PAUTA TAREA 3

Pregunta 1

a. - $Y_i(t) = K_i(t)^{\alpha}[A(t)L_i(t)]^{1-\alpha}$ $\quad A(t) = BK(t)$

- $n = 0$

- $\displaystyle\int_0^{\infty} e^{-\rho t}\, \frac{c(t)^{1-\theta} - \theta}{1-\theta}\, dt$

i) $Y_i(t) = K_i(t)^{\alpha}[BK(t)L_i(t)]^{1-\alpha}$

pmg. de K:
$$\frac{\partial Y_i(t)}{\partial K_i(t)} = \alpha K_i(t)^{\alpha-1}[BK(t)L_i(t)]^{1-\alpha} = \boxed{\alpha (BK(t))^{1-\alpha}(K_i(t)/L_i(t))^{\alpha-1}}$$

pmg. de L:
$$\frac{\partial Y_i(t)}{\partial L_i(t)} = (1-\alpha)L_i(t)^{-\alpha}K_i(t)^{\alpha}[BK(t)]^{1-\alpha} = \boxed{(1-\alpha)(BK(t))^{1-\alpha}(K_i(t)/L_i(t))^{\alpha}}$$

ii) Como mercados son competitivos, $\partial Y/\partial K$ y $\partial Y/\partial L = \forall i \Rightarrow$ para que sean iguales, $K_i(t)/L_i(t) = K(t)/L(t)$, como $n=0 \Rightarrow \boxed{K_i(t)/L_i(t) = K(t)/L}$

iii)
$$\boxed{r(t) = F_K(K_i(t), L_i(t)) - \delta}$$
$$= \partial Y_i(t)/\partial K_i(t) - \delta$$
$$= \alpha (K_i(t)/L_i(t))^{\alpha-1}(BK(t))^{1-\alpha} - \delta$$
$$= \alpha (K(t)/L)^{\alpha-1}(BK(t))^{1-\alpha} - \delta$$
$$\boxed{r = \alpha (BL)^{1-\alpha} - \delta} \rightarrow \text{es cte. en el tiempo}$$

$$\boxed{w(t) = F_L(K_i(t), L_i(t))}$$
$$= \partial Y_i(t)/\partial L_i(t)$$
$$= (1-\alpha)(K_i(t)/L_i(t))^{\alpha}[BK(t)]^{1-\alpha}$$
$$= (1-\alpha)(K(t)/L)^{\alpha}[BK(t)]^{1-\alpha}$$
$$= (1-\alpha)K(t)(1/L)^{\alpha}B^{1-\alpha}$$
$$= (1-\alpha)K(t)(L/L)(1/L)^{\alpha}B^{1-\alpha}$$
$$= (1-\alpha)K(t)L^{1-\alpha}B^{1-\alpha}(1/L)$$
$$\boxed{w(t) = (1-\alpha)(LB)^{1-\alpha}K(t)/L}$$

b. i)

$$\max \int_0^{\infty} e^{-\rho t}\, \frac{c(t)^{1-\theta} - \theta}{1-\theta}\, dt \quad \text{s.a.} \quad \dot{a}(t) = r a(t) + w(t) - c(t)$$

$$H = \frac{c(t)^{1-\theta} - \theta}{1-\theta} + \lambda(t)[r a(t) + w(t) - c(t)]$$

① $\frac{\partial H}{\partial c} = 0 \rightarrow c(t)^{-\theta} - \lambda(t) = 0 \rightarrow \lambda(t) = c(t)^{-\theta} \rightarrow \dot{\lambda}(t) = -\theta c(t)^{-\theta-1}\cdot \dot{c}(t)$

② $-\frac{\partial H}{\partial a} = \dot{\lambda}(t) - \rho\lambda(t) \rightarrow -\lambda(t)r(t) = \dot{\lambda}(t) - \rho\lambda(t)$

Reemplazamos $\lambda(t)$ y $\dot{\lambda}(t)$ en ②

$$-c(t)^{-\theta}r(t) = -\theta c(t)^{-\theta-1}\cdot \dot{c}(t) - \rho c(t)^{-\theta} \quad /\cdot c(t)^{\theta}$$
$$-r(t) = -\theta c(t)^{-1}\cdot \dot{c}(t) - \rho$$
$$\theta \frac{\dot{c}(t)}{c(t)} = r(t) - \rho$$
$$\boxed{\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\theta}}$$

ii) si $r(t) = r = \alpha(BL)^{1-\alpha} - \delta \rightarrow \boxed{\frac{\dot{c}(t)}{c(t)} = \frac{\alpha(BL)^{1-\alpha} - \delta - \rho}{\theta} \rightarrow \frac{\dot{c}(t)}{c(t)} \text{ es constante}}$

