Modelo de crecimiento AK

Crecimiento endógeno

- En el modelo de Ramsey, la tasa de crecimiento del ingreso per cápita en estado estacionario, x, es exógena.

- Esos modelos entregan un marco teórico interesante para analizar las dinámicas de transición. Sin embargo, no son tan útiles para entender las fuentes de crecimiento de largo plazo del ingreso per cápita.

- Una forma de solucionar este problema es considerando un concepto de capital más amplio, que incluya el capital humano, como veremos más adelante.

Crecimiento endógeno

- Otra visión del tema del crecimiento indica que la única forma de que la economía pueda escapar de los retornos decrecientes en el largo plazo viene dada por el progreso tecnológico en la forma de generación de nuevas ideas.

- Sin embargo, el modelo neoclásico asume la no rivalidad de las ideas que subyace en la tecnología. Lo anterior puede ser modificado para asumir que son al menos parcialmente excluibles (patentes?).

Sources of Growth

- It is convenient to explain this accounting using a Cobb–Douglas specification. More specifically, suppose final output Y_t is produced using stocks of physical capital K_t and human capital H_t:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

- Where A_t is total factor productivity that can be decompose between A'_t and M_t ($A_t = A'_t M_t$). In this case, A'_t denotes the economy's stock of knowledge, and M_t is anything else that influences total factor productivity (the letter "M" is reminiscent of the "measure of our ignorance" and also is suggestive of "misallocation".

Growth decomposition

- Labor adjusted for human capital can be rewritten as *Lh, where L is employment and h is a* measure of human capital per worker.
- Using lowercase letters for per-capita, we have that:

$$y_{jt} = A_{jt} k_{jt}^{\alpha} h_{jt}^{1-\alpha}$$

Growth decomposition

- Given that the capital-output ratio remains invariant to a productivity increase in the long run, we can write the previous relation in terms of the capital-output ratio:

$$y_{jt} = A_{jt}^{\frac{1}{1-\alpha}} \left(\frac{k_{jt}}{y_{jt}} \right)^{\frac{\alpha}{1-\alpha}} h_{jt}$$

Table 3 Growth accounting for the United States

Period	Output per hour	Contributions from: *K/Y*	Labor composition	Labor-Aug. TFP
1948–2013	**2.5**	**0.1**	**0.3**	**2.0**
1948–1973	3.3	−0.2	0.3	3.2
1973–1990	1.6	0.5	0.3	0.8
1990–1995	1.6	0.2	0.7	0.7
1995–2000	3.0	0.3	0.3	2.3
2000–2007	2.7	0.2	0.3	2.2
2007–2013	1.7	0.1	0.5	1.1

Note: Average annual growth rates (in percent) for output per hour and its components for the private business sector, following Eq. (3).
Source: Authors calculations using Bureau of Labor Statistics, *Multifactor Productivity Trends*, August 21, 2014.

Growth decomposition

- We can consider two countries and decompose the output per capita gap between them as:

$$\frac{y_{jt}}{y_{it}} = \left(\frac{A_{jt}}{A_{it}}\right)^{\frac{1}{1-\alpha}} \left(\frac{k_{jt}/y_{jt}}{k_{it}/y_{it}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{h_{jt}}{h_{it}}\right)$$

Growth decomposition

- The comparisons here are done with respect to the United States (country i).

- Therefore, output per capita differences can be explained by productivity differentials, differences in the capital-output ratio, and differences in human capital.

Development accounting

	GDP per worker, y	Capital/GDP $(K/Y)^{\alpha/(1-\alpha)}$	Human capital, h	TFP	Share due to TFP
United States	1.000	1.000	1.000	1.000	–
Hong Kong	0.854	1.086	0.833	0.944	48.9%
Singapore	0.845	1.105	0.764	1.001	45.8%
France	0.790	1.184	0.840	0.795	55.6%
Germany	0.740	1.078	0.918	0.748	57.0%
United Kingdom	0.733	1.015	0.780	0.925	46.1%
Japan	0.683	1.218	0.903	0.620	63.9%
South Korea	0.598	1.146	0.925	0.564	65.3%
Argentina	0.376	1.109	0.779	0.435	66.5%
Mexico	0.338	0.931	0.760	0.477	59.7%
Botswana	0.236	1.034	0.786	0.291	73.7%
South Africa	0.225	0.877	0.731	0.351	64.6%
Brazil	0.183	1.084	0.676	0.250	74.5%
Thailand	0.154	1.125	0.667	0.206	78.5%
China	0.136	1.137	0.713	0.168	82.9%
Indonesia	0.096	1.014	0.575	0.165	77.9%
India	0.096	0.827	0.533	0.217	67.0%
Kenya	0.037	0.819	0.618	0.073	87.3%
Malawi	0.021	1.107	0.507	0.038	93.6%
Average	0.212	0.979	0.705	0.307	63.8%
1/Average	4.720	1.021	1.418	3.260	69.2%

The product of the three input columns equals GDP per worker. The penultimate row, "Average," shows the geometric average of each column across 128 countries. The "Share due to TFP" column is computed as described in the text. The 69.2% share in the last row is computed looking across the columns, ie, as approximately 3.5/(3.5 + 1.5).
Source: Computed using the Penn World Tables 8.0 for the year 2010 assuming a common value of $\alpha = 1/3$.

