Probability cheatsheet

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1. Probability expresses uncertainty:

$P(X|C) = 0$ => "X definitely not true"

$P(X|C) = 1$ => "X definitely true"

$0 < P(X|C) < 1$ => we are uncertain about X

Extends standard logic

2. All probability is conditional:

$P(X)$ always means $P(X|C)$

"probability of X" "probability of X given C"

where C represents background information (i.e. a model).
C is often dropped from notation but is always there.

e.g. P(⚅ | die is fair) = 1/6 (Model of die) P(transmission | infection rates) (Model of infection) etc.

3. Probability is computable:

I. $P(!A|C) = 1 - P(A|C)$ ("not A") (A either occurs or doesn't.)

II. $P(A \text{ or } B|C) = P(A|C) + P(B|C) - P(A,B|C)$ ("A and B given C") (One of them must occur, but don't double-count)

III. $P(A,B|C) = P(A|B,C) \cdot P(B|C)$ (Split up joint probabilities into conditional ones.)

"A and B given C" "A given B and C" "B given C"

NB. "independence" happens when the middle term simplifies: P(A|B,C)=P(A|C)

4. These generate practically useful tools for computation:

$$P(A|C) = \sum_X P(A,X|C)$$

Computes a probability by summing (or integrating) over all the possible values of another variable.

Often seen as: $P(A|C) = \sum_X P(A|X,C) \cdot P(X|C)$ by applying rule III

Bayes' rule: $P(A|B,C) = \dfrac{P(B|A,C) \cdot P(A|C)}{P(B|C)}$

Computes P(A given B) in terms of P(B given A)