Macroeconomía I: Cátedra 2

Series de tiempo: Aplicaciones en macroeconomía

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Trend-Cycle Decomposition

Cost of Business Cycles

Rational Expectations

Trend-Cycle Decomposition

Cost of Business Cycles

Rational Expectations

Trend-Cycle Decomposition

We often want to remove the trend from a macroeconomic time series and work with a modified series that reflects only business cycle fluctuations. That is, we want to decompose a series into the sum of a trend and cyclical component:

$$x_t = x_t^{\text{tr}} + x_t^c \tag{1}$$

Of course, all we need is a methodology to estimate either the trend or the cyclical component. The other component then follows from identity (1).

One reason for doing this is that the cyclical component is stationary and therefore amenable to statistical inference.

Another is that often the model we are testing is designed to explain cyclical fluctuations, not long run trends.

Yet note that decomposing a series into the sum of a trend and a cyclical component does not make sense if the same model is supposed to explain long run growth and cyclical fluctuations.

Deterministic trends

Under this approach the main assumption is that

$$x_t^{\mathrm{tr}} = f(t; \beta_0, \beta_1, ..., \beta_n).$$

while x_t^c is covariance stationary (with no deterministic component). The deterministic trend x_t^{tr} usually is a low order polynomial (e.g., linear or cubic).

If we assume a linear deterministic trend, then we fit a linear trend via OLS:

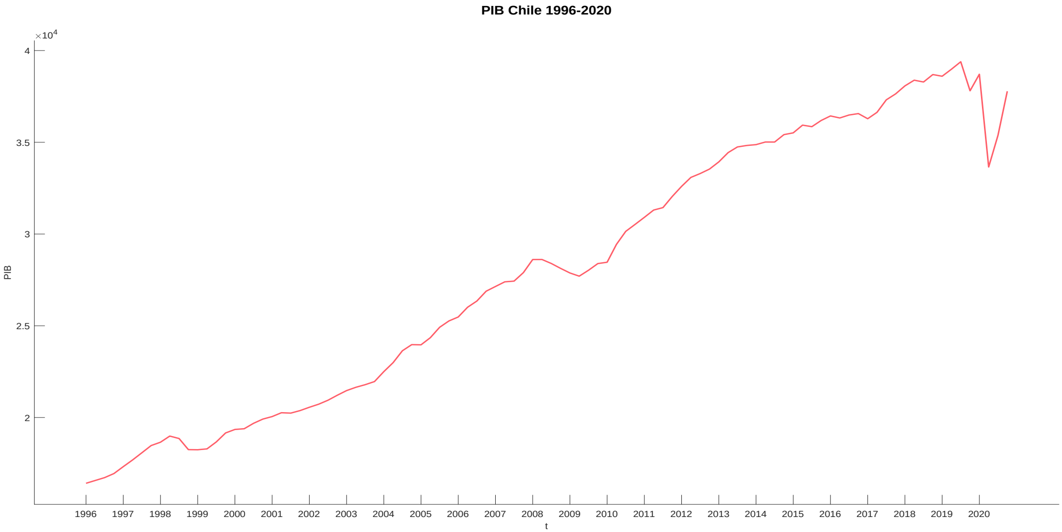
$$x_t = a_0 + a_1 t + e_t.$$

and then proxy the trend and cyclical components by:

$$\widehat{x}_t^{\mathrm{tr}} = \widehat{a}_0 + \widehat{a}_1 t, \qquad \widehat{x}_t^c = \widehat{e}_t = x_t - \widehat{x}_t^{\mathrm{tr}}.$$

