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1. **Hyperbolic discounting and procrastination**

   (a) This is the traditional case that we usually analyze. The utility from skipping the movie on any given Saturday is indicated in the following table:

Week	Utility from skipping movie in given week
1	5 + 8 + 13 = 26
2	3 + 8 + 13 = 24
3	3 + 5 + 13 = 21
4	3 + 5 + 8 = 16

   Therefore you skip the first movie.

   (b) The following table summarizes the calculation you make every Saturday when deciding whether to skip the movie that or the next Saturday[1]

Week	Utility from skipping movie this Saturday	Utility from skipping movie next Saturday
1	$\frac{1}{2}(5+8+13)=13$	$3+\frac{1}{2}(8+13)=13.5$
2	$\frac{1}{2}(8+13)=10.5$	$5+\frac{1}{2}13=11.5$
3	$\frac{1}{2}13=6.5$	8
4	0	$-\infty$

   It follows that in week 1 you prefer to wait until week 2; in week 2 you choose to wait an extra week; in week 3 you again procrastinate. Finally, in week 4 you have no other choice but to skip the Johnny Depp movie and work on the project.

   (c) You know that your future self will deviate and you take that into account today. This is what you think as of week 1.

      i. If I haven't skipped a movie by Week 4, I will have to skip the Johnny Depp movie that Saturday.

      ii. If I haven't skipped a movie by Week 3, at that point in time I will prefer to wait until Week 4 to work on the project and will end up skipping the Johnny Depp movie. Indeed, skipping the movie that week will entail a utility of $\frac{1}{2}13 = 6.5$, while skipping the following week leads to a utility of 8.

[1] Strictly speaking you should consider all future Saturdays, but as it turns out, given the particular valuations of this problem, considering next Saturday suffices.

iii. If I haven't skipped the movie by Week 2, that week I will have to choose between: (a) skip the second movie and (b) end-up skipping the Johnny Depp movie (here I am internalizing what I concluded above). Viewed from my Week 2 preferences, the first alternative entails a utility of $\frac{1}{2}(8+13) = 10.5$, the second a utility of $5 + \frac{1}{2}8 = 9$. Hence, when Week 2 arrives, I will choose to skip the movie on the second Saturday.

iv. Finally, I have the information (and calculations) I need to decide whether to view the first movie or not. If I do, my utility will be $\frac{1}{2}(5+8+13) = 13$. If I do not, from my digression above I know that I will view the movie the second Saturday, hence my utility, viewed from today, is $3 + \frac{1}{2}(8+13) = 14.5$. Conclusion: I choose to view the movie on the first Saturday and then skip the movie on the second Saturday.

2. **Hyperbolic discounting and saving**

(a) Note that, given the problem's statement, individuals are sophisticated, thus aware of the tension between their preferences and those of their future selves. So the correct solution uses backward induction: period 1 individual chooses C1 aware that period 2 individual saves less and spends more than period 1 individual would like her to do. (In the particular case considered in the problem, the consumption path is the same for sophisticated and naive individuals, but conceptually there's a fundamental difference).

Period 3 individual consumes all he has. Period 2 individual solves de problem:

$$\max_{C_2,C_3} \log(C_2) + \eta \log(C_3)$$

$$\text{s.t.} \quad C_2 + C_3 = \mathcal{W} - \bar{C}_1$$

where $\bar{C}_1$ is the consumption he chose in period 1 (thus it is take as given for the period 2 self). The solution of this problem is

$$C_2(\bar{C}_1) = \frac{\mathcal{W} - \bar{C}_1}{1+\eta} \quad \text{and} \quad C_3(\bar{C}_1) = \frac{\eta(\mathcal{W} - \bar{C}_1)}{1+\eta}$$

Therefore, the problem that the sophisticated period 1 individual has to solve is

$$\max_{C_1} \log(C_1) + \eta \log(C_2(C_1)) + \eta \log(C_3(C_1))$$

which implies and optimal consumption in period 1 of $C_1^* = \frac{\mathcal{W}}{1+2\eta}$. Finally, evaluating $C_2(.)$ and $C_3(.)$ in C_1^*, we get that the optimal path is

$$C_1^* = \frac{\mathcal{W}}{1+2\eta}, \quad C_2^* = \frac{2\eta\mathcal{W}}{(1+2\eta)(1+\eta)} \quad \text{and} \quad C_3^* = \frac{2\eta^2\mathcal{W}}{(1+2\eta)(1+\eta)}$$

