HOMEWORK 7

CARTER RHEA

1. Problem 1

Statement: Show that the Q8 element does not present the problem of shear locking. Proceed in the same manner as was shown for the Q4 element in class. That is, use the given continuous displacement field to compute displacements at nodes, calculate strains in the element using the element interpolation functions, and then compare these to the exact solution.
Solution:
From class we know:

$$U_x = xcy$$

$$U_y = \frac{-cx^2}{2}$$

$$\mathcal{E}_{xx} = \frac{\partial U_x}{\partial x} = cy$$

$$\mathcal{E}_{yy} = \frac{\partial U_y}{\partial y} = 0$$

$$\mathcal{E}_{xy} = \frac{1}{2}\Big[\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y}\Big]$$

Date: November 28, 2016.

We shall define U similarly as was done in class,

$$U = \begin{Bmatrix} 1 \\ -1/2 \\ -1 \\ -1/2 \\ 1 \\ -1/2 \\ -1 \\ -1/2 \\ 0 \\ 0 \\ 0 \\ -1/2 \\ 0 \\ 0 \\ 0 \\ -1/2 \end{Bmatrix}$$

The following is my Matlab code:

```
    x = sym('x'); y = sym('y'); c = sym('c');
    d = c*[1;-1/2;-1;-1/2;1;-1/2;-1;-1/2;0;0;0;-1/2;0;0;0;-1/2];
    N_1 = -(1/4)*(1-x)*(1-y)*(1+x+y);
    N_2 = -(1/4.)*(1+x)*(1-y)*(1-x+y);
    N_3 = -(1/4.)*(1+x)*(1+y)*(1-x-y);
    N_4 = -(1/4.)*(1-x)*(1+y)*(1+x-y);
    N_5 = (1/2.)*(1-x^2)*(1-y);
    N_6 = (1/2.)*(1+x)*(1-y^2);
    N_7 = (1/2.)*(1-x^2)*(1+y);
    N_8 = (1/2.)*(1-x)*(1-y^2);

%The following will create a vector of shape functions given a point
    N = zeros(2,16);
    N = sym('N');
    N(1,1) = N_1;
    N(1,3) = N_2;
    N(1,5) = N_3;
    N(1,7) = N_4;
    N(1,9) = N_5;
    N(1,11) = N_6;
    N(1,13) = N_7;
    N(1,15) = N_8;
    N(2,2) = N_1;
    N(2,4) = N_2;
    N(2,6) = N_3;
    N(2,8) = N_4;
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    N(2,10) = N_5;
    N(2,12) = N_6;
    N(2,14) = N_7;
    N(2,16) = N_8;

    U = N*d;
    U = simplify(U);
    E_xx = diff(U(1),x);
    E_yy = diff(U(2),y);
    E_xy = 1./2*(diff(U(2),x)+diff(U(1),y));
```

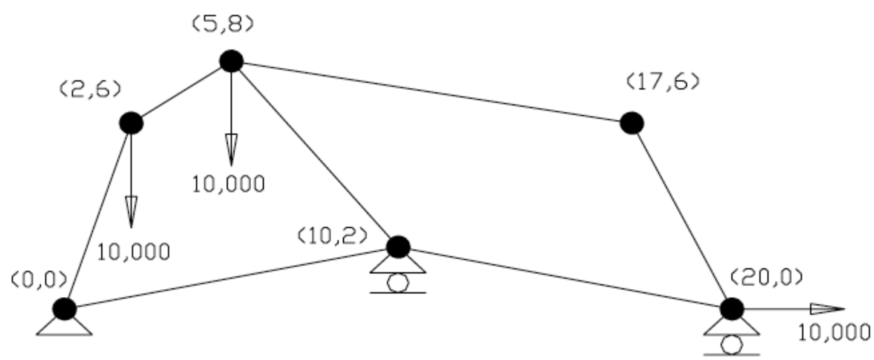
This will yield

$$U_x = c * x * y \ ; \ U_y = -(c * x^2)/2$$

$$\mathcal{E}_{xx} = c * y \ ; \ \mathcal{E}_{yy} = 0 \ ; \ \mathcal{E}_{xy} = 0$$

2. Problem 2

Complete the Matlab code provided with this homework and compute the nodal displacements for the assembly of elements given below. You are given the main script in a file called FEScript.m.



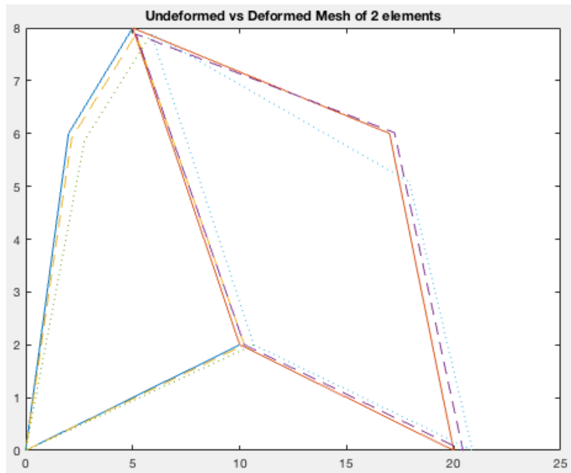
I wont include my code here because that would be insane, but I will include an image showing the final displacement of the nodes

In the configuration in Problem 2, determine the eigenvalues and eigenvectors of the stiffness matrix for these two cases.

(1) Full Integration
(2) Reduced Integration

Plot the eigenvectors and report the number of zero eigenvalues in each case. Is the number of zero eigenvalues as expected? Provide a brief justification to your answer.