Source: Jones (2016)

Growth decomposition

- For all countries considered, the most important gap is the TFP gap.

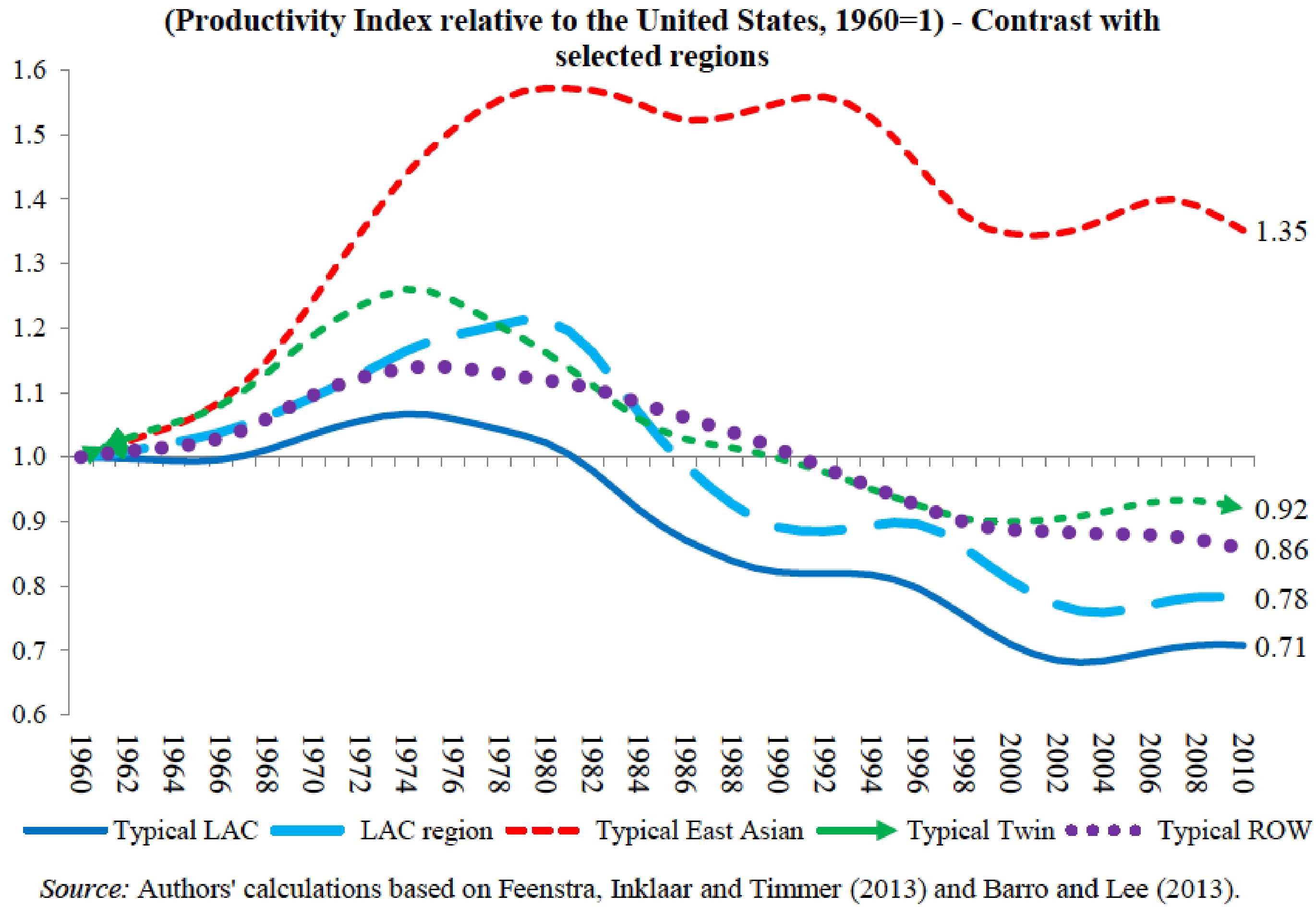
- It is also important to recall that the human capital gap is not corrected by quality and there is strong evidence that quality of education in the region is relatively poor, which may indicate that the human capital gap in previous table could be underestimated.

Growth decomposition

- Hence, the largest gains in terms of closing the income gap may be obtained by closing the productivity gap, that is, by increasing efficiency in the use of existing factors of production, in order to produce more with the same inputs.

- Was the Asian “miracle” more the result of capital deepening than of productivity enhancement? a point originally raised by Young (1995).

Productivity Catch-up: Latin America and selected regions



Source: Fernández-Arias and Rodríguez-Apolinar (2016)

Misallocation

- Hsieh and Klenow (2009): “Large differences in output per worker between rich and poor countries have been attributed, in no small part, to differences in total factor productivity (TFP).
- The natural question then is: What are the underlying causes of these large TFP differences?”
  - Howitt (2000) and Klenow and Rodríguez-Clare (2005) show how large TFP differences can emerge in a world with slow technology diffusion from advanced countries to other countries.

