Math 295: Take Home Test 3

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1. Let $A = \mathbb{R}$. Define $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = x^{3n} \text{ for some } n \in \mathbb{Q}\}$
   Prove that R is symmetric and transitive.
   Symmetric: Suppose $(x, y) \in R$. By the definition of R, this means $\exists n \in \mathbb{Q}(y = x^{3n})$. Then $\exists n \in \mathbb{Q}(x = y^{3n})$, thus $(y, x) \in R$ since there exists an n which satisfies the relation. Since (x, y) was an arbitrary element of R, R is symmetric.
   Transitive: Suppose $(x, y) \in R$ and $(y, x) \in R$. Then $\exists n \in \mathbb{Q}(y = x^{3n})$ and $\exists m \in \mathbb{Q}(z = y^{3m})$. Thus, $\exists n \in \mathbb{Q} \exists m \in \mathbb{Q}(x^{3n} = z^{\frac{m}{3}})$. Then, $\exists n \in \mathbb{Q} \exists m \in \mathbb{Q}(z = x^{3(m+n)})$. Substituting $s = m + n, \exists s \in \mathbb{Q}(z = x^{3s})$, so $(x, z) \in R$, as required. Q.E.D.

2. Suppose that R_1 and R_2 are equivalence relations on a set A. Prove that $R_1 \cap R_2$ is also an equivalence relation on A.
   $R_1 = \{(x, y) \in A \times A \mid xR_1y\}$
   $R_2 = \{(x, y) \in A \times A \mid xR_2y\}$
   $R_1 \cap R_2 = \{(x, y) \in A \times A \mid xR_1y \cap xR_2y\}$
   Proof: Reflexive: Let $a \in A$. Then $(a, a) \in R_1$ because R_1 is reflexive, and $(a, a) \in R_2$ because R_2 is reflexive. So $(a, a) \in R_1 \cap R_2$. Thus $R_1 \cap R_2$ is reflexive, as required.
   Symmetric: Let $(a, b) \in R_1$. Since R_1 is symmetric, $(b, a) \in R_1$. Also let $(a, b) \in R_2$. Since R_2 is symmetric, $(b, a) \in R_2$. So $(b, a) \in R_1 \cap R_2$. Thus, $R_1 \cap R_2$ is symmetric, as required.
   Transitive: Let $(a, b) \in R_1$ and $(b, c) \in R_1$. Since R_1 is transitive, $(a, c) \in R_1$.Also, let $(a, b) \in R_2$ and $(b, c) \in R_2$. Since R_2 is transitive, $(a, c) \in R_2$. So $(a, c) \in R_1 \cap R_2$. Thus $R_1 \cap R_2$ is transitive, as required.
   Therefore, $R_1 \cap R_2$ has been shown to be an equivalence class on A. Quod Erat Demonstratum.

3. Let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. Find:

   (a) $f([-1, 2]) = [1, 4]$.
   (b) $f^{-1}(9)$
   $f^{-1}(x) = \pm x^{1/2}$.
   So $f^{-1}(x) = \pm\{3\}$
   (c) $f^{-1}(\{x \mid x > 9\}) = (-\infty, -3) \cup (3, \infty)$