Figure 1. The solid line is the undeformed mesh, the dashed line is the mesh deformed by my code, and the dotted line is the mesh deformed by ANSYS.



3. Problem 3

3.1. Full Integration.

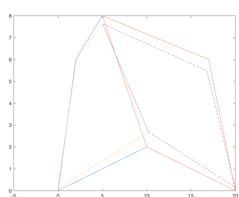


Figure 2. Eigenvalue 1

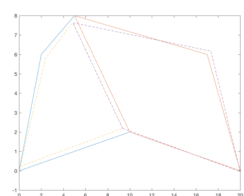


Figure 3. Eigenvalue 2

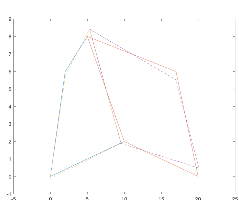


Figure 4. Eigenvalue 3

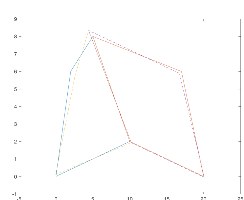


Figure 5. Eigenvalue 4

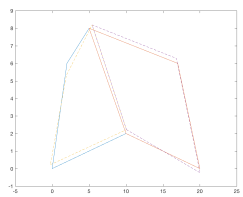


FIGURE 6. Eigenvalue 5

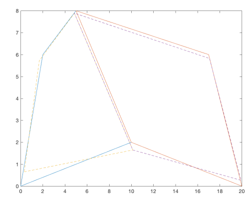


FIGURE 7. Eigenvalue 6

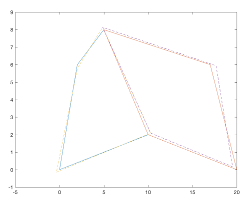


FIGURE 8. Eigenvalue 7

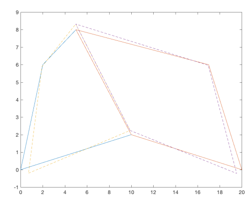


FIGURE 9. Eigenvalue 8

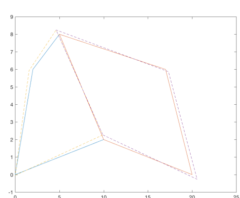


FIGURE 10. Eigenvalue 9

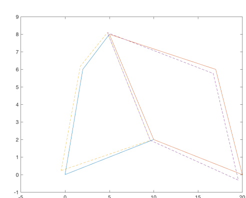


FIGURE 11. Eigenvalue 10

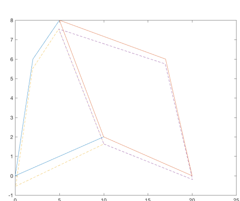


FIGURE 12. Eigenvalue 11

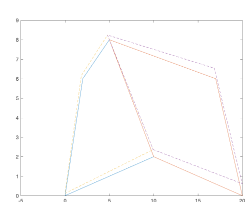


FIGURE 13. Eigenvalue 12

3.2. **Reduced Integration.**

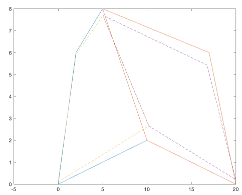


FIGURE 14. Eigenvalue 1

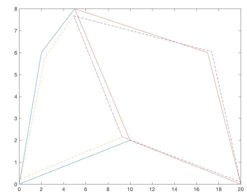


FIGURE 15. Eigenvalue 2

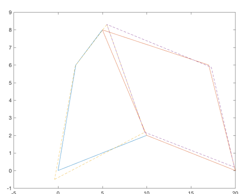


FIGURE 16. Eigenvalue 3

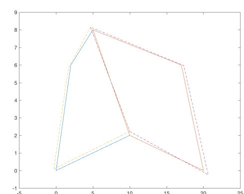


FIGURE 17. Eigenvalue 4

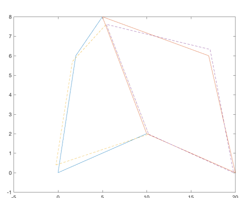


FIGURE 18. Eigenvalue 5

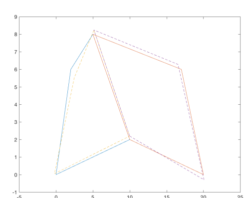


FIGURE 19. Eigenvalue 6

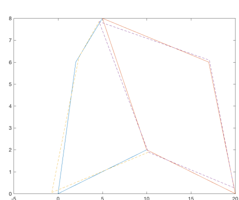


FIGURE 20. Eigenvalue 7

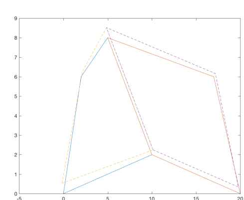


FIGURE 21. Eigenvalue 8

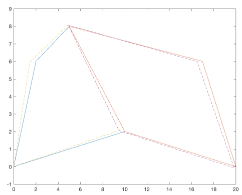


FIGURE 22. Eigenvalue 9

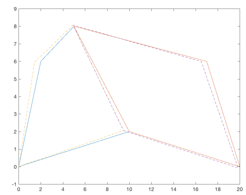


FIGURE 23. Eigenvalue 10

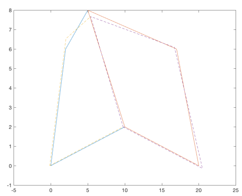


FIGURE 24. Eigenvalue 11

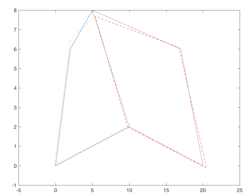


FIGURE 25. Eigenvalue 12

3.3. **Reduced Integration.**