Axiomatic Geometry Homework 1: Propositions 2.3,2.4,2.5

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1. Proposition 2.4 For every point there is at least one line not passing through it

    (a) Let P_0 be an arbitrary point.

    (b) Then there exists two other points, P_1 and P_2, such that P_0,P_1, and P_2 are not colinear. (I-3)

    (c) There exists a line, $\overleftrightarrow{P_1 P_2}$. (I-1)

    (d) Thus, $\overleftrightarrow{P_1 P_2}$ does not pass through P_0. (Steps 2 and 3).

    (e) Therefore, since P_0 was an arbitrary point, for every point there is at least one line not passing through it.

2. For every point, P, there exists at least two lines through it.

    (a) Let P_0 be an arbitrary point.

    (b) Then there exists two other points, P_1 and P_2, such that P_0,P_1, and P_2 are not colinear. (I-3)

    (c) There exist lines $\overleftrightarrow{P_0 P_1}$ and $\overleftrightarrow{P_0 P_2}$ (I-1)

    (d) Thus, since P_0 was arbritrary, for every point, P, there exists at least two lines through it.

3. 9d) Fix a sphere in Euclidean three-space. Two points on the sphere are called antipodal if they lie on a diameter of the sphere; e.g., the north and south poles are antipodal. Interpret a "point" to be a set P, PI consisting of two antipodal points on the sphere. Interpret a "line" to be a great circle C on the sphere. Interpret a "point" P, PI to "lie on" a "line" C if one of the points P, pI lies on the great circle C (then the other point also lies on C). Determine if I-1,I-2,and I-3 hold. Also determine the parallel property.

    (a) I-1: I-1does hold because it is possible to find two antipodal pairs which are incident on the same great circle. If you find two antipodal pairs and draw a great circle through only one point in each pair, you will end up drawing the line through the other point in the pair, thus connecting the two pairs with a line.

    (b) I-2: I-2 would hold because great circles would go through at least two antipodal points.

(c) I-3: I-3 hold because it is possible to find 3 points which are not colinear since you could pick 3 points lying on different diameters and then not be on the same great circle.

(d) Parallel: The space would have Elliptical parallel property. If you had two antipodal pairs, two points, and draw a great circle through them, which would be a diameter, then they would have to cross and thus, there would be no parallels.