

**ENECO 630 – MACROECONOMÍA I**

**DESEMPLEO**

**CÁTEDRAS D2**

**MODELO DE DIAMOND-MORTENSEN-PISSARIDES**

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Esta versión: Mayo 5, 2022.

Enfoque de Flujos a los Mercados Laborales

Modelo de Diamond-Mortensen-Pissarides (DMP)

Modelo DMP y la Evidencia

Eficiencia en el Modelo DMP

# Enfoque de Flujos a los Mercados Laborales

## ENFOQUE DE FLUJOS AL MERCADO LABORAL

En todo período hay, simultáneamente:

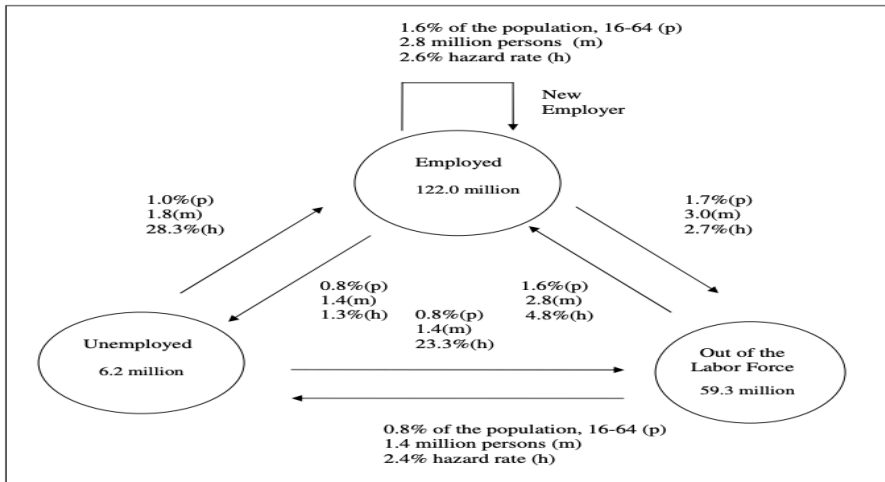
- ▶ Personas que encuentran empleo.
- ▶ Personas que pierden su empleo.

Luego:

$$\text{Cambio neto de empleo} = \underbrace{\text{Contrataciones} - \text{Separaciones}}_{\text{Flujos de trabajadores}} = \underbrace{\text{Creación} - \text{Destrucción}}_{\text{Flujos de empleos}}$$

El modelo que veremos considera flujos de trabajadores.

**Figure 1. Average Monthly Worker Flows, Current Population Survey, 1996-2003**



# **Flows US: Davis et al. (2006, JEP)**

**Job and Worker Flow Rates by Sampling Frequency and Data Source**

<i>Sampling Frequency and Data Source</i>	<i>Job creation</i>	<i>Job destruction</i>	<i>Hires</i>	<i>Separations</i>
<i>Monthly</i>				
JOLTS, continuous monthly units from microdata, Dec. 2000 to Jan. 2005	1.5	1.5	3.2	3.1
<i>Quarterly</i>				
JOLTS, continuous quarterly units from microdata, Dec. 2000 to Jan. 2005	3.4	3.1	9.5	9.2
BED, all private establishments, 1990:2–2005:1	7.9	7.6	—	—
LEHD, all transitions, ten selected states, 1993:2–2003:3	7.0	6.0	25.0	24.0
LEHD, “full-quarter” transitions, ten selected states, 1993:2– 2003:3	7.6	5.2	13.1	10.7
<i>Annual</i>				
BED, from Pinkston and Spletzer (2004), private establishments, 1998–2002	14.6	13.7	—	—

## FLOW APPROACH TO LABOR MARKETS

Davis and Haltiwanger (1999): job creation, job destruction

In representative agent models: either creation or destruction equals zero

In reality:

$$\begin{aligned} CR_t &\equiv \sum_i \text{creation}_{it} \gg 0; \\ DE_t &\equiv \sum_i \text{destruction}_{it} \gg 0. \end{aligned}$$

Reallocation =  $CR_t + DE_t$ .

**CUADRO 1**  
**CREACION, DESTRUCCION Y REASIGNACION DE EMPLEO EN CHILE**

Año	Creación	Destrucción	Reasignación	Variación Neta
1981	13.2%	20.3%	33.4%	-7.1%
1982	8.6%	27.3%	36.0%	-18.7%
1983	14.2%	15.6%	29.8%	-1.4%
1984	20.5%	9.9%	30.4%	10.6%
1985	14.9%	8.3%	23.3%	6.6%
1986	17.9%	9.8%	27.7%	8.1%
1987	24.4%	10.7%	35.1%	13.7%
1988	18.8%	12.7%	31.4%	6.1%
1989	23.2%	14.8%	38.0%	8.5%
1990	14.4%	12.1%	26.5%	2.4%
1991	13.4%	10.7%	24.1%	2.8%
1992	16.7%	10.7%	27.4%	6.0%
Promedio	16.7%	13.6%	30.2%	3.1%
Desv. Std.	4.3%	5.2%	4.5%	8.4%

**CUADRO 2**  
**COMPARACION INTERNACIONAL**

País	Período	Creación	Destrucción	Reasignación
Alemania	1983-1990	6,5%	5,6%	12,1%
Canadá	1983-1991	11,2%	8,8%	20,0%
Colombia	1977-1989	13,2%	13,0%	26,2%
Chile	1980-1992	16,7%	13,6%	30,2%
Dinamarca	1983-1989	9,9%	8,8%	18,7%
Estados Unidos	1973-1988	9,1%	10,2%	19,4%
Finlandia	1986-1991	6,5%	8,7%	15,2%
Francia	1984-1992	6,7%	6,3%	13,0%
Italia	1984-1992	8,4%	7,3%	15,7%
Marruecos	1984-1989	18,6%	12,1%	30,7%
Noruega	1976-1986	7,1%	8,2%	15,4%
Nueva Zelandia	1987-1992	8,3%	11,3%	19,6%
Reino Unido	1985-1991	6,0%	2,7%	8,7%
Suecia	1985-1992	8,0%	9,6%	17,6%

Fuente: Camhi, Engel y Micco (1997).



Enfoque de Flujos a los Mercados Laborales

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# Modelo de Diamond-Mortensen-Pissarides (DMP)

## OVERVIEW

Trade in the labor market is decentralized, uncoordinated, time-consuming and costly for both firms and workers.

Reasons: heterogeneities in skill supply and demand, frictions and information imperfections about skills, location, timing.

In contrast with Walrasian labor markets, existing jobs command rents in equilibrium.

We assume a well-behaved **matching function**: the number of jobs formed at any moment in time as a function of the number of workers looking for jobs, the number of firms looking for workers and possibly other variables.

The matching function is a modeling that plays an analogous role to the aggregate production function in competitive macro theory.

## MATCHING FUNCTION

Trade and production are completely separate activities:

- ▶ No on-the-job search.

$L$  workers in the labor force:

- ▶  $U$ : Number of unemployed workers.
- ▶  $V$ : Number of vacant jobs.
- ▶  $u \equiv U/L$ : unemployment rate. Defined as the fraction of unmatched workers.
- ▶  $v \equiv V/L$ : vacant jobs as a fraction of the labor force, the vacancy rate.

Matches are 1:1. One firm is one job, or equivalently CRS.

Only the  $U = uL$  unemployed workers and the  $V = vL$  job vacancies engage in matching.