Or we could fit a cubic trend instead:

$$x_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + e_t.$$



PIB Chile 1996-2020
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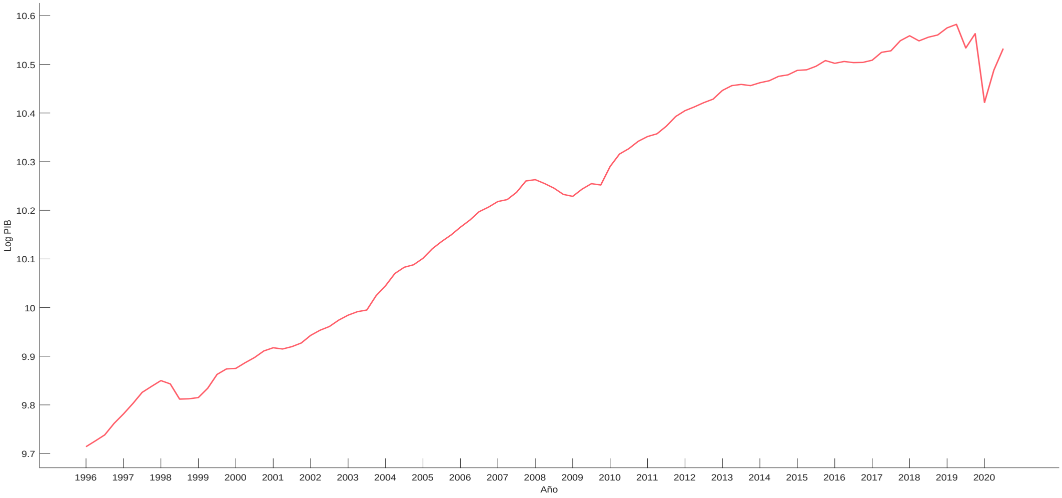
Log PIB para Chile 1996-2020



Why logarithms?

It is common to work with the logarithm of economic variables:

- $\log$ GDP, $\log$ Consumption, $\log$ Employment

Of course, there this is not always the case:

- Unemployment, Investment, ...

The reason for this is that usually economic models explain fluctuations in growth rates of economic variables, not fluctuations of levels. We want to understand what determines whether GDP will grow by 1, 2, 3 or 4%, not what determines whether GDP will grow by $120, $240, $360 or $480 billion.

Why logarithms?

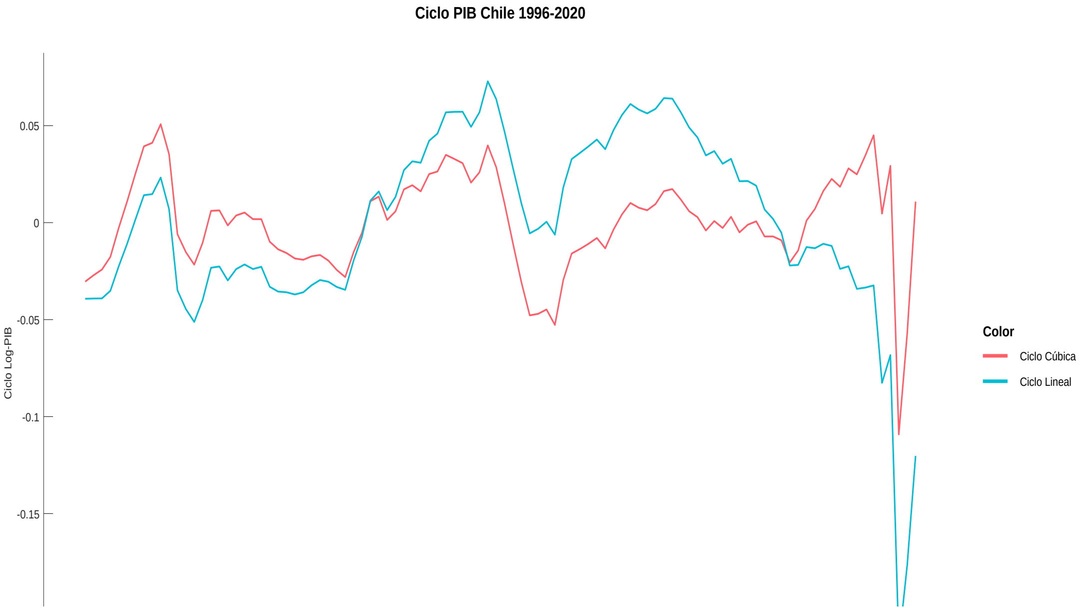
There are three obvious choices to capture growth rates for a series x_t:

$$
\begin{aligned}
\widehat{x}_t &= (x_t - x_{t-1})/x_{t-1}, \\
\widehat{x}_t &= (x_t - x_{t-1})/x_t, \\
\widehat{x}_t &= \log x_t - \log x_{t-1} = \log(x_t/x_{t-1})
\end{aligned}
$$

The first option is the one that is most used by the press or in non technical settings. It has the problem that a 10% increase followed by a 10% decrease does **not** get you back to where you started.

