## ENECO 630 - MACROECONOMÍA I

INVERSIÓN. Costos no convexos de ajuste. Cátedras 13

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Costos de ajuste: Evidencia

El modelo más simple con ajuste abultado

Modelos Ss e inversión

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# COSTOS DE AJUSTE: QUÉ DICE LA EVIDENCIA

#### Types of adjustment:

- 1. Ongoing: maintenance
- 2. Gradual: refinements, training
- 3. Major and infrequent

#### Neoclassical and *q*-theory:

- ▶ Only capture, albeit imperfectly, 1 and 2.
- ▶ We need models that capture 3

#### **EVIDENCE OF LUMPY MICRO: INVESTMENT**

#### Doms and Dunne (1998)

- ▶ 12,000 continuous plants from US manufacturing (LRD)
- ▶ 1972–1989, annual frequency
- ► Consider both micro and macro findings that suggest the relevance of lumpy micro capital adjustment

#### Micro findings

For more than half of the plants:

$$\begin{aligned} & \max_t I_{i\,t} & > & 0.3 \sum_t I_{i\,t}, \\ & \max_t \Delta K_{i\,t}/K_{i\,t} & > & 0.5. \end{aligned}$$

#### Macro relevance

▶ For every plant i the pair  $(t_i, S_i)$  defined via

$$S_i = \max_t I_{it}, \qquad t_i = \operatorname{argmax}_t I_{it}.$$

characterizes the plant specific spike

▶ Let  $\mathcal{N}_t$  denote the number of spikes in period t and  $\mathcal{S}_t$  their average investment. We then have:

$$I_t \simeq \mathcal{N}_t \mathcal{S}_t. \tag{1}$$

- $\triangleright$   $\mathcal{S}_t$ : intensive margin average adjustment of (spike) adjusters
- $\triangleright$   $\mathcal{N}_t$ : extensive margin fraction that adjust
- Finding: extensive margin explains aggregate investment much better than the intensive margin.
- Conclusion: extensive margin more important then intensive margin for investment.

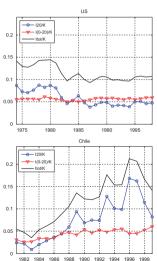
### **EVIDENCE OF LUMPY MICRO: INVESTMENT**

Gourio-Kashyap (JME, 2007)

- ▶ Data from the U.S. and Chile
- ▶ Define spike: I/K > 20%
- ▶ Aggregate investment dynamics very similar to that of investment accounted for by spikes

## A. EVIDENCE OF LUMPY MICRO: INVESTMENT





## EVIDENCIA SOBRE COSTOS DE AJUSTE: EVIDENCIA

La evidencia para inversión sugiere:

- ▶ Ajuste abultado, no suave como se infiere de un modelo con costos convexos de ajuste.
- ► La mayor parte del tiempo las firmas no ajustan mayormente su capital y existen unos pocos períodos, de grandes ajustes, estos últimos explican la mayor parte de los ajustes del stock de capital de una firma específica.

La evidencia es similar para el comportamiento de otras variables de interés en macro a nive de los agentes económicos:

- ► Consumo de durables.
- ► Empleo.
- ▶ Precios.

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Costos de ajuste: Evidencia

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## THE SIMPLEST MODEL WITH LUMPY ADJUSTMENT

The simplest model with lumpy adjustment is the Calvo model: economic agents adjust with an exogenous probability that is fixed across agents and over time. Also, adjustments are independent across agents and over time.

#### In what follows:

- ▶ We first cover a generic version of the quadratic adjustment cost model (*q* theory with quadratic adjustment costs is particular case).
- ▶ Next we cover the Calvo model.

After deriving aggregate dynamics for both models, we show that, despite their differences at the micro level, the aggregate dynamics they lead to are the same.

## THE SIMPLEST MODEL WITH LUMPY ADJUSTMENT

Since this implies that under Calvo the relation between shocks and aggregate investment will be linear, we conclude that we must move beyond Calvo to capture non-linearities.