Continuous time.

## GENERALIZED POISSON PROCESS

Matches “arrive” according to a Poisson process, with instantaneous rate

$$m_t \equiv m(U, V) = m(uL, vL)$$

that is allowed to vary over time ( $u$  and  $v$  vary over time).

If we denote by  $N_{[t, t+\Delta t]}$  the number of matches in  $[t, t+\Delta t]$  and  $\Delta t > 0$  is small, following the result derived in Lecture D1 for Poisson processes, an extension of the Poisson process that allows for time-varying arrival rates, can be defined via

$$\Pr(N_{[t, t+\Delta t]} = 0) \simeq 1 - m_t \Delta t,$$

$$\Pr(N_{[t, t+\Delta t]} = 1) \simeq m_t \Delta t,$$

$$\Pr(N_{[t, t+\Delta t]} \geq 2) \simeq 0.$$

## MATCHING FUNCTION

Less formally but more intuitive: the “number” of new job matches per unit time is given by

$$m_t = m(U, V) = m(uL, vL).$$

$m(U, V)$  is increasing and concave in each argument.

Estimated matching functions are close to constant returns to scale:

$$m(tu, tv) = tm(u, v).$$

Differentiating both sides of the above equality w.r.t.  $t$  and evaluating at  $t = 1$  yields **Euler's Theorem**:

$$m(u, v) = m_1(u, v)u + m_2(u, v)v.$$

## ARRIVAL RATES AND TIGHTNESS

Job vacancies and unemployed workers matched at  $t$ :

- ▶ Randomly selected from the sets  $vL$  and  $uL$ .

**Labor market tightness:**

$$\theta \equiv \frac{v}{u}.$$

Interpretation:

- ▶ Large  $\theta$ : higher values of  $v$ , lower values of  $u$ , a more buoyant labor market.
- ▶ Low  $\theta$ : the opposite.

Rate at which vacant jobs become filled  $q(\theta)$ , satisfies

$$q(\theta) \equiv \frac{m(uL, vL)}{vL} = m\left(\frac{u}{v}, 1\right) = m\left(\frac{1}{\theta}, 1\right).$$

Rate at which unemployed workers move into employment:

$$\frac{m(uL, vL)}{uL} = \theta q(\theta).$$

As tightness  $\theta$  increases, it becomes harder to fill a vacancy and easier to find a job:

$$\begin{aligned} q'(\theta) &= \frac{dm\left(\frac{1}{\theta}, 1\right)}{d\theta} = -\frac{1}{\theta^2} m_1\left(\frac{1}{\theta}, 1\right) < 0, \\ \frac{d[\theta q(\theta)]}{d\theta} &= q(\theta) + \theta q'(\theta) = q(\theta)(1 - \eta(\theta)) \geq 0. \end{aligned}$$



Where  $\eta(\theta)$  is the elasticity of the job filling rate w.r.t. labor market tightness:

$$\eta(\theta) \equiv -\frac{\theta q'(\theta)}{q(\theta)}.$$

When analyzing efficiency of the model, we will show that the equilibrium is constrained efficient only when  $\eta$  takes one particular value.

At this point we show that  $0 < \eta < 1$ .

Using Euler's Theorem at the crucial step:

$$\eta(\theta) = -q'(\theta) \frac{\theta}{q(\theta)} = \frac{1}{\theta^2} m_1\left(\frac{1}{\theta}, 1\right) \frac{\theta}{m\left(\frac{1}{\theta}, 1\right)} = \frac{\frac{1}{\theta} m_1\left(\frac{1}{\theta}, 1\right)}{m\left(\frac{1}{\theta}, 1\right)} = \frac{m\left(\frac{1}{\theta}, 1\right) - m_2\left(\frac{1}{\theta}, 1\right)}{m\left(\frac{1}{\theta}, 1\right)} \in (0, 1).$$

## THE BEVERIDGE CURVE

Job-specific ('idiosyncratic') exogenous shocks that arrive to occupied jobs at the Poisson rate  $\lambda$  separate matches. Worker returns to unemployment, firm returns to vacancy,  $\lambda$  constant over time.

Evolution of unemployment rate

$$\frac{du}{dt} = \dot{u}_t = \lambda(1 - u_t) - \theta_t q(\theta_t) u_t,$$

where the first and second terms denote number of separations and number of matches, both divided by  $L$ .

It follows that in steady state ( $\dot{u}_t = 0$ ):

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

This relation is the **Beveridge Curve**. It defines a relation between  $u$  and  $v$  or  $u$  and  $\theta$  in our model.

## THE BEVERIDGE CURVE: THEORY



The Beveridge curve is downward sloping in  $(u, v)$  space.

To prove this, we differentiate implicitly w.r.t.  $u$  both sides of

$$\lambda(1 - u) = m(u, v).$$

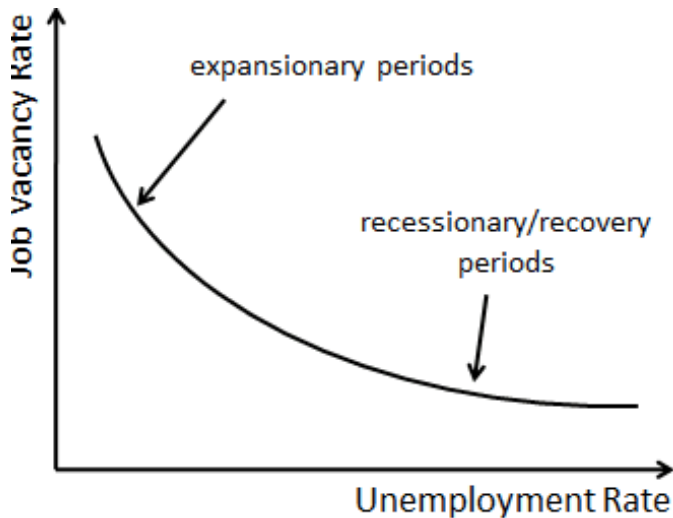
and since  $m_1, m_2 > 0$ , we obtain:

$$\frac{du}{dv} = -\frac{m_2}{\lambda + m_1} < 0.$$

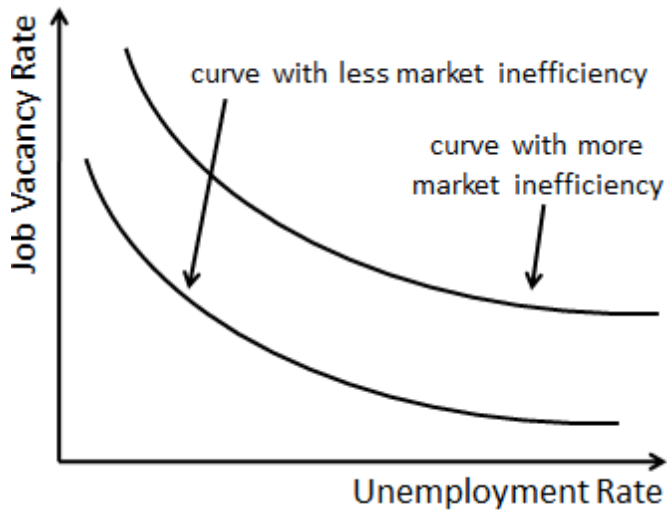
Intuition:

- ▶ We start at  $(u_0, v_0)$  on the Beveridge Curve, that is, a point at which job creation equals job destruction.
- ▶ Next  $v$  increases to  $v_1$ ,  $u$  remains at  $u_0$ .
- ▶  $v \uparrow \Rightarrow \theta q(\theta) \uparrow \Rightarrow \theta q(\theta)u \uparrow \Rightarrow \text{Job creation} \uparrow$ . That is, more vacancies increase job creation.
- ▶ To reestablish job creation equals job destruction,  $u$  must change to increase job destruction,  $\lambda(1 - u)$ .
- ▶ This requires a reduction in  $u$ .

