"In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?"

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ABSTRACT

Most children learn through experience that bottles can be source of sounds when they blow over the top, depending on the size and the content of the bottle. This phenomenon is known as Helmholtz resonance and is what is going to be investigated in this essay.

When air passes the top of a wine bottle it makes the bottle act as a Helmholtz resonator creating resonance in the form of a measurable, audible sound frequency. The frequency of this resonance is determined, roughly, by the cross sectional area of the opening port (A), the volume of the cavity (V) and the length of the port (L).

It is possible to change the frequency and consequently the audible sound by adding water to the wine bottle; creating a change in one the determining components (V). The interesting part, however, comes when we introduce interferences in the resonator in form of pebbles blocking parts of the volume in the chamber. These pebbles are added *in place* of the water, leaving us able to measure frequencies from identical *volumes* of air in the chamber, but with differently shaped volumes.

Thus the Research Question for this essay is **"In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?"**

Through the course of the lab described in this essay and collection of the corresponding results a theory could be composed;

Referring to the research question the introduction of interferences to the volume gave a smaller rate of change to the results found, meaning that a smaller portion of the volume was utilized when there were interferences present.

Further analysis of the results showed that the interferences created a different slope of the frequency function contra volume, but did not shift it vertically or horizontally.

-Word Count: 298-

CONTENTS:

1. INTRODUCTION

In the 1860's Hermann von Helmholtz introduced the Helmholtz resonator[1]. This was a cavity resonator with a given natural frequency that he used when he wrote and researched *On the Sensations of tone as a physiological basis for the theory of music* which he published in 1863[2]. In this essay I will be following the research and concepts introduced by Helmholtz and Rayleigh following this publication and I will be using their formulas for calculating the resonance frequency, and density and pressure change of water in a cavity resonator. My procedure for this will be altering the nature of the cavity of the resonator and observing the effects. This can be expressed more precisely in my research question:

"In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?"

1.1 Relevance of Helmholtz resonance:

The concept of pressure changes to create resonance is used both directly as Helmholtz studied in many forms of music instruments and loudspeakers such as sub woofers, but also indirectly and in other forms such as in architectural acoustics (noise reduced rooms) and even in some combustion engines[3]. One of the most relevant in the world today is the research done on aeroplanes. These concepts could be used to reduce aeroplane fuel consumption by 20% which would be extremely beneficial towards reducing the damage aeroplanes emissions inflict upon the environment. The reason for this reduction in fuel consumption is a possible 40 % reduction in the mid-air parasitic drag. This 40% reduction would be obtained by something called the principle of *wings that waggle*; utilizing the principles of the Helmholtz resonator, having small jets pushing air back and forth over the wings over small cavities which pull in and push out air by similar principles to the ones explained above.

1.2 Choice of resonator and decisions when obtaining data:

To obtain the research data needed to answer the research question through lab work, I needed a resonator where I could keep all conditions constant except for one variable factor. The best way I saw to solve this problem would be to use a wine bottle as a resonator where all orifice conditions would be kept constant and adding water to vary the internal spacial volume of the chamber as a reference.

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[1]Helmholtz Resonators. Psych.utoronto.ca. 8 September 2011 <http://www.psych.utoronto.ca/museum/helmholtz.htm>
[2]Kursell, Julia. Ear and Instrument - Hermann v. Helmholtz's "On the Sensations of Tone as a Physiological Basis for the Theory of Music". Mpiwg-berlin.ppg.de. 8 December 2011 <http://www.mpiwgberlin.mpg.de/en/research/projects/DeptIII_Julia_Kursell>
[3]Airbox. Wikipedia.org. 4 January 2012. <http://en.wikipedia.org/wiki/Airbox>

Water was in this case used as a non-pressure changing compound to decrease the spacial volume in the resonator according to Rayleigh's and several others' research as explained in Rayleigh's publishing *The theory of sound*[4] through fluid statics also known as hydrostatics that when the pressure in the air over the water changes, there will be an insignificant or even negligible pressure change in the liquid.

Using this equipment and the aforementioned formulas it is possible to use varying volumes of pebbles to see the effect of obstacles in the volume of a resonator. When collecting data, the ability to keep all factors constants except the volume will allow for an easy investigation of the practical relationship between volume changes contra frequency changes, as well as being able to compare this relationship between pebbles and water in the chamber of the resonator.

1.3 How can we measure the frequency of the resonance?

As the mass of air inside the neck of the bottle oscillates it will emit wavelengths defined by the forces acting on it, with a wave speed which is constant. The frequency can be found very easily as $\lambda f = v$, where λ is wavelength, v is wave speed and f is frequency. The sound program can detect the time between peaks of the amplitude of individual sounds also known as the wavelengths. The program also knows the wave speed to be $v = c \approx 340\ m\,s^{-1}$, where c is the speed of sound, making the calculation of the frequency very simple[5].

Looking at Helmholtz resonators more specifically we can see how they produce these frequencies and try to explain them both quantitatively and qualitatively.

Qualitative explanation of a Helmholtz resonator:

Resonance is essentially the phenomenon where an external force is applied to a system; with or without potential for some sort of oscillation inside, with equal or similar frequency to the natural frequency of the system to create higher amplitude of oscillations[6].

Resonators are normally described as walls with traveling waves between them. These waves are reflected off the walls and when some external force gets applied with the right frequency, standing waves are formed inside the resonator to create resonance. A good example of mechanical resonance would be pushing someone on a swing set; if you try to push outside the frequency of the swing it will be destructive in regards to the

[4] Rayleigh, Baron John William Strutt. The Theory of Sound: Volume One. (MacMillan & co. 1894). <http://books.google.com/books?id=EGQSAAAAIAAJ,> Rayleigh, Baron John William Strutt. The Theory of Sound: Volume Two.(MacMillan &co. 1894) <http://books.google.com/books?id=Zm9LAAAAMAAJ>

[5] It is important to note that the sound program gives the frequency directly from the soundscape of the resonance.

[6] Tsokos, K.A. Physics for the IB Diploma: Fifth Edition. (Cambridge. 1998), p. 209

height or *amplitude* of the system, but this description of a resonator does not directly apply in the case of Helmholtz resonators.

The basic concept of the Helmholtz resonator is that air inside the neck of a bottle acts as a mass and the air inside the chamber acts as a spring. When air is accelerated over the top it acts as an external force pushing the *mass* down, again compressing the *spring*. To be more exact we can define this as an increase in pressure as pressure is defined by *the collision of the gas molecules with the walls of its container*[7]. As the volume of the container gets smaller, there would be more collisions with the same amount of gas molecules. At this point we have potential energy stored as a high pressure inside the chamber. When this pressure overcomes the force on the air in the neck, or our *mass*, it is no longer in equilibrium and, to compensate, pushes the mass in the opposite direction. As a natural reaction the resonator overcompensates and pushes the *mass* out of equilibrium in the other direction. In the absence of the external force the system would be damped to a halt, but if the force is continuous the two forces will continue to push the mass of air back and forth which would display simple harmonic motion, emitting audible sound frequencies[8].

Quantitative explanation of the Helmholtz resonator:

The physical formula for this phenomenon was defined by Helmholtz and can be found through some fairly simple steps:

(1) Firstly we say that there is a mass of air inside the neck and that we can find the force pushing down on the mass by defining pressure as earlier by *the collision of the gas molecules with the walls of its container*, mathematically giving us:

$P = \frac{F_1}{A}$, now isolating force gives us $F_1 = PA \quad where\ P\ is\ pressure, A\ is\ cross\ sectional\ area\ of\ the\ neck\ and\ F\ is\ the\ force\ pushing\ down\ on\ the\ mass.$

This seems both logical and reasonable as the number of collisions into area gives force exerted on the mass of air.

(2) Secondly if we define the force need to compress a spring by a distance x as: $F_2 = -kx, \quad where\ F_2\ is\ force\ in\ Newtons, k\ is\ the\ spring\ constant\ and\ x\ is\ displacement\ in\ meters,$

[7]Ibid p. 170
[8]Helmholtz Resonator. Wikipedia.org. 1 December 2011 <http://en.wikipedia.org/wiki/Helmholtz_resonator>

(3) Finally we can express the sum of forces on a mass of air moving in the neck as $\Sigma F = ma,\ or\ \Sigma F = m\frac{d^2x}{dt^2}$

$giving\ a\ mathematical\ definition\ of\ a\ in\ terms\ of\ x,$
$where\ t\ is\ time, a\ is\ acceleration, x\ is\ displacement$
$and\ \Sigma F\ is\ the\ sum\ of\ all\ forces\ acting\ on\ the\ mass\ of\ air$

(4) Now we are able to express all forces of our system mathematically:

$$\Sigma F = F_1 + F_2 \Rightarrow m\frac{d^2x}{dt^2} = PA - kx,$$

As this displacement of the mass is defined by two opposing and varying forces we can say that the mass in fact does undergoes simple harmonic motion through mathematical interpretation of the system.

Now we can continue the quantitative explanation of the Helmholtz resonator to derive the formula for the frequency emitted.