The above results are useful in two ways:

- ► They motivate considering non-convex adjustment cost models, which are more complex than the Calvo model, but end up providing a better fit for aggregate investment dynamics (and better micro foundations for investment theory).
- ▶ The Calvo model provides a useful benchmark, as a particular (and extreme, in a sense to be made precise shortly) case of a model with stochastic adjustment costs.

# **QUADRATIC ADJUSTMENT COSTS**

### **Agent's Optimization Problem**

The objective function:

$$\min_{y_{i,t}} \mathsf{E}_t \sum_{j \ge 0} \rho^j \left[ c(y_{i,t+j} - y_{i,t+j-1})^2 + \left( y_{i,t+j} - \widehat{y}_{i,t+j} \right)^2 \right] \tag{2}$$

with:

- $\triangleright$  E<sub>t</sub>: expected value conditional on information available at the beginning of period t.
- ▶ Information available at t:

$$y_{i,t-1}, y_{i,t-2}, y_{i,t-3}, \dots; \widehat{y}_{i,t}, \widehat{y}_{i,t-1}, \widehat{y}_{i,t-2}, \widehat{y}_{i,t-3}, \dots$$

 $\rho \in (0,1)$  discount factor.

#### **OPTIMAL POLICY**

Using standard optimization methods, it can be shown that the optimal policy is characterized by:

$$y_{i,t} - y_{i,t-1} = (1 - \alpha) \left( y_{i,t}^* - y_{i,t-1} \right)$$
(3)

with:

$$y_{i,t}^* = (1 - \delta) \sum_{k \ge 0} \delta^k \mathsf{E}_t \left[ \widehat{y}_{i,t+k} \right] \tag{4}$$

where:

$$\delta = \frac{1 + c(1 + \rho) - \sqrt{\left[1 + c(1 + \rho)\right]^2 - 4c^2\rho}}{2c} \in (0, 1),$$

$$\frac{1}{\alpha} \quad = \quad \frac{1+c(1+\rho)+\sqrt{\left[1+c(1+\rho)\right]^2-4c^2\rho}}{2c} \in (1,\infty).$$

#### Names:

**▶** Dynamic target: *y*\*

▶ Static target:  $\hat{y}$ 

#### Conclusion:

▶ Quadratic adjustment costs ⇒ partial adjustment model with dynamic target equal to a weighted average of expected present and future static targets.

#### AGGREGATION

Let's assume shocks are common across agents. We may then write  $y_t^* = y_{i,t}^*$  for all i.

Defining aggregate y via

$$y_t = \frac{1}{N} \sum_{i=1}^{N} y_{i,t}$$
 (5)

we will have that, adding over i in (3) leads to

$$y_t - y_{t-1} = (1 - \alpha) (y_t^* - y_{t-1}).$$

Aggregation of the model with quadratic adjustment costs is straightforward: the relation derived for an individual agent holds as well for the aggregate.

This justifies using. a representative agent model when adjustment costs are quadratic.

#### AN IMPORTANT PARTICULAR CASE

Assume  $\hat{y}$  follows a random walk:

$$\widehat{y}_t = g + \widehat{y}_{t-1} + \nu_t,$$

with  $v_t$  i.i.d.  $(0, \sigma_v^2)$ .

Then:

$$y_t^* = (1 - \delta)[\widehat{y}_t + \sum_{k > 1} \delta^k (\widehat{y}_t + kg)] = \widehat{y}_t + \frac{\delta}{1 - \delta} g.$$

Hence:

$$y_t = \alpha y_{t-1} + (1-\alpha)\widehat{y}_t + (1-\alpha)\frac{\delta}{1-\delta}g \implies \Delta y_t = \alpha \Delta y_{t-1} + \frac{(1-\alpha)g}{1-\delta} + (1-\alpha)\nu_t.$$

And we conclude that  $\Delta y \sim AR(1)$  with the first-order autocorrelation  $\alpha$  a function of the adjustment cost

### CALVO MODEL

Probability of adjusting in any given period denoted by  $\pi$ , independent across agents and over time.