## MOVING ALONG THE BEVERIDGE CURVE



## MOVING across BEVERIDGE CURVES ( $\lambda \downarrow$ )



## THE ECONOMY

Employment contract specifies only a wage rule, that gives the wage rate at any moment of time as a function of some commonly observed variables.

Hours of work are fixed (and normalized to unity) and either side can break the contract at any time.

Value of a job's output is some constant  $p > 0$ .

When the job is vacant, flow fixed cost  $pc > 0$ .

When the worker is unemployed, enjoys flow payoff  $z$ .

Infinitely-lived, risk neutral agents, wealth maximizers, discount payoffs at rate  $r$ , perfect capital markets.

Large number of firms and finite measure of workers  $\Rightarrow$  free entry at all times.

## BELLMAN EQUATIONS: FIRM

$V_t$ : Value of a vacancy.

$J_t$ : Value of a filled job at  $t$ .

Bellman equations for  $V$  and  $J$ :

$$rV_t = -pc + \dot{V}_t + q(\theta_t) \max\langle J_t - V_t, 0 \rangle, \quad (1)$$

$$rJ_t = p - w_t + \dot{J}_t + \lambda(V_t - J_t). \quad (2)$$

Firm can choose between accepting and rejecting a match: explains presence of max in last term in (1).

No choices involved when job separation occurs: explains why last term in (2) does not involve a max.

Arbitrage interpretation for both equation: assumes a job is an asset owned by the firm.



## BELLMAN EQUATIONS: WORKER

Employed worker has human wealth  $\max\langle W_t, U_t \rangle$  and unemployed worker has wealth  $U_t$ , where

$$rW_t = \dot{W}_t + w_t + \lambda(U_t - W_t), \quad (3)$$

$$rU_t = \dot{U}_t + z + \theta_t q(\theta_t) \max\langle W_t - U_t, 0 \rangle. \quad (4)$$

## CLOSING THE MODEL: WAGES

Match surplus

$$S_t = (W_t - U_t) + (J_t - V_t).$$

Bilateral monopoly (bargaining) problem. Unresolved in game theory.

We assume Generalized Nash Bargaining solution (axiomatic, cooperative). We then have that the wage  $w_t$  solves at each  $t$

$$w_t = \operatorname{argmax}(W_t - U_t)^\beta (J_t - V_t)^{1-\beta}$$

where  $0 \leq \beta \leq 1$  captures worker's bargaining power and  $W$ ,  $U$ ,  $J$  and  $V$  depend on  $w$ .

FOC:

$$\beta(W_t - U_t)^{\beta-1}(J_t - V_t)^{1-\beta} \frac{d(W_t - U_t)}{dw_t} + (1 - \beta)(W_t - U_t)^{\beta}(J_t - V_t)^{-\beta} \frac{d(J_t - V_t)}{dw_t} = 0.$$

Wage is bargained at individual match level, so cannot affect continuation values (threat point):

$$\frac{dU_t}{dw_t} = 0 = \frac{dV_t}{dw_t}.$$

Wage is a pure transfer between parties

$$\frac{dW_t}{dw_t} = -\frac{dJ_t}{dw_t}.$$

Plugging derivatives and rearranging

$$W_t - U_t = \beta(J_t + W_t - V_t - U_t) = \beta S_t$$

and therefore

$$J_t - V_t = (1 - \beta)S_t, \quad W_t - U_t = \beta S_t.$$

Hence:

$$W_t - U_t = \frac{\beta}{1 - \beta} (J_t - V_t). \quad (5)$$

The bargaining solution is privately Pareto efficient:

$$W_t - U_t \geq 0 \iff J_t - V_t \geq 0 \iff S_t \geq 0.$$

Also

$$\dot{W}_t - \dot{U}_t = \frac{\beta}{1 - \beta} (\dot{J}_t - \dot{V}_t). \quad (6)$$

## SOLVING THE MODEL

We have 4 Bellman equations, (1) – (4), two wage equations, (5) – (6), and a free entry condition ( $V_t = 0$ ). We want to solve for  $(v_t, u_t, w_t)$ .

Assume that parameter values are such that matches do form,  $S_t > 0$ .

Then,  $\beta[(2) - (1)] + (1 - \beta)[(4) - (3)]$  and using (6) to get rid of  $(\dot{W}_t - \dot{U}_t)$  and (5) to get rid of  $(W_t - U_t)$ :

$$w_t = \beta p + \beta p c + (1 - \beta)z + \beta q(\theta_t)(\theta_t - 1)(J_t - V_t). \quad (7)$$

## FREE ENTRY

At all points in time firms enter the market and open vacancies until value of the vacancy is driven to zero

$$V_t = 0 \quad (8)$$

so that substituting in (1)

$$J_t = \frac{pc}{q(\theta_t)} \quad (9)$$

Wage equation (7) becomes

$$w_t = (1 - \beta)z + \beta p(1 + c\theta_t) \quad (10)$$

## INTERPRETACIÓN DE LA ECUACIÓN DE SALARIO

La expresión (10) de la página anterior es conveniente para entender la intuición en las aplicaciones que veremos.

La podemos reescribir

$$w_t = (1 - \beta)z + \beta p + \beta pc\theta_t.$$

El término  $(1 - \beta)z + \beta p = z + \beta(p - z)$  es la suma del salario de reserva y la renta que obtiene el trabajador al compartir la renta que genera una fuente de trabajo ocupada.

Como  $pc\theta = pcv/u$ , tenemos que  $pc\theta$  es el costo promedio de contratación por trabajador desempleado.

El término  $\beta pc\theta$  en la expresión para  $w_t$  entonces dice que el trabajador se lleva una fracción  $\beta$  del ahorro de la firma en costos de contratación cuando se llena una vacante.

## FREE ENTRY

Sustituyendo (10) en (1):

$$(r + \lambda) J_t = \dot{J}_t + p - (1 - \beta)z - \beta p (1 + c\theta_t).$$

de modo que

$$J_t = \frac{p - w_t}{r + \lambda}. \quad (11)$$

Si reemplazamos  $w_t$  por la expresión de la página anterior, tendremos que cuando  $\beta$  es suficientemente cercano a uno,  $J_t < 0$ . Como debemos tener  $J_t > 0$  en equilibrio, esto significa que en equilibrio  $\theta < (1 - \beta)(p - z)/\beta pc$ .

Tenemos dos expresiones para  $J_t$ : (9) y (11). La primera captura el costo del dueño de una vacante, hasta encontrar un trabajador. La segunda el beneficio una vez que se llena la vacante. La condición de libre entrada,  $V_t = 0$ , significa que las dos deben ser iguales.



## STEADY STATE EQUILIBRIUM

$r$ ,  $p$ ,  $c$  and  $z$  given.

Wage has no allocative role, just a rent-sharing rule. Only real economic decision is free entry.