Restuccia and Rogerson (2008)

- Instead of focusing on the efficiency of a representative firm, they suggest that misallocation of resources across firms can have important effects on aggregate TFP.

- For example, imagine an economy with two firms that have identical technologies but in which the firm with political connections benefits from subsidized credit (say from a state-owned bank) and the other firm (without political connections) can only borrow at high interest rates from informal financial markets.

Restuccia and Rogerson (2008)

- Assuming that both firms equate the marginal product of capital with the interest rate, the marginal product of capital of the firm with access to subsidized credit will be lower than the marginal product of the firm that only has access to informal financial markets.

- This is a clear case of capital misallocation: aggregate output would be higher if capital was reallocated from the firm with a low marginal product to the firm with a high marginal product. The misallocation of capital results in low aggregate output per worker and TFP.

Hsieh and Klenow (2009)

- They find that there is greater dispersion of TFP in India and China than in the United States.

- For example, for TFP, the 90-10 ratio is 1.59 in China, 1.60 in India and 1.19 in the United States.

- They estimate that this could account for lower aggregate productivity. In particular, there estimates suggest that this type of misallocation could increase TFP in China by 30%-50% and in India by 40%-60%.

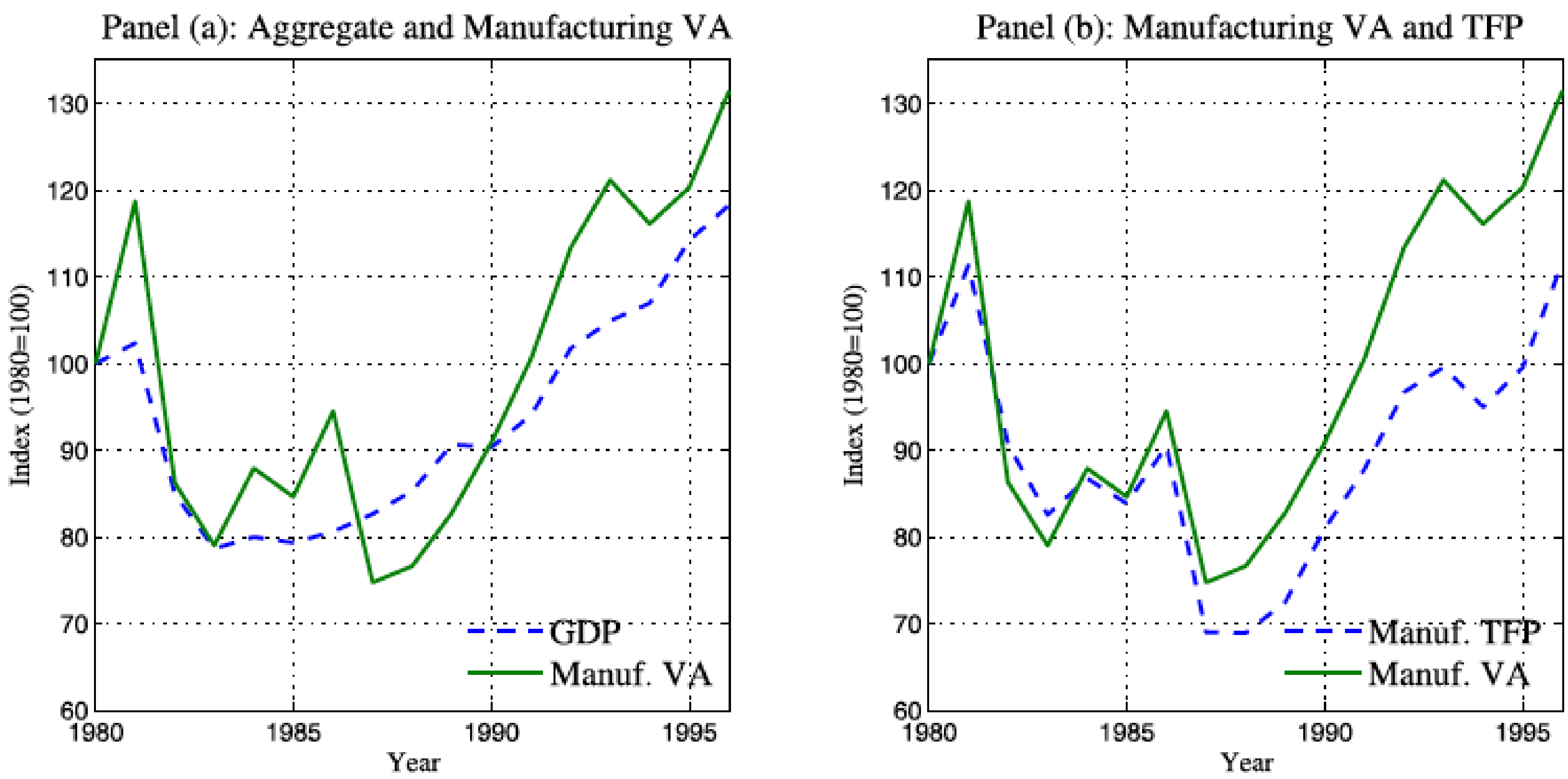
Chile: TFP Trends

TFP Comisión Nacional de Productividad

	1990-2000	2000-2010	2010-2015	2000-2015
TFP	2.3%	0.3%	-0.2%	0.1%
TFP excluding mining	2.3%	1.6%	0.8%	1.4%

