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Guia 5: Soluciones

1. Liquidity Constraints

(a) The idea is that the maximum you can consume in period 1 is your income Y_1 plus the maximum you can borrow on the account of Y_2, which is $Y_2/(1+r_D)$, implying a consumption of $Y_1 + Y_2/(1+r_D)$. On the other hand, if you do not consume in period 1, and thus save all your income Y_1, then in period 2 your consumption will be equal to $Y_2 + Y_1(1+r_S)$. Therefore, the budget constraint (in the (C_1, C_2) space) will be made up of two lines: one that goes from $(Y_1 + Y_2/(1+r_D), 0)$ until (Y_1, Y_2), and thus has a slope of $-(1+r_D)$. The other line goes from (Y_1, Y_2) until $Y_2 + Y_1(1+r_S)$, and thus has a slope of $-(1+r_S)$.

(b) Let $S \geq 0$ represent savings and $B \geq 0$ borrowings, then $C_1 = Y_1 - S + B$ and $C_2 = Y_2 + (1+r_S)S - (1+r_D)B$. Then the problem we have to solve is

$$\max_{S,B} U(Y_1 - S + B, Y_2 + (1+r_S)S - (1+r_D)B) \quad \text{s.t.} \quad S, B \geq 0$$

The FOC of this problem are

$$B: \quad \frac{\partial U}{\partial C_1}(C_1, C_2) - \frac{\partial U}{\partial C_2}(C_1, C_2)[1+r_D] \leq 0$$

$$S: \quad -\frac{\partial U}{\partial C_1}(C_1, C_2) + \frac{\partial U}{\partial C_2}(C_1, C_2)[1+r_S] \leq 0$$

If we want consumption in period 1 and 2 to be Y_1 and Y_2, we need $S = B = 0$ to satisfy these inequalities, and thus what we need is that

$$\frac{\partial U}{\partial C_1}(Y_1, Y_2) \leq \frac{\partial U}{\partial C_2}(Y_1, Y_2)[1+r_D] \quad \text{and} \quad \frac{\partial U}{\partial C_1}(Y_1, Y_2) \geq \frac{\partial U}{\partial C_2}(Y_1, Y_2)[1+r_S]$$

Thus, given that U is increasing and concave, necessary and sufficient conditions for consumption in periods 1 and 2 to be Y_1 and Y_2 respectively, are that

$$\left(\frac{\left.\frac{\partial U}{\partial C_1}\right|_{(Y_1,Y_2)}}{\left.\frac{\partial U}{\partial C_2}\right|_{(Y_1,Y_2)}} \right) \in [1+r_S, 1+r_D]$$

(c) In this part we assume that $U(C_1, C_2) = u_1(C_1) + u_2(C_2)$, and the inequalities are now:

$$u_1'(Y_1) \leq u_2'(Y_2)[1+r_D] \quad \text{and} \quad u_1'(Y_1) \geq u_2'(Y_2)[1+r_S]$$

(d) You all did the graph correctly: you have to shift the BC to the right in Δy units. The idea here is that as the derivatives are continuous and the conditions hold with strict inequality, if Δy is small enough the conditions will still hold for the new period 1 income $Y_1 + \Delta y$, and thus the individual will eat his entire income each period. Therefore he will consume $Y_1 + \Delta y$ in the first period implying $\Delta C_1 = \Delta y$.

(e) As we can not borrow, but we can still save, what changes in the BC is that the line that went from $(Y_1 + Y_2/(1+r_D), 0)$ until (Y_1, Y_2), now is replaced by the straight line that goes from $(Y_1, 0)$ until (Y_1, Y_2), and thus the slope of this line is infinite. The other line is the same.

The condition for $C_1 = Y_1$ and $C_2 = Y_2$ in this case is that

$$\frac{\partial U}{\partial C_1}(Y_1, Y_2) \geq \frac{\partial U}{\partial C_2}(Y_1, Y_2)[1 + r_S]$$

and in the case of separable utility

$$u_1'(Y_1) \geq u_2'(Y_2)[1 + r_S]$$

The same arguments as before are used when analysing a small increase in period 1 income.