If the agent adjusts at t, she chooses  $y_{i,t}$  keeping in mind that it may be a long time before she adjusts again.

Hence, if the agent adjusts in t she solves:

$$\min_{y_{i,t}} \mathsf{E}_t \left[ \sum_{k \ge 0} \left\{ \rho(1-\pi) \right\}^k \left( y_{i,t} - \widehat{y}_{i,t+k} \right)^2 \right] \tag{6}$$

where the discount factor is  $\rho(1-\pi)$  because the probability that  $y_{i,t}$  is still relevant at time (t+j) is  $(1-\pi)^j$ .

#### SOLVING THE MODEL

It follows from (6) that the function to be minimized is quadratic in  $y_{i,t}$ ,  $Ay_{i,t}^2 - 2By_{i,t} + C$ , with

$$A = \sum_{k \geq 0} \left\{ \rho(1-\pi) \right\}^k = [1-\rho(1-\pi)]^{-1}, \qquad B = \sum_{k \geq 0} \left\{ \rho(1-\pi) \right\}^k \mathsf{E}_t[\widehat{y}_{i,t+k}].$$

Resolviendo la cuadrática concluimos que  $y_{i,t}^* = B/A$ , where  $y_{i,t}^*$  denotes the value of  $y_{i,t}$  that solves (6).

Therefore:

$$y_{i,t}^* = [1 - \rho(1 - \pi)] \sum_{k \ge 0} \{\rho(1 - \pi)\}^k \,\mathsf{E}_t \left[ \hat{y}_{i,t+k} \right]. \tag{7}$$

## AGGREGATION

We define aggregate y via (5) and assume a very large number of agents, N.

Since N is large, by the Law of Large Numbers we will have that a fraction  $1-\pi$  of agents won't adjust while a fraction  $\pi$  will adjust.

Because of the independence assumption, those that adjust are representative of the entire population of agents. It follows that their contribution to aggregate y will be  $(1-\pi)y_{t-1}$ .

We assume that shocks are common across agents.

Equation (7) then implies:

$$.y_{t}^{*} = [1 - \rho(1 - \pi)] \sum_{k \ge 0} \{\rho(1 - \pi)\}^{k} \mathsf{E}_{t} [\widehat{y}_{t+k}].$$
(8)

Adjusters will choose  $y_{i,t} = y_t^*$  and it follows that:

$$y_t = (1 - \pi)y_{t-1} + \pi y_t^*,$$

that is

$$y_t - y_{t-1} = \pi(y_t^* - y_{t-1}). \tag{9}$$

## A USEFUL EQUIVALENCE RESULT

We show next that, based on aggregate data, it is impossible to distinguish between a model with quadratic adjustment costs, where all agents adjust continuously at the micro level, and a model with lumpy microeconomic adjustment, where a fraction of agents does not adjust in any given period.

This implies that showing that lumpy adjustment matters for aggregate dynamics may be far from straightforward and motivates the models with fixed (and non-convex) adjustment costs we cover later in this section.

# ROTEMBERG (1987) EQUIVALENCE RESULT

Comparing (8) and (9) with the results in Section 2, we conclude that, by assigning

$$\alpha \longleftrightarrow 1-\pi$$

$$\delta \longleftrightarrow \rho(1-\pi)$$

the aggregate dynamics of both models are indistinguishable.

An econometrician that only observes aggregate data *cannot* distinguish between a quadratic adjustment and a Calvo model.