Source: Productivity Commission Chile (2016)

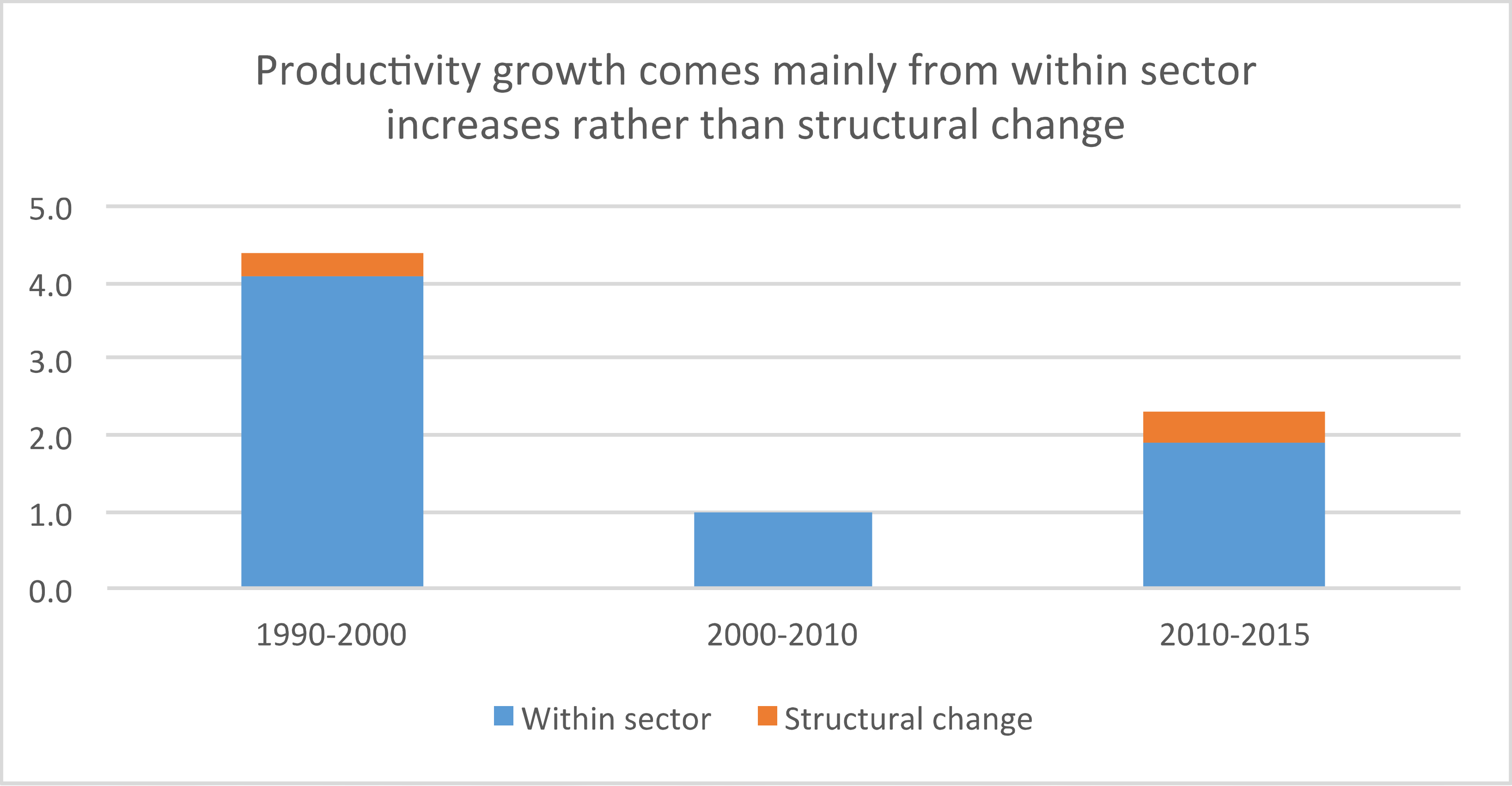
Chilean manufacturing value-added and TFP:1980-1995



Note: Panel (a) shows Chilean GDP and value-added (referred to as "VA") for the manufacturing sector, while panel (b) shows value added and TFP for the manufacturing sector. Measured TFP is $\frac{VA}{K^{\alpha}L^{1-\alpha}}$ with $\alpha = 0.3$. Both GDP and the value-added for manufacturing sector are detrended by 2 percent per year and normalized such that their 1980 values equal to 100. The manufacturing TFP is detrended by 1.4 percent per year and normalized in a similar way.