<u>Finding the frequency formula:</u>

We can mathematically find the frequency emitted by looking at four other physical relationships;

(1) Firstly the only considerable oscillating mass is the mass of the air inside the resonators neck. This mass can be found through:

a. $m = \rho_0 V_0, \quad where\ m\ is\ the\ mass\ of\ the\ air,$
$\rho_0 is\ the\ density\ of\ air\ and\ V\ is\ the\ Volume\ of\ air$

b. $V = L \times A, \quad where\ L\ is\ the\ length\ of\ the\ neck$
$and\ A\ is\ the\ cross\ sectional\ Area\ of\ the\ nec$

c. $Substituting\ for\ V, we\ get: \quad m = \rho_0 LA$

(2) As we defined our system as containing a mass of air acting as a spring we need to find an expression for the spring constant of air. It is possible to derive the unit of the spring constant which we will use later from:

a. $F = -kx$

b. giving us:
$k = \frac{F}{x} \Rightarrow k = \frac{N}{m} \Rightarrow k = \frac{kg \times m \times s^{-2}}{m} \Rightarrow k = kg \times s^{-2}$

c. The *spring constant* of air is in this case defined by[9]:

$$k = \rho_0 c^2 \frac{A^2}{V_c}$$

$where\ c\ is\ constant;\ the\ speed\ of\ sound\ and,$
$V_c\ is\ the\ volume\ of\ the\ chamber$

This expression is not from the IB-textbook and is harder to imagine, but it makes sense as a higher density means greater force needed to compress it, a higher area meaning more force needed to compress it and a bigger volume of the spring indirectly meaning a less dense gas to be compressed and if we calculate the units it seems to add up.

$$kg \times s^{-2} = kg \times m^{-3} \times (m \times s^{-1})^2 \times \frac{(m^2)^2}{m^3}$$
$$\Rightarrow kg \times s^{-2} = kg \times m^{-3} \times m^2 \times s^{-2} \times \frac{m^4}{m^3}$$
$$\Rightarrow kg \times s^{-2} = kg \times m^{-1} \times s^{-2} \times m$$
$$\underline{\underline{\Rightarrow kg \times s^{-2} = kg \times s^{-2}}}$$

(3) Thirdly we defined the sum of forces on the oscillating mass in the neck as an example of a system showing simple harmonic motion (SHM) and we can use SHM's definition of angular frequency as:

$$\omega = \sqrt{\frac{k}{m}}$$ [10]

$where\ \omega\ is\ the\ angular\ frequency, k\ is\ the\ spring\ constant\ and$
$m\ is\ the\ mass\ of\ the\ air\ in\ the\ neck.$

(4) As the frequency f is related to ω by a factor of 2π[11]:

a. $f = \frac{\omega}{2\pi},\quad or\ f = \frac{1}{2\pi} \times \omega$

b. Now to substitutions for ω:

$$f = \frac{1}{2\pi} \times \omega, \qquad \omega = \sqrt{\frac{k}{m}}$$

[9] Patton, Kelly. Studying the Effects of Filling a Helmholtz Resonator with Spheres. (Physics Department, The Col lege of Wooster, Wooster, Ohio 44691, USA, May 9, 2007) <http://www3.wooster.edu/physics/jris/Files/Patton_Web_article.pdf>

[10] Tsokos, K.A. Physics for the IB Diploma: Fifth Edition. (Cambridge. 1998), p. 197

[11] Ibid p. 200

$$Thus: f = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}}$$

c. Now substitute for *k* and *m:*

$$k = \rho_0 c^2 \frac{A^2}{V_c} \ and\ m = \rho_0 LA$$

$$f = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}}, Thus: f = \frac{1}{2\pi} \times \sqrt{\frac{\rho_0 c^2 \frac{A^2}{V_c}}{\rho_0 LA}} \Rightarrow f = \frac{1}{2\pi} \times \sqrt{\frac{\cancel{\rho_0} c^2 \frac{A^{\cancel{2}}}{V_c}}{\cancel{\rho_0} L \cancel{A}}}$$

$$\Rightarrow f = \frac{1}{2\pi} \times \sqrt{\frac{c^2 \frac{A}{V_c}}{L}} \Rightarrow f = \frac{1}{2\pi} \times c\sqrt{\frac{A}{V_c} \times \frac{1}{L}}$$

$$\Rightarrow f = \frac{c}{2\pi}\sqrt{\frac{A}{L \times V_c}}$$

This is in fact the same relationship that Helmholtz found for his resonators[12].

So what this tells us is that frequency is proportional to one upon the square root of the volume of the chamber and one upon the square root of the length of the neck, and proportional to the square root of the cross sectional area of the neck. So now I was intrigued and started to wonder about what extent the shape of this volume and the introduction of obstacles to said volume would affect the frequency resonated by the resonator. This is lead to designing a lab which is explained below in order to experimentally investigate this.

[12] Helmholtz Resonator. Wikipedia.org. 1 December 2011 <http://en.wikipedia.org/wiki/Helmholtz_resonator>

2. HYPOTHESIS

Following the formulas and conclusions presented in the introduction, as well as the equipment mentioned; a hypothesis for the experiment can be made:

"Using a wine bottle as a Helmholtz resonator, pebbles to achieve interferences in the volume and water to reduce the volume of the reference resonator I will now attempt to compare the frequencies resonated by a resonator with interferences and one with an open chamber. Doing this I am expecting to find a higher frequency in the results with the canals and cavities as I expect the utilized volume air in the resonator to be less. Due to the inverse relationship between volume and frequency, as volume goes down I am expecting the frequency to go up."

3. APPARATUS

The following apparatus was utilized in the experiment.
(2) Beakers (250 ml, 600 ml)
(3) Measuring cylinders (1000 ml, 500 ml, 100 ml)
(100+) Glass Pebbles (1.5 – 1.6 mm. in diameter)
(2) Wine bottles (750 ml-whole), (375 ml-half)
(1) Soda bottle (1500 ml)
(1) Funnel (small, diameter 1 cm – 7 cm)
(1) Ruler (20 cm)
(1) Calliper (7 inches, 18 cm)
(1) Computer with sound recording program

Distilled water

3.1 APPARATUS ERROR:

The following errors were recorded for the apparatus by taking the smallest interval on the apparatus as the possible reading error, other than this the apparatus is thought to be perfect.
Ruler - ± 1 mm.
Calliper - ± 0.1 mm.
100 ml. beaker - ± 10 ml.
250 ml. beaker - ± 25 ml.
500 ml. beaker - ± 50 ml.
1000 ml. cylinder - ± 100 ml.
500 ml. cylinder - ± 50 ml.
100 ml. cylinder - ± 1 ml.
Computer program (Audacity) - ± 1 Hz

4. METHOD

Measuring the diameter of bottle neck, pebbles and the exact total volume of the bottle

1) Fill the entire bottle to the top with distilled water
2) Accurately empty the water into a beaker and note the results
3) Use the ruler to measure the diameter of the opening of the bottle
4) Use the calliper to measure the diameter of a pebble

Collecting data when reducing the bottle to 1.5 l of air with pebbles

1) Use the readings from the previous step to calculate how many cl. of pebbles are needed for there to be 1.5 l of air in the bottle,
2)Fill a beaker with 3dl of distilled water
3) Pour pebbles into the beaker until 3 dl plus the calculated value is reached
4) Pour out the water through the sieve to isolate pebbles
5) Pour the pebbles into the wine bottle
6) Use your mouth to blow over the top of the bottle until a constant tone is reached
7) Record in sound recording program

Use the same method to reduce the volume of the bottle by multiples of 1 dl, and repeat five times for every volume until the bottle either is full or no audible frequency is achievable,

Collecting data when reducing the chamber to 1.5 l. this time using water to get a reference to the reading with pebbles

1) Use the readings from the previous step to calculate how many cl. of water are needed for there to be 1.5 l of air in the bottle,
2) Fill beaker with this calculated amount of distilled water
3) Pour the beaker into the bottle
4) Use your mouth to blow over the top of the bottle until a constant tone is reached
5) Record in sound recording program

Use the same method to reduce the volume of the bottle by multiples of 1 dl, and repeat five times for every volume until the bottle either is full or no audible frequency is achievable,

(Repeat all steps with 2 more bottles and reduce with multiples of 5 cl. and initial reading as 75 cl for the regular wine-bottle and initial readings as 37.5 cl for the half-wine bottle. This repetition is done to get more readings to base any theories regarding a trend on)

5. RESULTS:

The main focus of the results gathered were the results from bottle 1[13], but the results were recorded for two other bottles to get more accurate and a wider range of results, making it easier to make a theory about the trend observed. Error calculation can be found in **5.1 Error Calculation** and the full results with errors can be found in Appendix A and C; the results listed below are averages of five readings.

