Fuente: Examen de Econometría II 2021

c) (10 puntos) La log-likelihood es:

$$\updownarrow(\theta; Y) = \sum_{i=1}^{T} \updownarrow(\theta; y_i) = -T \ln \theta - \frac{1}{\theta} \sum_{i=1}^{T} y_i. \tag{1}$$

La FONC es:

$$\frac{\partial \updownarrow(\theta; Y)}{\partial \theta} = -\frac{T}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{T} y_i \Rightarrow \frac{\partial \updownarrow(\theta; Y)}{\partial \theta}|_{\hat{\theta}} = 0 \Rightarrow \hat{\theta} = \frac{\sum_{i=1}^{T} y_i}{T}. \tag{2}$$

La SOSC es:

$$\frac{\partial^2 \updownarrow(\theta; Y)}{\partial \theta^2} = \frac{T}{\theta^2} - \frac{2}{\theta^3} \sum_{i=1}^{T} y_i \Rightarrow \frac{\partial^2 \updownarrow(\theta; Y)}{\partial \theta^2}|_{\hat{\theta}} = -\frac{T}{\hat{\theta}^2} < 0, \tag{3}$$

que indica que el estimador es máximo (global).
Es fácil demostrar que $\hat{\theta}$ es consistente y que su distribución asintótica es:

$$\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{D} N(0, \theta_0^2). \tag{4}$$