Source: Chen and Irarrazabal (2015)

Chile: Decomposition of productivity growth



Source: Productivity Commission Chile (2016)

Crecimiento endógeno

- The phrase "endogenous growth" embraces a diverse body of theoretical and empirical work that emerged in the 1980s.

- This work distinguishes itself from neoclassical growth by emphasizing that economic growth is an endogenous outcome of an economic system, not the result of forces that impinge from outside.

- The origins of the work on endogenous growth are two. The first concerns what has been called the convergence controversy. The second concerns the struggle to construct a viable alternative to perfect competition in aggregate-level theory.

The convergence controversy

- Why is it that the poor countries as a group are not catching up with the rich countries in the same way that, for example, the low-income states in the United States have been catching up with the high-income states?

- Robert Lucas (1988) and Romer (1986) cited the failure of cross-country convergence to motivate models of growth that drop the two central assumptions of the neoclassical model: that technological change is exogenous and that the same technological opportunities are available in all countries of the world.

The convergence controversy

- Consider the following production function:

$$Y = A(t)K^{1-\beta}L^{\beta}$$

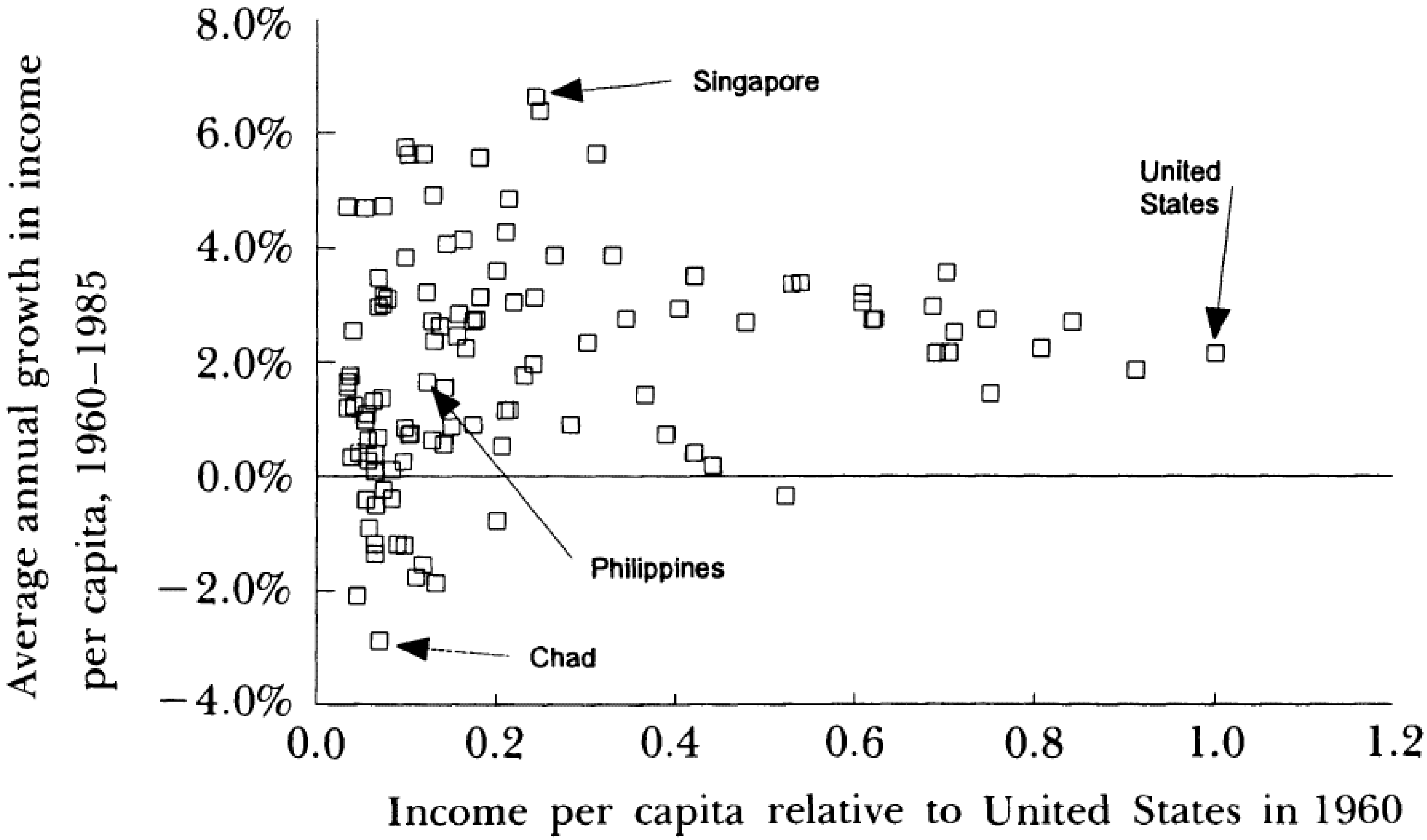
- Let y = Y/L denote output per worker and let k = K/L denote capital per worker. Let n denote the rate of growth of the labor force. Then the behavior of the economy can be summarized by the following equation:

$$\hat{y} = (1-\beta)\hat{k} + \hat{A}$$

$$= (1-\beta)\left[sA(t)^{1/(1-\beta)} y^{(-\beta)/(1-\beta)} - n \right] + \hat{A}$$

