Writing Assignment #NUMBER   due DATE

AUTHORS

Suppose that ℓ is a line with a slope of m that passes through the point (x_0, y_0). Traditionally, we write lines in "slope-intercept" form, $y = mx + b$, where m the ordered pair $(0, b)$ is the point on the line where it passes through the y-axis. The point-slope formula for our line can be manipulated to produce the slope-intercept form using the properties of real number addition and multiplication. Indeed, $y - y_1 = m(x - x_1)$ implies $y = mx + (y_1 - mx_1)$, from which we see that $b = y_1 - mx_1$.

Consider the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$. Many people know of the existence of the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{1}$$

for finding the solutions to this equation.

Proposition (Chapter 1, #36). For all positive integers n, three divides $4^n - 1$.

Proof. □

*The next problems involve **binomial coefficients**, which are defined in our book right around where these problems are stated. They're the numbers that show up in Pascal's Triangle (still attributed to Blaise Pascal for some reason even though it was known millenia before by at least 3 distinct civilizations…*

Lemma (Chapter 1, #41). For all positive integers n and r satisfying $0 < r \leq n$, the binomial coefficients satisfy the equality

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

(where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ and $0! = 1$ by definition).

You can use the above lemma without proving it. You can also use the result that binomial coefficients are integers without proof.

do the assigned problems here

Proposition (Chapter 1, #43).

Proof. □

Proposition (Chapter 1, #44).

Proof. □

Complete the text of the example below.

Example (Chapter 1, #44). The Binomial Theorem helps us expand $(x+1)^5$. Indeed,

$$
\begin{aligned}
(x+1)^5 &= \sum_{k=0}^{5} \binom{5}{k} x^{5-k} 1^k \\
&= \binom{5}{0} x^5 + \binom{5}{1} x^4 + \ldots + \binom{5}{5} \\
&= x^5 + 5x^4 + \ldots + 1.
\end{aligned}
$$

Tell your reader what you notice about the middle coefficients; use #43 for inspiration here. Hint: 5 is prime.

Lemma (Ch. 1, #39)**.** For all nonzero integers m and n, if $\gcd(m, n) = 1$ and n divides mk for some integer k, then n divides k.

You don't have to prove this lemma, but it's a good practice problem before the first exam.

Proposition (Ch. 3, #53)**.** ...