Volume of water (volume of air) [1]	Water	Pebbles
0.16 l. (1.40 l.) ± 0.03 l.	118 Hz	116 Hz
0.26 l. (1.30 l). ± 0.05 l.	117 Hz	117 Hz
0.36 l. (1.20 l.) ± 0.08 l.	123 Hz	121 Hz
0.46 l. (1.10 l.) ± 0.10 l.	130 Hz	128 Hz
0.56 l. (1.00 l.) ± 0.13 l.	134 Hz	132 Hz
0.66 l. (0.90 l.) ± 0.15 l.	143 Hz	137 Hz
0.76 l. (0.80 l.) ± 0.18 l.	147 Hz	137 Hz
0.86 l. (0.70 l.) ± 0.20 l.	155 Hz	Full Bottle
0.96 l. (0.60 l.) ± 0.23 l.	167 Hz	Full Bottle

[1] – Bottle 1

Volume of water (volume of air) [2]	Water	Pebbles
0.16 l. (0.65 l.) ± 0.03 l.	118 Hz	120 Hz
0.21 l. (0.60 l.) ± 0.04 l.	123 Hz	122 Hz
0.26 l. (0.55 l.) ± 0.05 l.	133 Hz	129 Hz
0.31 l. (0.50 l.) ± 0.06 l.	135 Hz	136 Hz
0.36 l. (0.45 l.) ± 0.07 l.	143 Hz	140 Hz
0.41 l. (0.40 l.) ± 0.08 l.	152 Hz	143 Hz
0.46 l. (0.35 l.) ± 0.09 l.	161 Hz	146 Hz

[2] – Bottle 2

Volume of water (volume of air) [3]	Water	Pebbles
0.75 dl. (0.30 l.) ± 0.03 l.	182 Hz	188 Hz
1.25 dl. (0.25 l.) ± 0.04 l.	204 Hz	208 Hz
1.75 dl. (0.20 l.) ± 0.05 l.	232 Hz	Full Bottle
2.25 dl. (0.15 l.) ± 0.06 l.	267 Hz	Full Bottle

[3] – Bottle 3

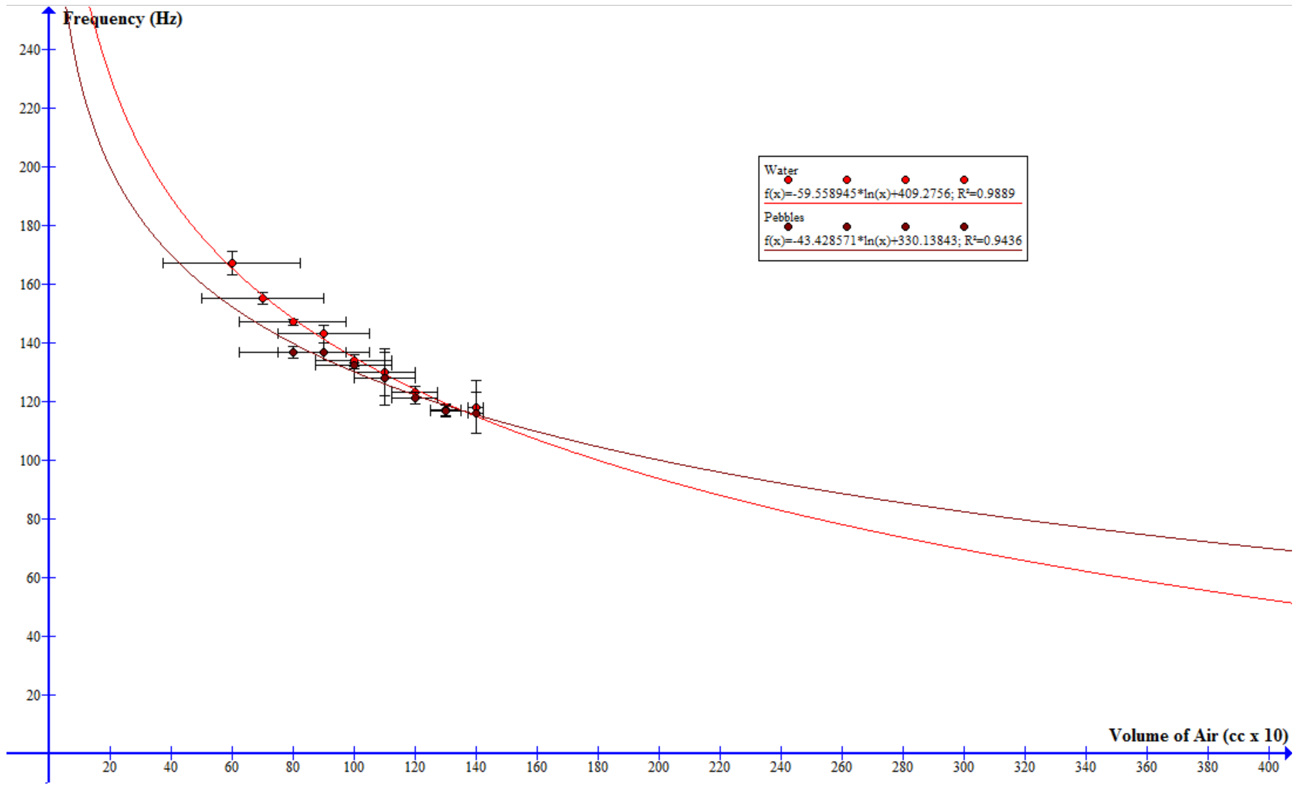
From the results it appears that the readings with pebbles generally have a lower frequency; but as displayed in early volumes in bottle 2 and 3, the results start at a higher frequency than the reference (Water).

To further investigate this, the results can be displayed graphically.

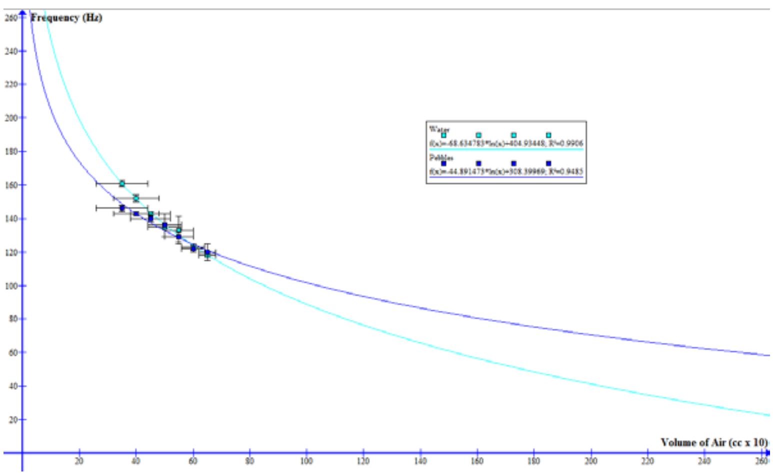
[13] See appendix A for exact scale of the bottles 1, 2, 3,

The lighter shaded line is the bottle's chamber filling up with water and the darker shaded line is the bottle's chamber filling up with pebbles. There are no max/min-lines included; this is because the average lines of the two results for each bottle are being compared[14].

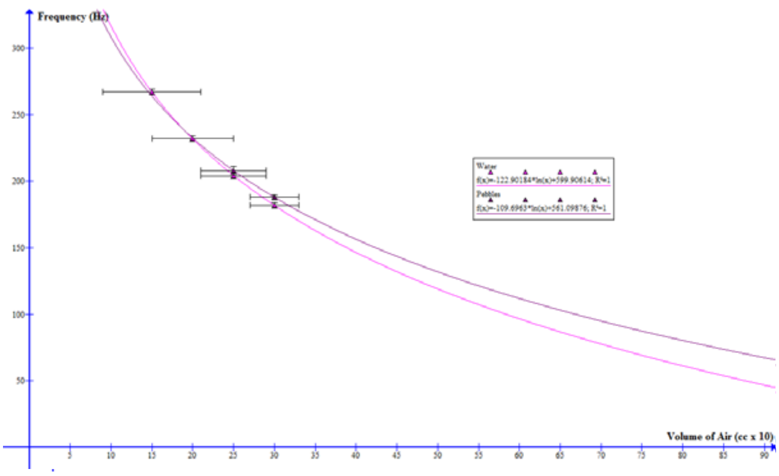
Bottle 1



Bottle 2



Bottle 3



Now it is clear that there is a very prominent trend where the lines cross for all the bottles and that if the readings were taken at a small enough volume the recorded results could be predicted to have crossed for bottle 1 as well. The main differences

