Homework 2

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1 Problem 1

$$\sigma,_x + f = 0$$

$$u = g \ at \ x = 0$$

$$\sigma = h \ at \ x = L$$

Weak form:

$$\int_0^L w * (\sigma,_x + f)dx = 0$$

By applying the divergence theorem we obtain,

$$-\int w,_x \sigma dx + (w\sigma)\Big|_0^L + \int_0^L wf dx = 0$$

Now applying the boundary conditions and noting the $\forall w \in V \ w(0) = 0$ yields,

$$\int_0^L w,_x \sigma dx = \int_0^L wf dx + w(L)h$$

As usual, let us discretize:

$$u^h = sum_{A=2}^{n_{el}+1} N_A(x)d_A + gN_1(x)$$

$$w^h = sum_{A=2}^{n_{el}+1} N_A(x)c_A$$

Plugging into the weak form gives us,

$$\int_0^L w^h,_x \hat{\sigma}(u^h,_x)dx = \int_0^L w^h f dx + w^h(L)h$$

Hence,

$$F^{int} = \int_0^L w^h,_x \hat{\sigma}(sum_{B=1}^{n_{el}+1} N_B,_x (x)d_B)dx$$

$$F^{ext} = \int_0^L N_A(x) f dx + N_A(x)(L)h$$

Note that $A = 2, ...n_{en} + 1$ and d_i is such that $d_1 = g$.

As for the tangent stiffness matrix:

$$\begin{aligned} K^e_{ab} &= \frac{\partial f^{int\ e}}{\partial d^e_b} \\ &= \frac{\partial}{\partial d^e_b}\Big[\int_{x_1}^{x_2} N_{a,x}(x)\hat{\sigma}\big(\sum_{c=1}^{N_{en}} N_{c,x}\, d^e_c\big)dx\Big] \\ &= \int_{x_1^e}^{x_2^e} N_{a,x}\frac{\partial\hat{\sigma}}{\partial\epsilon}\big(\sum_{c=1}^{N_{en}} N_{c,x}\, d^e_c\big)\frac{\partial}{\partial d^e_b}\big(\sum_{c=1}^{N_{en}} N_{c,x}\, d^e_c\big)d^e_b dx \\ &= \int_{x_1^2}^{x_2^e} N_{a,x}\, E(\epsilon\Big|_{d=d^e})N_{b,x}\, dx \end{aligned}$$

Although it at first appears that the consistent tangent is symmetric, we actually are unable to claim such due to the nonlinearity of E.

2 Problem 2

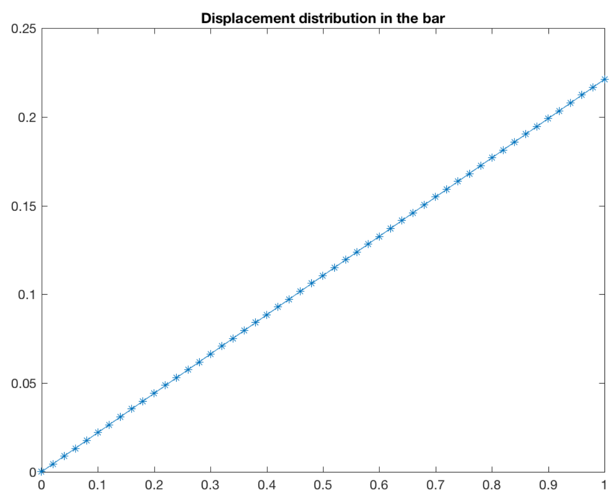


Figure 1: final solution with 100 loading steps

3 Problem 3

Show (using Taylor's theorem) that Newton-Raphson itertaion converges and that the order of convergence is 2.
Solution: Let R be the residual.
By Taylor's theorem truncated at the second derivative we have,

$$0 = R + R'\Delta d + \frac{R''}{2}\Delta d^2$$

subtracting both sides by R and dividing by R'^{-1} yields,

$$R'^{-1}R = \Delta d + R'^{-1}R''\frac{\Delta d}{2}$$

Further isolating Δd and noting that it is equal to $d - d^i$ we have,

$$(d^i - R'^{-1}R) - d = R'^{-1}R''\frac{(d^i - d)^2}{2}$$

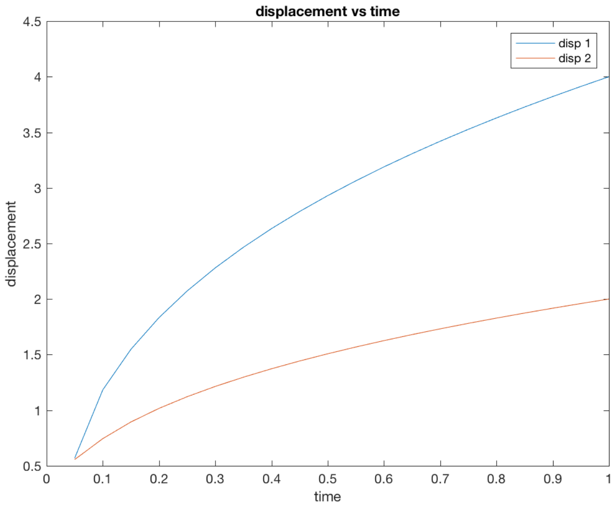
Note that $d^i - R'^{-1}R = d^{i+1}$ and $d^i - d = e_i$. Hence we arrive at the following:

$$e_{i+1} = e_i^2\frac{R'^{-1}R''}{2}$$

We are given that $\frac{R'^{-1}R''}{2} \leq c$ hence we have the following quadratic convergence:

$$\frac{e_{i+1}}{e_i^2} < c$$

4 Problem 4



displacement vs time
disp 1
disp 2
displacement
time
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1