Steady state equilibrium is triple  $(u, \theta, w)$  that satisfies

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}, \quad (12)$$

$$w = (1 - \beta)z + \beta p(1 + c\theta), \quad (13)$$

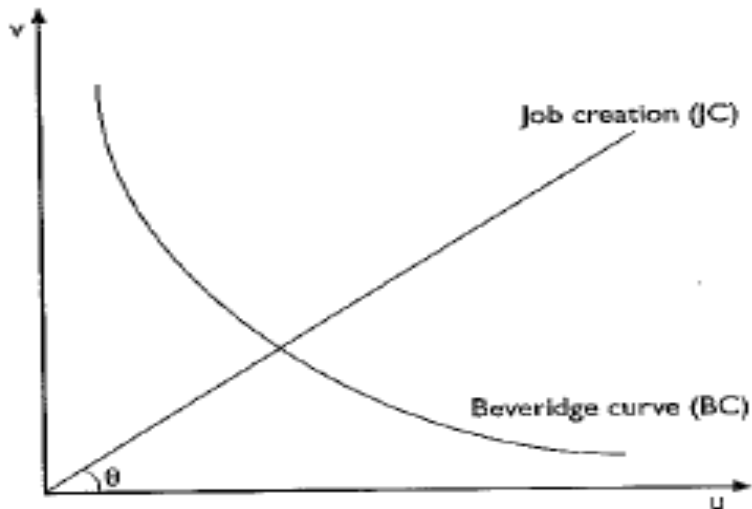
$$\frac{pc}{q(\theta)} = \frac{p - w}{r + \lambda} (= J). \quad (14)$$

Combine the last two into the following equation in  $\theta$ :

$$\frac{pc}{q(\theta)} = \frac{p - (1 - \beta)z - \beta p(1 + c\theta)}{r + \lambda} \quad (15)$$

solved by a unique  $\theta^*$  consistent with Nash bargaining and free entry. Next substitute  $\theta^*$  in (12) and (13) to obtain  $u^*$  and  $w^*$ . Finally,  $v^* = \theta^* u^*$ .

## STEADY STATE



## COMPARATIVE STATICS: $p$ SHOCK AND $\lambda$ SHOCK

We focus on steady states and ignore the dynamics.

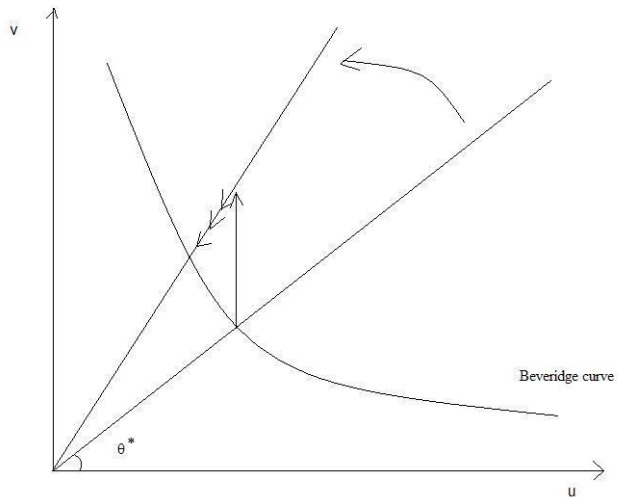
### $p$ -shock:

- ▶ Figure on the following slide shows the change in Job Creation (JC) curve after an increase in  $p$ .
- ▶ Beveridge Curve (BC) unchanged in this case, so that  $u^* \downarrow$ ,  $v^* \uparrow$ .
- ▶ Analogous analysis for comparative statics in  $z$ ,  $c$ ,  $r$  and  $\beta$

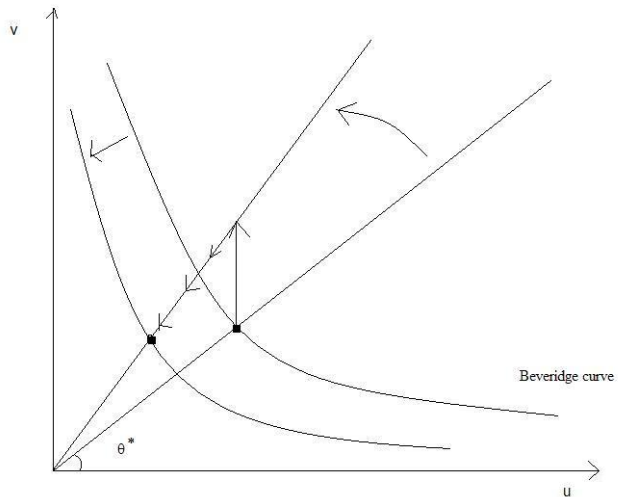
### $\lambda$ shock:

- ▶ Figure on slide 37 depicts change in Job Creation and Beveridge Curve after a decrease in  $\lambda$ .
- ▶ The JC curve moves up and the BC curve moves inward.
- ▶  $u^*$  decreases and the effect on  $v^*$  is ambiguous.

## UNANTICIPATED PRODUCTIVITY SHOCK (IGNORE DYNAMICS)



## UNANTICIPATED SEPARATION SHOCK (IGNORE DYNAMICS)



## OUT-OF-STEADY-STATE DYNAMICS\*

- ▶ Question: for fixed parameters, how does the economy converge to the steady state?
- ▶ Log-differentiating the free entry condition  $J = pc/q(\theta)$  w.r.t. time

$$\frac{\dot{J}_t}{J_t} = -\frac{q'(\theta_t)}{q(\theta_t)}\dot{\theta}_t$$

- ▶ Using  $\dot{J}_t$  from (2) and expression for  $J_t$  from free entry condition

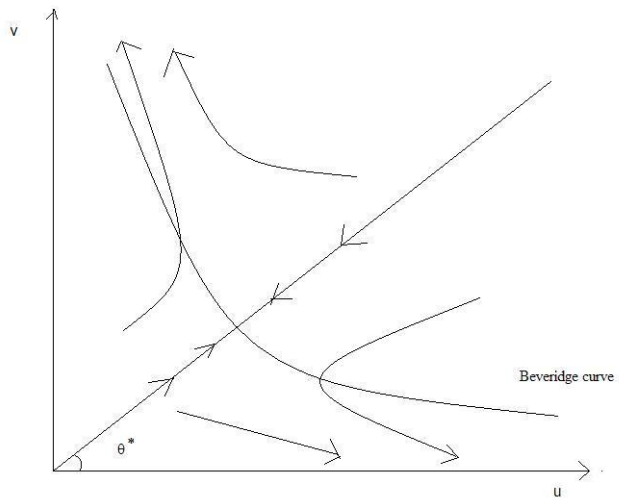
$$\frac{\dot{J}_t}{J_t} = r + \lambda - \frac{(1-\beta)(p-z) - \beta pc\theta_t}{pc/q(\theta_t)}$$

- ▶ Using both expressions above to get rid of  $\dot{J}_t/J_t$  yields an ODE for  $\dot{\theta}_t$ :

$$\dot{\theta}_t = -\frac{q(\theta_t)}{q'(\theta_t)} \left[ r + \lambda - \frac{(1-\beta)(p-z)}{pc} q(\theta_t) + \beta \theta_t q(\theta_t) \right] := Q(\theta_t)$$

- ▶ Other dynamic equation is the familiar  $\dot{u}_t = \lambda(1-u_t) - \theta_t q(\theta_t) u_t$ .
- ▶ Free entry implies that  $\theta$  is a jump variable, while  $u$  is predetermined, a state variable, due to search frictions.

