

# El Modelo Schumpeteriano

# Introduction

- Here we will develop an alternative model of endogenous growth, in which growth is generated by a random sequence of quality-improving (or vertical) innovations.
- The model grew out of modern industrial organization theory, which portrays innovation as an important dimension of industrial competition.
- This model is Schumpeterian in that: (i) it is about growth generated by innovations; (ii) innovations result from entrepreneurial investments that are themselves motivated by the prospects of monopoly rents; and (iii) new innovations replace old technologies: in other words, growth involves creative destruction.

# Introduction

- Schumpeterian growth theory has developed into an integrated framework for understanding not only the macroeconomic structure of growth but also the many microeconomic issues regarding incentives, policies and organizations that interact with growth:
  - who gains and who loses from innovations, and what the net rents from innovation are; these ultimately depend on characteristics such as property right protection, competition and openness, education, democracy and so forth and to a different extent in countries or sectors at different stages of development.

# Introduction

- This model of growth with vertical innovations has the natural property that new inventions make old technologies or products obsolete.
- This obsolescence (or creative destruction) feature in turn has both positive and normative consequences.
- On the positive side it implies a negative relationship between current and future research, which results in the existence of a unique steady state (or balanced growth) equilibrium.

# Introduction

- On the normative side, although current innovations have positive externalities for future research and development, they also exert a negative externality on incumbent producers.
- This business-stealing effect in turn introduces the possibility that growth be excessive under laissez-faire, a possibility that did not arise in the endogenous growth model previously studied.

# A (very) simple model

- We develop a simple version of the Schumpeterian growth model with discrete time and where individuals and firms live for one period.
- The basic model abstracts from capital accumulation completely.
- There is a unique final good in this economy,  $Y_t$ ; which is used for consumption  $C_t$ ; intermediate good production  $X_t$ ; and R&D  $R_t$ .
- Therefore, the resources constraint of this economy is simply

$$Y_t = C_t + X_t + R_t.$$

# Production technology

- There is a sequence of discrete time periods  $t = 1, 2, \dots$
- Each period there is a fixed number  $L$  of individuals, each of whom lives for just that period and is endowed with one unit of labor services which she supplies inelastically.
- Her utility depends only on her consumption, and she is risk-neutral, so she has the single objective of maximizing expected consumption.

# Production technology

- People consume only one good, called the final good, which is produced by perfectly competitive firms using two inputs - labor and a single intermediate product - according to the Cobb-Douglas production function:

$$Y_t = (A_t L_t)^{1-\alpha} y_t^\alpha$$

- where  $Y_t$  is output of the final good in period  $t$ ,  $A_t$  is a parameter that reflects the productivity of the intermediate input that period and  $y_t$  is the amount of intermediate product used.
- The coefficient  $\alpha$  lies between zero and one. The economy's entire labor supply  $L$  is used in final-good production. As in the neoclassical model, we refer to the product  $A_t L$  as the economy's effective labor supply.
- We normalize the price of the final good to unity without loss of any generality.



# Production technology

- The intermediate product is produced by a monopolist each period, using the final good as an input, one for one.
- Let us denote the amount of final good used for intermediate-good production by  $X_t$ .
- Then the production function is simply

$$y_t = X_t.$$

- That is, for each unit of intermediate product, the monopolist must use one unit of final good as input.

# Innovation

- Growth results from innovations that raise the productivity parameter  $A_t$  by improving the quality of the intermediate product.
- Each period there is one person (the entrepreneur) who has an opportunity to attempt an innovation.
- If she succeeds, the innovation will create a new version of the intermediate product, which is more productive than previous versions.

# Innovation

- Let us denote last period's productivity as  $A_{t-1}$
- Specifically, the productivity of the intermediate good in use will go from last period's value  $A_{t-1}$  up to the level  $A_t = \gamma A_{t-1}$ , where  $\gamma > 1$ .
- On the other hand, if she fails then there will be no innovation at  $t$  and, in this case, another randomly chosen monopolist will produce the intermediate good with the old productivity that was used in  $t-1$ , so  $A_t = A_{t-1}$ .