También sabemos que: $K_i(t)/L_i(t) = K(t)/L \;\forall i \Rightarrow k_i(t) = k(t)$ con $k_i(t) = K_i(t)/L_i(t)$ y $k(t) = K(t)/L$

$\dot{k}(t) = \frac{\partial K(t)/L}{\partial t} = \frac{\dot{K}(t)}{L} \therefore \frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{L} \cdot \frac{L}{K(t)} \Rightarrow \boxed{\frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{K(t)}}$

En ec. cerrada $a(t) = k(t) \therefore$

$\boxed{\dot{k}(t) = rk(t) + w(t) - c(t)}$ con $k(t) = K(t)/L$ y $c(t) = C(t)/L$

$\dot{k}(t) = (\alpha(BL)^{1-\alpha} - \delta)\,k(t) + (1-\alpha)(LB)^{1-\alpha} K(t)/L - c(t) \quad /\cdot 1/k(t)$ con $k(t) = K(t)/L$

$\frac{\dot{k}(t)}{k(t)} = \alpha(BL)^{1-\alpha} - \delta + (1-\alpha)(BL)^{1-\alpha} - \frac{C(t)}{\cancel{L}} \cdot \frac{\cancel{L}}{K(t)}$ $\quad \frac{\dot{k}(t)}{k(t)} = (BL)^{1-\alpha} - \delta - \frac{C(t)}{K(t)}$

$\frac{\dot{k}(t)}{k(t)} = (BL)^{1-\alpha} - \delta - \frac{C(t)}{K(t)} \rightarrow$ recordando que $\dot{k}(t)/k(t) = \dot{K}(t)/K(t) \Rightarrow \boxed{\frac{\dot{K}(t)}{K(t)} = (BL)^{1-\alpha} - \delta - \frac{C(t)}{K(t)}}$

como $\dot{K}(t)/K(t)$ es cte en EE, $\therefore C(t)/K(t)$ es cte $\therefore \boxed{\frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)}}$

Dado que mercados son competitivos: $Y(t) = (r+\delta)K(t) + w(t)L(t)$

$Y(t) = (\alpha(BL)^{1-\alpha} - \delta + \delta)K(t) + (1-\alpha)(LB)^{1-\alpha} K(t)/\cancel{L} \cdot \cancel{L}$

$= \alpha(BL)^{1-\alpha} K(t) + (1-\alpha)(LB)^{1-\alpha} K(t)$

$= (LB)^{1-\alpha} K(t)$

$\frac{Y(t)}{K(t)} = (LB)^{1-\alpha}$ cte $\therefore \boxed{\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)}}$

Ahora bien, tenemos que: $\frac{\dot{C}(t)}{C(t)} = \frac{\dot{K}(t)}{K(t)}$ y $\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} \Rightarrow \boxed{\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{C}(t)}{C(t)}}$

Nos falta encontrar el crecimiento del consumo agregado, a partir del per cápita

$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{C}(t)/L}{C(t)/L} = \frac{\dot{C}(t)}{C(t)} \Rightarrow \boxed{\frac{\dot{C}(t)}{C(t)} = \frac{\alpha(BL)^{1-\alpha} - \delta - \rho}{\theta}} \therefore \boxed{\frac{\dot{Y}(t)}{Y(t)} = \frac{\alpha(BL)^{1-\alpha} - \delta - \rho}{\theta}}$

c. i. $\frac{\partial \dot{Y}/Y}{\partial B} = \frac{\alpha(1-\alpha)B^{-\alpha}L^{1-\alpha}}{\theta} > 0$

↳ $\uparrow B = \uparrow$ productividad $\therefore \uparrow$ crecimiento del producto

ii. $\frac{\partial \dot{Y}/Y}{\partial \rho} = -\frac{1}{\theta} < 0$

↳ $\uparrow \rho$, $\uparrow$ impaciencia hogares y $\therefore \downarrow \dot{c}/c = \dot{Y}/Y$ $\therefore \downarrow \dot{Y}/Y$

iii. $\frac{\partial \dot{Y}/Y}{\partial L} = \frac{\alpha(1-\alpha)B^{1-\alpha}L^{-\alpha}}{\theta} > 0$