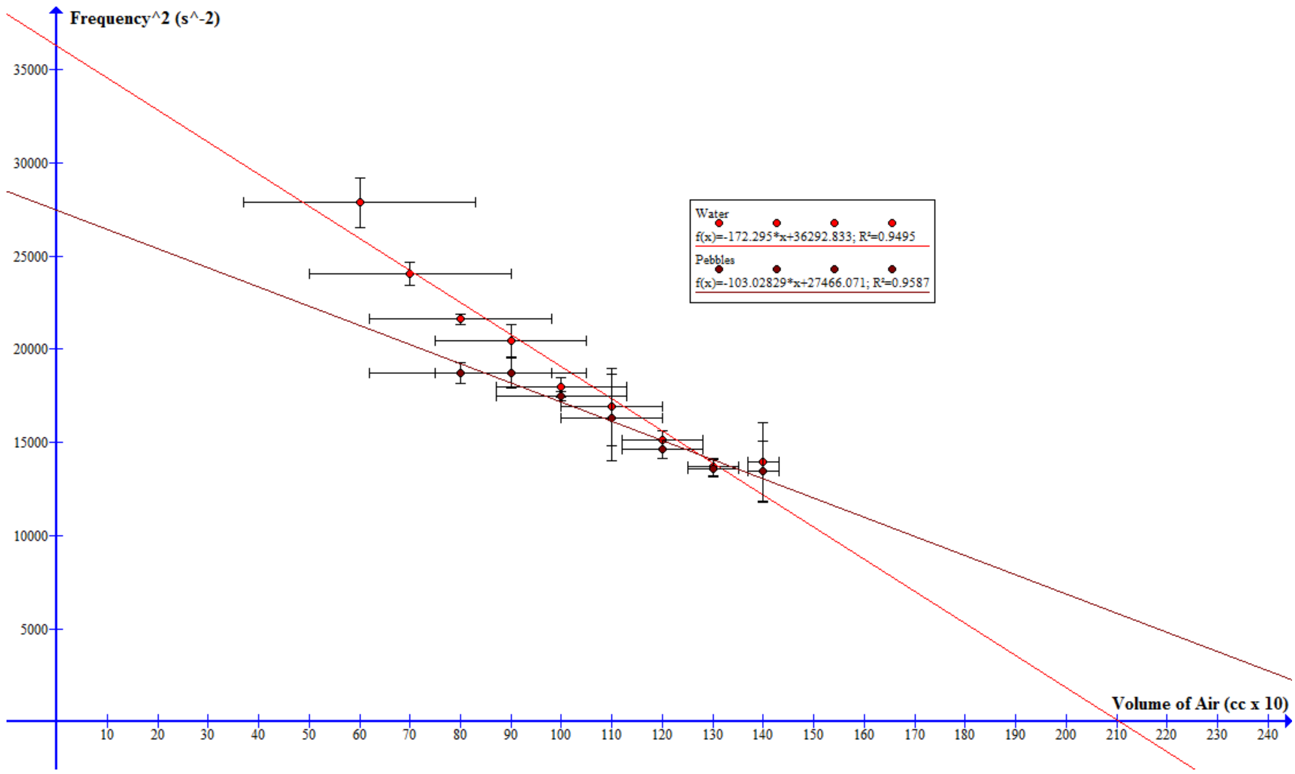
[14] Full size graphs without error bars are in Appendix E if there are any issues with visibility of the trend or "crossing".

between the three bottles is the slopes and so are displayed with a different scale on their x-axes, other than this they display the same basic relationship between the two lines, but to get a clear linear relationship we can square the readings for the frequency.

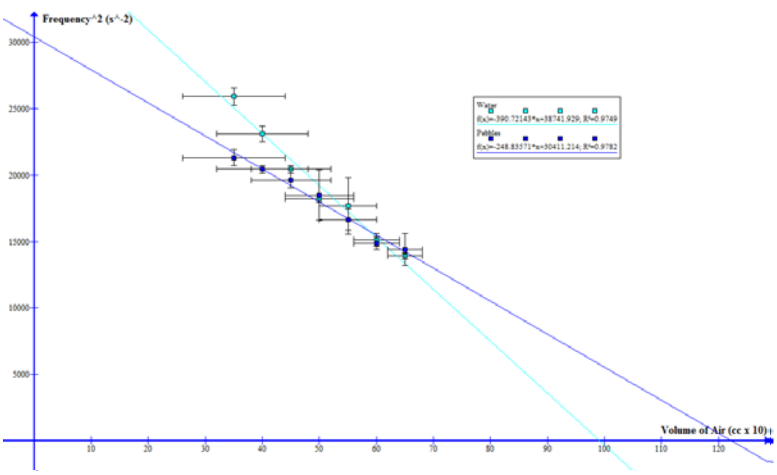
The relationship between the frequency and volume is: $f \propto \sqrt{\frac{1}{V_c}} \quad \Rightarrow \quad f^2 \propto \frac{1}{V_c}$

This can be displayed graphically as shown below[15]; once again these graphs to not have max/min- lines due to their comparative nature separate sets of results.

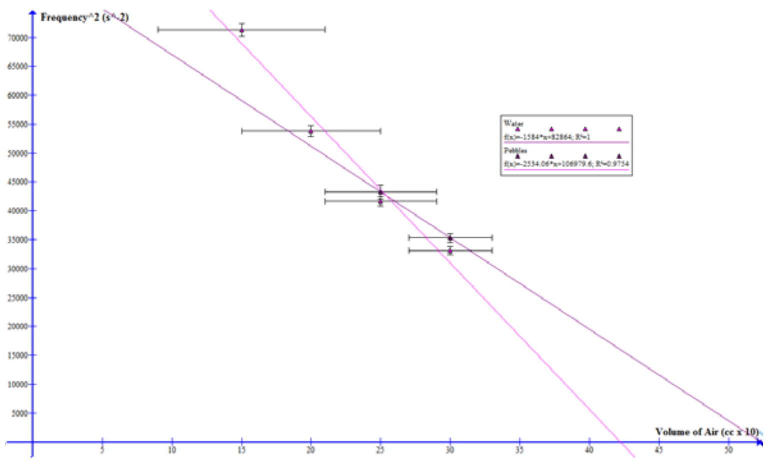
Bottle 1



Bottle 2



Bottle 3



[15] Full size graphs without error bars are in Appendix F if there are any issues with visibility of the trend or "crossing".

Note that once again the different bottles make the scales different on the axes, but that the individual lines have different slopes and a cross can still be seen clearly.

Now to refer to the hypothesis of the lab:

"Using a wine bottle as a Helmholtz resonator, pebbles to achieve interferences in the volume and water to reduce the volume of the reference resonator I will now attempt to compare the frequencies resonated by a resonator with interferences and one with an open chamber. Doing this I am expecting to find a higher frequency in the results with the canals and cavities as I expect the utilized volume air in the resonator to be less. Due to the inverse relationship between volume and frequency, as volume goes down I am expecting the frequency to go up."

The results contradict the hypothesis, in the way that it does not simply shift the curve horizontally or vertically, but it affects the slope of the curves. This would mean that the relationship is slightly more complex than first expected when interferences are introduced to the chamber of this type of resonator. Further discussion around this effect is in the conclusion part of this essay.

5.1 ERROR CALCULATION:

The volume taken for the bottles is including the neck, but used as only for the chamber, this is a source of systematic error. As this is done both when pebbles and water is added to the bottles, it should not affect the comparative results, but it is a source of systematic error nonetheless.

As apparatus error has been found, a good estimate for the error on the average would be to find the standard deviation and add this to the apparatus error for the frequency found. This shows the reading error as well as the random error occurring when performing the experiment.
To find the standard deviation, use the formula;

$$S_n = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$$ [16]

An example of this calculation is shown below:

0.16 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
128 Hz	12.0	144.00
113 Hz	-3.0	9.00
113 Hz	-3.0	9.00
113 Hz	-3.0	9.00
113 Hz	-3.0	9.00
580 Hz	Total	180.00

$\bar{x} = \frac{588}{5} = 117.6\ Hz \approx 118\ Hz$

$S = \sqrt{\frac{331.20}{5}} = 8.14\ Hz \approx 8\ Hz$

These calculations were taken from the first five readings of the first volume of air in the first bottle[17]. Now to find the total error this is added to the ± 1Hz apparatus error found in **3.1**.

When calculating new data by altering previously recorded pieces of data, the new errors have to be calculated using specific formulae. In the previous section the results for frequency were squared and so to find the correct new errors more calculations need to be done. These new errors are shown both as error bars above as well as a table in Appendix D[18].

[16] See appendix B for full Standard Deviation Calculations.
[17] Appendix B – first table.
[18] See appendix D

When calculating f^2 the new errors are found using the formula:

$If\ n^2 = m, \quad then\ \frac{\Delta m}{m} = 2 \times \frac{\Delta n}{n}, \quad where\ \Delta\ is\ the\ total\ error.$

E.g.: $f = 118\ \pm 9$

$118^2 = 13924$, error in f is ± 9, thus error in $f^2$is found through $2 \times \frac{9}{118} = \frac{\Delta f^2}{13924}$ which gives $2 \times \frac{9}{118} \times 13924 = \Delta f^2 = 2124$, thus $f^2 = 13924 \pm 2124$ [19]

[19] For all error calculations for frequency squared see Appendix D

6. CONCLUSION:

Referring to the research question of this essay:
In what way does the introduction of interferences to the volume of a Helmholtz resonator effect the frequency resonated?
The results were contradictory to the hypothesis, but the detected trend was that the slopes of the graphs were increased when the interferences in form of pebbles were introduced. This difference in slopes is quite easy to see and is linear when the frequency is squared.
As the relationship between frequency squared and the other factors is:

$$f^2 = k \times \frac{A}{L \times V}$$

where k is the constant $(\frac{c}{2\pi})^2$, A is area of the neck, L is length of the neck and V is the volume of the chamber; the introduction of pebbles must make one of the following changes, assuming to a certain extent that the existing theory is correct.