## PHASE DIAGRAM\*

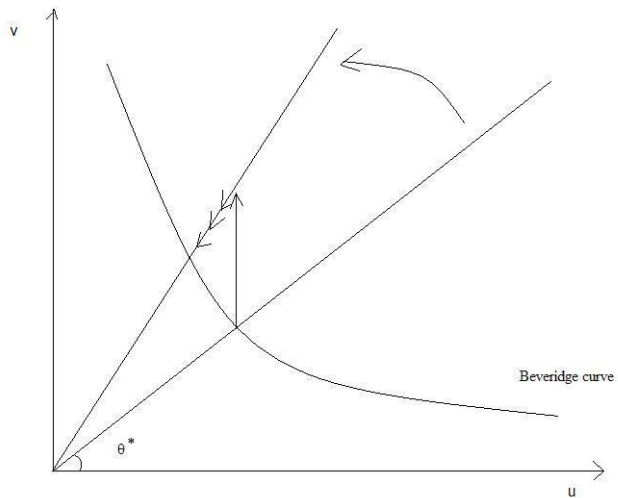


## DYNAMICS: UNANTICIPATED PRODUCTIVITY SHOCKS\*

- ▶ Positive and unanticipated productivity shock:  $p \uparrow$
- ▶ Economy goes through counterclockwise loop around Beveridge curve (see following slide).
- ▶ Value of vacancy,  $V_t$ , increases at time of shock:  $V_t > 0$
- ▶ Firms enter (immediately), enough firms for  $V_t$  to return to zero
- ▶ Over time:  $u_t \downarrow$ ,  $v \downarrow$ , with  $V_t = 0$  at all time.
- ▶ Shows that productivity shocks lead to movements **along** the Beveridge curve (if convergence to new steady state is fast)
- ▶ Also note that the number of vacancies grows above its new equilibrium value immediately after the shock ('overshooting')



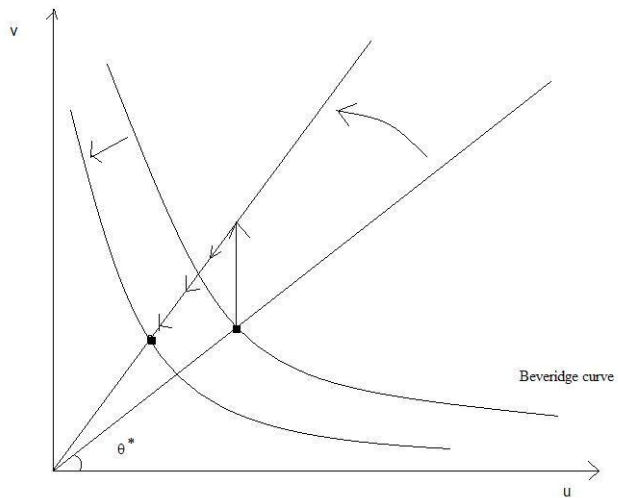
## UNANTICIPATED PRODUCTIVITY SHOCK: DYNAMICS\*



## UNANTICIPATED SEPARATION SHOCK: DYNAMICS\*

- ▶ Unanticipated  $\lambda \downarrow$
- ▶ Shocks to separations  $\lambda$ : vacancies move much less than unemployment, if at all, which is contrary to empirical evidence.
- ▶ Unemployment lower in new steady state.
- ▶ Effect on vacancies ambiguous.
- ▶ Also, these shocks shift the Beveridge curve

## UNANTICIPATED SEPARATION SHOCK\*



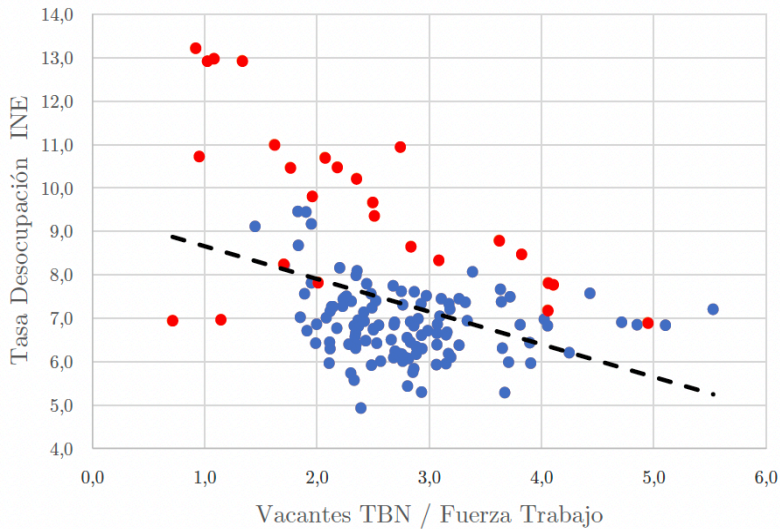
## CURVA DE BEVERIDGE: EVIDENCIA

Concepto teórico: relación de estado estacionario.

En la práctica, se grafica  $V$  vs.  $U$ , de los datos.

No (necesariamente) son estados estacionarios.

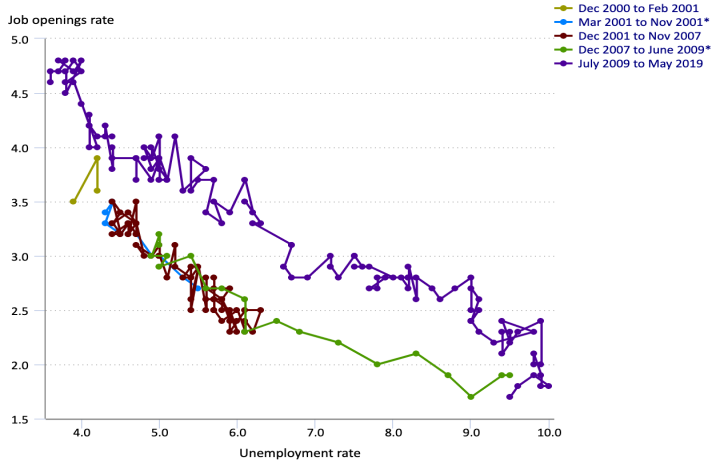
## CURVA DE BEVERIDGE PARA CHILE: VILLENA (1992)



# BEVERIDGE CURVE: UNITED STATES

## The Beveridge Curve (job openings rate vs. unemployment rate), seasonally adjusted

Click and drag within the chart to zoom in on time periods



## INTERPRETANDO LA CURVA DE BEVERIDGE

Consideremos primero shocks de productividad,  $p$ . La curva de Beveridge no cambia y a curva (recta) de creación de empleo, que impone  $V_t = 0$ , rota hacia arriba si  $p$  sube y hacia abajo si baja.

Por lo visto al estudiar la dinámica de estos shocks, a un aumento de  $p$  le sigue un movimiento de  $(u, v)$  a lo largo de la curva de Beveridge, en sentido contrario a las agujas del reloj. Y a una caída de  $p$  le sigue un movimiento en el mismo sentido que se mueven las agujas del reloj, también a lo largo de la curva de Beveridge.

En cambio, un incremento de la tasa de separación  $\lambda$  lleva a un desplazamiento de la curva de Beveridge.

Una interpretación posible de los datos de la evolución de la curva de Beveridge (ver página 46 es que el 2007-08 hubo un gran shock negativo de productividad, que fue seguido de una trayectoria en sentido contrario a las agujas del reloj. Eventualmente, este shock se revierte y la economía retorna a niveles bajos de desempleo, pero en una curva de Beveridge que parece haberse desplazado hacia arriba.

