Analyzing The 15 Puzzle and Others

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Introduction

We are exploring the configurations of the 15 puzzle. This game was invented in late 1800's. We analyzed the game using concepts such as groups, modules, configurations and graph theory; even though the game looks simple, the mathematics behind the game are complex. The total number of configurations is 10461394944000.

Main Problem

The purpose of our research is to study the game by finding all the possible configurations. We are to analyze and describe the game in mathematical form. For the 15 Puzzle, we know that a position τ can be achieved starting from a position σ, according to

Theorem

$$\tau \in G^{\sigma} \Leftrightarrow \sigma \in G^{\tau}$$

Figure 1. The 15 Puzzle

Definitions

A game is **linear** if there are two squares adjacent to only one square and any other square is adjacent to two squares. A **non-linear** game is non-linear if and only if every pair of squares is adjacent to more than one square or if there is a square that is adjacent to exactly one square or more than two squares.

Main Topic

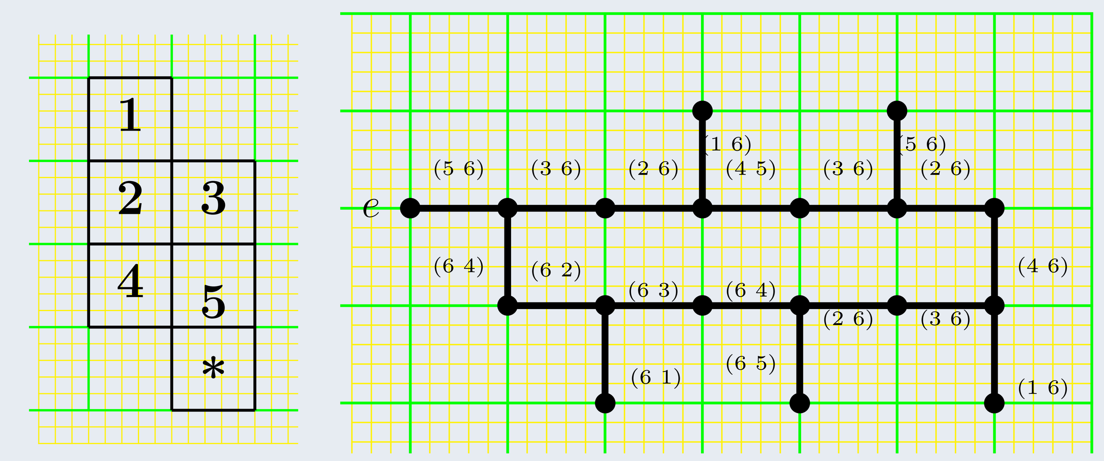

Figure 2. A made up puzzle and its corresponding graph

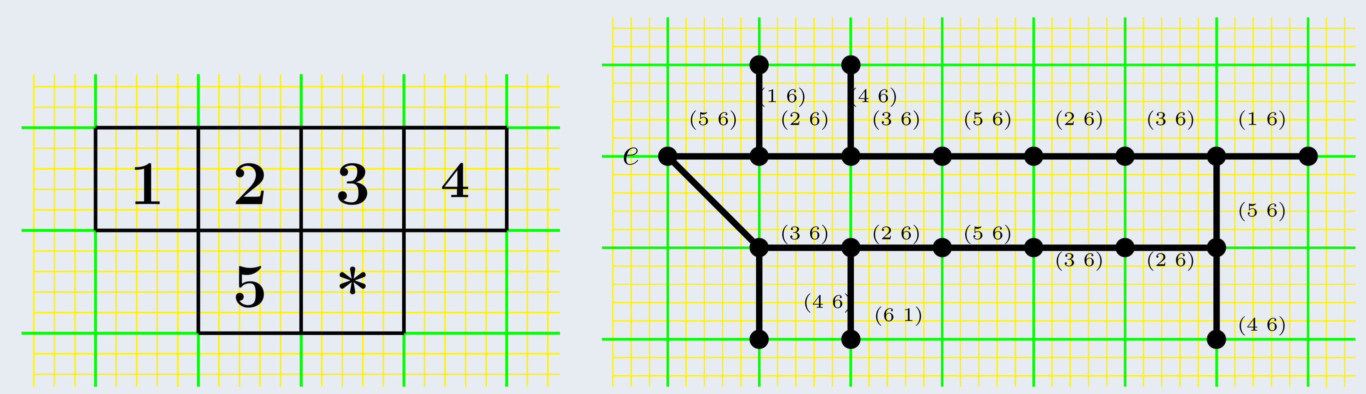

Figure 3. Another made up puzzle and its corresponding graph

Game Description

The **15 Puzzle** is a a 4x4 square. Each square contains a number from one to fifteen, and there is a blank square, number 16. This game consists of sliding little squares into the blank square. In this way, each move of this game may be regarded as a permutations of the integers 1 through 15.

Future Work

Which positions are possible to achieve and which are not starting from an initial configuration? What particular types of games can be achieved such that their associating graphs are isomorphic? How can we generalize similar games as the 15 Puzzle in more than two-dimensions?