↳ Efecto escala

d. La tasa de crecimiento es menor a la socialmente óptima (problema planner) porque los agentes no internalizan que existen spillovers de aprendizaje.

b.ii. solución alternativa

Sabemos que: $\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\theta} = \frac{\alpha(BL)^{1-\alpha} - \delta - \rho}{\theta}$, también tenemos: $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{C}(t)/L}{C(t)/L} = \frac{\dot{C}(t)}{C(t)}$

$\therefore \frac{\dot{C}(t)}{C(t)} = \frac{\alpha(BL)^{1-\alpha} - \delta - \rho}{\theta} \rightarrow$ crecimiento consumo es constante

Usamos que $Y(t) = (r(t)+\delta)K(t) + w(t)L(t) = (r+\delta)K(t) + w(t)L$

$$= \alpha(BL)^{1-\alpha}K(t) + (1-\alpha)(BL)^{1-\alpha}\frac{K(t)}{\cancel{L}}\cdot\cancel{L}$$

$Y(t) = (BL)^{1-\alpha}K(t)$ cte $\therefore$ $\boxed{\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)}}$

Podemos escribir $I(t) = Y(t) - C(t) = s(t)Y(t)$, es decir: $\dot{K}(t) = s(t)(BL)^{1-\alpha}K(t) - \delta K(t)$ donde s es la tasa de ahorro

$$\therefore \frac{\dot{K}(t)}{K(t)} = s(t)(BL)^{1-\alpha} - \delta = \frac{\dot{Y}(t)}{Y(t)}$$

Sabemos que $s(t)$ viene de $C(t) = (1-s)Y(t) \rightarrow s(t) = 1 - \frac{C(t)}{Y(t)}$

$$\therefore \frac{\dot{Y}(t)}{Y(t)} = (BL)^{1-\alpha}\left(1 - \frac{C(t)}{Y(t)}\right) - \delta$$

si $\dot{Y}/Y < \dot{c}/c$, c/Y crecería a tal punto que $\dot{Y}/Y$ se volvería negativa $\rightarrow$ no es plausible

si $\dot{Y}/Y > \dot{c}/c$, c/Y caería hasta 0 tal que $\dot{Y}/Y = (BL)^{1-\alpha} - \delta > r$ $\rightarrow$ tampoco es plausible

$\therefore$ el único equilibrio plausible es uno donde $\dot{Y}/Y = \dot{c}/c$

Pregunta 2

a. El equilibrio competitivo va a estar dado por la secuencia de precios y asignaciones $[K(t), C(t), a(t), r(t), w(t)]_{t=0}^{\infty}$ tal que: 1) se resuelve el problema del consumidor y de las firmas, y 2) se vacía el mercado de activos y bienes finales.

b. hogar: $\max \int_0^{\infty} e^{-(\rho-n)t}\frac{c(t)^{1-\theta}-1}{1-\theta}\,dt$ s.a. $\dot{a}(t) = (r(t)-n)a(t) + w(t) - c(t)$

$$H: \frac{c(t)^{1-\theta}-1}{1-\theta} + \lambda(t)\left[(r(t)-n)a(t) + w(t) - c(t)\right]$$

① $\frac{\partial H}{\partial c} = 0 \rightarrow \frac{(1-\cancel{\theta})}{(\cancel{1-\theta})}c(t)^{-\theta} - \lambda(t) = 0 \Rightarrow \lambda(t) = c(t)^{-\theta} \rightarrow \dot{\lambda}(t) = -\theta\cdot c(t)^{-\theta-1}\cdot\dot{c}(t)$

② $-\frac{\partial H}{\partial a} = \dot{\lambda}(t) + (n-\rho)\lambda(t) \rightarrow -\lambda(t)[r(t)-n] = \dot{\lambda}(t) + (n-\rho)\lambda(t)$

$$\therefore -c(t)^{-\theta}(r(t)-n) = -\theta\cdot c(t)^{-\theta-1}\cdot\dot{c}(t) + (n-\rho)\cdot c(t)^{-\theta} \quad /\cdot c(t)^{\theta}$$

$$-r(t) + \cancel{n} = -\theta\cdot\frac{\dot{c}(t)}{c(t)} + \cancel{n} - \rho$$

$$\boxed{\frac{\dot{c}(t)}{c(t)} = \frac{r(t)-\rho}{\theta}}$$

Firmas: $r(t) = f'(k(t)) - \delta$

$w(t) = f(k(t)) - k(t)f'(k(t))$

$f(k(t)) = A\frac{K(t)}{L(t)} + B = Ak(t) + B$

$$\therefore \boxed{\begin{aligned} r(t) &= A - \delta \\ w(t) &= Ak(t) + B - k(t)A \rightarrow w(t) = B \end{aligned}}$$