2. Hyperbolic discounting and procrastination

(a) This is the traditional case that we usually analyze. The utility from skipping the movie on any given Saturday is indicated in the following table:

Week	Utility from skipping movie in given week
1	$5 + 8 + 13 = 26$
2	$3 + 8 + 13 = 24$
3	$3 + 5 + 13 = 21$
4	$3 + 5 + 8 = 16$

Therefore you skip the first movie.

(b) The following table summarizes the calculation you make every Saturday when deciding whether to skip the movie that or the next Saturday[1]

Week	Utility from skipping movie	
	this Saturday	next Saturday
1	$\frac{1}{2}(5 + 8 + 13) = 13$	$3 + \frac{1}{2}(8 + 13) = 13.5$
2	$\frac{1}{2}(8 + 13) = 10.5$	$5 + \frac{1}{2}13 = 11.5$
3	$\frac{1}{2}13 = 6.5$	8
4	0	$-\infty$

It follows that in week 1 you prefer to wait until week 2; in week 2 you choose to wait an extra week; in week 3 you again procrastinate. Finally, in week 4 you have no other choice but to skip the Johnny Depp movie and work on the project.

(c) You know that your future self will deviate and you take that into account today. This is what you think as of week 1.

i. If I haven't skipped a movie by Week 4, I will have to skip the Johnny Depp movie that Saturday.

ii. If I haven't skipped a movie by Week 3, at that point in time I will prefer to wait until Week 4 to work on the project and will end up skipping the Johnny Depp movie. Indeed, skipping the movie that week will entail a utility of $\frac{1}{2}13 = 6.5$, while skipping the following week leads to a utility of 8.

iii. If I haven't skipped the movie by Week 2, that week I will have to choose between: (a) skip the second movie and (b) end-up skipping the Johnny Depp movie (here I am internalizing what I concluded above). Viewed from my Week 2 preferences, the first alternative entails a utility of $\frac{1}{2}(8+13) = 10.5$, the second a utility of $5 + \frac{1}{2}8 = 9$. Hence, when Week 2 arrives, I will choose to skip the movie on the second Saturday.

[1] Strictly speaking you should consider all future Saturdays, but as it turns out, given the particular valuations of this problem, considering next Saturday suffices.

iv. Finally, I have the information (and calculations) I need to decide whether to view the first movie or not. If I do, my utility will be $\frac{1}{2}(5+8+13)=13$. If I do not, from my digression above I know that I will view the movie the second Saturday, hence my utility, viewed from today, is $3+\frac{1}{2}(8+13)=14.5$. Conclusion: I choose to view the movie on the first Saturday and then skip the movie on the second Saturday.

3. Optimal Consumption with Stone-Geary Utility

(a) We can write the problem of the consumer as,

$$\begin{aligned}\max_{\{c_t\}_{t=0}^{\infty}} \quad & \sum_{t\geq 0}\gamma^t\frac{\sigma}{\sigma-1}(c_t-m)^{(\sigma-1)/\sigma} \\ s.t. \quad & \sum_{t\geq 0}R^{-t}c_t=\mathcal{W}_0\end{aligned}$$

It is convenient (though not necessary) to perform a change of variable. Let $\hat{c}_t=(c_t-m)$. Then, the problem is,

$$\begin{aligned}\max_{\{\hat{c}_t\}_{t=0}^{\infty}} \quad & \sum_{t\geq 0}\gamma^t\frac{\sigma}{\sigma-1}\hat{c}_t^{(\sigma-1)/\sigma} \\ s.t. \quad & \sum_{t\geq 0}R^{-t}\hat{c}_t+\sum_{t\geq 0}R^{-t}m=\mathcal{W}_0\end{aligned}$$