# Innovation

- Hence,

$$(*) \quad A_t = \begin{cases} \gamma A_{t-1} & \text{if entrepreneur is successful,} \\ A_{t-1} & \text{if entrepreneur fails.} \end{cases}$$

- In order to innovate, the entrepreneur must conduct research, a costly activity that uses the final good as its only input.
- As indicated above, research is uncertain, for it may fail to generate any innovation. But the more the entrepreneur spends on research the more likely she is to innovate.

# Innovation

- Specifically, we assume to innovate from  $A_{t-1}$  up to  $A_t = \gamma A_{t-1}$  with probability  $z_t$ ; one must spend the amount

$$R_t = c(z_t)A_{t-1}$$

- For simplicity, assume a quadratic R&D cost:

$$c(z_t) = \delta z_t^2 / 2,$$

- where  $\delta$  is a parameter which inversely measures the productivity of the research sector.

# Timing of events

- step 0: Period  $t$  begins with the initial productivity  $A_{t-1}$  which is inherited from the previous period (cohort).
- step 1: a randomly chosen entrepreneur invests in R&D by choosing  $(z_t; R_t)$
- step 2: innovation (success/failure) is realized, and productivity evolves according to equation \*
- step 3: production of the intermediate good ( $y_t$ ) takes place
- step 4: production of the final good ( $Y_t$ ) takes place
- step 5: consumption ( $C_t$ ) takes place, and period  $t$  ends.

# Solving the model

- The model is solved by backward induction: in each period  $t$ ; we first compute the equilibrium production and profit of a successful innovator; then, we move back one step and compute the optimal innovation intensity by the firm selected to be an innovator.

# Equilibrium production and profits

- We start from step 4. The final good producer maximizes the following objective function

$$\max_{y_t, L_t} \left\{ (A_t L)^{1-\alpha} y_t^\alpha - w_t L_t - p_t y_t \right\}.$$

- Therefore, we can express the inverse demand for intermediate good  $y_t$  and labor as

$$p_t = \partial Y_t / \partial y_t = \alpha (A_t L)^{1-\alpha} y_t^{\alpha-1}, \quad (**)$$

and

$$w_t = (1 - \alpha) A_t^{1-\alpha} L^{-\alpha} y_t^\alpha,$$

where we already imposed that  $L_t = L$ .



# Equilibrium production and profits

- Now we move to step 3. The monopolist with productivity  $A_t$ , taking the demand in previous slide as given, maximizes her expected profit ( $A_t$ ), measured in units of the final good:

$$\Pi(A_t) = \max_{y_t, p_t} \{p_t y_t - y_t\}$$

- Subject to demand (\*\*)

# Equilibrium production and profits

- In equilibrium

$$y_t = \alpha^{\frac{2}{1-\alpha}} A_t L,$$

and equilibrium price

$$p_t = p = \frac{1}{\alpha}.$$

- Finally, the equilibrium profit is:

$$\Pi(A_t) = \pi A_t L, \text{ where } \pi \equiv (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$$

# Equilibrium production and profits

- Final output will be proportional to  $A_t L$

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} A_t L$$

- Therefore, the growth rate of this economy will be equal to the growth rate of the productivity  $A_t$ :

# Equilibrium innovation intensity

- In step 2, for any given innovation rate  $z_t$

$$A_t = \begin{cases} \gamma A_{t-1} & \text{with probability } z_t, \\ A_{t-1} & \text{with probability } 1 - z_t. \end{cases}$$

- We now move back to step 1 and consider the innovation investment decision of the entrepreneur who has the opportunity to innovate at date  $t$ .

# Equilibrium innovation intensity

- If the entrepreneur at  $t$  successfully innovates, she will become the intermediate monopolist that period, because she will be able to produce a better product than anyone else.
- Otherwise, the monopoly will pass to someone else chosen at random, who is able to produce last period's product.
- Thus, the entrepreneur will choose the innovation intensity  $z_t$

$$\text{to } \max_{z_t} \{z_t \Pi(\gamma A_{t-1}) - c(z_t) A_{t-1}\}$$

# Equilibrium innovation intensity

- Which is equivalent to

$$\max_{z_t} \{z_t \pi \gamma L - c(z_t)\}.$$

- The first order condition implies

$$c'(z_t) = \pi \gamma L,$$

- Using the quadratic form for  $c$  we obtain

$$z_t = z = \pi \gamma L / \delta.$$

# Equilibrium innovation intensity

- The following assumption ensures that the innovation rate  $z$  is between zero and one.
- Assumption 1 The parameters of the model satisfies the following condition

$$(1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}} \gamma L < \delta.$$

# Growth

- The rate of economic growth is the proportional growth rate of final good, which according to equation for final output is also the proportional growth rate of the productivity parameter  $A_t$ :