Figure 1

Testing for Convergence



The convergence controversy

- The key parameter is the exponent β on labor in the Cobb-Douglas expression for output. Under the neoclassical assumption that the economy is characterized by perfect competition, β is equal to the share of total income that is paid as compensation to labor.
- Perform the following calculation. Pick a country like the Philippines that had output per worker in 1960 that was equal to about 10 percent of output per worker in the United States.
- Because $0{,}1^{-1{,}5}$ is equal to about 30, the equation suggests that the United States would have required a savings rate that is about 30 times larger than the savings rate in the Philippines for these two countries to have grown at the same rate.
- The evidence shows that these predicted saving rates for the United States are orders of magnitude too large.

The convergence controversy

- The way to reconcile the data with the theory is to reduce β so that labor is relatively less important in production and diminishing returns to capital accumulation set in more slowly.

- The theoretical challenge in constructing a formal model with a smaller value for β lies in justifying why labor is paid more than its marginal product and capital is paid less.

- To explain these divergences between private and social returns, Romer (1987) proposed a model in which A was determined locally by knowledge spillovers.

The convergence controversy

- After imposing the constraint implied by the equation, Romer (1987) estimated the value of β to be in the vicinity of 0.25. With this value, it would only take a doubling of the investment rate- rather than a 30- or 100-fold increase-to offset the negative effect that a ten-fold increase in the level of output per worker would have on the rate of growth.

- As a possible explanation of the slow rate of convergence, Barro and Sala i Martin (1992) propose an alternative to the neoclassical model that is somewhat less radical than the spillover model that Romer (1987) proposed. As in the endogenous growth models, they suggest that the level of the technology A(t) can be different in different states or countries and try to model its dynamics. They take the initial distribution of differences in A(t) as given by history and suggest that knowledge about A diffuses slowly from high A to low A regions.

The passing of perfect competition

- The evidence about growth that economists have long taken for granted and that poses a challenge for growth theorists can be distilled to five basic facts.
- Fact #1: There are many firms in a market economy
- Fact #2: Discoveries differ from other inputs in the sense that many people can them at the same time
- Fact #3: It is possible to replicate physical activities.
- Fact #4: Technological advance comes from things that people do.
- Fact #5: Many individuals and firms have market power and earn monopoly rents on discoveries.

The passing of perfect competition

- The neoclassical model captured facts 1, 2, and 3, but postponed consideration of facts 4 and 5.

- From a theoretical point of view, a key advantage of the model is its treatment of technology as a pure public good. This makes it possible to accommodate fact 2 - that knowledge is a nonrival good-in a model that retains the simplicity of perfect competition.

- The public good assumption also implies that knowledge is nonexcludable, and this is clearly inconsistent with the evidence summarized in fact 5- that individuals and firms earn profits from their discoveries.

The passing of perfect competition

- Endogenous growth models try to take the next step and accommodate fact 4.

- Both Romer (1986) and Robert Lucas's model (1988) included fact 4 without taking the final step and including step 5.

- In both of these models, the technology is endogenously provided as a side effect of private investment decisions. From the point of view of the users of technology, it is still treated as a pure public good, just as it is in the neoclassical model. As a result, firms can be treated as price takers and an equilibrium with many firms can exist.

The passing of perfect competition

- Romer (1990) developed endogenous growth theory, emphasizing that technological change is the result of efforts by researchers and entrepreneurs who respond to economic incentives. Anything that affects their efforts, such as tax policy, basic research funding, and education, for example, can potentially influence the long-run prospects of the economy.

Modelo AK

- Modelo más simple de crecimiento endógeno.
- Con una función de producción AK.
- Una interpretación del modelo AK es que el capital debiese ser visto de manera amplia. Incorporando capital humano y otros bienes como gasto de gobierno productivo.

Modelo AK: comportamiento de los hogares

- Los hogares maximizan:

$$U = \int_0^{\infty} e^{-(\rho - n)t} \cdot \left[\frac{c^{(1-\theta)} - 1}{(1-\theta)} \right] dt$$

- Sujeto a la restricción:

$$\dot{a} = (r - n) \cdot a + w - c$$

- Donde a son los activos por persona, r es la tasa de interés, w es el salario y n es la tasa de crecimiento de la población.

Modelo AK: comportamiento de los hogares

- Nuevamente imponemos la restricción:

$$\lim_{t\to\infty}\left\{a(t)\cdot\exp\left[-\int_0^t [r(v)-n]\,dv\right]\right\}\geq 0$$

- Las condiciones de optimización son nuevamente:

$$\dot{c}/c = (1/\theta)\cdot(r-\rho)$$

$$\lim_{t\to\infty}\left\{a(t)\cdot\exp\left[-\int_0^t [r(v)-n]\,dv\right]\right\}= 0$$

Comportamiento de las empresas

- Asumiremos ahora que las empresas tienen una función de producción lineal.

$$y = f(k) = Ak$$

- Donde A>0. Esta ecuación difiere de la ecuación de producción neoclásica en que el producto marginal del capital no es decreciente ($f''(k) = 0$) y las condiciones de Inada no se cumplen. En particular, $f'(k) = A$ cuando k va a cero o infinito.