(b) Period 1 individual could try to use the illiquid asset to achieve his optimal consumption path with commitment, that is, the one obtained by solving

$$\max_{C_1,C_2,C_3} \log(C_1) + \eta \log(C_2) + \eta \log(C_3)$$

$$\text{s.t.} \quad C_1 + C_2 + C_3 = \mathcal{W}$$

This problem's optimal solution is $C_1 = \mathcal{W}/(1+2\eta)$ and $C_2 = C_3 = \eta\mathcal{W}/(1+2\eta)$. For future selves to follow this path, the agent can put C_3 into the illiquid asset in period 1. Notice that from period 1's perspective, she would never put more money than C_3 in the illiquid asset because then in all circumstances (taking the money out in period two or not) the agent is less well off because if the period 2 person takes the money out then he has to pay the penalty and from period 1's perspective she should have put less money in to begin with if this is an equilibrium. Likewise, from period 1's perspective you would never put less than C_3 in because you know you will consume more than you should in period 2. This possible policy is definitely welfare improving to the individual of period 1, but we also need this to be optimal from period 2 individual's perspective, in particular, what we need is period 2 individual to choose to keep the asset in period 2 (rather than sell it). Therefore, what we want is that

$$\log(C_2) + \eta\log(C_3) > \log(\tilde{C}_2) + \eta\log(\tilde{C}_3)$$

where $(\tilde{C}_2, \tilde{C}_3)$ denote the consumption path when the illiquid asset is sold in period 2 (and thus period 2 individual receives $(1-\rho)C_3$ back). This means we can follow an analogous procedure to the first part of part (a) by noting that given the log preferences, from period 2's perspective you consume your wealth in shares equal to $\frac{1}{1+\eta}\text{vs}\frac{\eta}{1+\eta}$. If the period 2 self takes the money out of illiquid asset, her wealth equals

$$\begin{aligned}
\widetilde{\mathcal{W}} &= \mathcal{W} - C_1 - C_3 + (1-\rho)C_3 \\
&= C_2 + (1-\rho)C_3 \\
&= \frac{\eta\mathcal{W}}{(1+2\eta)} + \frac{(1-\rho)\eta\mathcal{W}}{(1+2\eta)} \\
&= \frac{(2-\rho)\eta\mathcal{W}}{(1+2\eta)}
\end{aligned}$$

which is the same as the total wealth minus what she consumed in period 1 and the money she put away in the illiquid asset plus the money they took out with penalty. Therefore we have the period 2's self decides to consume

$$\tilde{C}_2 = \frac{(2-\rho)\eta\mathcal{W}}{(1+2\eta)(1+\eta)} \quad \text{and} \quad \tilde{C}_3 = \frac{(2-\rho)\eta^2\mathcal{W}}{(1+2\eta)(1+\eta)}$$

and thus what we need is that

$$\rho > 2 - (1+\eta)\eta^{-\eta/(1+\eta)}$$

(c) As we saw in part (b), if ρ is large enough, then all selves are going to follow the optimal path that period 1 individual determined as optimal, and therefore, as seen from period 1, the illiquid asset is welfare improving. This comes from the fact that the illiquid asset is acting as a commitment device, making optimal for future selves to stick to period 1 optimal path, selves that otherwise, given their time inconsistency, would deviate from it.

