non-Examples

linear system of equations example
non-example

Group Excercises, Chapter 1, Sections 1-3

We'll review some of the basic concepts from Chapter 1, sections 1-3 in groups, with some exercises to turn in.

Exploring Linear Systems. For each of the following prompts, first try to construct an example (or decide that one cannot exist). Then reflect on what the question reveals about linear systems. Use vocabulary from the chapter in your reflections.

Worksheet Exercise 1 Try These!

(a) Write down an inconsistent *homogeneous* linear system in two unknowns, or explain why this is impossible.

(b) Find the solution(s) to the linear system, if any exist.

$$\begin{aligned} x + 4y &= 12 \\ 3x + 8y &= 4 \end{aligned}$$

(c) Write down a nonhomogeneous linear system with the trivial solution, or explain why this is impossible.

(d) Write down a linear system with three equations that is equivalent to the system below.

$$\begin{aligned} x + 4y &= 12 \\ 3x + 8y &= 4 \end{aligned}$$

(e) ***Turn-In!***

Find a and b so that the linear system below is consistent with infinitely many solutions.

$$\begin{aligned} ax + by &= 12 \\ 3x + 8y &= 4 \end{aligned}$$

Worksheet Exercise 2 Reflect. Which of these questions ask for a specific example, and which are really asking you to think about what *must* be true of certain kinds of linear systems? What properties distinguish homogeneous and nonhomogeneous systems?

Is it possible for a linear system to have exactly three solutions? Why or why not?

Worksheet Exercise 3 Computation Questions.. *Turn-In!*

For each linear system below, solve using the method of elimination to find its solutions or show that it has no solutions.

First system.

$$\begin{aligned} 3x + 4y - z &= 8 \\ 6x + 8y - 2z &= 3 \end{aligned}$$

Second system.

$$\begin{aligned} x + y &= 1 \\ 2x - y &= 5 \\ 3x + 4y &= 2 \end{aligned}$$

Third system.

$$\begin{aligned} 2x - y &= 5 \\ 4x - 2y &= t \end{aligned}$$

where t is any real number. For what value(s) of t is the system consistent? For what value(s) of t is the system inconsistent?

Worksheet Exercise 4 Special Forms.. For each linear system below, try to solve *without* using the method of elimination.

First system.

$$\begin{aligned} 2x + y - 2z &= -5 \\ 3y + 2z &= 7 \\ z &= 4 \end{aligned}$$

Second system.

$$\begin{aligned} 4x &= 8 \\ -2x - 3y &= -1 \\ 3x + 5y - 2z &= 11 \end{aligned}$$

Adding Matrices and Scaling Matrices.. In Section 2, we see the definition of an $m \times n$ matrix with entries $a_{i,j}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m\times 1} & a_{m\times 2} & \cdots & a_{m\times n} \end{bmatrix} = [a_{i,j}] \in M_{m\times n}.$$

This matrix has m rows and n columns, and $M_{m\times n}$ is the shorthand for the set of all such matrices (we're defaulting to real entries now).

Worksheet Exercise 5 True or False? Determine whether each statement is true or false.

(a) It is possible that

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} g & h \\ k & \ell \end{bmatrix}.$$

(b) The matrices below are equal.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5^0 & 6/3 \\ 4-1 & 2^2 \end{bmatrix}.$$

(c) There exist real numbers a and b such that

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

(d) There exists a real number x such that

$$\begin{bmatrix} x & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & x \end{bmatrix}.$$

Worksheet Exercise 6 Isolating Rows and Columns. Sometimes it's helpful to talk about a row or column of a matrix instead of the whole thing. Fill out the entries for the following based on the matrix A above. A row is sometimes called a **row vector** and a column is sometimes called a **column vector**. In this class, if we say "vector", we (almost always) mean a column vector.

$$\mathrm{row}_i(A) = [\qquad\qquad\qquad\qquad]$$

and

$$\mathrm{col}_j(A) = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

This is a good time to review the definitions of matrix addition ("entrywise" is how people talk about this sometimes), scalar multiplication, and subtraction (a combination of addition and multiplication, if you like), and transposes. What does it mean to be a zero matrix from the point of view of matrix addition? Can a matrix have an additive inverse?

Theorem 7 Existence of the Additive Identity. *The $m \times n$ matrix of all zeros, Z, is the additive identity for the set of $m \times n$ matrices.*

That is for each $A \in M_{m \times n}$,

$$A + Z = Z + A = A.$$

Proof. Let m, n be positive integers. Assume $A \in M_{m \times n}$, and assume $Z \in M_{m \times n}$ is the matrix of all zeros.