Then, the FOC of the problem is:

$$\gamma^t\hat{c}_t^{-1/\sigma}=\lambda R^{-t}$$

Then, $\hat{c}_t=\lambda^{-\sigma}(\gamma R)^{\sigma t}$. Using the constraint

$$\begin{aligned}\sum_{t\geq 0}R^{-t}\hat{c}_t &= \mathcal{W}_0-\sum_{t\geq 0}R^{-t}m \\ \lambda^{-\sigma}\sum_{t\geq 0}(\gamma R)^{\sigma t}R^{-t} &= \mathcal{W}_0-\frac{1}{1-R^{-1}}m \\ \lambda^{-\sigma} &= \left(1-\gamma^\sigma R^{\sigma-1}\right)\left[\mathcal{W}_0-\frac{1}{1-R^{-1}}m\right]\end{aligned}$$

and therefore

$$\begin{aligned}\hat{c}_t &= (\gamma R)^{\sigma t}(1-\gamma^\sigma R^{\sigma-1})\left[\mathcal{W}_0-\frac{R}{r}m\right] \\ c_t &= (\gamma R)^{\sigma t}(1-\gamma^\sigma R^{\sigma-1})\left[\mathcal{W}_0-\frac{R}{r}m\right]+m \qquad (1)\end{aligned}$$

An alternative derivation starts from the Euler equation:

$$u'(c_t)=\gamma R u'(c_{t+1})$$

which leads to

$$c_t-m=(\gamma R)^{\sigma t}(c_0-m).$$

Multiplying both sides by R^{-t}, summing over $t\geq 0$ and recalling that the No Ponzi condition implies that

$$\mathcal{W}_0=\sum_{t\geq 0}R^{-t}c_t$$

then leads to (1).

(b) From equation (1), we have that

$$c_0 = (1 - \gamma^\sigma R^{\sigma-1}) \left[\mathcal{W}_0 - \frac{R}{r} m\right] + m \tag{2}$$

then,

$$\frac{\partial c_0}{\partial A_0} = (1 - \gamma^\sigma R^{\sigma-1}),$$

which is the same as the case with $m = 0$.

(c) The condition $r\mathcal{W}_0 > Rm$ ensures that the consumer can sustain $c_t > m$ for all t. It follows that if $m > y$, the consumer can sustain the same consumption path as in (b) without having to borrow against future income. This path then has to be optimal, since it is feasible here and was optimal in part (b) which had fewer constraints.

What happens is that the existence of a subsistence consumption level makes the consumer act as if she were more patient than in the case where $m = 0$, where the borrowing constraint is binding (we saw this in class). Thus $\partial c_0/\partial A_0$ is larger when $m > 0$.

4. Evidencia sobre compartición de riesgo (30 puntos)

(a) Sabemos que bajo equivalencia cierta tenemos

$$\begin{aligned}
\Delta C_t &= \frac{r}{1+r} \sum_{u \geq 0} \beta^u \left\{\mathrm{E}_t[Y_{t+u}] - \mathrm{E}_{t-1}[Y_{t+u}]\right\} \\
&= \frac{r}{1+r} \sum_{u \geq 0} \beta^u \left\{Y_t + \mathrm{E}_t[v_{t+1} + \ldots + v_{t+u}] - Y_{t-1} - \mathrm{E}_{t-1}[v_t + \cdots + v_{t+u}]\right\} \\
&= \frac{r}{1+r} \sum_{u \geq 0} \beta^u \left\{Y_t - Y_{t-1}\right\} \\
&= v_t \frac{r}{1+r} \sum_{u \geq 0} \beta^u = v_t \frac{r}{1+r} \frac{1}{1-\beta} = v_t.
\end{aligned}$$

(b) La pérdida de desviarse del óptimo sin fricciones, C^*_{t+k}, es asumida cuadrática y se escribe como $(C_t - C^*_{t+k})^2$. Esto se puede justificar con una expansión de Taylor de segundo orden alrededor del óptimo sin fricciones: el término de primer orden es igual a cero en el óptimo C^*_{t+k} porque la derivada de la utilidad es cero. Esto es, la pérdida es de segundo orden en la desviación del óptimo.

