HOMEWORK 4

CARTER RHEA

1. Problem 2

Problem statement: Show that the element stiffness matrix defined as $[K^e] = \int_{\Omega_e} [B^e]^T E [B^e] dx$ is not invertible by providing one example of a non-zero vector that is in the kernel of $[K^e]$.

So we want to find a v such that $[K^e]v = 0$. This yields,

$$[K^e]v = \int_{\Omega_e} [B^e]^T E [B^e] v dx$$

Just looking at the $[B^e]v$ portion we can expand this out to see that we have

$$[N_1' \dots N_q'][v^e]$$

where v is a column vector of dimension q, so $v = \begin{pmatrix} v_1 \\ \vdots \\ v_q \end{pmatrix}$.

There we get

$$[N_1' \dots N_q'] \begin{pmatrix} v_1 \\ \vdots \\ v_q \end{pmatrix} = N_1' v_1 + \dots N_q' v_1$$

Here we are summing up the N_i's which we calculated earlier to be equal to zero. Hence if we allow v to simply be a vector of a single constant $v = \begin{pmatrix} c \\ \vdots \\ c \end{pmatrix}$

Thus we have $c \sum N_i' = 0$.

Therefore any v vector that is just constants will be in the kernel of $[K]$.

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