Resumiendo:

- ▶ Los shocks de productividad explican la dinámica de corto plazo, la cual en primera aproximación es a lo largo de la curva de Beveridge.
- ▶ Un incremento de la tasa de separación podría explicar el desplazamiento de la curva de Beveridge hacia arriba en una perspectiva de mediano plazo.



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# Modelo DMP y la Evidencia

## MODELO DMP Y LA EVIDENCIA

Shimer (2005, AER) se toma en serio el modelo DMP y lo confronta con los datos de EE.UU. para ver en qué medida puede explicar el comportamiento cíclico del desempleo, vacantes, creación y destrucción de empleo.

¿Puede el modelo capturar la volatilidad de desempleo, vacantes, tasa a la cual se encuentran empleos y tasa de separación?

Tablas que siguen: los datos y el modelo que calibra Shimer con shocks a  $p$

## TASA DE DESMPLEO

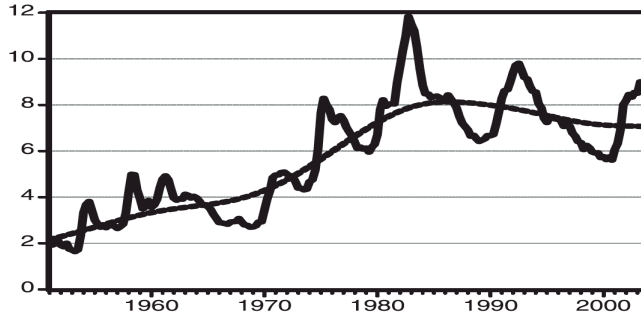


FIGURE 1. QUARTERLY U.S. UNEMPLOYMENT (IN MILLIONS) AND TREND, 1951–2003

*Notes:* Unemployment is a quarterly average of the seasonally adjusted monthly series constructed by the BLS from the CPS, survey home page <http://www.bls.gov/cps/>. The trend is an HP filter of the quarterly data with smoothing parameter  $10^5$ .

## INDICE DE VACANTES

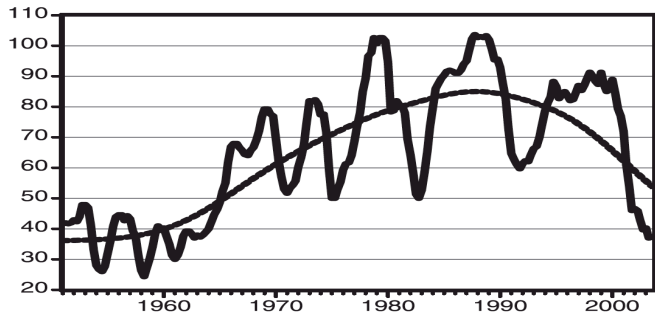


FIGURE 3. QUARTERLY U.S. HELP-WANTED ADVERTISING INDEX AND TREND, 1951–2003

*Notes:* The help-wanted advertising index is a quarterly average of the seasonally adjusted monthly series constructed by the Conference Board with normalization 1987 = 100. The data were downloaded from the Federal Reserve Bank of St. Louis database at <http://research.stlouisfed.org/fred2/data/helpwant.txt>. The trend is an HP filter of the quarterly data with smoothing parameter  $10^5$ .

## TASA DE SEPARACIÓN

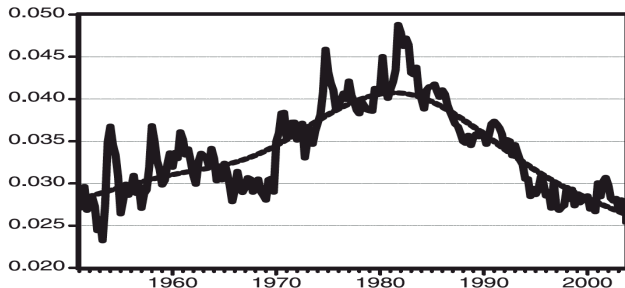


FIGURE 7. MONTHLY SEPARATION PROBABILITY FOR EMPLOYED WORKERS, 1951–2003

*Notes:* The separation rate is computed using equation (2), with employment, unemployment, and short-term unemployment data constructed and seasonally adjusted by the BLS from the CPS, survey home page <http://www.bls.gov/cps/>. It is expressed as a quarterly average of monthly data. The trend is an HP filter of the quarterly data with smoothing parameter  $10^5$ .

## CURVA DE BEVERIDGE

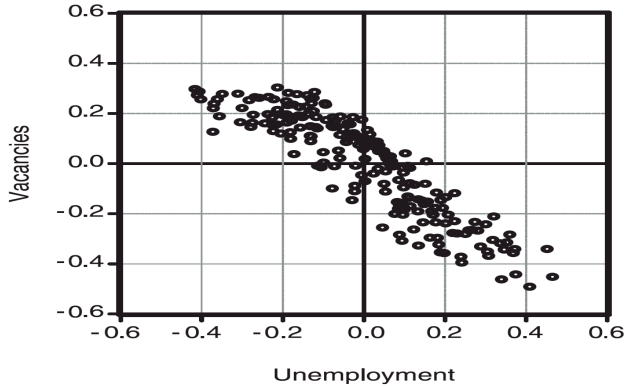


FIGURE 4. QUARTERLY U.S. BEVERIDGE CURVE,  
1951–2003

*Notes:* Unemployment is constructed by the BLS from the CPS. The help-wanted advertising index is constructed by the Conference Board. Both are quarterly averages of seasonally adjusted monthly series and are expressed as deviations from an HP filter with smoothing parameter  $10^5$ .

# LOS MOMENTOS EN LOS DATOS

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		$u$	$v$	$v/u$	$f$	$s$	$p$
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation matrix	$u$	1	−0.894	−0.971	−0.949	0.709	−0.408
	$v$	—	1	0.975	0.897	−0.684	0.364
	$v/u$	—	—	1	0.948	−0.715	0.396
	$f$	—	—	—	1	−0.574	0.396
	$s$	—	—	—	—	1	−0.524
	$p$	—	—	—	—	—	1

*Notes:* Seasonally adjusted unemployment  $u$  is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index  $v$  is constructed by the Conference Board. The job-finding rate  $f$  and separation rate  $s$  are constructed from seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS, as explained in equations (1) and (2).  $u$ ,  $v$ ,  $f$ , and  $s$  are quarterly averages of monthly series. Average labor productivity  $p$  is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter  $10^5$ .



# EL AJUSTE CON SHOCKS DE PRODUCTIVIDAD

TABLE 3—LABOR PRODUCTIVITY SHOCKS

	$u$	$v$	$\varepsilon/u$	$f$	$p$
Standard deviation	0.009 (0.001)	0.027 (0.004)	0.035 (0.005)	0.010 (0.001)	0.020 (0.003)
Quarterly autocorrelation	0.939 (0.018)	0.835 (0.045)	0.878 (0.035)	0.878 (0.035)	0.878 (0.035)
Correlation matrix	$u$	1	-0.927 (0.020)	-0.958 (0.012)	-0.958 (0.012)
	$v$	—	1	0.996 (0.001)	0.995 (0.001)
	$\varepsilon/u$	—	—	1	0.999 (0.001)
	$f$	—	—	—	1 (0.001)
	$p$	—	—	—	—
					1

*Notes:* Results from simulating the model with stochastic labor productivity. All variables are reported in logs as deviations from an HP trend with smoothing parameter  $10^5$ . Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for productivity.