1. The introduction interferences could result in some external factor not taken into account in the existing relationship

2. The introduction of interferences could affect one of the other factors inconstantly. These changes being; *k* or *A* being made larger, or *L* or *V* being made smaller.

Weighing the two alternatives up against each other it seems quite easy to make an alteration to the original hypothesis of the lab to make it fit the second scenario. It seems as though the hypothesis was wrong in assuming that the alteration of the effective volume would shift the results linearly by a constant. As the volume in reality is being affected by the pebbles which are added increasingly, the slope would naturally be changed and it seems as though the reasoning behind the hypothesis was almost accurate. This alternative is not only accompanied by an alteration to the hypothesis, but also follows previous discovery regarding Helmholtz resonators, excluding the first alternative.

To answer the Research Question, the introduction of interferences in the form of air pockets and canals to the effective volume of a Helmholtz resonator will make the effective volume indeed smaller, causing a positive shift of the rate of change for the frequency, as there is an inverse relationship between the volume and the frequency emitted squared.

6.1 Limitations:

There were some limitations present in the methodology of the lab, including the airstream added as the external oscillating force on the bottle. This could have been done by a machine, but it was simply more feasible for a person to do it as the equipment in the school lab was limited. This should not be a big source of error as an audible frequency only exists when the right amount of force is supplied to the system, and if the readings are taken multiple times the accuracy of the readings should be quite good.

Using Helmholtz resonators instead of wine bottles *as* Helmholtz resonators more accurate results could have been achieved, but as shown above in the results none of these limitations limited the experiment to such an extent that a well based conclusion was unobtainable.

6.2 Further Investigation

This topic could be further investigated in a numerous ways. In order to acquire more accurate results more readings could be taken. The effects of the shape of the volume could be relevant, as well as the density of the substance added to the resonator. As the relevance of resistance is discussed above[20], these seem like beneficial aspects of the resonance creation to investigate.

[20] Section *1.1* of the introduction.

7. EVALUATION:

It seems as though the research question of this essay has been answered and that the original hypothesis was oversimplified due to the expectations of a simple change in the nature of the resonator. When the relationship for the frequency was properly analyzed, a better fitting solution was achieved.

The results taken from bottle number 3 were very limited and did not give much insight in any other way than being a point of comparison for the other gathered results. The extensive amount of results collected made the weak points; the low number of readings from bottle 3, systematic error in volume of the chambers and the source of error in human lungs providing the resonating force, less determining when obtaining a conclusion for the essay as a whole.

The fact that the hypothesis made the assumption that adding the of pebbles would make a constant change in the numbers and not a varying change according to the volume of pebbles made the hypothesis flawed, but turned out to be a good centre of discussion as well as helping to answer the research question as a whole.

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9. APPENDICES:

Appendix A:

Results from experiment with equipment errors.

				Water:					
Bottle 1	Plastic	Volume of water (volume of air)		1	2	3	4	5	Average:
Total Volume	1.56 l. ± 0.03 l.	0.16 l. (1.40 l.) ± 0.03 l.	→	111 Hz	129 Hz	111 Hz	111 Hz	126 Hz	117.6 Hz
Diameter O	21.0 ± 0.1 mm.	0.26 l. (1.30 l). ± 0.05 l.	→	117 Hz	119 Hz	117 Hz	117 Hz	117 Hz	117.4 Hz
Length O	30 ± 1 mm.	0.36 l. (1.20 l.) ± 0.08 l.	→	123 Hz	124 Hz	123 Hz	122 Hz	123 Hz	123.0 Hz
		0.46 l. (1.10 l.) ± 0.10 l.	→	127 Hz	125 Hz	127 Hz	145 Hz	128 Hz	130.4 Hz
		0.56 l. (1.00 l.) ± 0.13 l.	→	135 Hz	135 Hz	135 Hz	132 Hz	134 Hz	134.2 Hz
		0.66 l. (0.90 l.) ± 0.15 l.	→	145 Hz	144 Hz	141 Hz	142 Hz	142 Hz	142.8 Hz
		0.76 l. (0.80 l.) ± 0.18 l.	→	148 Hz	147 Hz	147 Hz	147 Hz	147 Hz	147.2 Hz
		0.86 l. (0.70 l.) ± 0.20 l.	→	154 Hz	155 Hz	155 Hz	154 Hz	156 Hz	154.8 Hz
		0.96 l. (0.60 l.) ± 0.23 l.	→	165 Hz	166 Hz	165 Hz	169 Hz	172 Hz	167.4 Hz

				Pebbles:					
Bottle 1	Plastic	Volume of pebbles (volume of air)		1	2	3	4	5	Average:
Total Volume	1.56 l. ± 0.03	0.16 l. (1.40 l.) ± 0.03 l.	→	128 Hz	113 Hz	113 Hz	113 Hz	113 Hz	116.0 Hz
Diameter O	21.0 ± 0.1 mm.	0.26 l. (1.30 l.) ± 0.05 l.	→	116 Hz	116 Hz	117 Hz	117 Hz	117 Hz	116.6 Hz
Length O	30 ± 1 mm.	0.36 l. (1.20 l.) ± 0.08 l.	→	122 Hz	121 Hz	121 Hz	120 Hz	121 Hz	121.0 Hz
		0.46 l. (1.10 l.) ± 0.10 l.	→	128 Hz	129 Hz	128 Hz	127 Hz	127 Hz	127.8 Hz
		0.56 l. (1.00 l.) ± 0.13 l.	→	132 Hz	132 Hz	132 Hz	132 Hz	133 Hz	132.2 Hz
		0.66 l. (0.90 l.) ± 0.15 l.	→	135 Hz	135 Hz	139 Hz	138 Hz	137 Hz	136.8 Hz
		0.76 l. (0.80 l.) ± 0.18 l.	→	136 Hz	137 Hz	136 Hz	137 Hz	138 Hz	136.8 Hz
		0.86 l. (0.70 l.) ± 0.20 l.	→	Full bottle					
		0.96 l. (0.60 l.) ± 0.23 l.	→	Full bottle					

				Water:					
Bottle 2	Glass	Volume of water (volume of air)		1	2	3	4	5	Average:
Total Volume	0.76 l. ± 0.03 l.	0.16 l. (0.65 l.) ± 0.03 l.	→	118 Hz	118 Hz	118 Hz	118 Hz	118 Hz	118.0 Hz
Diameter O	21.0 ± 0.1 mm.	0.21 l. (0.60 l.) ± 0.04 l.	→	124 Hz	123 Hz	122 Hz	123 Hz	122 Hz	122.8 Hz
Length O	89 ± 1 mm.	0.26 l. (0.55 l.) ± 0.05 l.	→	130 Hz	129 Hz	130 Hz	147 Hz	129 Hz	133.0 Hz
		0.31 l. (0.50 l.) ± 0.06 l.	→	136 Hz	135 Hz	135 Hz	135 Hz	135 Hz	135.2 Hz
		0.36 l. (0.45 l.) ± 0.07 l.	→	143 Hz	143 Hz	142 Hz	143 Hz	143 Hz	142.8 Hz
		0.41 l. (0.40 l.) ± 0.08 l.	→	151 Hz	152 Hz	152 Hz	151 Hz	152 Hz	151.6 Hz
		0.46 l. (0.35 l.) ± 0.09 l.	→	161 Hz	162 Hz	161 Hz	161 Hz	162 Hz	161.4 Hz

				Pebbles:					
Bottle 2	Glass	Volume of pebbles (volume of air)		1	2	3	4	5	Average:
Total Volume	0.76 l. ± 0.03 l.	0.16 l. (0.65 l.) ± 0.03 l.	→	128 Hz	118 Hz	118 Hz	119 Hz	119 Hz	120.4 Hz
Diameter O	21.0 ± 0.1 mm.	0.21 l. (0.60 l.) ± 0.04 l.	→	122 Hz	122 Hz	122 Hz	121 Hz	121 Hz	121.6 Hz
Length O	89 ± 1 mm.	0.26 l. (0.55 l.) ± 0.05 l.	→	129 Hz	133 Hz	128 Hz	128 Hz	129 Hz	129.4 Hz
		0.31 l. (0.50 l.) ± 0.06 l.	→	133 Hz	147 Hz	133 Hz	133 Hz	133 Hz	135.8 Hz
		0.36 l. (0.45 l.) ± 0.07 l.	→	140 Hz	140 Hz	140 Hz	141 Hz	139 Hz	140.0 Hz
		0.41 l. (0.40 l.) ± 0.08 l.	→	143 Hz	143 Hz	143 Hz	143 Hz	143 Hz	143.0 Hz
		0.46 l. (0.35 l.) ± 0.09 l.	→	146 Hz	146 Hz	147 Hz	146 Hz	145 Hz	146.0 Hz

In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?