$\therefore$ en equilibrio:

1) $\boxed{\frac{\dot{c}(t)}{c(t)} = \frac{A-\delta-\rho}{\theta}}$

2) $\dot{a}(t) = \dot{k}(t) \rightarrow \boxed{\dot{k}(t) = (A-\delta-n)k(t) + B - c(t)}$

3) $\lim_{t\to\infty} a(t)e^{-\int_0^t r(v)-n\,dv} = 0$

$\lim_{t\to\infty} k(t)e^{-\int_0^t A-\delta-n\,dt} = \boxed{\lim_{t\to\infty} k(t)e^{-(A-\delta-n)t} = 0 \quad \therefore \quad A > \delta + n}$

c. Share del trabajo : $1 - \frac{K(t) f'(K(t))}{f(K(t))} = 1 - \frac{K(t) \cdot A}{A K(t) + B} = \frac{A K(t) + B - K(t) A}{A K(t) + B} = \frac{B}{A K(t) + B}$

A medida que aumenta el capital, disminuye el share del trabajo.

d. Problema planner :

$$\max \int_0^\infty e^{-(\rho-n)t} \frac{c(t)^{1-\theta} - 1}{1-\theta} \, dt \quad \text{s.a.} \quad \dot{k}(t) = f(k(t)) - c(t) - (\delta+n) k(t)$$
$$\dot{k}(t) = A k(t) + B - c(t) - (\delta+n) k(t)$$

$$H: \frac{c(t)^{1-\theta} - 1}{1-\theta} + \lambda(t) \left[(A-\delta-n) k(t) + B - c(t) \right]$$

1) $\frac{\partial H}{\partial c} = 0 \rightarrow \frac{1-\theta}{1-\theta} \cdot c(t)^{-\theta} - \lambda(t) = 0 \rightarrow \lambda(t) = c(t)^{-\theta}$

2) $-\frac{\partial H}{\partial k} = \dot{\lambda}(t) + (n-\rho) \lambda(t) \rightarrow -\lambda(t)(A-\delta-n) = \dot{\lambda}(t) + (n-\rho)\lambda(t)$

$$-c(t)^{-\theta} (A-\delta-n) = -\theta \cdot c(t)^{-\theta-1} \cdot \dot{c}(t) + (n-\rho) c(t)^{-\theta} \cdot / c(t)^{\theta}$$
$$-(A-\delta-n) = -\theta \cdot \frac{\dot{c}(t)}{c(t)} + n - \rho$$

$$\boxed{\frac{\dot{c}(t)}{c(t)} = \frac{A-\delta-\rho}{\theta}} \rightarrow$$ llegamos a la misma tasa de crecimiento. $\therefore$ el equilibrio descentralizado es pareto eficiente

Pregunta ayudantía

- $d_i \leq \lambda K_i$, si $d_i = K_i - a_i \Rightarrow a_i \geq (1-\lambda)K_i$
- $\rho_i + \theta_i x_i > r$

a Hogares: $\max \int_0^\infty e^{-(\rho_i - n_i)t} \dfrac{c_i(t)^{1-\theta_i} - 1}{1-\theta_i} dt$ s.a. $\dot{a_i}(t) = (r - n_i) a_i(t) + w_i(t) - c_i(t)$

resolviendo el típico hamiltoniano, llegamos a: $\boxed{\dfrac{\dot{c_i}(t)}{c_i(t)} = \dfrac{r - \rho_i}{\theta_i}}$

Para pasarlo a trabajo efectivo: $\hat{c_i}(t) = \dfrac{C_i(t)}{A_i(t) L_i(t)}$

$$\hat{c_i}(t) = \frac{C_i(t)}{L_i(t)} \cdot e^{-x_i t}$$

$$\hat{c_i}(t) = c_i(t) \cdot e^{-x_i t}$$

$$\ln(\hat{c_i}(t)) = \ln(c_i(t)) - x_i t$$

$$\ln(\dot{\hat{c_i}}(t)) = \ln(\dot{c_i}(t)) - x_i$$

$$\boxed{\frac{\dot{\hat{c_i}}(t)}{\hat{c_i}(t)} = \frac{\dot{c_i}(t)}{c_i(t)} - x_i}$$

$$\therefore \boxed{\frac{\dot{\hat{c_i}}(t)}{\hat{c_i}(t)} = \frac{r - \rho_i - \theta_i x_i}{\theta_i}}$$