Try find a way to use the definition of addition to get to the end of the proof (the next line). Since matrix equality happens at the entry level, you will probably have define $A = [a_{i,j}]$ and $Z = [z_{i,j}]$ for all $1 \le i \le m$, $1 \le j \le n$, and note that $z_{i,j} = 0$ for each entry. Now use the definition of addition: what is the i, j-th entry of $A + Z$?

Therefore, $A + Z = Z + A = A$. ■

Is $Z^T = Z$? How would you prove it?

Once you work those questions out, talk about the defintion of *matrix multiplication*. That is, explain why the following is the definition; what is the point of defining matrix multiplication this way? Probably has something to do with linear systems, right?

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \end{bmatrix}$$

Practice: Matrix Operations. In Sections 2 and 3 we saw how the basic operations on matrices are defined. Let's practice a few examples involving addition, scalar multiplication, transposes, and matrix multiplication.

Worksheet Exercise 8 Matrix Operations.

(a) Add the matrices below.

$$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ -1 & 2 \end{bmatrix}$$

(b) Decide whether the following sum is defined. If it is, compute it.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}$$

(c) Compute the scalar multiple below.

$$3 \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$$

(d) Simplify the expression below.

$$-\tfrac{1}{2} \begin{bmatrix} 4 & -6 \\ -2 & 8 \end{bmatrix}$$

(e) Compute the product below, if it is defined.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

(f) Decide whether the following product is defined. If it is, compute it.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

(g) ***Turn-In!***

What does the matrix multiplication below represent?

$$\begin{bmatrix} 1 & 3 & 2 \\ -1 & 9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

Which Matrix Products Are Well-Defined?

Consider the matrices below.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 3 \\ -2 & 0 \\ 4 & -4 \end{bmatrix}, \qquad C = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

Worksheet Exercise 9 Determine which of the following matrix products are *well-defined.* If a product is well-defined, compute it. If it is not, explain why not.

1. AB
2. BA
3. AC
4. BC
5. CB^T
6. $C^2 = CC$ (Do you have any conjectures about C^n?)
7. $(AB)C$
8. $A(BC)$

Worksheet Exercise 10 Based on the previous example, determine whether each statement below is true or false. Justify your answers.

1. "Matrix multiplication is *commutative.*"
2. "Matrix multiplication is *associative.*"

Worksheet Exercise 11 Preparing for the Matrix–Vector Product Theorem. *Theorem (Matrix–Vector Product Theorem, MVP).* Let

$$A = \begin{bmatrix} a_{i,j} \end{bmatrix} \in M_{m\times n}, \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in M_{n\times 1}.$$

Then the product $A\mathbf{x}$ is well-defined, and

$$A\mathbf{x} = \sum_{i=1}^{n} x_i \operatorname{col}_i(A).$$

Worksheet Exercise 12 *Assumptions:*

- $A \in M_{m\times n}$
- $\mathbf{x} \in M_{n\times 1}$

Relevant definitions:

- Matrix multiplication
- Linear combinations
- Matrix equality

Goal: Starting from the assumptions and using the relevant definitions, show that $A\mathbf{x} = \sum_{i=1}^{n} x_i \operatorname{col}_i(A)$.

Worksheet Exercise 13 Finishing the Proof of the Matrix–Vector Product Theorem. *Turn-In!*
The proof below is partially completed. Work together to finish it. Be explicit about where each definition is used.

Proof. By the definition of matrix multiplication, the product $A\mathbf{x}$ is an $m \times 1$ matrix, and

$$A\mathbf{x} = \begin{bmatrix} x_1a_{11} \\ x_1a_{21} \\ \vdots \\ x_1a_{m1} \end{bmatrix} + \begin{bmatrix} x_2a_{12} \\ x_2a_{22} \\ \vdots \\ x_2a_{m2} \end{bmatrix} + \cdots + \begin{bmatrix} x_na_{1n} \\ x_na_{2n} \\ \vdots \\ x_na_{mn} \end{bmatrix}.$$

Using the definition of matrix addition, this can be written as

$$\sum_{i=1}^{n} \begin{bmatrix} x_ia_{1i} \\ x_ia_{2i} \\ \vdots \\ x_ia_{mi} \end{bmatrix}.$$

By the definition of scalar multiplication, we may factor out x_i from each term, leaving $\text{col}_i(A)$. Therefore,

$$A\mathbf{x} = \sum_{i=1}^{n} x_i\text{col}_i(A),$$

as desired. QED. ■

More Turn-In!

In addition to the problems marked as ***Turn-In*** above, you should do the following questions and turn them in with your group on ONE piece of paper with EVERYONE's name. It's okay if you don't quite finish all of these in class.

I recommend trying them outside of class if you don't finish, though you don't need to turn them in.

Section 1.1 14, 34

Section 1.2 4, 6 and 8 (see the description of the matrices before this problem on the previous page), 13

Section 1.3 2, 11, 22, 24