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$$

- It follows that growth will be random. Each period, with probability  $z$  the entrepreneur will innovate, resulting in  $g_t = \frac{\gamma A_{t-1} - A_{t-1}}{A_{t-1}} = \gamma - 1$  and with probability  $1-z$  she will fail, resulting in zero growth.



# Growth

- The growth rate will be governed by this probability distribution every period:

$$g = \mathbb{E}(g_t) = z \cdot (\gamma - 1)$$

- will also be the economy's long-run average growth rate.
- To interpret this formula, note that  $z$  is not just the probability of an innovation each period but also the long-run frequency of innovations; that is, the fraction of periods in which an innovation will occur.
- Also,  $\gamma - 1$  is the proportional increase in productivity resulting from each innovation.
- Thus, the formula expresses a simple but important result of Schumpeterian growth theory:
  - In the long run, the economy's average growth rate equals the frequency of innovations times the size of innovations.

# Growth

- The average growth rate is then:

$$g = \frac{\pi \gamma L}{\delta} (\gamma - 1)$$

where  $\pi \equiv (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$ .

# Comparative statics $g = \frac{\pi\gamma L}{\delta}(\gamma - 1)$

- 1. Growth increases with the productivity of innovations inversely measured by  $\delta$ . This result points to the importance of education, and particularly higher education, as a growth-enhancing device. Countries that invest more in higher education should achieve a higher productivity of research and should also reduce the opportunity cost of research by increasing the aggregate supply of skilled labor.
- 2. Growth increases with the size of innovations, as measured by the productivity improvement factor  $\gamma$ . The frequency of innovation is increasing in  $\gamma$ . The result in turn points to a feature.

# Comparative statics $g = \frac{\pi\gamma L}{\delta}(\gamma - 1)$

- The previous results points to a feature that is important when we discuss cross-country convergence.
- A country that lags behind the world technology frontier has what Gerschenkron (1962) called an advantage of backwardness. That is, the further it lags behind the frontier, the bigger the productivity improvement it will get if it can implement the frontier technology when it innovates, and hence the faster it can grow.

# Comparative statics $g = \frac{\pi\gamma L}{\delta}(\gamma - 1)$

- 3. An increase in the size of population should also bring about an increase in growth by raising the supply of labor  $L$ .

# Welfare analysis

- In this simplified framework, we can now ask the following question: What is the socially optimal level of production and R&D investment in this economy?
- To answer this question, we first need to take a stand on the objective function of the social planner.
- Consider a myopic social planner that maximizes the period-by-period utility which is equivalent to maximizing the per-period consumption.
- Recall that the level of consumption from the resource constraint is simply:

$$C_t = Y_t - X_t - R_t.$$

# Welfare analysis

- In step 4, therefore, the social planner maximizes consumption subject to final good and intermediate good production technologies.
- Note that in step 4, the R&D investment is already made, hence  $R_t$  is taken as constant at this stage.
- The planner's maximization can be rewritten as

$$\tilde{C}(A_t, R_t) \equiv \max_{y_t} \left\{ (A_t L)^{1-\alpha} y_t^\alpha - y_t - R_t \right\}.$$

# Welfare analysis

- From this maximization, the socially optimal level of intermediate good production is

$$y_t^{sp} = A_t L \alpha^{\frac{1}{1-\alpha}}$$

- and the resulting maximum consumption is

$$\tilde{C}(A_t, R_t) \equiv A_t L \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha) - R_t.$$