Our Conjectures and Analysis

If the empty spot of the game is adjacent to an odd number of squares and the graph has a cycle, then the cycle does pass by e.	*True*
If the game is linear with n squares then its graph is a path with n vertices.	*True*
If a game is non-linear its graph must have a vertex degree 2 or more.	*True*
If a game is non-linear its graph must have at least one vertex with degree three.	*False*
There is a non-linear game whose graph is almost a path.	*True*
Any game with 3 or less squares is linear.	*True*

Conclusions

We have learned that mathematical representations of games such as the 15 Puzzle can be used to comprehend why certain configurations are possible or not and more. Analyzing the game was kind of fun.

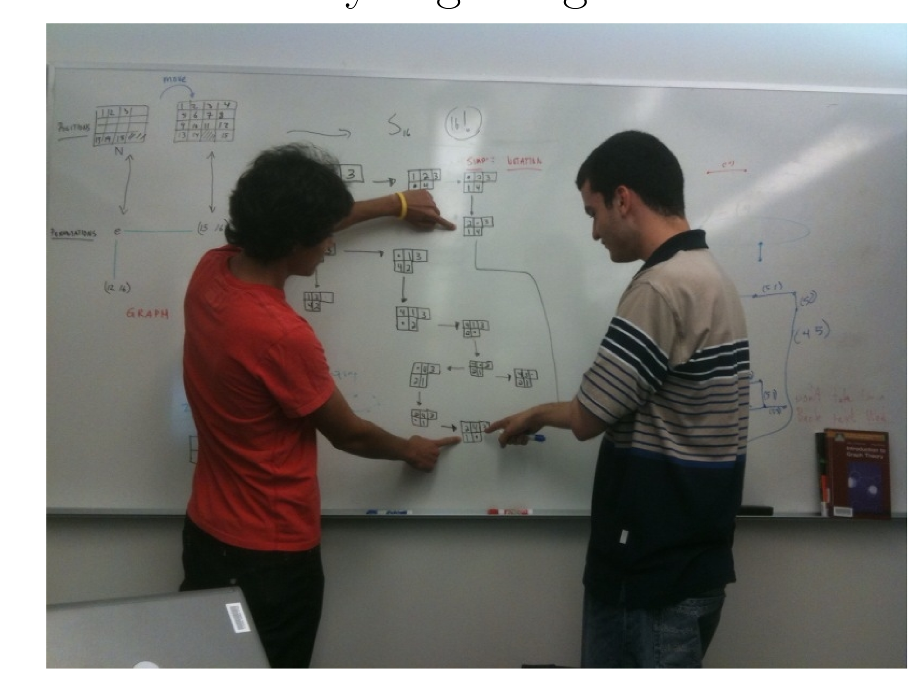

Figure 4. Cesar Lopez and Eduardo Reynoso

Acknowledgements

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References

[1] Gary Chartrand & Ping Zhang, Introduction to Graph Theory, McGraw-Hill Companies,Inc; 2005.

[2] David S. Dummit & Richard M. Foote, Abstract Algebra, John Wiley and Sons, Inc; 2004.

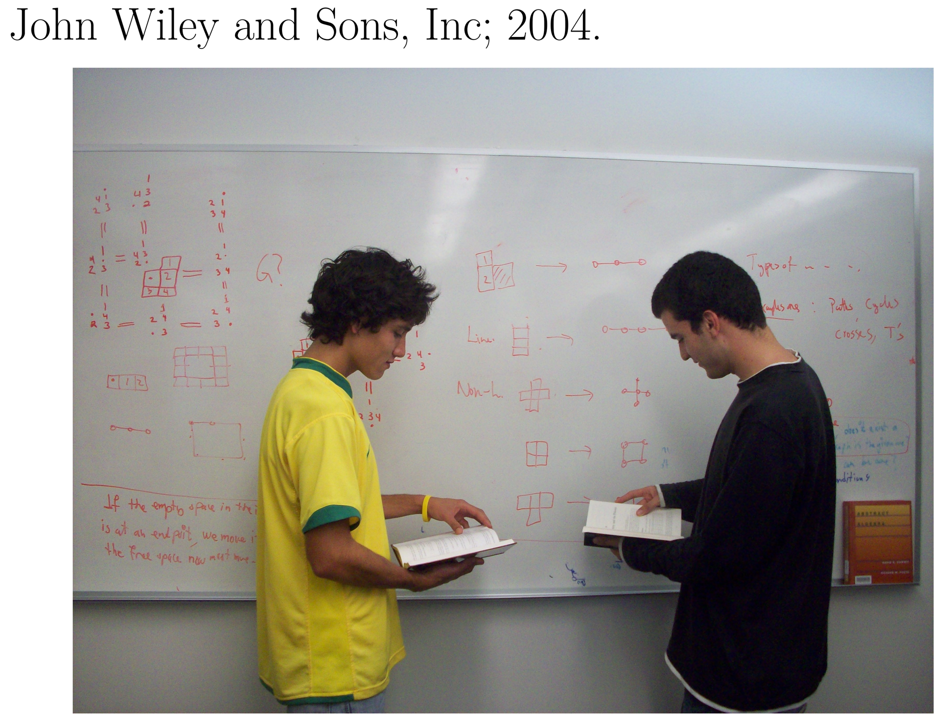

Figure 5. Team JEC