# Computational Project: Numerical Methods for ODE Initial Value Problems

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We consider ordinary differential equations of the form

$$\frac{du}{dt} = f(t, u), \qquad a \leq t \leq b$$

where the unknown function $u = u(t)$ is initially real valued, but later both $u(t)$ and $f(t, u)$ can be vector valued. The associated initial value problem is

$$u(a) = u_0$$

where again, $u_0$ is initially a single number, but later will be a vector.

The simplest method, discussed in most textbook on DE's, is Euler's method, so this should be implemented and tested first, starting with

### Test Case 1

$\frac{du}{dt} = -u, \quad 0 \leq t \leq 2, \quad u(0) = 2$, with the solution $u(t) = 2e^{-t}$. When an error tolerance is used, start with $10^{-2}$.

Errors can be estimated by comparing the results with two different step sizes, one twice the other, so a slow but fairly reliable can be written based on repeatedly halving the step size until the estimated error satisfies the specified tolerance. The program should also measure cost in terms f time taken and floating point [arithmetic] operations performed.

Euler's method is very inefficient, so successive improvements will later be tried, such as the modified Euler and "classical" Runge-Kutta methods.

Some methods considered should also be extended to systems of equations, at least systems of two equations.

**Reference:** *Numerical Analysis* by Sauer, Sections 6.1 to 6.4.