## LA CALIBRACION DE SHIMER

Volatilidad de  $u$  y  $v$  del modelo es muchísimo menor que en los datos.

Incorpora otros shocks:

- ▶ tasa de separación
- ▶ productividad laboral (con persistencia)
- ▶ poder de negociación

Conclusión:

- ▶ El problema no está en el modelo de pareo (matching) sino en el supuesto de Nash bargaining.
- ▶ Cambios en los salarios absorben los cambios de productividad, de modo que  $u$  y  $v$  no necesitan moverse mucho.
- ▶ Se deben explorar otros mecanismos de determinación de salarios.

## HALL, 2005

No hay buenos motivos para suponer Nash bargaining.

Cualquier repartición de rentas que le da una fracción del excedente al empleador y trabajador es una solución válida.

Reemplaza negociaciones a la Nash por un supuesto de salarios rígidos (sticky wages):

- ▶ el salario no varía mientras las partes obtienen rentas positivas
- ▶ se ajusta lo mínimo cuando el excedente que le toca a una de las partes se vuelve negativo

La rigidez se podría justificar por costos de negociar.

Hall considera shocks pequeños, donde nunca se requiere ajustar salarios.

## HALL (2005): EFECTO DE PRODUCTIVIDAD SOBRE $u$ Y $v$

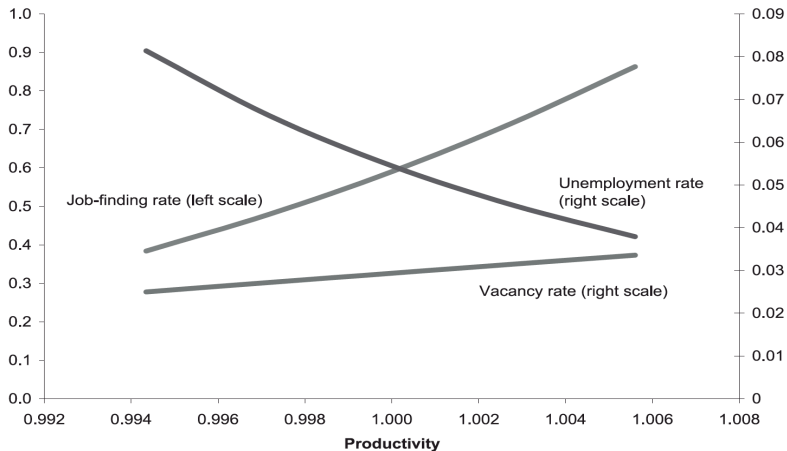


FIGURE 2. JOB FINDING, VACANCY, AND UNEMPLOYMENT RATES, FIXED WAGE

## HALL (2005): CON NASH BARGAINING

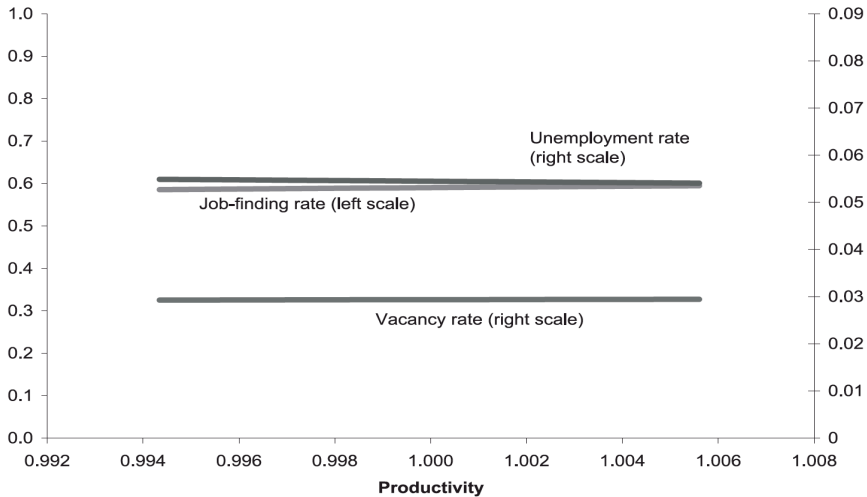


FIGURE 4. JOB FINDING, VACANCY, AND UNEMPLOYMENT RATES, NASH BARGAIN WAGE

## OTRAS RESPUESTAS A SHIMER

Hagedorn and Manovskii (2008):

- ▶ Cambio de parámetros en la calibración.

Eyigungor (2009).

- ▶ Productividad que varía con cohortes y capital específico.

Nagypal (2005):

- ▶ Permite búsqueda de empleos mientras se está empleado.

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# Eficiencia en el Modelo DMP



## EFICIENCIA DEL EQUILIBRIO EN MODELOS DE BÚSQUEDA

En modelos de búsqueda el desempleo es un insumo productivo en la creación de nuevos empleos. Luego no existe ninguna presunción de que el nivel óptimo de desempleo sea cero.

En efecto, para tener una tasa de desempleo baja se requiere de un gran número de vacantes, lo cual consume recursos. Luego es perfectamente posible tener demasiadas vacantes y un desempleo inferior al socialmente óptimo.

Por otra parte, los desempleados también representan un costo social en estos modelos, pues no producen. Luego también se puede tener una tasa de desempleo más alta de la socialmente óptima.

En general, el rol de vacantes y desempleados es simétrico al igual que el rol de trabajos activos y trabajadores empleados.

## **EFICIENCIA: DOS FUENTES DE INEFICIENCIA**

Dos fuentes de ineficiencia:

1. Externalidad de congestión.
2. Problemas de apropiabilidad.

Las estudiamos en detalle a continuación.

## EXTERNALIDAD DE CONGESTIÓN

Un incremento en la búsqueda induce una externalidad positiva en el otro lado del mercado:

- ▶ Más desempleados reduce la duración de las vacantes.
- ▶ Más vacantes reduce la duración del desempleo.

Como  $m(uL, vL)$  es homogénea de grado uno, un incremento en la búsqueda induce una externalidad negativa en el mismo lado del mercado:

- ▶ Más desempleo incrementa el tiempo esperado de un desempleado para encontrar trabajo:  
 $m(uL, vL)/uL = \theta q(\theta)$  decreciente en  $u$ .
- ▶ Más vacantes aumenta el tiempo esperado para llenar una vacante:  $m(uL, vL)/vL = q(\theta)$  es decreciente en  $v$ .

## PROBLEMA DE APROPIABILIDAD

La firma paga todo el costo de postear una vacancia pero se apropia de solo parte del beneficio de llenarla.

Luego los incentivos para postear vacancias son menores que los óptimos.

Se trata del problema de hold-up descrito en Grout (Econometrica, 1984).

De manera análoga, un trabajador paga todos los costos de buscar trabajo (el costo de oportunidad:  $w - z$ ) pero recibe sólo parte del valor social que se crea cuando lo encuentra.

## ASIGNACIÓN EFICIENTE

Asignación eficiente ignorando fricciones de búsqueda: todos trabajan si y sólo si  $p > z$ . Obvio y de poco interés.

Asignación eficiente sujeta a fricciones: interesante, la derivamos a continuación. Será aquella que iguala externalidades positivas y negativas.

Consideramos primero el caso en que  $r \rightarrow 0$ , lo cual interpretamos como el planificador maximizando sobre las variables de estado estacionario s.a. que pertenecen a la curva de Beveridge.