The second option has the same problem.

The third option does not have the above mentioned problem, which is the main reasons why it is often used. Yet this option also has its problems, for example, when x can take values equal to zero (e.g., firm level employment with entry and exit). In this case an often used option is $(x_t - x_{t-1})/\overline{x}_{t,t-1}$ with $\overline{x}_{t,t-1} \equiv (x_t + x_{t-1})/2$.


Ciclo PIB Chile 1996-2020
Ciclo Log-PIB
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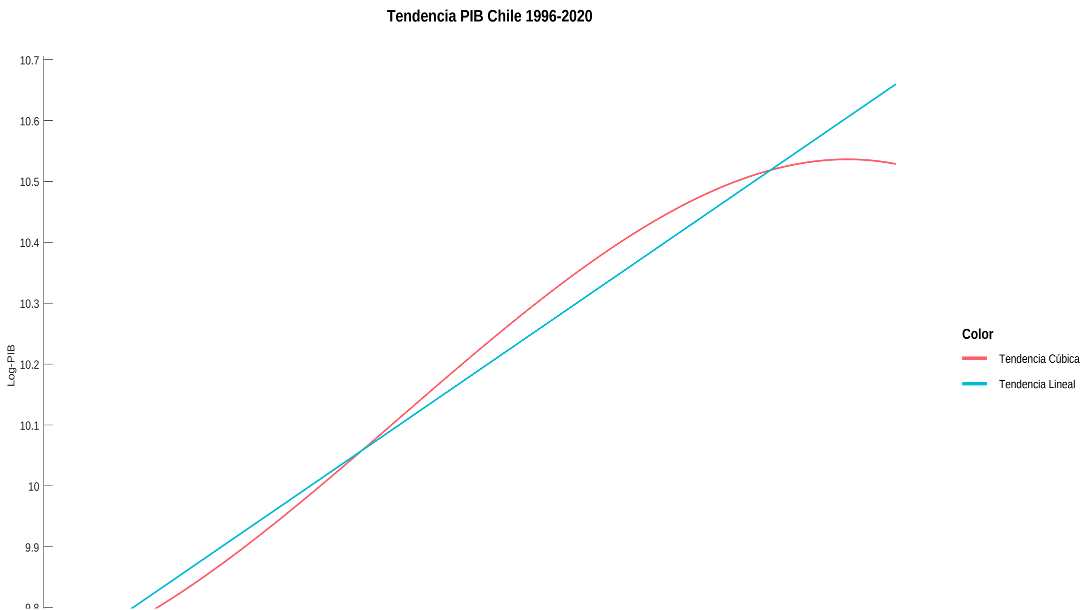
Ciclo Cúbica
Ciclo Lineal



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Log-PIB
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Tendencia Cúbica
Tendencia Lineal

Deterministic Trends

The cubic trend leads to a cyclical component with less volatility than the linear trend: $\sigma_{\text{lin}} = 0.0476$, $\sigma_{\text{cubic}} = 0.0247$.

This is to be expected, with more degrees of freedom the trend will capture a larger part of fluctuations and the cyclical component will be less volatile.

A cubic trend allows for changes in the underlying growth rate, which according to many analysts happened in Chile in the late 1990s.

Deterministic Trends

Assume we have observations $x_1, x_2, ..., x_T$ from a series that is the sum of a deterministic trend and a stationary series.

Then, when k is sufficiently large, the forecast of x_{T+k} will satisfy:

$$\widehat{x}_{T+k|T} \cong f(T+k; \widehat{\beta}_0, \widehat{\beta}_1, ..., \widehat{\beta}_n) + \widehat{\mu}(x^c).$$

Then the mean-squared error of the k-step ahead forecast satisfies:

$$\lim_{k\to\infty} \mathrm{MSE}_k \cong \mathrm{Var}(x_t^c) < \infty,$$

since we forecast the trend pretty well and long run forecasts of a stationary process are trivial (equal to the mean).