## EVIDENCE OF (PARTIAL) IRREVERSIBILITY IN CAPITAL

#### Ramey and Shapiro (JPE, 2001):

- ► Consider aerospace plants that closed during the early 90's.
- ► Closed because of end of cold war (avoids selection bias).
- Firms in aerospace sector overrepresented among buyers of capital.
- ► Capital sold at significant discount compared with replacement cost: discount larger when sold to firms outside the aerospace sector.
- ▶ Non-trivial part: calculate replacement price of installed capital (equipment).
- ► Conclusion: average sales price 72% below the replacement price.

This evidence suggest the term  $p_K I_t$  we had in the q-theory model needs to be replaced by a term that is  $p_K^+ I_t$  when  $I_t > 0$  and equal to  $p_K^- I_t$  when  $I_t < 0$ , with  $p_K^+ \gg p_K^-$ .

The extreme case where  $p_K^- = 0$  is referred to as **irreversible** investment. When the resale cost is (significantly) smaller than the purchase cost, we talk about partial irreversibility.

The fact that selling capital occurs at a significant loss, that is, the presence of irreversibility, leads firms to be cautious when investing. That is, the **option to wait** becomes more valuable as the gap between both prices increases.

There are two strands of the recent literature on investment that departs from the assumptions on adjustment costs made by q theory.

One stresses the importance of irreversibilities, the other the importance of non convex adjustment costs.

In this introduction we have presented micro evidence suggesting the relevance of both.

We will cover the latter in the remainder of this lecture. For the former, see Pindyck (1988, AER) and Abel and Eberly (1994, AER).

#### SUMMING UP

Evidence suggesting smooth adjustment to shocks, as implied by q-theory, does not capture aggregate investment dynamics.

Evidence suggesting fluctuations of the extensive margin play an important role in aggregate investment dynamics

Calvo model as the simplest model with lumpy adjustment ... yet too simple, since it is equivalent to the quadratic adjustment cost model and therefore cannot improve upon q-theory (nor can it capture fluctuations in the extensive margin)

Motivate moving beyond the Calvo model, to consider models with fixed (and, more generally, non-convex) adjustment costs, which is what we do in the remainder of this section.

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#### **COSTOS NO CONVEXOS**

El caso más simple, para un nivel de *K* dado:

$$C(I,K) = \begin{cases} F, & \text{si } I \neq 0, \\ 0, & \text{si } I = 0. \end{cases}$$
 (10)

Se conoce como costos fijos de ajuste.

La diferencia entre ajustarse y no ajustarse pasa a ser muy importante.

- Ya no habrá ajustes pequeños.
- En muchos periodos será óptimo no ajustarse.
- Y cuando la firma invierte, su inversión será relativamente grande.

La constante F puede depender de K, por ejemplo, ser proporcional a K (o a alguna potencia de K).

#### Lo relevante es:

- ► Que los costos de ajuste crezcan con el tamaño de la firma (vemos luego cómo definir "tamaño"). Si los costos no dependen del tamaño de la empresa y estas crecen en el tiempo, entonces los costos de ajuste pasan a ser irrelevantes, lo cual no es razonable.
- ▶ También es importante que la función  $I \longrightarrow C(I,K)$  tiene una discontinuidad en I = 0 que hace que no see convexa. Por eso se habla de costos no convexos de ajuste.

La ecuación (10) se puede generalizar a:

$$C(I,K) = F\{I \neq 0\} + C_0(I,K),$$

donde  $\{I \neq 0\}$  es igual a 1 si  $I \neq 0$  e igual a cero si no y  $C_0(I,K)$  es una función convexa con las propiedades que vimos en la teoría q.

En el trabajo aplicado se trabaja habitualmente con (10), porque el poder estadístico es limitado y estimar con precisión muchos parámetros de costo de ajuste es difícil.

## MODELO SS SIN COSTOS DE AJUSTE

Tiempo discreto.

Función de beneficio de la firma:

$$\Pi(K,\theta) = K^{\beta}\theta - (r+\delta)K; \tag{11}$$

con

- $\triangleright$  0 <  $\beta$  < 1
- $\triangleright$   $\theta$ : shocks de demanda/productividad/salarios.