3. **Evidencia sobre compartición de riesgo**

(a) Sabemos que bajo equivalencia cierta tenemos

$$\begin{aligned}
\Delta C_t &= \frac{r}{1+r}\sum_{u\geq 0}\beta^u \left\{\mathrm{E}_t[Y_{t+u}] - \mathrm{E}_{t-1}[Y_{t+u}]\right\} \\
&= \frac{r}{1+r}\sum_{u\geq 0}\beta^u \left\{Y_t + \mathrm{E}_t[v_{t+1} + \ldots + v_{t+u}] - Y_{t-1} - \mathrm{E}_{t-1}[v_t + \cdots + v_{t+u}]\right\} \\
&= \frac{r}{1+r}\sum_{u\geq 0}\beta^u \left\{Y_t - Y_{t-1}\right\} \\
&= v_t \frac{r}{1+r}\sum_{u\geq 0}\beta^u = v_t \frac{r}{1+r}\frac{1}{1-\beta} = v_t.
\end{aligned}$$

(b) La pérdida de desviarse del óptimo sin fricciones, C^*_{t+k}, es asumida cuadrática y se escribe como $(C_t - C^*_{t+k})^2$. Esto se puede justificar con una expansión de Taylor de segundo orden alrededor del óptimo sin fricciones: el término de primer orden es igual a cero en el óptimo C^*_{t+k} porque la derivada de la utilidad es cero. Esto es, la pérdida es de segundo orden en la desviación del óptimo.

En el momento t, el consumidor escoge C_t para maximizar su utilidad esperada descontada de toda su vida desde el momento t en adelante, tomando en cuenta que con probabilidad $(1-\pi)^k$ no habrá cambiado su consumo entre el período t y $t+k$. Cuando cambie su consumo de nuevo (por ejemplo en el momento $s > t$), el valor que había escogido en el momento t, C_t, se vuelve irrelevante y no se hace parte de la utilidad esperada descontada de toda su vida desde el momento s en adelante. Por esto se escribe la función objetivo considerando solo los eventos donde *no* se cambia el nivel de consumo (i.e. se mantiene C_t y se tiene esa utilidad k períodos en el futuro descontado de acuerdo al factor de descuento cominado con la probabilidad que la decisión corriente sea relevante $\{\gamma(1-\pi)\}^k$).

Como conclusión, el consumidor escoge su consumo en el momento t para minimizar (una aproximación de) la pérdida de la utilidad esperada de no ser posible ajustar su consumo en períodos futuros.

(c) La función objetivo puede ser escrita como cuadrática en C_t:

$$\sum_{k=0}^{\infty}\{\gamma(1-\pi)\}^k C_t^2 - 2\left[\sum_{k=0}^{\infty}\{\gamma(1-\pi)\}^k \mathrm{E}_t C^*_{t+k}\right] C_t + \sum_{k=0}^{\infty}\{\gamma(1-\pi)\}^k \mathrm{E}_t (C^*_{t+k})^2.$$

Del resultado de camino aleatorio (martingale) de Hall tenemos que

$$\sum_{k=0}^{\infty}\{\gamma(1-\pi)\}^k \mathrm{E}_t C^*_{t+k} = \sum_{k=0}^{\infty}\{\gamma(1-\pi)\}^k C^*_t = \frac{C^*_t}{1-\gamma(1-\pi)}.$$

Se sigue que resolviendo la función objetivo original del problema es equivalente a resolver

$$\min_{C_t}[1-\gamma(1-\pi)]^{-1}C_t^2 - 2[1-\gamma(1-\pi)]^{-1}C^*_t C_t + \text{constante}$$

y la CPO implica:

$$C_t = C^*_t.$$

(d) Los individuos que no cambian su consumo en el período t forman una distribución aleatoria de la población dado el supuesto que si el consumidor ajusta su consumo es independiente entre consumidores. Estos individuos mantienen el mismo nivel de consumo en el momento $t-1$ y por lo tanto su consumo agregado es $(1-\pi)C_{t-1}$, dado que forman una fracción representativa $(1-\pi)$ de la población. Aquellos que cambian su nivel de consumo escogen consumir C_t^* y por lo tanto el consumo agregado de este sub-grupo en el período t es πC_t^*. Obtenemos por lo tanto

$$C_t = (1-\pi)C_{t-1} + \pi C_t^*.$$

(e) De (d) obtenemos

$$\Delta C_t = (1-\pi)\Delta C_{t-1} + \pi \Delta C_t^* = (1-\pi)\Delta C_{t-1} + \pi v_t.$$

Se sigue que

$$[1-(1-\pi)L]\Delta C_t = \pi v_t$$

de tal manera que

$$\Delta C_t = \frac{1}{1-(1-\pi)L}\pi v_t = \pi v_t + \pi(1-\pi)v_{t-1} + \pi(1-\pi)^2 v_{t-2} + ...$$