- The socially optimal GDP is

$$Y_t^{sp} = A_t L \alpha^{\frac{\alpha}{1-\alpha}}.$$



# Welfare analysis

- And we have that

$$y_t^{sp} > y_t \text{ and } Y_t^{sp} > Y_t$$

- Now we go one step back in social planner's problem and specify the maximization problem for the optimal innovation decision as max:

$$C_t^{sp} \equiv \max_{z_t} \{z_t \tilde{C}(\gamma A_{t-1}, R_t) + (1 - z_t) \tilde{C}(A_{t-1}, R_t)\}$$

- subject to the R&D technology.
- We can express the planner's innovation problem as

$$C_t^{sp} \equiv \max_{z_t} \{z_t \gamma A_{t-1} L \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha) + (1 - z_t) A_{t-1} L \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha) - \frac{\delta z_t^2}{2} A_{t-1}\}.$$

# Welfare analysis

- Note here the social planner compares the consumption level upon a successful innovation to consumption in the case of a failure, which implies that the innovation size plays a crucial role for the social planner.
- However, in the decentralized economy, the entrepreneur cares only about the success since in the case of a failure, the market is served by another firm.
- Therefore, the entrepreneur does not internalize the size of the innovation as the social planner does and therefore creates an innovation externality.

# Welfare analysis

- Now we can find the socially optimal innovation rate by taking the first-order condition

$$z_t^{sp} = (\gamma - 1) \frac{L \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha)}{\delta}$$

- If  $\alpha^{\frac{1}{1-\alpha}} < \frac{\gamma-1}{\gamma}$ , we have underinvestment in R&D ( $z_t < z_t^{sp}$ ).
- In this economy, there are two types inefficiencies: (i) Monopoly distortion, and (ii) innovation externality.

# Welfare analysis

- The monopoly externality is governed by the parameter  $\alpha$  which determines the equilibrium markups.
- The innovation externality is governed by  $\gamma$  as described above.
- Therefore, if the innovation externalities are above a threshold (high  $\gamma$ ), then the economy features underinvestment in R&D and vice versa.

# Industrial policy

- A policymaker can improve the welfare in this economy by using standard policy tools such as production subsidy or R&D subsidy.
- Assume that the government subsidizes production and R&D at the rates  $\tau_p$  and  $\tau_R$ ; respectively.
- Now we will find the optimal (welfare maximizing) rates of  $\tau_p$  and  $\tau_R$ .
- For simplicity, we assume that the government finances these subsidies through lump-sum taxes on the household.

# Production subsidy

$$\Pi^{\tau}(A_t) = \max_{y_t} \left\{ \alpha (A_t L)^{1-\alpha} y_t^{\alpha} - (1 - \tau^P) y_t \right\}.$$

- The resulting output is

$$y_t^{\tau} = \left[ \frac{\alpha^2}{1 - \tau^P} \right]^{\frac{1}{1-\alpha}} A_t L.$$

- The optimal subsidy is the one that equates the previous equation to  $y_t^{sp} = A_t L \alpha^{\frac{1}{1-\alpha}}$  :

$$\tau^P = 1 - \alpha.$$

# R&D subsidy

- Now assume that the government is already imposing the optimal production subsidy  $\tau_p = 1 - \alpha$  and also subsidizes R&D at the rate  $\tau_R$ .

- Then the innovation decision of the entrepreneur is

$$\max_{z_t} \{ z_t \gamma A_{t-1} L \alpha^{\frac{1}{1-\alpha}} (1 - \alpha) - (1 - \tau^R) \frac{\delta z_t^2}{2} A_{t-1} \}.$$

- The optimal choice is:

$$z_t^\tau = \frac{\gamma L \alpha^{\frac{1}{1-\alpha}} (1 - \alpha)}{(1 - \tau^R) \delta}$$

# R&D subsidy

- The optimal rate is (remember that  $z_t^{sp} = (\gamma - 1) \frac{L\alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha)}{\delta}$  )

$$\tau^R = 1 - \frac{\alpha\gamma}{(\gamma - 1)}$$

$$\frac{\partial \tau^R}{\partial \gamma} > 0 \text{ and } \frac{\partial \tau^R}{\partial \alpha} < 0.$$

- This implies the optimal R&D subsidy rate is increasing in the size of the innovation. This is intuitive because as we saw, one of the main inefficiencies in R&D spending is the uninternalized contribution of each innovation, which is measured by  $\gamma$ .