For the same reason, an increase of x_T has a negligible impact on the forecast of x_{T+k} when k is large.

Stochastic trends

Assuming an underlying deterministic trend is a strong assumption, with little economic basis.

A more attractive option is to assume a stochastic trend, that is, that the trend component is I(1) and the cyclical component I(0).

We say that the trend is stochastic, because Δx_t, which can be interpreted as the growth rate of the macroeconomic series of interest, follows a stationary process. The average growth rate over a given period then is random.

Hodrick-Prescott (HP) Filter

Hodrick and Prescott (1980).

Given a series $x_1, x_2, ..., x_T$ find $x_1^{\text{tr}}, x_2^{\text{tr}}, ..., x_T^{\text{tr}}$ that minimize:

$$\sum_{t=1}^{T} (x_t - x_t^{\text{tr}})^2 + \lambda \sum_{t=2}^{T-1} (\Delta x_{t+1}^{\text{tr}} - \Delta x_t^{\text{tr}})^2. \tag{2}$$

This program assumes that the trend is "smooth" by penalizing variations in its second difference.

The parameter λ captures the relative importance of having a smooth trend, versus fitting the observed series.

- As λ increases, the trend becomes smoother.
- Obtain a linear trend as $\lambda \to \infty$.
- As λ tends to zero we obtain that the trend approaches the actual series.

In practice, for quarterly series, Hodrick and Prescott suggested setting λ to 1600. This is done by most practitioners, often without giving this choice a second thought.

Kydland and Prescott (1990) justify their choice of λ (and, more generally, using the HP filter in macro) arguing that you want a simple linear filter, well defined, judgement free, cheaply available, and that x_t^{tr} should "approximate the curve that students of business cycles would draw through a time plot..."

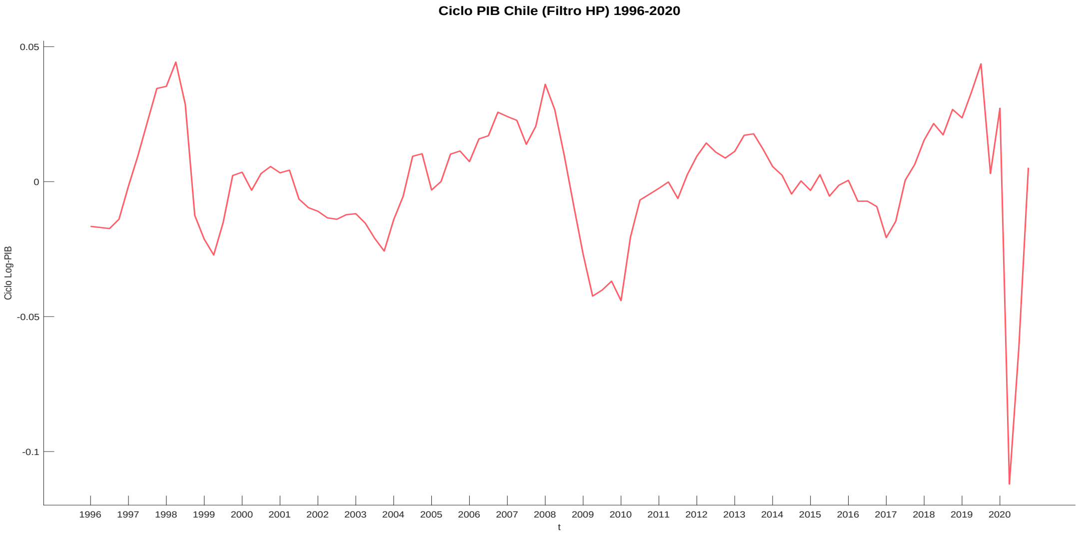
Wabha (1980) shows that if the cyclical component and the second difference of the trend components are independent white noises, with variances σ_c^2 and σ_{tr}^2, then program (2) provides the best estimates for the trend if $\lambda = \sigma_c^2/\sigma_{\mathrm{tr}}^2$. Thus setting $\lambda = 1600$ means that the standard deviation of cyclical innovations is 40 times the standard deviation of innovations for the (second difference of the) trend.