Comportamiento de las empresas

- ¿Qué tan realista es suponer que no hay retornos decrecientes del capital? Si K representa capital humano, infraestructura pública, bienes públicos productivos…puede ser más realista. Estudiaremos estas dimensiones.
- La condición de maximización de utilidades requiere que el producto marginal del capital ser igual al precio del arriendo del capital $R = r + \delta$.
- Pero en este caso, la productividad marginal del capital es constante, y por lo tanto

$$r = A - \delta$$

Equilibrio

- Asumimos una economía cerrada por lo que $a = k$.
- Lo anterior implica que $\dot{a} = (r - n) \cdot a + w - c$ pasa a ser:

$$\dot{k} = (A - \delta - n) \cdot k - c$$

- Y por su parte $\dot{c}/c = (1/\theta)(r - \rho)$ pasa a ser:

$$\dot{c}/c = (1/\theta)(A - \delta - \rho)$$

Equilibrio

- Finalmente, la condición de transversalidad pasa a ser:

$$\lim_{t\to\infty}\left\{k(t)\cdot e^{-(A-\delta-n)t}\right\} = 0$$

- La característica notable de esta economía es que el crecimiento del consumo no depende del stock de capital por persona. La trayectoria del consumo viene dada por:

Equilibrio

$$c(t) = c(0) \cdot e^{(1/\theta)(A-\delta-\rho)t}$$

- Aun tenemos que determinar C(0)…
- Asumiremos que la función de producción es lo suficientemente productiva como para asegurar que c crezca pero no tan productiva como para generar utilidad sin límites…

$$A > \rho + \delta > (1-\theta)(A-\delta) + \theta n + \delta$$

Equilibrio

- Ahora, dividiendo $\dot{k} = (A - \delta - n) \cdot k - c$ por k, obtenemos:

$$c/_k = (A - \delta - n) - \dot{k}/_k$$

- En el estado estacionario, donde todas las variables por definición crecen a una tasa constante, la tasa de crecimiento de k es constante. Por lo tanto c/k es constante. En consecuencia, k crece a la misma tasa que c.

Dinámica de transición

- ¿Cómo crece el capital fuera del estado estacionario?
- Sabemos que $c(t) = c(0) \cdot e^{(1/\theta)(A-\delta-\rho)t}$. Si reemplazamos esta ecuación en la dinámica de acumulación de capital obtenemos:

$$\dot{k} = (A - \delta - n) \cdot k - c(0) \cdot e^{(1/\theta)(A-\delta-\rho)t}$$

- Podemos solucionar esta ecuación diferencial…

Dinámica de transición

$$k(t) = (\text{constant}) \cdot e^{(A-\delta-n)\cdot t} + [c(0)/\varphi] \cdot e^{(1/\theta)\cdot(A-\delta-\rho)\cdot t}$$

- Donde $\varphi \equiv (A - \delta - n) - \gamma$. Con $\gamma = (1/\theta)(A - \delta - \rho)$ que es la tasa de crecimiento consumo.
- De la condición que se debe cumplir para que la utilidad tenga límite sabemos que $\varphi > 0$.

Dinámica de transición

- Si reemplazamos la solución para la trayectoria del capital per cápita en la condición de transversalidad tenemos que:

$$\lim_{t\to\infty} \{\text{constant} + [c(0)/\varphi] \cdot e^{-\varphi t}\} = 0$$

- Dado que c(0) es finito y $\varphi > 0$, entonces el término constante tiene que ser igual a cero.

Dinámica de transición

- Lo anterior implica que:

$$c(t) = \varphi \cdot k(t)$$

$$\dot{k}/k = \dot{c}/c = (1/\theta)(A - \delta - \rho)$$

- Y dado que $y = Ak$,

$$\dot{y}/y = \dot{k}/k = \dot{c}/c$$

Dinámica de transición

- En consecuencia, el modelo no tiene dinámica de transición: las variables k(t), c(t) e y(t) comienzan en sus valores $k(0)$, $c(0) = \varphi \cdot k(0)$, y $y(0) = A \cdot k(0)$ y luego crecen a una tasa $\gamma = (1/\theta)(A - \delta - \rho)$.