			Water:						
Bottle 3	Glass	Volume of water (volume of air):		1	2	3	4	5	Average:
Total Volume	3.75 dl ± 0.03 l	0.75 dl. (0.30 l.) ± 0.03 l.	→	182 Hz	180 Hz	181 Hz	182 Hz	183 Hz	181.6 Hz
Diameter O	21.0 ± 0.1 mm.	1.25 dl. (0.25 l.) ± 0.04 l.	→	205 Hz	205 Hz	204 Hz	204 Hz	204 Hz	204.4 Hz
Length O	68 ± 1 mm.	1.75 dl. (0.20 l.) ± 0.05 l.	→	231 Hz	232 Hz	231 Hz	232 Hz	232 Hz	231.6 Hz
		2.25 dl. (0.15 l.) ± 0.06 l.	→	266 Hz	266 Hz	269 Hz	267 Hz	267 Hz	267.0 Hz

			Pebbles:						
Bottle 3	Glass	Volume of pebbles (volume of air)		1	2	3	4	5	Average:
Total Volume	3.75 dl ± 0.03 l	0.75 dl. (0.30 l.) ± 0.03 l.	→	188 Hz	188 Hz	188 Hz	190 Hz	188 Hz	188.4 Hz
Diameter O	21.0 ± 0.1 mm.	1.25 dl. (0.25 l.) ± 0.04 l.	→	206 Hz	209 Hz	208 Hz	207 Hz	208 Hz	207.6 Hz
Length O	68 ± 1 mm.	1.75 dl. (0.20 l.) ± 0.05 l.	→	Full bottle					
		2.25 dl. (0.15 l.) ± 0.06 l.	→	Full bottle					

Appendix B:

Standard deviation calculations for results displayed in Appendix A and listed in Appendix C.

Standard deviation for Bottle 1, Water:

0.16 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
111 Hz	- 6.6	43.56
129 Hz	11.4	129.96
111 Hz	- 6.6	43.56
111 Hz	- 6.6	43.56
126 Hz	8.4	70.56
588 Hz	Total	331.20

$\bar{x} = \frac{588}{5} = 117.6\ hz$

S = $\sqrt{\frac{331.20}{5}}$ = 8.14

0.26 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
117 Hz	- 0.4	0.16
119 Hz	1.6	2.56
117 Hz	- 0.4	0.16
117 Hz	- 0.4	0.16
117 Hz	- 0.4	0.16
587 Hz	Total	3.20

$\bar{x} = \frac{587}{5} = 117.4\ hz$

S = $\sqrt{\frac{3.20}{5}}$ = 0.80

0.36 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
123 Hz	0.0	0.00
124 Hz	1.0	1.00
123 Hz	0.0	0.00
122 Hz	- 1.0	1.00
123 Hz	0.0	0.00
615 Hz	Total	2.00

$\bar{x} = \frac{615}{5} = 123.0\ hz$

S = $\sqrt{\frac{2.0}{5}}$ = 0.63

0.46 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
127 Hz	- 3.4	11.56
125 Hz	- 5.4	29.16
127 Hz	- 3.4	11.56
145 Hz	14.6	213.16
128 Hz	- 2.4	5.76
652 Hz	Total	271.20

$\bar{x} = \frac{652}{5} = 130.4\ hz$

S = $\sqrt{\frac{271.20}{5}}$ = 7.36

0.56 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
135 Hz	0.8	0.64
135 Hz	0.8	0.64
135 Hz	0.8	0.64
132 Hz	- 2.2	4.84
134 Hz	- 0.2	0.04
671 Hz	Total	6.80

$\bar{x} = \frac{671}{5} = 134.2\ hz$

S = $\sqrt{\frac{6.80}{5}}$ = 1.17

0.66 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
145 Hz	1.0	1.00
144 Hz	0.0	0.00
141 Hz	- 3.0	9.00
145 Hz	1.0	1.00
145 Hz	1.0	1.00
720 Hz	Total	12.00

$\bar{x} = \frac{720}{5} = 144.0\ hz$

S = $\sqrt{\frac{12.00}{5}}$ = 1.55

0.76 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
148 Hz	0.8	0.64
147 Hz	- 0.2	0.04
147 Hz	- 0.2	0.04
147 Hz	- 0.2	0.04
147 Hz	- 0.2	0.04
736 Hz	Total	0.80

$\bar{x} = \frac{736}{5} = 147.2\ hz$

S = $\sqrt{\frac{0.80}{5}}$ = 0.40

0.86 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
154 Hz	- 0.8	0.64
155 Hz	0.2	0.04
155 Hz	0.2	0.04
154 Hz	- 0.8	0.64
156 Hz	1.2	1.44
774 Hz	Total	2.80

$\bar{x} = \frac{744}{5} = 154.8\ hz$

S = $\sqrt{\frac{2.80}{5}}$ = 0.75

0.96 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
165 Hz	- 2.4	5.76
166 Hz	- 1.4	1.96
165 Hz	- 2.4	5.76
169 Hz	1.6	2.56
172 Hz	4.6	21.16
837 Hz	Total	37.20

$\bar{x} = \frac{837}{5} = 167.4\ hz$

S = $\sqrt{\frac{37.20}{5}}$ = 2.73

Standard deviation calculations for Bottle 1, Pebbles:

0.16 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
128 Hz	12.0	144.00
113 Hz	-3.0	9.00
113 Hz	-3.0	9.00
113 Hz	-3.0	9.00
113 Hz	-3.0	9.00
580 Hz	Total	180.00

$\bar{x} = \frac{580}{5} = 116.0\ hz$

S = $\sqrt{\frac{180.00}{5}}$ = 6.00

0.26 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
116 Hz	-0.6	0.36
116 Hz	-0.6	0.36
117 Hz	0.4	0.16
117 Hz	0.4	0.16
117 Hz	0.4	0.16
583 Hz	Total	1.20

$\bar{x} = \frac{583}{5} = 116.6\ hz$

S = $\sqrt{\frac{1.20}{5}}$ = 0.49

0.36 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
122 Hz	1.0	1.00
121 Hz	0	0.00
121 Hz	0	0.00
120 Hz	-1.0	1.00
121 Hz	0	0.00
605 Hz	Total	2.00

$\bar{x} = \frac{605}{5} = 121.0\ hz$

S = $\sqrt{\frac{2.00}{5}}$ = 0.63

0.46 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
128 Hz	0.2	0.04
129 Hz	1.2	1.44
128 Hz	0.2	0.04
127 Hz	-0.8	0.64
127 Hz	-0.8	0.64
639 Hz	Total	272.80

$\bar{x} = \frac{639}{5} = 127.8\ hz$

S = $\sqrt{\frac{272.80}{5}}$ = 7.39

0.56 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
132 Hz	-0.2	0.04
132 Hz	-0.2	0.04
132 Hz	-0.2	0.04
132 Hz	-0.2	0.04
133 Hz	0.8	0.64
661 Hz	Total	0.80

$\bar{x} = \frac{661}{5} = 132.2\ hz$

S = $\sqrt{\frac{0.80}{5}}$ = 0.40

0.66 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
135 Hz	-1.8	3.24
135 Hz	-1.8	3.24
139 Hz	2.2	4.84
138 Hz	1.2	1.44
137 Hz	0.2	0.04
684 Hz	Total	12.80

$\bar{x} = \frac{684}{5} = 136.8\ hz$

S = $\sqrt{\frac{12.80}{5}}$ = 1.60

0.76 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
136 Hz	-0.8	0.64
137 Hz	0.2	0.04
136 Hz	-0.8	0.64
137 Hz	0.2	0.04
138 Hz	-1.2	1.44
684 Hz	Total	2.80

$\bar{x} = \frac{684}{5} = 136.8\ hz$

S = $\sqrt{\frac{2.80}{5}}$ = 0.75

Standard deviation calculations for Bottle 2, Water:

0.16 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
118 Hz	0.0	0.00
118 Hz	0.0	0.00
118 Hz	0.0	0.00
118 Hz	0.0	0.00
118 Hz	0.0	0.00
590 Hz	Total	0.00

$\bar{x} = \frac{590}{5} = 118.0\ hz$

S = $\sqrt{\frac{0.00}{5}} = 0.00$

0.21 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
124 Hz	1.2	1.44
123 Hz	0.2	0.04
122 Hz	-0.8	0.64
123 Hz	0.2	0.04
122 Hz	-0.8	0.64
614 Hz	Total	2.80

$\bar{x} = \frac{614}{5} = 122.8\ hz$

S = $\sqrt{\frac{2.80}{5}} = 0.75$

0.26 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
130 Hz	-3.0	9.00
129 Hz	-4.0	16.00
130 Hz	-3.0	9.00
147 Hz	14.0	196.00
129 Hz	-4.0	16.00
665 Hz	Total	246.00

$\bar{x} = \frac{665}{5} = 133.0\ hz$

S = $\sqrt{\frac{246.00}{5}} = 7.01$

0.31 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
136 Hz	0.8	0. 64
135 Hz	-0.2	0.04
135 Hz	-0.2	0.04
135 Hz	-0.2	0.04
135 Hz	-0.2	0.04
676 Hz	Total	0.80