Firmas: $\max \pi_i = F(K_i(t), L_i(t) A_i(t)) - (r + \delta_i) K_i(t) - w_i(t) L_i(t) \;/\; {}^{L_i(t) A_i(t)}\!/\!_{L_i(t) A_i(t)}$

$$= L_i(t) A_i(t) [f(\hat{k_i}(t)) - (r + \delta_i) \hat{k_i}(t) - w_i(t) e^{-x_i t}]$$

$$\frac{\partial \pi_i}{\partial \hat{k_i}} = L_i(t) A_i(t) [f'(\hat{k_i}(t)) - (r + \delta_i)] = 0 \rightarrow \boxed{f'(\hat{k_i}(t)) = r + \delta_i}$$

$$\frac{\partial \pi_i}{\partial L_i} = \cancel{A_i(t)} [f(\hat{k_i}(t)) - (r + \delta_i) \hat{k_i}(t) - w_i(t) e^{-x_i t} + \cancel{L_i(t)} \left(- \frac{f'(\hat{k_i}(t)) K_i(t)}{L_i(t)^{\cancel{2}} A_i(t)} + \frac{(r + \delta_i) K_i(t)}{L_i(t)^{\cancel{2}} A_i(t)}\right)] = 0$$

$$f(\hat{k_i}(t)) - \cancel{(r + \delta_i) \hat{k_i}(t)} - w_i(t) e^{-x_i t} - f(\hat{k_i}(t)) \cdot \hat{k_i}(t) + \cancel{(r + \delta_i) \hat{k_i}(t)} = 0$$

$$\boxed{w_i(t) = e^{x_i t} [f(\hat{k_i}(t)) - \hat{k_i}(t) f'(\hat{k_i}(t))]}$$

b. Asumimos que $r < \rho_i + \theta_i x_i$, es decir que la economía no acumula tantos activos para dejar de ser un país pequeño, lo que implica:

$$\frac{\dot{\hat{c_i}}(t)}{\hat{c_i}(t)} = \frac{r - \rho_i - \theta_i x_i}{\theta_i} \leq 0$$ ∴ el país es impaciente y consumo es decreciente

Como la restricción no está activa al comienzo: $d_i(t) < \lambda K_i(t)$, sin embargo, como son impacientes, sus niveles de deuda comienzan a aumentar hasta que $d_i(t) = \lambda K_i(t) \Rightarrow a_i(t) = (1-\lambda) K_i(t)$, lo que activa la restricción

Para encontrar $\dot{\hat{k_i}}(t)$ con la restricción activa, usamos:

$\dot{a_i}(t) = (r - n_i) a_i(t) + w_i(t) - c_i(t)$ con $\dot{\hat{a_i}}(t) = \hat{a_i}(t) [\dot{a_i}(t) / a_i(t) - x_i]$

$$\therefore \frac{\dot{a_i}(t)}{a_i(t)} = (r - n_i) + \frac{w_i(t)}{a_i(t)} - \frac{c_i(t)}{a_i(t)} \rightarrow \boxed{\dot{\hat{a_i}}(t) = \hat{a_i}(t)(r - n_i) + \frac{\hat{a_i}(t)}{a_i(t)} w_i(t) - \frac{\hat{a_i}(t)}{a_i(t)} c_i(t) - \hat{a_i}(t) x_i}$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t)(r-n_i) + \frac{\hat{a}_i(t)}{a_i(t)} w_i(t) - \frac{\hat{a}_i(t)}{a_i(t)} c_i(t) - \hat{a}_i(t) x_i$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t)(r-n_i-x_i) + \frac{\cancel{A_i(t)}}{A_i(t)\cancel{L_i(t)}} \cdot \frac{\cancel{L_i(t)}}{\cancel{A_i(t)}} w_i(t) - \frac{\cancel{A_i(t)}}{A_i(t)\cancel{L_i(t)}} \cdot \frac{\cancel{L_i(t)}}{\cancel{A_i(t)}} \cdot c_i(t)$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t)(r-n_i-x_i) + e^{-x_i t} w_i(t) - \hat{c}_i(t)$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t)(r-n_i-x_i) + f(\hat{k}_i(t)) - \hat{k}_i(t) f'(\hat{k}_i(t)) - \hat{c}_i(t)$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t)(r-n_i-x_i) + f(\hat{k}_i(t)) - \hat{k}_i(t)(r+\delta_i) - \hat{c}_i(t)$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t) r + \hat{a}_i(t)\delta_i - \hat{a}_i(t)\delta_i - \hat{a}_i(t)(n_i+x_i) - \hat{k}_i(t)(r+\delta_i) + f(\hat{k}_i(t)) - \hat{c}_i(t)$$