(f) De (e) tenemos que

$$\Delta C_t = \pi v_t + \sum_{k\geq 1} \pi(1-\pi)^k v_{t-k}$$

Estimar la regresión que la investigadora propone arrojará el valor de $\phi = \pi$ porque el término de error $\sum_{k\geq 1}\pi(1-\pi)^k v_{t-k}$ no está correlacionado con el regresor v_t. Esto implica que el valor estimado de ϕ será significativamente menor que 1. Este resultado es, por lo tanto, no necesariamente dado por un mayor monto de compartir riesgo que como lo sugieren modelos de mercados incompletos. Puede ser la consecuencia de ser desatento, como lo muestra este problema.

4. **Premio accionario y concentración de shocks agregados**

(a) Si disminuyo en dA_b mi posición de activo bueno, la pérdida en utilidad esperada en el estado bueno es de $U'(1)\cdot dA_b \cdot \frac{1}{2}$. Luego, si esa disminución la gasto en activo malo, la ganancia en utilidad esperada en el estado malo será de $\left[\lambda U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) + (1-\lambda)U'(1)\right]\cdot \frac{dA_b}{p}\cdot\frac{1}{2}$.

El hecho de que la utilidad esperada no cambie, implica que las ganancias son iguales a las pérdidas, es decir:

$$U'(1)\cdot dA_b \cdot \frac{1}{2} = \left[\lambda U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) + (1-\lambda)U'(1)\right]\cdot \frac{dA_b}{p}\cdot\frac{1}{2}$$

(b) Simplificando la expresión anterior, y despejando p, tenemos que:

$$p = \frac{\left[\lambda U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) + (1-\lambda)U'(1)\right]}{U'(1)}$$

(c) Derivando la expresión recién encontrada, llegamos a:

$$\frac{\partial p}{\partial \lambda} = \frac{1}{U'(1)}\left[U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) - U'(1) + \frac{\phi}{\lambda}U''\left(1-\left(\frac{\phi}{\lambda}\right)\right)\right]$$

(d) Si la función de utilidad es cuadrática, entonces la primera derivada será lineal en C y la segunda será constante. Eso llevará a que la pendiente de $U'(\cdot)$ (que a su vez es $U''(\cdot)$) pueda calcularse como la pendiente de una recta. Así, tendremos que:

$$U''\left(1-\left(\frac{\phi}{\lambda}\right)\right) = \frac{U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) - U'(1)}{1-\frac{\phi}{\lambda}-1} = \frac{U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) - U'(1)}{-\frac{\phi}{\lambda}}$$

Reordenando términos, llegamos a que $U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) - U'(1) + \frac{\phi}{\lambda}U''\left(1-\left(\frac{\phi}{\lambda}\right)\right) = 0$, lo que implica que $\frac{\partial p}{\partial \lambda} = 0$. Esto queda más claro con el gráfico visto en ayudantía.

(e) Si $U''' > 0$, tendremos entonces que U' es convexa en C. Así, replicando el gráfico de la parte anterior para este caso (visto en ayudantía), se concluye que:

$$U''\left(1-\left(\frac{\phi}{\lambda}\right)\right) < \frac{U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) - U'(1)}{-\frac{\phi}{\lambda}}$$

Reordenando términos se concluye que $U'\left(1-\left(\frac{\phi}{\lambda}\right)\right) - U'(1) + \frac{\phi}{\lambda}U''\left(1-\left(\frac{\phi}{\lambda}\right)\right) < 0$, lo que implica que $\frac{\partial p}{\partial \lambda} < 0$. La intuición de este resultado es que cuando $U''' > 0$, el agente realiza ahorro por precaución. Luego, si aumenta λ, por un lado aumenta la probabilidad de que en el estado malo pierda parte de mis ingresos, pero también cae el monto en el que se reduce mi ingreso en ese caso. Dado eso, como de caer en el estado malo perderé menos ingreso, caerá el ahorro por precaución. En este contexto, eso significará una menor demanda por el activo malo, lo que llevará a que su precio caiga.