Usually x_t does not satisfy the conditions that make (2) optimal:

- Innovations to trend and cycle independent?
- Trend I(2)?

Hodrick-Prescott (HP) Filter

- Nonetheless, the HP filter works reasonably well when the variance of the innovations of the trend component is much smaller than the variance of the innovations of the cyclical component.
- The following slide shows the cyclical component for Chile's GDP obtained via the HP filter.
- Note that the cyclical component was 5% under its mean value during the 2009 recession. and 12% in the recession of 2020.


Ciclo PIB Chile (Filtro HP) 1996-2020
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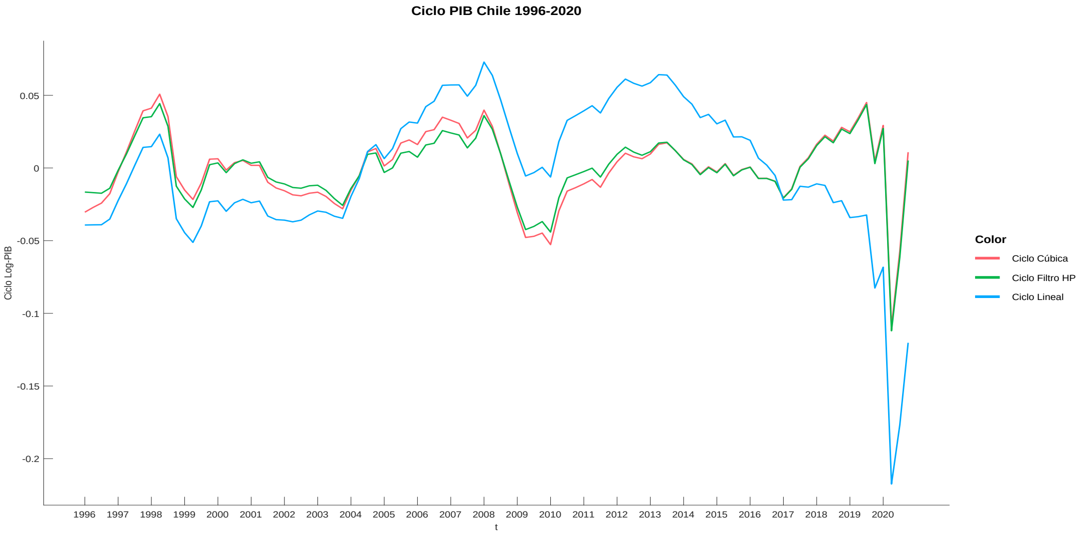
Consideramos:

- Tendencia lineal
- Tendencia cúbica
- Filtro HP

La lámina que sigue muestra las tres componentes cíclicas que se obtienen.

Llama la atención lo similares que son las componentes obtenidas con el filtro HP y la cúbica.



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Ciclo Log-PIB
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Ciclo Filtro HP
Ciclo Lineal

Hodrick-Prescott (HP) Filter

- Solving (2) leads to $T \times T$ matrix F s.t.:

$$\left[\begin{array}{c} x_1 \\ \vdots \\ x_T \end{array}\right] = F \left[\begin{array}{c} x_1^{\mathrm{tr}} \\ \vdots \\ x_T^{\mathrm{tr}} \end{array}\right].$$

where F depends on T and λ. F is invertible. Thus, using the inverse of F we obtain the trend component. It turns out that x_t^{tr} depends on $x_{t-1}, x_{t-1}, x_t, x_{t+1}, x_{t+2}$ (except at the beginning and end of the series, where some of these values are not observed and the filter is not symmetric).

GDP: Deterministic or Stochastic Trend?

As we saw earlier in these slides the variance of the forecast errors converges to the variance of the cyclical component, as the forecast horizon grows, for a deterministic trend, while it tends to infinity for an I(1) process.