En el momento t, el consumidor escoge C_t para maximizar su utilidad esperada descontada de toda su vida desde el momento t en adelante, tomando en cuenta que con probabilidad $(1-\pi)^k$ no habrá cambiado su consumo entre el período t y $t+k$. Cuando cambie su consumo de nuevo (por ejemplo en el momento $s > t$), el valor que había escogido en el momento t, C_t, se vuelve irrelevante y no se hace parte de la utilidad esperada descontada de toda su vida desde el momento s en adelante. Por esto se escribe la función objetivo considerando solo los eventos donde *no* se cambia el nivel de consumo (i.e. se mantiene C_t y se tiene esa utilidad k períodos en el futuro descontado de acuerdo al factor de descuento cominado con la probabilidad que la decisión corriente sea relevante $\{\gamma(1-\pi)\}^k$).

Como conclusión, el consumidor escoge su consumo en el momento t para minimizar (una aproximación de) la pérdida de la utilidad esperada de no ser posible ajustar su consumo en períodos futuros.

(c) La función objetivo puede ser escrita como cuadrática en C_t:

$$\sum_{k=0}^{\infty} \{\gamma(1-\pi)\}^k C_t^2 - 2\left[\sum_{k=0}^{\infty} \{\gamma(1-\pi)\}^k \mathrm{E}_t C^*_{t+k}\right] C_t + \sum_{k=0}^{\infty} \{\gamma(1-\pi)\}^k \mathrm{E}_t (C^*_{t+k})^2.$$

Del resultado de camino aleatorio (martingale) de Hall tenemos que

$$\sum_{k=0}^{\infty} \{\gamma(1-\pi)\}^k \mathrm{E}_t C^*_{t+k} = \sum_{k=0}^{\infty} \{\gamma(1-\pi)\}^k C^*_t = \frac{C^*_t}{1-\gamma(1-\pi)}.$$

Se sigue que resolviendo la función objetivo original del problema es equivalente a resolver

$$\min_{C_t}[1-\gamma(1-\pi)]^{-1}C_t^2 - 2[1-\gamma(1-\pi)]^{-1}C_t^* C_t + \text{constante}$$

y la CPO implica:

$$C_t = C_t^*.$$

(d) Los individuos que no cambian su consumo en el período t forman una distribución aleatoria de la población dado el supuesto que si el consumidor ajusta su consumo es independiente entre consumidores. Estos individuos mantienen el mismo nivel de consumo en el momento $t-1$ y por lo tanto su consumo agregado es $(1-\pi)C_{t-1}$, dado que forman una fracción representativa $(1-\pi)$ de la población. Aquellos que cambian su nivel de consumo escogen consumir C_t^* y por lo tanto el consumo agregado de este sub-grupo en el período t es πC_t^*. Obtenemos por lo tanto

$$C_t = (1-\pi)C_{t-1} + \pi C_t^*.$$

(e) De (d) obtenemos

$$\Delta C_t = (1-\pi)\Delta C_{t-1} + \pi \Delta C_t^* = (1-\pi)\Delta C_{t-1} + \pi v_t.$$

Se sigue que

$$[1-(1-\pi)L]\Delta C_t = \pi v_t$$

de tal manera que

$$\Delta C_t = \frac{1}{1-(1-\pi)L}\pi v_t = \pi v_t + \pi(1-\pi)v_{t-1} + \pi(1-\pi)^2 v_{t-2} + ...$$

(f) De (e) tenemos que

$$\Delta C_t = \pi v_t + \sum_{k\geq 1}\pi(1-\pi)^k v_{t-k}$$

Estimar la regresión que la investigadora propone arrojará el valor de $\phi = \pi$ porque el término de error $\sum_{k\geq 1}\pi(1-\pi)^k v_{t-k}$ no está correlacionado con el regresor v_t. Esto implica que el valor estimado de ϕ será significativamente menor que 1. Este resultado es, por lo tanto, no necesariamente dado por un mayor monto de compartir riesgo que como lo sugieren modelos de mercados incompletos. Puede ser la consecuencia de ser desatento, como lo muestra este problema.