$$\dot{\hat{a}}_i(t) = \hat{a}_i(t)(r+\delta_i) - \hat{a}_i(t)(\delta_i+n_i+x_i) - \hat{k}_i(t)(r+\delta_i) + f(\hat{k}_i(t)) - \hat{c}_i(t)$$

$$\dot{\hat{a}}_i(t) = f(\hat{k}_i(t)) - (\hat{k}_i(t) - \hat{a}_i(t))(r+\delta_i) - \hat{a}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)$$

- Como $a_i(t) = (1-\lambda) k_i(t) \Leftrightarrow \hat{a}_i(t) = (1-\lambda)\hat{k}_i(t) \rightarrow \dot{\hat{a}}_i(t) = (1-\lambda)\dot{\hat{k}}_i(t) \rightarrow \dot{\hat{k}}_i(t) = \frac{\dot{\hat{a}}_i(t)}{1-\lambda}$

$$\therefore \dot{\hat{k}}_i(t) = \left(\frac{1}{1-\lambda}\right)\left[f(\hat{k}_i(t)) - (\hat{k}_i(t) - \hat{a}_i(t))(r+\delta_i) - \hat{a}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)\right]$$

- $\hat{k}_i(t) - \hat{a}_i(t) = \hat{k}_i(t) - (1-\lambda)\hat{k}_i(t) = \lambda \hat{k}_i(t)$

$$\therefore \boxed{\dot{\hat{k}}_i(t) = \left(\frac{1}{1-\lambda}\right)\left[f(\hat{k}_i(t)) - \lambda\hat{k}_i(t)(r+\delta_i) - (1-\lambda)\hat{k}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)\right]}$$

Para encontrar $\dot{\hat{c}}/\hat{c}$ cuando la restricción está activa, resolvemos el problema por trabajador efectivo:

Antes: $\max \int_0^\infty e^{-(\rho_i-n_i)t} \frac{c_i(t)^{1-\theta_i} - 1}{1-\theta_i} dt$, ahora $\hat{c}_i(t) = e^{-x_i t} c_i(t)$, $c_i(t) = e^{x_i t}\hat{c}_i(t)$ $= \max \int_0^\infty e^{-(\rho_i-n_i)t} \frac{(e^{x_i t}\hat{c}_i(t))^{1-\theta_i} - 1}{1-\theta_i} dt$

$$= \max \int_0^\infty e^{-(\rho_i-n_i)t}\left[\frac{e^{x_i t(1-\theta_i)} \cdot \hat{c}_i(t)^{1-\theta_i}}{1-\theta_i} - \frac{1}{1-\theta_i}\right] dt$$

$$\therefore \boxed{\begin{aligned} &\max \int_0^\infty e^{[(1-\theta_i)x_i - \rho_i + n_i]t} \frac{\hat{c}_i(t)^{1-\theta_i}}{1-\theta_i} - \frac{e^{-(\rho_i-n_i)t}}{1-\theta_i} dt \quad \text{s.a.} \\ &\dot{\hat{k}}_i(t) = \left(\frac{1}{1-\lambda}\right)\left[f(\hat{k}_i(t)) - \lambda\hat{k}_i(t)(r+\delta_i) - (1-\lambda)\hat{k}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)\right]\end{aligned}}$$

$$H: \frac{\hat{c}_i(t)^{1-\theta_i}}{1-\theta_i} + \mu(t)\left[\left(\frac{1}{1-\lambda}\right)\left[f(\hat{k}_i(t)) - \lambda\hat{k}_i(t)(r+\delta_i) - (1-\lambda)\hat{k}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)\right]\right]$$

