CEE 630 / ME 525
Nonlinear Finite Element Analysis
Spring 2017
Professor Guglielmo Scovazzi
Homework # 2
Due Thursday, February 16 2017.

1. Consider a variation of the 1D nonlinear elastic rod problem, in particular

$$\left\{ \begin{array}{c} \sigma_{,x} + f = 0 \quad \text{for } x \in [0, L] \\ u = g \quad \text{at } x = 0 \\ \sigma = h(u) \quad \text{at } x = L \end{array} \right\} \qquad (\mathcal{S})$$

where h is a nonlinear function of the displacement u. Derive the weak form for this case, as well as the expressions for the (element-level) internal and external force vectors, and tangent stiffness matrix. State whether or not the resulting tangent stiffness matrix is symmetric. You can assume (as in class) that the stress $\sigma = \hat{\sigma}(\epsilon)$, is a nonlinear function of strain.

2. Download the Matlab routines posted on Sakai for the nonlinear elastic rod. The code is written to complete a NR-iteration for a single load step. Modify the code to incorporate an incremental loading scheme, where the number of loading steps is an input. Apply the incremental scheme to the boundary conditions as well as the body force. You will need to modify the main code as well as a few of the subroutines. Submit (via email) any modified routines.

3. Consider the nonlinear scalar equation $N(d) = F$. Assume:

- $N(d)$ is smooth (all derivatives can be computed)
- $N'(d) \neq 0$ for all d
- $|N'(x)^{-1} N''(y)| \leq 2c$, where c is a constant, for all x, y
- $|ce^{(0)}| < 1$, where $e^{(0)} = d^{(0)} - d$ (i.e., our first iterate for d is sufficiently close)

Show (using Taylor's theorem) that Newton-Raphson iteration converges and that the order of convergence is 2.

4. Consider the following system of nonlinear equations, containing two equations and two unknowns:

$$\mathbf{N}(\mathbf{d}(t)) = \mathbf{F}^{ext}$$

where $\mathbf{d}$ represents ths unknown solution vector (parameterized by t), and $\mathbf{F}^{ext}$ is prescribed by

$$\mathbf{F}^{ext} = \left\{ \begin{array}{c} 56t \\ 132t \end{array} \right\}$$

Take $\mathbf{N}$ to be a vector-valued nonlinear function of $\mathbf{d}$ as

$$\mathbf{N}(\mathbf{d}) = \left\{ \begin{array}{c} d_1 + 4d_2^3 + 5d_2^2 \\ d_1^3 + 3d_1^2 + 10d_2 \end{array} \right\}$$

Write a Newton-Raphson solver for this problem, using an incremental loading procedure. Have the solver plot d_1 and d_2 as functions of the load parameter t, for $t \in [0, 1]$. Submit (via email) your solver.