El problema del planificador entonces es

$$\begin{aligned} \max_{u, \theta} \quad & [p(1-u) + zu - pcv] \\ \text{s.a.} \quad & u = \frac{\lambda}{\lambda + \theta q(\theta)}. \end{aligned}$$

Como  $v = \theta u$ , el Lagrangiano en función de  $u$  y  $\theta$  es:

$$\mathcal{L} = p(1-u) + zu - pc\theta u + \gamma \left\{ u - \frac{\lambda}{\lambda + \theta q(\theta)} \right\}.$$

## ASIGNACIÓN EFICIENTE: CASO $r = 0$

Las CPO respecto de  $u$  y  $\theta$ :

$$\begin{aligned} -p + z - pc\theta + \gamma &= 0, \\ -pcu + \gamma\lambda \frac{q(\theta)(1 - \eta(\theta))}{[\lambda + \theta q(\theta)]^2} &= 0. \end{aligned}$$

Despejando  $\gamma$  de la primera condición y sustituyendo en la segunda, y usando la Curva de Beveridge:

$$\frac{pc}{q(\theta)} = \frac{[p(1 + c\theta) - z](1 - \eta(\theta))}{\lambda + \theta q(\theta)}.$$

Realizando las multiplicaciones cruzadas (numerador de uno por denominador del otro), cancelando expresiones que aparecen a los dos lados y reagrupando:

$$\frac{pc}{q(\theta)} = \frac{p - (1 - \eta(\theta))z - \eta(\theta)p(1 + c\theta)}{\lambda}.$$

Comparando esta expresión con la condición de equilibrio al final de la página 33 que reproducimos a continuación

$$\frac{pc}{q(\theta)} = \frac{p - (1 - \beta)z - \beta p(1 + c\theta)}{r + \lambda}$$

y recordando que tomamos  $r = 0$ , concluimos que una condición necesaria para que el equilibrio descentralizado sea socialmente óptimo es que  $\theta$  cumpla

$$\beta = \eta(\theta)$$

(16)

Este resultado se conoce como la **Condición de Hosios**.

## ASIGNACIÓN EFICIENTE: CASO GENERAL

El problema dinámico que resuelve el planificador, restringido por las fricciones de búsqueda, dado un nivel inicial de desempleo  $u_0$ , es :

$$\begin{aligned} \max_{v_t} \quad & \int_0^{\infty} [p(1 - u_t) + zu_t - v_t pc] e^{-rt} dt \\ \text{s.a.} \quad & \dot{u}_t = \lambda(1 - u_t) - v_t q\left(\frac{v_t}{u_t}\right). \end{aligned}$$

Hamiltoniano corriente:

$$H = p(1 - u_t) + zu_t - v_t pc + \mu_t [\lambda(1 - u_t) - v_t q\left(\frac{v_t}{u_t}\right)]$$

Condiciones de optimalidad:

$$\begin{aligned} 0 &= \frac{\partial H}{\partial v_t} = -pc - \mu_t q\left(\frac{v_t}{u_t}\right) - \mu_t v_t q'\left(\frac{v_t}{u_t}\right) \frac{1}{u_t}, \\ \dot{\mu}_t - r\mu_t &= -\frac{\partial H}{\partial u_t} = p - z + \mu_t \lambda + \mu_t v_t q'\left(\frac{v_t}{u_t}\right) \left(-\frac{v_t}{u_t^2}\right), \\ \lim_{t \rightarrow \infty} e^{-rt} \mu_t u_t &= 0. \end{aligned}$$



- ▶  $\mu_t$  is shadow social value of one more unemployed, negative because unemployment is worth less than employment, but not  $-\infty$  because unemployment makes it easier to fill new vacancies, which are necessary to compensate for attrition at rate  $\lambda$
- ▶ Focus on stationary efficient allocation. TVC is satisfied and

$$\begin{aligned} -pc &= \mu q(\theta) + \mu \theta q'(\theta) \\ [r + \lambda - \theta^2 q'(\theta)] \mu &= z - p \end{aligned}$$

- ▶ Eliminate  $\mu$  to get a condition for socially efficient  $\theta^{**}$

$$pc[r + \lambda - \theta^{**2} q'(\theta^{**})] = (p - z)[q(\theta^{**}) + \theta^{**} q'(\theta^{**})]$$

$$\begin{aligned} pc(r + \lambda) &= (p - z) q(\theta^{**}) [1 - \eta(\theta^{**})] + pc \theta^{**} [\theta^{**} q'(\theta^{**})] \\ &= (p - z) q(\theta^{**}) [1 - \eta(\theta^{**})] - pc \theta^{**} q(\theta^{**}) \eta(\theta^{**}) \end{aligned}$$

## HOSIOS CONDITION

- Finally

$$\frac{pc}{q(\theta^{**})} = \frac{(p-z)[1-\eta(\theta^{**})] - pc\theta^{**}\eta(\theta^{**})}{r+\lambda}$$

- Compare to free entry equilibrium condition (optimal tightness  $\theta^*$ )

$$\frac{pc}{q(\theta^*)} = \frac{(p-z)(1-\beta) - \beta pc\theta^*}{r+\lambda}$$

- $\theta^* = \theta^{**}$  and the equilibrium is efficient if and only if the Hosios condition holds

$$\beta = \eta(\theta^{**}).$$

- This condition is feasible since both  $\beta$  and  $\eta(\theta^{**})$  belong to  $(0,1)$ . Yet nothing ensures that it is satisfied in the decentralized equilibrium.
- If  $\beta > \eta(\theta^{**})$  workers are paid too much, equilibrium  $\theta^*$  is too low, unemployment is too high.
- If  $\beta < \eta(\theta^{**})$  workers are paid too little, equilibrium  $\theta^*$  is too high, too many expensive vacancies, unemployment is too low.

## HOSIOS CONDITION\*

- ▶ Note that  $\eta(\theta)$  is the elasticity of vacancy duration wrt the vacancy rate and  $1 - \eta(\theta) =$  elasticity of unemployment duration wrt the unemployment rate (size of negative externalities).
- ▶ If  $\eta(\theta)$  is high: firms are causing more negative externalities to other firms than workers are doing to other workers. Efficiency requires that firms should be “taxed”, and a larger share of surplus ( $\beta$ ) goes to workers
- ▶ Can show that private opportunity cost of labor is  $\beta \frac{m(u,v)}{u} W$  while social opportunity cost is  $\mu \frac{\partial m}{\partial u}$ . Using this result can also show that will have  $W = \mu$  and  $\theta^* = \theta^{**}$  if and only if both opportunity costs are the same, which is Hosios condition.
- ▶ Thus the Hosios condition can be interpreted as equating the social and private opportunity cost of labor.

## FUENTES Y BIBLIOGRAFÍA

Este documento se basa en el ppt de Guiseppe Moscarini de Yale, quien a su vez sigue de cerca el capítulo 1 del libro de Pissarides, *Unemployment Equilibrium Theory*, MIT Press, 2nd Ed., 2000. Se ha posteado este capítulo.

**ENECO 630 – MACROECONOMÍA I**

**DESEMPLEO**

**CÁTEDRAS D2**

**MODELO DE DIAMOND-MORTENSEN-PISSARIDES**

**Eduardo Engel**

ENECO 630. Macroeconomía I

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Esta versión: Mayo 5, 2022.