One family of formal tests to decide whether a time series has a deterministic or stochastic trend is based on this difference.

For example, visual inspection of the next slide (taken from Stock and Watson, 1988) strongly suggests that the underlying trend is stochastic:

- lower line: forecast based on the 50s, assuming deterministic trend
- upper line: forecast based on the 60s, assuming deterministic trend
- dotted lines: ± 2 standard deviations confidence band
- conclusion: **not** a deterministic trend; forecast errors are consistent with a stochastic trend, since the forecast error grows with the forecast horizon

Formal tests rejecting the deterministic trend hypothesis are based on this idea. They show, conclusively, that this assumption does not hold for macroeconomic series.

REJECTING DETERMINISTIC TRENDS

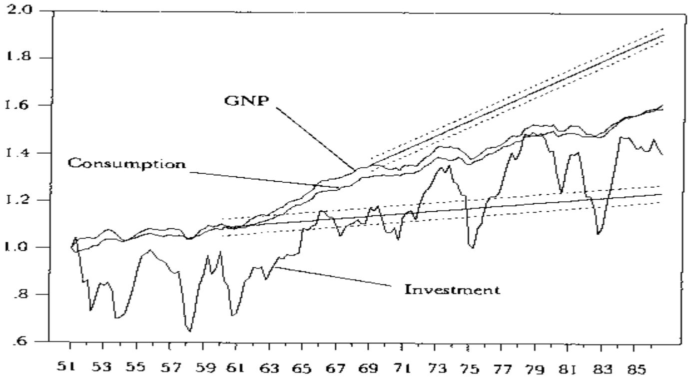


Fig. 1. Postwar real per capita U.S. GNP, total consumption, and gross private domestic investment (in logarithms)

Trend-Cycle Decomposition

Cost of Business Cycles

Rational Expectations

Trend-Cycle Decomposition

Cost of Business Cycles

Rational Expectations

Cost of Business Cycles

How much would welfare increase if business cycles were completely eliminated?

- Great question.
- Will need strong assumptions and simplifications to come up with numbers.
- Lucas (1988) provided the answer we study next.

Cost of Business Cycles

Assume that the log of consumption per capita (which we interpret as log-consumption of the representative household) can be decomposed into the sum of a linear trend (with growth rate g) and an i.i.d. cyclical component that is normal with mean $-\frac{1}{2}\sigma_D^2$ and variance σ_D^2:

$$C_t = (1+\lambda)(1+g)^t e^{\varepsilon_t^D},$$

where $\mathrm{E}e^{\varepsilon_t^D} = 1$. Here we used that if $\log X$ is $\mathcal{N}(\mu, \sigma^2)$ then $\mathrm{E}[X] = e^{\mu + \frac{1}{2}\sigma^2}$.

Cost of Business Cycles

Define:

$$W(\lambda, \beta, g, \gamma, \sigma) = \mathrm{E}_0 \left[\sum_{t \geq 0} \beta^t u(C_t) \right],$$

with $u(c) = c^{1-\gamma}/(1-\gamma)$, where γ denotes the coefficient of relative risk aversion,

Lucas proposes to gauge the cost of business cycles by determining the value of λ that solves

$$W(\lambda, \beta, g, \gamma, \sigma_D) = W(0, \beta, g, \gamma, 0). \tag{3}$$

That is, λ is the fraction by which consumption must increase (in all periods) to compensate the representative household for consumption volatility. The r.h.s. of (3) is the household's discounted expected welfare when consumption fluctuations are completely eliminated. The l.h.s. of (3) equals discounted expected welfare with consumption volatility compensated by a proportional increase of λ.

Cost of Business Cycles

A straightforward calculation leads to:

$$\lambda = e^{\frac{1}{2}\gamma\sigma_D^2} - 1.$$

We estimate σ_D as the standard deviation of the residuals obtained from regressing $\log C$ on a constant and time. We then have $\lambda = 0.068\%$ for $\gamma = 1$.

Legend has it that Lucas stopped working on business cycles and shifted to growth after dong this calculation. A follow-up legend has it that Lucas reconsidered his decision in 2009.