Definimos el óptimo estático en ausencia de costos de ajuste:

$$K^* = \operatorname{argmax}_K \Pi(K, \theta) = \left(\frac{\beta \theta}{r + \delta}\right)^{1/(1-\beta)}.$$

Suponemos que  $\log \theta$  sigue un camino aleatorio. Como la expresión que derivamos para  $K^*$  implica que  $\Delta \log K^*$  es lineal en  $\Delta \log \theta$ , concluimos que  $k \equiv \log K$  también sigue un camino aleatorio de modo que los  $\Delta k$  son i.i.d.

## INTRODUCIENDO COSTOS DE AJUSTE

Ahora introducimos costos de ajuste no convexos.

Suponemos que cuando la firma ajusta su capital, deja de percibir una fracción  $\omega$  de los ingresos del periodo en cuestión.

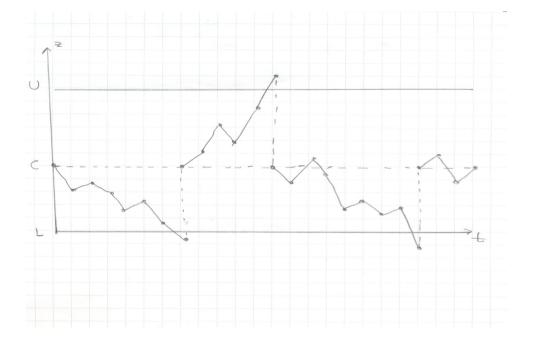
Como los ingresos no percibidos solo incluyen la primera expresión al lado derecho de (11), tenemos

Costo de ajuste =  $\omega K^{\beta} \theta$ .

Resultados generales permiten mostrar que la política óptima será del tipo (L,C,U), en la variable

$$z = \log(K^*/K) = \log K^* - \log K \equiv k^* - k,$$

tal como se ilustra en la lámina que sigue.



Es decir, existen umbrales L y U y un valor C con L < C < U, tales que:

- ▶ Si  $z_t > U$  invierto de modo que  $z_t$  cae a C.
- ▶ Si  $z_t < L$  desinvierto de modo que  $z_t$  sube a C.
- ▶ Si  $L \le z_t \le U$ , no hago nada.

#### Cuestión de nombres:

- ▶ Rango de inacción (inaction range): L < x < U
- ▶ Target para la desviación de capital luego de ajusterse: C
- ightharpoonup Gatillos (triggers): L and U

Las reglas (L, C, U) son un caso particular de una regla Ss.

- ► Estas nacen en teoría de inventarios en la década de 1950. Lo que las caracteriza es que existe un rango de inacción (mientras la variable de estado está en esta región el agente económico no hace nada) y umbrales que gatillan acciones.
- ▶ En el caso de reglas (L,C,U), cuando se ajuste el capital se elige k de modo que  $k^* k = c$ . Este ajuste se puede dar de dos maneras: un aumento del stock de capital (si al momento de ajustar, z > U) o una reducción del stock (si al momento de ajustar z < L).

## ECUACIÓN DE BELLMAN\*

Esbozamos la matemática para resolver el problema de la firma.

Por lo que vimos en la lámina 34 y siguientes, como  $K^*$  es función de  $\theta$ , las variables de estado se pueden elegir de varias maneras:  $(K,\theta)$ ,  $(K,K^*)$ ,  $(K^*,z)$ , etc. Usamos  $(K^*,z)$ .

Definiendo  $\xi = (r + \delta)/\beta$ , tenemos

$$\theta = \xi K^{*(1-\beta)}$$

lo cual, junto a  $K = K^* e^{-z}$ , lleva a

$$\Pi(z,K^*) = K^{\beta}\theta - (r+\delta)K = K^{*\beta}e^{-\beta z}\xi K^{*(1-\beta)} - (r+\delta)K^{*\beta}e^{-\beta z} = \xi[e^{-\beta z} - \beta e^{-z}]K^* \equiv \pi(z)K^*.$$

Y el costo de ajuste se puede expresar como

Costo de ajuste = 
$$\omega K^{\beta} \theta = \omega \xi e^{-\beta z} K^*$$
.