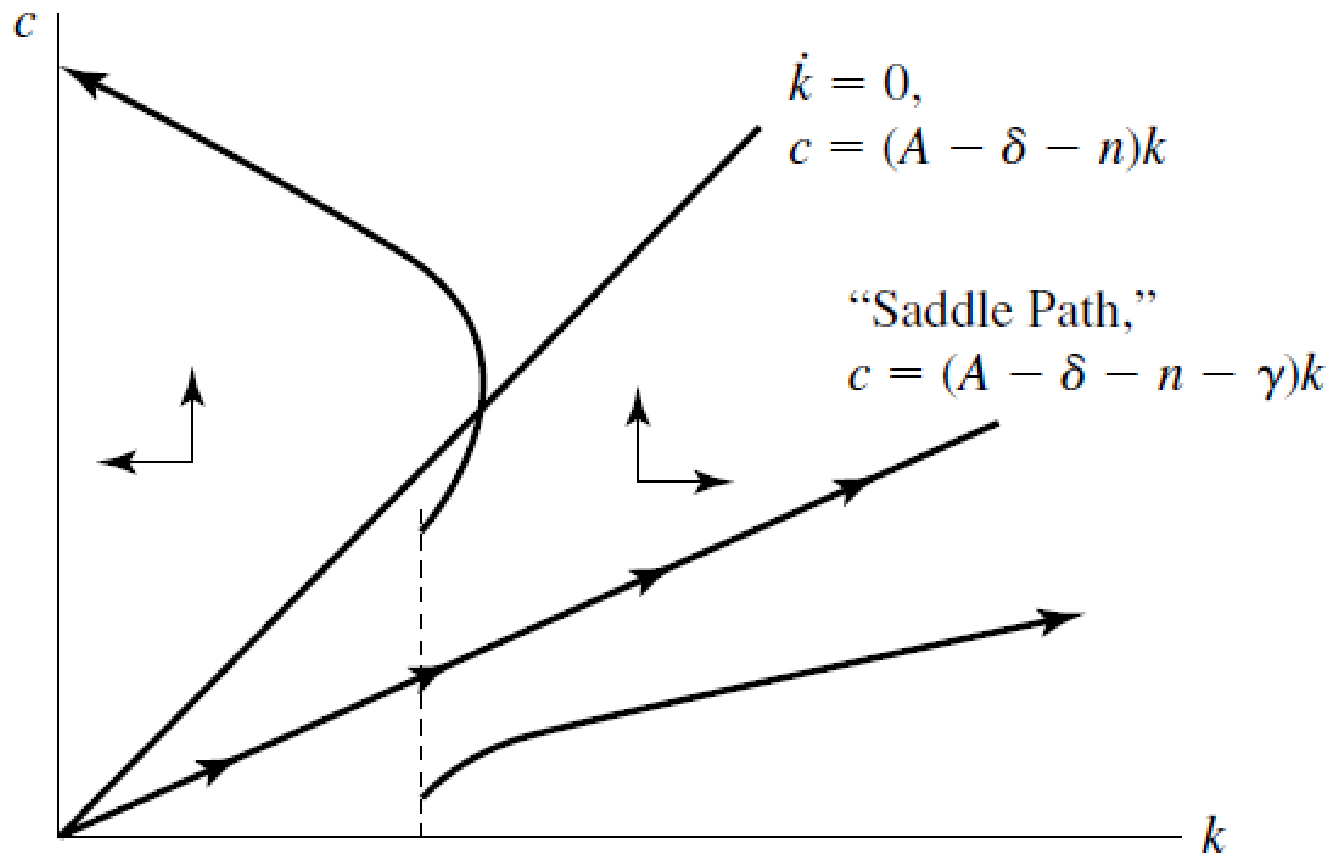
- En este modelo, cambios en los parámetros de la economía pueden afectar los niveles y las tasas de crecimiento de la economía.

Dinámica de transición

- En esta economía, la tasa de ahorro viene dada por:

$$s = (\dot{K} + \delta K)/Y = (1/A) \cdot (\dot{k}/k + n + \delta) = \left[\frac{A - \rho + \theta n + (\theta - 1) \cdot \delta}{\theta A}\right]$$

Diagrama de fase



Determinantes del crecimiento

- En el modelo AK, la tasa de crecimiento de largo plazo depende de los parámetros que determinan la disposición a ahorrar y la productividad del capital.
- Menores valores de ρ y θ, los cuales aumentan la disposición a ahorrar, implican un tasa de ahorro mayor y una tasa de crecimiento del ingreso per cápita mayor.
- Un aumento en el nivel de tecnología, el cual incrementa el producto marginal del capital, incrementa también la tasa de crecimiento.

Determinantes del crecimiento

- En el modelo de Ramsey, una mayor disposición a ahorrar o un aumento en el nivel de la tecnología, incrementaba los niveles de largo plazo del capital y del producto por trabajador. Pero no en un cambio en las tasas de crecimiento de estado estacionario.

- La diferencia viene dada por la productividad marginal decreciente del capital en el modelo neoclásico.

Determinantes del crecimiento

- Cuantitativamente, la diferencia entre estos modelos depende de cuan rápida sea la “aparición” de los retornos marginales decrecientes, un elemento central en la velocidad de convergencia hacia el estado estacionario.
- Si los retornos marginales decrecientes aparecen muy lentamente, el periodo de convergencia es largo.
- En este caso, la disposición a ahorrar o el nivel de la tecnología afectan el crecimiento por un periodo de tiempo prolongado en el modelo neoclásico.