Cost of Business Cycles: Stochastic Trend

Now we assume that consumption has a stochastic trend. More precisely, we assume that consumption per capita follows a random walk.

$$C_t = (1+\lambda)(1+g)^t[\epsilon_t \epsilon_{t-1} \cdots \epsilon_0] C_0,$$

with the ε i.i.d. normal with mean $-\frac{1}{2}\sigma_S^2$ and variance σ_S^2.

Now we obtain:

$$\lambda = \left[\frac{1-\beta(1+g)^{1-\gamma} e^{-\frac{1}{2}\gamma(1-\gamma)\sigma_S^2}}{1-\beta(1+g)^{1-\gamma}} \right]^{1/(1-\gamma)} - 1.$$

Where we estimate σ_S as the standard deviation of $\Delta \log C_t$. We consider annual $\beta = 0.96$. We then obtain $\lambda = 0.241\%$ for $\gamma = 1$.

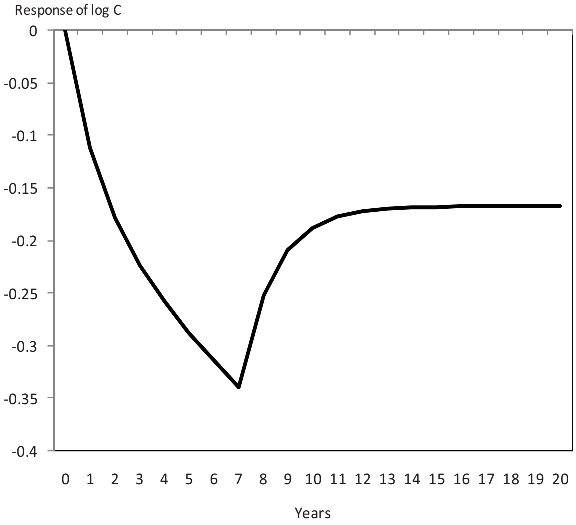
Cost of Business Cycles: Consumption disasters

In a series of papers, Robert Barro and coauthors have documented consumption disasters in a large number of countries. In Nakamura, Steinsson, Barro and Ursua (2010) they consider consumption disasters in 24 countries over a period of more than 100 years.

They find:

- Prob. of disaster: 1.7% (annual)
- Avge. length of disaster: 6.5 years
- Avge. fall in consumption: 30%
- Only half of the fall is reversed in the long run (see figure on following slide for a typical disaster episode).
- Large increase in consumption volatility during disasters
- Incorporating consumption disasters increases λ a lot.

Cost of Business Cycles: Consumption disasters



Cost of Business Cycles: Beyond Lucas

Additional reasons why Lucas's calculation may be underestimating the true cost of business cycles:

- Higher risk aversion: Obstfeld (1994).
- Use Epstein-Zin utility to break relation between risk aversion and intertemporal smoothing. Tallarini (2000).
- The Lucas calculation assumes idiosyncratic risk can be diversified. This isn't true in an incomplete markets framework.
- Consider distributional issues: not all households affected the same, some are unemployed, others are not.

Coffee Break

Trend-Cycle Decomposition

Cost of Business Cycles

Rational Expectations

Trend-Cycle Decomposition

Cost of Business Cycles

Rational Expectations

Rational Expectations

Usually rational expectations means that economic agents know the true model.

Yet rational expectations is much more than that.

And, in solving the Calvo model in the preceding section, we have used the broader meaning for rational expectations.

We illustrate the main ideas with a simple example, close to the examples used by Muth (Econometrica, 1961)

And then explain where and how we used rational expectations in its more nuanced sense in the preceding section.

Rational Expectations and Commodity Markets

Supply of an agricultural good in period t, q_t^S, depends on the price expected by producers when they make production decisions, p_t^e, as captured by the following supply function:

$$q_t^S = c p_t^e + v_t \tag{4}$$

where v_t is i.i.d. and independent of previous prices and quantities.

Demand is given by:

$$q_t^D = 1 - p_t. \tag{5}$$

To determine equilibrium prices and quantities, we need to make assumptions about how producers form expectations about future prices.