1) $\frac{\partial H}{\partial \hat{c}} = 0 \rightarrow \frac{(1-\theta_i)\cdot \hat{c}_i(t)^{-\theta_i}}{1-\theta_i} - \frac{\mu(t)}{1-\lambda} = 0 \rightarrow \mu(t) = (1-\lambda)\hat{c}_i(t)^{-\theta_i}$

2) $-\frac{\partial H}{\partial \hat{k}} = \dot{\mu}(t) + [(1-\theta_i)x_i - \rho_i + n_i]\mu(t) \rightarrow -\mu(t)\left(\frac{1}{1-\lambda}\right)\left[f'(\hat{k}_i(t)) - \lambda(r+\delta_i) - (1-\lambda)(\delta_i+n_i+x_i)\right] = \dot{\mu}(t) + [(1-\theta)x_i - \rho_i + n_i]\mu(t)$

$$\dot{\mu}(t) = -\theta_i(1-\lambda)\hat{c}_i(t)^{-\theta_i-1} \cdot \dot{\hat{c}}_i(t)$$

$$\therefore -\frac{\cancel{(1-\lambda)}\hat{c}_i(t)^{-\theta_i}}{\cancel{1-\lambda}}\left[f'(\hat{k}_i(t)) - \lambda(r+\delta_i) - (1-\lambda)(\delta_i+n_i+x_i)\right] = -\theta_i(1-\lambda)\hat{c}_i(t)^{-\theta_i-1} \cdot \dot{\hat{c}}_i(t) + [(1-\theta_i)x_i - \rho_i + n_i](1-\lambda)\hat{c}_i(t)^{-\theta_i} \quad /\cdot\ \hat{c}_i(t)^{\theta_i}$$

$$-\left[f'(\hat{k}_i(t)) - \lambda(r+\delta_i) - (1-\lambda)(\delta_i+n_i+x_i)\right] = -\theta_i(1-\lambda)\cdot\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} + (1-\lambda)[(1-\theta_i)x_i - \rho_i + n_i]$$

$$-[f'(\hat{k}_i(t)) - \lambda(r+\delta_i) - (1-\lambda)(\delta_i + n_i + x_i)] = -\theta_i(1-\lambda)\cdot\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} + (1-\lambda)[(1-\theta_i)x_i - \rho_i + n_i]$$

$$\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \frac{1}{\theta_i}\cdot\frac{1}{1-\lambda}\Big[f'(\hat{k}_i(t)) - \lambda(r+\delta_i) - (1-\lambda)(\delta_i+n_i+x_i) + (1-\lambda)[(1-\theta_i)x_i - \rho_i + n_i]\Big]$$

$$= \frac{1}{\theta_i}\cdot\frac{1}{1-\lambda}\Big[f'(\hat{k}_i(t)) - \lambda r - \cancel{\lambda\delta_i} - \delta_i + \cancel{\lambda\delta_i} - \cancel{(1-\lambda)n_i} - (1-\lambda)x_i + (1-\lambda)(1-\theta_i)x_i - (1-\lambda)\rho_i + \cancel{(1-\lambda)n_i}\Big]$$

$$= \frac{1}{\theta_i}\left[\frac{f'(\hat{k}_i(t)) - \lambda r - \delta_i}{1-\lambda} - \cancel{x_i} + \cancel{x_i} - \theta_i x_i - \rho_i\right]$$

$$\boxed{\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \frac{1}{\theta_i}\left[\frac{f'(\hat{k}_i(t)) - \lambda r - \delta_i}{1-\lambda} - \theta_i x_i - \rho_i\right]}$$

$\therefore$ cuando la restricción está activa, las ecuaciones de equilibrio son:

$$\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \frac{1}{\theta_i}\left[\frac{f'(\hat{k}_i(t)) - \lambda r - \delta_i}{1-\lambda} - \theta_i x_i - \rho_i\right]$$

$$\dot{\hat{k}}_i(t) = \left(\frac{1}{1-\lambda}\right)\left[f(\hat{k}_i(t)) - \lambda\hat{k}_i(t)(r+\delta_i) - (1-\lambda)\hat{k}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)\right]$$

c. $\lambda = 0 \rightarrow$

$$\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \frac{1}{\theta_i}\left[f'(\hat{k}_i(t)) - \delta_i - \theta_i x_i - \rho_i\right]$$

$$\dot{\hat{k}}_i(t) = f(\hat{k}_i(t)) - \hat{k}_i(t)(\delta_i + n_i + x_i) - \hat{c}_i(t)$$

si $\lambda = 0 \rightarrow \hat{d}_i = \lambda\hat{k}_i \rightarrow \hat{d}_i = 0$, no me puedo endeudar, por lo tanto, llegamos a las mismas dinámicas de ec. cerrada y $\hat{a}_i = \hat{k}_i$

$\lambda \in (0,1) \rightarrow$ mantenemos las dinámicas de b).