La ecuación de Bellman en problemas con costo de ajuste no convexo, se escriben en función de dos variables auxiliares: el valor si la firma se ajusta y el valor si no se ajusta, de modo que

$$V(K_t^*, z_t) = \max\{V_a(K_t^*), V_{na}(K_t^*, z_t)\}$$

con

$$\begin{array}{rcl} V_a(K_t^*) & = & V(c,K_t^*) - \omega \xi e^{-\beta z_t} K_t^*, \\ V_{na}(K_t^*,z_t) & = & \pi(z_t) K_t^* + \frac{1}{1+r} \mathrm{E} V(z_{t+1},K_{t+1}^*). \end{array}$$

Donde el valor esperado se toma sobre los  $\Delta z_{t+1}$  que son i.i.d. porque supusimos que los  $\Delta \log \theta$  son i.i.d. Y donde, usando que  $K_{t+1} = (1-\delta)K_t$  cuando no hay ajuste, expresamos  $K^{*t+1}$  en función de  $\Delta z_{t+1}$  y  $K_t^*$  como sigue:

$$K_{t+1}^* = K_t^* \frac{K_{t+1}^*}{K_t^*} = K_t^* e^{\Delta z_{t+1}} \frac{K_{t+1}}{K_t} = (1 - \delta) K_t^* e^{\Delta z_{t+1}}.$$

# LIMITACIONES DE LAS REGLAS (L, C, U)

Cuando las firmas aumentan su capital, es siempre en el mismo porcentaje, U-C. Y cuando lo bajan también: C-L.

Lo anterior no es realista y lleva a que las funciones objetivo que se maximizan al estimar el modelo no se comporten bien cuando varía el costo de ajuste.

Lo cual lleva a preguntarse qué es lo esencial que captura una regla (L, C, U).

Respuesta: capturan que la probabilidad de ajustarse es creciente en la brecha de la firma, es decir, en su variable de desequilibrio.

## LA FUNCIÓN DE PROBABILIDAD DE AJUSTE

En lo que sigue, sin pérdida de generalidad, suponemos C = 0. Es cosa de trabajar con x = z - C.

Lo razonable es buscar modelos donde la probabilidad de ajustarse es creciente en |x|.

Denotamos la probabilidad de ajuste (adjustment hazard en inglés) por  $\Lambda(x)$ .

Si el costo de ajuste es fijo,  $\Lambda(x)$  pasa de 0 a 1 cuando x llega a L o U.

Es más realista que  $\Lambda(x)$  crezca lentamente con |x|, lo cual será el caso si en lugar de tener un costo fijo F de ajuste, el costo de ajuste de cada firma es una realización  $\omega$  de una v.a. con cumulativa  $G(\omega)$ .

Donde los costos de ajuste son independientes entre firmas y para una misma firma en el tiempo.

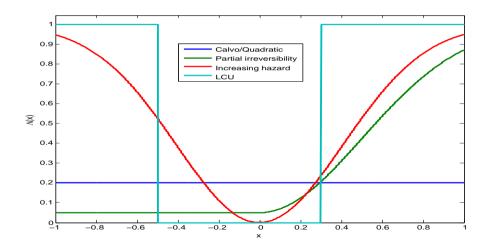
Entonces la política óptima de las firmas es seguir reglas (L, C, U) con valores de L y U que dependen de x:

- ▶ El rango de inacción es mas grande mientras más grande es |x|.
- ightharpoonup L es decreciente en |x|.
- ightharpoonup U es creciente en |x|.