$\bar{x} = \frac{676}{5} = 135.2\ hz$

S = $\sqrt{\frac{0.80}{5}} = 0.40$

0.36 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
143 Hz	0.2	0.04
143 Hz	-0.2	0.04
142 Hz	-0.8	0.64
143 Hz	0.2	0.04
143 Hz	0.2	0.04
714 Hz	Total	0.80

$\bar{x} = \frac{714}{5} = 142.8\ hz$

S = $\sqrt{\frac{0.80}{5}} = 0.40$

0.41 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
151 Hz	-0.6	0.36
152 Hz	0.4	0.16
152 Hz	0.4	0.16
151 Hz	-0.6	0.36
152 Hz	0.4	0.16
758 Hz	Total	1.20

$\bar{x} = \frac{758}{5} = 151.6\ hz$

S = $\sqrt{\frac{1.20}{5}} = 0.49$

0.46 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
161 Hz	-0.4	0.16
162 Hz	0.6	0.36
161 Hz	-0.4	0.16
161 Hz	-0.4	0.16
162 Hz	0.6	0.36
807 Hz	Total	1.20

$\bar{x} = \frac{807}{5} = 161.4\ hz$

S = $\sqrt{\frac{1.20}{5}} = 0.49$

Standard deviation calculations for Bottle 2, Pebbles:

0.16 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
128 Hz	7.6	57.76
118 Hz	-2.4	5.76
118 Hz	-2.4	5.76
119 Hz	-1.4	1.96
119 Hz	-1.4	1.96
602 Hz	Total	73.20

$\bar{x}=\frac{602}{5}=120.4\ hz$

S = $\sqrt{\frac{73.20}{5}}$ = 3.83

0.21 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
122 Hz	0.4	0.16
122 Hz	0.4	0.16
122 Hz	0.4	0.16
121 Hz	-0.6	0.36
121 Hz	-0.6	0.36
608 Hz	Total	1.20

$\bar{x}=\frac{608}{5}=121.6\ hz$

S = $\sqrt{\frac{1.20}{5}}$ = 0.49

0.26 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
129 Hz	-0.4	0.16
133 Hz	3.6	12.96
128 Hz	-1.4	1.96
128 Hz	-1.4	1.96
129 Hz	-0.4	0.16
647 Hz	Total	17.20

$\bar{x}=\frac{647}{5}=129.4\ hz$

S = $\sqrt{\frac{17.20}{5}}=1.85$

0.31 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
133 Hz	-2.8	7.84
147 Hz	11.2	125.44
133 Hz	-2.8	7.84
133 Hz	-2.8	7.84
133 Hz	-2.8	7.84
679 Hz	Total	156.80

$\bar{x}=\frac{679}{5}=135.8\ hz$

S = $\sqrt{\frac{156.80}{5}}$ = 5.60

0.36 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
140 Hz	0.0	0.00
140 Hz	0.0	0.00
140 Hz	0.0	0.00
141 Hz	1.0	1.00
139 Hz	-1.0	1.00
700 Hz	Total	2.00

$\bar{x}=\frac{700}{5}=140.0\ hz$

S = $\sqrt{\frac{2.00}{5}}$ = 0.63

0.41 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
143 Hz	0.00	0.00
143 Hz	0.00	0.00
143 Hz	0.00	0.00
143 Hz	0.00	0.00
143 Hz	0.00	0.00
715 Hz	Total	0.00

$\bar{x}=\frac{715}{5}=143.0\ hz$

S = $\sqrt{\frac{0.00}{5}}=0.00$

0.46 l.		
x	$x-\bar{x}$	$(x-\bar{x})^2$
146 Hz	0.0	0.00
146 Hz	0.0	0.00
147 Hz	1.0	1.00
146 Hz	0.0	0.00
145 Hz	-1.0	1.00
730 Hz	Total	2.00

$\bar{x}=\frac{730}{5}=146.0\ hz$

S = $\sqrt{\frac{2.00}{5}}$ = 0.63

Standard deviation calculations for Bottle 3, Water:

0.075 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
182 Hz	0.4	0.16
180 Hz	-1.6	2.56
181 Hz	-0.6	0.36
182 Hz	0.4	0.16
183 Hz	1.4	1.96
908 Hz	Total	5.20

$\bar{x} = \frac{908}{5} = 181.6\ hz$

S = $\sqrt{\frac{5.20}{5}}$ = 1.02

0.125 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
205 Hz	0.6	0.36
205 Hz	0.6	0.36
204 Hz	-0.4	0.16
204 Hz	-0.4	0.16
204 Hz	-0.4	0.16
1022 Hz	Total	1.20

$\bar{x} = \frac{1022}{5} = 204.4\ hz$

S = $\sqrt{\frac{1.20}{5}}$ = 0.49

0.175 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
231 Hz	-0.6	0.36
232 Hz	0.4	0.16
231 Hz	-0.6	0.36
232 Hz	0.4	0.16
232 Hz	0.4	0.16
1158 Hz	Total	1.20

$\bar{x} = \frac{1158}{5} = 231.6\ hz$

S = $\sqrt{\frac{1.20}{5}}$ = 0.49

0.225 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
266 Hz	-1.0	1.00
266 Hz	-1.0	1.00
269 Hz	2.00	4.00
267 Hz	0.0	0.00
267 Hz	0.0	0.00
1335 Hz	Total	6.00

$\bar{x} = \frac{1335}{5} = 267.0\ hz$

S = $\sqrt{\frac{6.00}{5}}$ = 1.09

Standard deviation calculations for Bottle 3, Pebbles:

0.075 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
188 Hz	-0.4	0.16
188 Hz	-0.4	0.16
188 Hz	-0.4	0.16
190 Hz	1.6	2.56
188 Hz	-0.4	0.16
942 Hz	Total	3.20

$\bar{x} = \frac{942}{5} = 188.4\ hz$

S = $\sqrt{\frac{3.20}{5}}$ = 0.80

0.125 l.		
x	$x - \bar{x}$	$(x - \bar{x})^2$
206 Hz	-1.6	2.56
209 Hz	2.4	5.76
208 Hz	0.4	0.16
207 Hz	-0.6	2.56
208 Hz	0.4	0.16
1038 Hz	Total	11.20

$\bar{x} = \frac{1038}{5} = 207.6\ hz$

S = $\sqrt{\frac{11.20}{5}}$ = 1.50

Appendix C:

Results with total errors and other apparatus errors.

Bottle 1	Plastic	Volume of air:		Water: Average Frequency	Average Error	Standard Deviation	Total Error
Total Volume	1.56 l. ± 0.03 l.	1.40 l. ± 0.03	→	118 Hz	± 1 Hz	8 Hz	± 9 Hz
Diameter O	21.0 ± 0.1 mm.	1.30 l. ± 0.05	→	117 Hz	± 1 Hz	1 Hz	± 2 Hz
Length O	30 ± 1 mm.	1.20 l. ± 0.08	→	123 Hz	± 1 Hz	1 Hz	± 2 Hz
		1.10 l. ± 0.10	→	130 Hz	± 1 Hz	7 Hz	± 8 Hz
		1.00 l. ± 0.13	→	134 Hz	± 1 Hz	1 Hz	± 2 Hz
		0.90 l. ± 0.15	→	143 Hz	± 1 Hz	2 Hz	± 3 Hz
		0.80 l. ± 0.18	→	147 Hz	± 1 Hz	0 Hz	± 1 Hz
		0.70 l. ± 0.20	→	155 Hz	± 1 Hz	1 Hz	± 2 Hz
		0.60 l. ± 0.23	→	167 Hz	± 1 Hz	3 Hz	± 4 Hz

Bottle 1	Plastic	Volume of air:		Pebbles: Average Frequency	Average Error	Standard Deviation	Total Error
Total Volume	1.56 l. ± 0.03 l.	1.40 l. ± 0.03	→	116.0 Hz	± 1 Hz	6 Hz	± 7 Hz
Diameter O	21.0 ± 0.1 mm.	1.30 l. ± 0.05	→	116.6 Hz	± 1 Hz	1 Hz	± 2 Hz
Length O	30 ± 1 mm.	1.20 l. ± 0.08	→	121.0 Hz	± 1 Hz	1 Hz	± 2 Hz
		1.10 l. ± 0.10	→	127.8 Hz	± 1 Hz	8 Hz	± 9 Hz
		1.00 l. ± 0.13	→	132.2 Hz	± 1 Hz	0 Hz	± 1 Hz
		0.90 l. ± 0.15	→	136.8 Hz	± 1 Hz	2 Hz	± 3 Hz
		0.80 l. ± 0.18	→	136.8 Hz	± 1 Hz	1 Hz	± 2 Hz
		0.70 l. ± 0.20	→		Full bottle		
		0.60 l. ± 0.23	→		Full bottle		