$\lambda = 1 \rightarrow$

$$\lim_{\lambda\to 1}\dot{\hat{k}}_i(t) = \lim_{\lambda\to 1}\left(\frac{1}{1-\lambda}\right)\left[f(\hat{k}_i(t)) - \lambda\hat{k}_i(t)(r+\delta_i) - (1-\lambda)\hat{k}_i(t)(\delta_i+n_i+x_i) - \hat{c}_i(t)\right]$$

$$\lim_{\lambda\to 1}\dot{\hat{k}}_i(t) = \frac{f(\hat{k}_i(t)) - \hat{k}_i(t)(r+\delta_i) - \hat{c}_i(t)}{0}$$

$$\lim_{\lambda\to 1}\dot{\hat{k}}_i(t) = \infty$$

$\rightarrow$ todo el capital es colaterizable $\therefore$ no hay restricción de crédito y el capital se ajusta de forma instantánea

$$\lim_{\lambda\to 1}\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \lim_{\lambda\to 1}\frac{1}{\theta_i}\left[\frac{f'(\hat{k}_i(t)) - \lambda r - \delta_i - (1-\lambda)(\theta_i x_i + \rho_i)}{1-\lambda}\right]$$

$$= \frac{1}{\theta_i}\left[\frac{f'(\hat{k}_i(t)) - r - \delta_i}{1-\lambda}\right] \rightarrow f'(\hat{k}_i(t)) = r + \delta_i$$

$$= \frac{1}{\theta_i}\left[\frac{0}{0}\right] = \frac{0}{0}$$

aplicamos L'Hopital: $\dfrac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \dfrac{1}{\theta_i}\left[\dfrac{f'(\hat{k}_i(t)) - \lambda r - \delta_i - (1-\lambda)(\theta_i x_i + \rho_i)}{1-\lambda}\right]$

$$\text{L'Hopital} \rightarrow \frac{1}{\theta_i}\left[\frac{-r + \theta_i x_i + \rho_i}{-1}\right]$$

$$\lim_{\lambda\to 1}\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \lim_{\lambda\to 1}\left[\frac{1}{\theta_i}\left[\frac{-r + \theta_i x_i + \rho_i}{-1}\right]\right]$$

$$\lim_{\lambda\to 1}\frac{\dot{\hat{c}}_i(t)}{\hat{c}_i(t)} = \frac{1}{\theta_i}\left[r - (\theta_i x_i + \rho_i)\right] < 0$$

$\therefore$ consumo es decreciente porque $r < \theta_i x_i + \rho_i$ (hogares son impacientes)

d. $\dot{\hat{c}}_i(t) = 0 \quad \therefore \quad \frac{1}{\theta_i}\left[\frac{f'(\hat{k}_i(t)) - \lambda r - \delta_i}{1-\lambda} - \theta_i x_i - \rho_i\right] = 0$

$$f'(\hat{k}_i(t)) - \lambda r - \delta_i - (1-\lambda)(\theta_i x_i + \rho_i) = 0$$

$$\boxed{f'(\hat{k}_{ee}) = \lambda r + (1-\lambda)(\theta_i x_i + \rho_i) + \delta_i} \rightarrow$$ Si conocemos $f()$ podemos encontrar $\hat{k}_{ee}$.

$\frac{\partial f'(\hat{k}_{ee})}{\partial \lambda} = r - \theta_i x_i - \rho_i < 0 \rightarrow$ Como $f''(\hat{k}_{ee}) < 0 \Rightarrow \boxed{\frac{\partial \hat{k}_{ee}}{\partial \lambda} > 0} \rightarrow$ ↑ proporción colateralizable, ↑ límite max de deuda

∴ ↑ demanda de k

$\frac{\partial f'(\hat{k}_{ee})}{\partial r} = \lambda > 0 \rightarrow$ como $f''(\hat{k}_{ee}) < 0 \Rightarrow \boxed{\frac{\partial \hat{k}_{ee}}{\partial r} < 0} \rightarrow$ ↑ costo capital, disminuye su demanda