La econometrista puede estimar  $\Lambda(x)$  directamente, suponiendo una familia de funciones con la propiedad de increasing hazard:  $\Lambda(x)$  es creciente en |x|.

Alternativamente, la econometrista puede postular una familia de distribuciones  $G(\omega)$ , obtener la distribución que mejor ajusta los datos y luego inferir la función  $\Lambda(x)$  correspondiente a la distribución óptima.

# FUNCIÓN DE PROBABILIDAD DE AJUSTE $\Lambda(x)$ PARA DIVERSOS MODELOS



### COMENTARIOS A LA FIGURA ANTERIOR

- ▶ Por el resultado de Rotemberg, la  $\Lambda(x)$  constante corresponde tanto a Calvo como a ajuste cuadrático.
- La  $\Lambda(x)$  verde corresponde al caso de irreversibilidad parcial: la probabilidad de reducir el stock de capital es baja (porque recupero poco) pero la probabilidad de aumentar mi capital crece rápido con x cuando x > 0.
- La  $\Lambda(x)$  roja corresponde al caso en que la probabilidad de ajustar el stock de capital también crece para valores negativos de x.

## MODELO SS GENERALIZADO\*

Same assumptions as in (L, C, U) case, but now the fixed adjustment cost in a given period are i.i.d. draws from a known distribution  $G(\omega)$ , with probability density  $g(\omega)$ .

The optimal policy now depends on the firm's gap before adjusting, x, and on the current adjustment cost draw,  $\omega$ 

Assuming stochastic fixed adjustment costs combines two literatures:

- ► Ss: fixed cost leads to infrequent and lumpy adjustment
- ▶ Search: a distribution of adjustment costs means that it may be attractive to wait for a better draw.

## SOLVING THE DYNAMIC PROGRAMMING PROBLEM\*

See Caballero and Engel (1999) for details

State variable:

$$u = z - C = \log(K^*/K) - C.$$

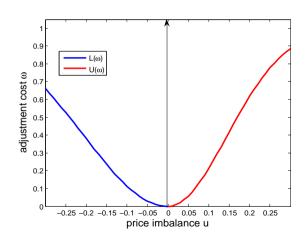
corresponds to mandated investment, mandated by the neoclassical model

#### Result

Conditional on  $\omega$ , the optimal policy is of the (L,C,U) type, with  $L=L(\omega)$  and  $U=U(\omega)$ . C does not depend on  $\omega$ . We also have:  $L'(\omega) < 0$ ,  $U'(\omega) > 0$  and  $L(\omega) < C < U(\omega)$ .

### Figures that follow:

- ▶ Imbalance: *u* instead of *x*.
- ▶ Price imbalance should be capital imbalance

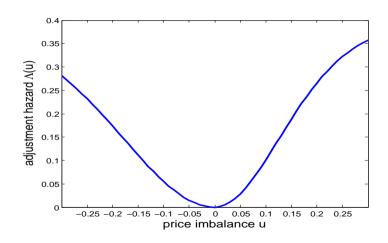


We can now summarize the optimal policy via a state-dependent adjustment hazard,  $\Lambda(u)$ .

Given a price deviation u, we denote by  $\Lambda(u)$  the probability of adjusting the price **before** observing the current draw of the adjustment cost.

Then:

$$\Lambda(u) \equiv \Pr(\omega \leq \Omega(u)) = G(\Omega(u)).$$



It is straightforward to prove that  $\Lambda(u)$  is decreasing for u < 0, increasing for u > 0 and that  $\Lambda(0) = 0$ .

It follows that:

$$u \neq 0 \Longrightarrow u\Lambda'(u) > 0.$$

This is referred to as the increasing hazard property: the hazard increases with the absolute value of the price-deviation u.

The standard (L, C, U) policy is obtained when  $G(\omega)$  has all its mass at one point.

The Calvo model  $(\Lambda(u) = \lambda \text{ for all } u)$  is obtained when  $G(\omega)$  has mass  $\lambda$  at  $\omega = 0$  and mass  $1 - \lambda$  at a very large value of  $\omega$ .