Naive expectations

Assume $p_t^e = p_{t-1}$. That is, people expect the current price to prevail in the next period.

Solving for $q_t^S = q_t^d$ leads to

$$p_t = 1 - cp_{t-1} - v_t.$$

and p_t follows an AR(1).

Two problems (the first reason is more important):

- People assume $p_t^e = p_{t-1}$, yet when period t price occurs, they should realize that their expectation was wrong, which they do not. That is, expectations are not rational in the sense that producers do not learn for their forecast mistakes.
- The process p_t will be explosive if $|c| > 1$.

Rational expectations

We assume that when forming expectations about period t prices, producers use all information available in the best possible way, that is:

$$p_t^e = \mathrm{E}_{t-1} p_t \equiv \mathrm{E}[p_t | \mathscr{I}_{t-1}],$$

with $\mathscr{I}_{t-1} = \{p_{t-1}, q_{t-1}, p_{t-2}, q_{t-2}, ...\}$.

We also assume producers know the model and the corresponding parameters (c in our case).

Equating supply and demand:

$$c\mathrm{E}_{t-1} p_t + v_t = 1 - p_t.$$

Taking E_{t-1} on both sides yields

$$\mathrm{E}_{t-1} p_t = \frac{1}{1+c}.$$

Substituting this expression in (5) and (4):

$$
\begin{aligned}
p_t &= \frac{1}{1+c} - v_t, \\
q_t &= \frac{c}{1+c} + v_t.
\end{aligned}
$$

That is:

- producers assume $p_t = \frac{1}{1+c} - v_t$,
- therefore at time $t-1$ they set $p_t^e = \mathrm{E}_{t-1} p_t = 1/(1+c)$,
- the process of prices, p_t, they observe will follow the process they assumed,
- therefore they will not regret having made the expectations they made

Rational Expectations

Next we relax the i.i.d. assumption for v_t and assume an AR(1) process instead:

$$v_t = \phi v_{t-1} + e_t,$$

with $|\phi| < 1$ and e_t is i.i.d.

Assuming rational expectations and equating supply and demand yields

$$c\mathrm{E}_{t-1} p_t + v_t = 1 - p_t.$$

Taking E_{t-1} on both sides:

$$(1+c)\mathrm{E}_{t-1} p_t + \phi v_{t-1} = 1$$

and we obtain

$$\mathrm{E}_{t-1} p_t = \frac{1}{1+c} - \frac{\phi}{1+c} v_{t-1}.$$

Rational Expectations

Substituting the expression above in (4):

$$p_t = \frac{1}{1+c} - v_t + \frac{\phi c}{1+c} v_{t-1}.$$

Applying $1-\phi L$ on both sides:

$$(1-\phi L)\, p_t = \frac{1-\phi}{1+c} - e_t + \frac{\phi c}{1+c} e_{t-1}.$$

Letting $\widetilde{e}_t \equiv -e_t$, we trivially have that $\widetilde{e}_t$ will also be i.i.d. and

$$(1-\phi L)\, p_t = \frac{1-\phi}{1+c} + \widetilde{e}_t - \frac{\phi c}{1+c} \widetilde{e}_{t-1}.$$

We conclude that p_t follows an ARMA(1,1) with AR coefficient ϕ and MA coefficient $-\phi c/(1+c)$.

Rational Expectations

Using the ARMA(1,1) representation for p_t and (5) we conclude that q_t also follows an ARMA(1,1):

$$(1-\phi L)q_t = \frac{c+\phi}{1+c} + e_t - \frac{\phi c}{1+c}e_{t-1}.$$

As in the i.i.d. case, producers assume prices and quantities follow the ARMA(1,1) processes derived above, they form their expectations about future prices accordingly, and once prices and quantities are realized they see confirmation for the processes they assumed.

That is, producers have **model consistent expectations**, which over time became known as rational expectations.

Rational expectations and commodity markets

Summary

Agents assume
a process for p_t

Against set
$p_t^e = \mathrm{E}_{t-1} p_t$

Market clears:
p_t and q_t determined

Actual process
for p_t determined

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