This is a state-dependent adjustment hazard, to distinguish it from the usual, time-dependent hazard:

- ▶ time-dependent: probability of adjusting, conditional on having last adjusted t periods ago
- state-dependent: probability of adjusting given current state (in our case: mandated investment and current adjustment cost)

In an adjustment hazard model, the size of upward and downward adjustments can vary over time, which does not happen with the standard (L, C, U) model

This added realism also is useful when estimating the model

## **OPTIMAL MICROECONOMIC POLICY: SUMMARY\***

#### Conditional on current draw $\omega$ :

- ▶ two-sided Ss policy (LCU policy)
- ightharpoonup inaction range larger for larger values of  $\omega$

### Conditional on current gap x:

► Probability of adjusting:

$$\Lambda(x) \equiv G(\Omega(z-c)) = G(\Omega(x)).$$

#### Particular cases:

- ▶ Calvo:  $G(\omega)$  with mass  $\lambda$  at 0 mass  $1-\lambda$  at  $\omega_M$ , with  $\omega_M \to \infty$ .
- ▶ Quadratic adjustment: equivalent to Calvo
- ▶ Ss:  $G(\omega)$  mass one at K. In this case  $\Lambda(x) = 0$  or 1.

## AGREGACIÓN\*

Muchas firmas que sigue la regla óptima.

El shock a los  $z_{i,t}$  se descompone en la suma de una componente común a todas las firmas y una componente específica a cada firma:

$$\Delta z_{it} = v_t^A + v_{it}$$

### Donde:

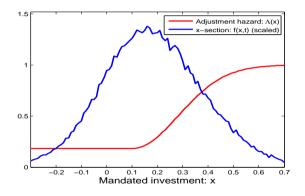
- ▶ Los  $v_t^A$  son i.i.d.  $N(\mu_A, \sigma_A^2)$ .
- ▶ Los  $v_{it}$  i.i.d.  $N(0, \sigma_I^2)$ , independientes entre firmas y de los  $v^A$ .

#### Nombres:

- $\triangleright v_t^A$ : Shocks agregados.
- v<sub>it</sub>: Shocks idiosincrásicos.

Lo que se tiene, entonces, en todo momento del tiempo, es una distribución de los valores de las variables de estado,  $x_{it}$ .

# FROM THE X-SECTION TO AGGREGATE INVESTMENT\*



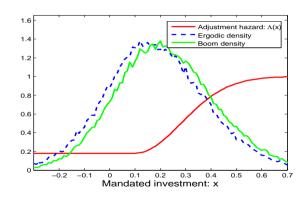
## FROM THE X-SECTION TO AGGREGATE INVESTMENT\*

To track aggregate investment we must keep track of the evolution of the x-section of mandated investment, f(x,t)

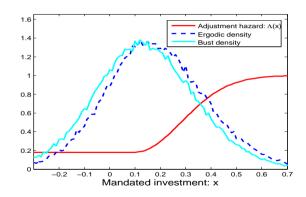
To go from this x-section to the aggregate of interest, we note that:

$$\frac{I_t}{K_t} \cong \int x \Lambda(x) f(x, t) \, \mathrm{d}x. \tag{12}$$

# THE EFFECT OF A BOOM



# THE EFFECT OF A RECESSION



### TIME-VARYING IRFS

In this setting, an aggregate shock is a shock that shifts mandated investment for all firms by the same amount.

The preceding pair of figures suggest that:

- 1. Investment responds more to an additional impulse after a sequence of positive shocks than after a sequence of negative shocks
- 2. When you need it most is when a stimulus is least effective.
- 3. The size of a stimulus will be too small if use linear models to gauge size of the stimulus

# ENECO 630 - MACROECONOMÍA I

INVERSIÓN. Costos no convexos de ajuste. Cátedras 13

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