Bottle 2	Glass	Volume of air:		Water Average Frequency	Average Error	Standard Deviation	Total Error
Total Volume	7.6 dl. ± 0.03 l.	0.65 l. ± 0.03	→	118 Hz	± 1 Hz	0 Hz	± 1 Hz
Diameter O	21.0 ± 0.1 mm.	0.60 l. ± 0.04	→	123 Hz	± 1 Hz	1 Hz	± 2 Hz
Length O	89 ± 1 mm.	0.55 l. ± 0.05	→	133 Hz	± 1 Hz	7 Hz	± 8 Hz
		0.50 l. ± 0.06	→	135 Hz	± 1 Hz	0 Hz	± 1 Hz
		0.45 l. ± 0.07	→	143 Hz	± 1 Hz	0 Hz	± 1 Hz
		0.40 l. ± 0.08	→	152 Hz	± 1 Hz	1 Hz	± 2 Hz
		0.35 l. ± 0.09	→	161 Hz	± 1 Hz	1 Hz	± 2 Hz

Bottle 2	Glass	Volume of air:		Pebbles: Average Frequency	Average Error	Standard Deviation	Total Error
Total Volume	7.6 dl. ± 0.03 l.	0.65 l. ± 0.03	→	120 Hz	± 1 Hz	4 Hz	± 5 Hz
Diameter O	21.0 ± 0.1 mm.	0.60 l. ± 0.04	→	122 Hz	± 1 Hz	1 Hz	± 2 Hz
Length O	89 ± 1 mm.	0.55 l. ± 0.05	→	129 Hz	± 1 Hz	2 Hz	± 3 Hz
		0.50 l. ± 0.06	→	136 Hz	± 1 Hz	6 Hz	± 7 Hz
		0.45 l. ± 0.07	→	140 Hz	± 1 Hz	1 Hz	± 2 Hz
		0.40 l. ± 0.08	→	143 Hz	± 1 Hz	0 Hz	± 1 Hz
		0.35 l. ± 0.09	→	146 Hz	± 1 Hz	1 Hz	± 2 Hz

In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?

			Water:			
Bottle 3	Glass	Volume of air:	Average Frequency	Average Error	Standard Deviation	Total Error
Total Volume	3.75 dl ± 0.03 l	0.30 l. ± 0.03 →	182 Hz	± 1 Hz	1 Hz	± 2 Hz
Diameter O	21.0 ± 0.1 mm.	0.25 l. ± 0.04 →	204 Hz	± 1 Hz	1 Hz	± 2 Hz
Length O	68 ± 1 mm.	0.20 l. ± 0.05 →	232 Hz	± 1 Hz	1 Hz	± 2 Hz
		0.15 l. ± 0.06 →	267 Hz	± 1 Hz	1 Hz	± 2 Hz

			Pebbles:			
Bottle 3	Glass	Volume of air:	Average Frequency	Average Error	Standard Deviation	Total Error
Total Volume	3.75 dl ± 0.03	0.30 l. ± 0.03 →	188 Hz	± 1 Hz	1 Hz	± 2 Hz
Diameter O	21.0 ± 0.1 mm.	0.25 l. ± 0.04 →	208 Hz	± 1 Hz	2 Hz	± 3 Hz
Length O	68 ± 1 mm.	0.20 l. ± 0.05 →	Full bottle			
		0.15 l. ± 0.06 →	Full bottle			

APPENDIX D:

Error calculations for processed graphs with frequency squared contra volume. (p.15)

New error is found through $2 \times \frac{\Delta f}{f} = \frac{\Delta f^2}{f^2} \Rightarrow 2 \times \frac{\Delta f}{f} \times f^2 = \Delta f^2$, where Δ is total error.

Volume of Water (volume of air) [1]	Frequency	Error (frequency)	Frequency Squared	Error (Frequency squared)
0.16 l. (1.40 l.) ± 0.03 l.	118 Hz	± 9 Hz	13924 Hz^2	± 2124 Hz^2
0.26 l. (1.30 l). ± 0.05 l.	117 Hz	± 2 Hz	13689 Hz^2	± 468 Hz^2
0.36 l. (1.20 l.) ± 0.08 l.	123 Hz	± 2 Hz	15129 Hz^2	± 492 Hz^2
0.46 l. (1.10 l.) ± 0.10 l.	130 Hz	± 8 Hz	16900 Hz^2	± 2080 Hz^2
0.56 l. (1.00 l.) ± 0.13 l.	134 Hz	± 2 Hz	17956 Hz^2	± 536 Hz^2
0.66 l. (0.90 l.) ± 0.15 l.	143 Hz	± 3 Hz	20449 Hz^2	± 858 Hz^2
0.76 l. (0.80 l.) ± 0.18 l.	147 Hz	± 1 Hz	21609 Hz^2	± 294 Hz^2
0.86 l. (0.70 l.) ± 0.20 l.	155 Hz	± 2 Hz	24025 Hz^2	± 620 Hz^2
0.96 l. (0.60 l.) ± 0.23 l.	167 Hz	± 4 Hz	27889 Hz^2	± 1336 Hz^2

[1] – Bottle 1

Volume of Pebbles (volume of air) [1]	Frequency	Error (frequency)	Frequency Squared	Error (Frequency squared)
0.16 l. (1.40 l.) ± 0.03 l.	116 Hz	± 7 Hz	13456 Hz^2	± 1624 Hz^2
0.26 l. (1.30 l). ± 0.05 l.	117 Hz	± 2 Hz	13689 Hz^2	± 468 Hz^2
0.36 l. (1.20 l.) ± 0.08 l.	121 Hz	± 2 Hz	14641 Hz^2	± 484 Hz^2
0.46 l. (1.10 l.) ± 0.10 l.	128 Hz	± 9 Hz	16384 Hz^2	± 2304 Hz^2
0.56 l. (1.00 l.) ± 0.13 l.	132 Hz	± 1 Hz	17424 Hz^2	± 264 Hz^2
0.66 l. (0.90 l.) ± 0.15 l.	137 Hz	± 3 Hz	18769 Hz^2	± 822 Hz^2
0.76 l. (0.80 l.) ± 0.18 l.	137 Hz	± 2 Hz	18769 Hz^2	± 548 Hz^2
0.86 l. (0.70 l.) ± 0.20 l.	Full Bottle	Full Bottle	Full Bottle	Full Bottle
0.96 l. (0.60 l.) ± 0.23 l.	Full Bottle	Full Bottle	Full Bottle	Full Bottle

[1] – Bottle 1

Volume of Water (volume of air) [2]	Frequency	Error (frequency)	Frequency Squared	Error (Frequency squared)
0.16 l. (0.65 l.) ± 0.03 l.	118 Hz	± 1 Hz	13924 Hz^2	± 236 Hz^2
0.21 l. (0.60 l.) ± 0.04 l.	123 Hz	± 2 Hz	15129 Hz^2	± 492 Hz^2
0.26 l. (0.55 l.) ± 0.05 l.	133 Hz	± 8 Hz	17689 Hz^2	± 2128 Hz^2
0.31 l. (0.50 l.) ± 0.06 l.	135 Hz	± 1 Hz	18225 Hz^2	± 270 Hz^2
0.36 l. (0.45 l.) ± 0.07 l.	143 Hz	± 1 Hz	20449 Hz^2	± 286 Hz^2
0.41 l. (0.40 l.) ± 0.08 l.	152 Hz	± 2 Hz	23104 Hz^2	± 608 Hz^2
0.46 l. (0.35 l.) ± 0.09 l.	161 Hz	± 2 Hz	25921 Hz^2	± 644 Hz^2

[2] – Bottle 2

In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?

Volume of Pebbles (volume of air) [2]	Frequency	Error (frequency)	Frequency Squared	Error (Frequency squared)
0.16 l. (0.65 l.) ± 0.03 l.	120 Hz	± 5 Hz	14400 Hz^2	± 1200 Hz^2
0.21 l. (0.60 l.) ± 0.04 l.	122 Hz	± 2 Hz	14884 Hz^2	± 488 Hz^2
0.26 l. (0.55 l.) ± 0.05 l.	129 Hz	± 3 Hz	16641 Hz^2	± 774 Hz^2
0.31 l. (0.50 l.) ± 0.06 l.	136 Hz	± 7 Hz	18496 Hz^2	± 1904 Hz^2
0.36 l. (0.45 l.) ± 0.07 l.	140 Hz	± 2 Hz	19600 Hz^2	± 560 Hz^2
0.41 l. (0.40 l.) ± 0.08 l.	143 Hz	± 1 Hz	20449 Hz^2	± 286 Hz^2
0.46 l. (0.35 l.) ± 0.09 l.	146 Hz	± 2 Hz	21316 Hz^2	± 584 Hz^2

[2] – Bottle 2

Volume of Water (volume of air) [3]	Frequency	Error (frequency)	Frequency Squared	Error (Frequency squared)
0.75 dl. (0.30 l.) ± 0.03 l.	182 Hz	± 2 Hz	33124 Hz^2	± 728 Hz^2
1.25 dl. (0.25 l.) ± 0.04 l.	204 Hz	± 2 Hz	41616 Hz^2	± 816 Hz^2
1.75 dl. (0.20 l.) ± 0.05 l.	232 Hz	± 2 Hz	53824 Hz^2	± 928 Hz^2
2.25 dl. (0.15 l.) ± 0.06 l.	267 Hz	± 2 Hz	71289 Hz^2	± 1068 Hz^2

[3] – Bottle 3

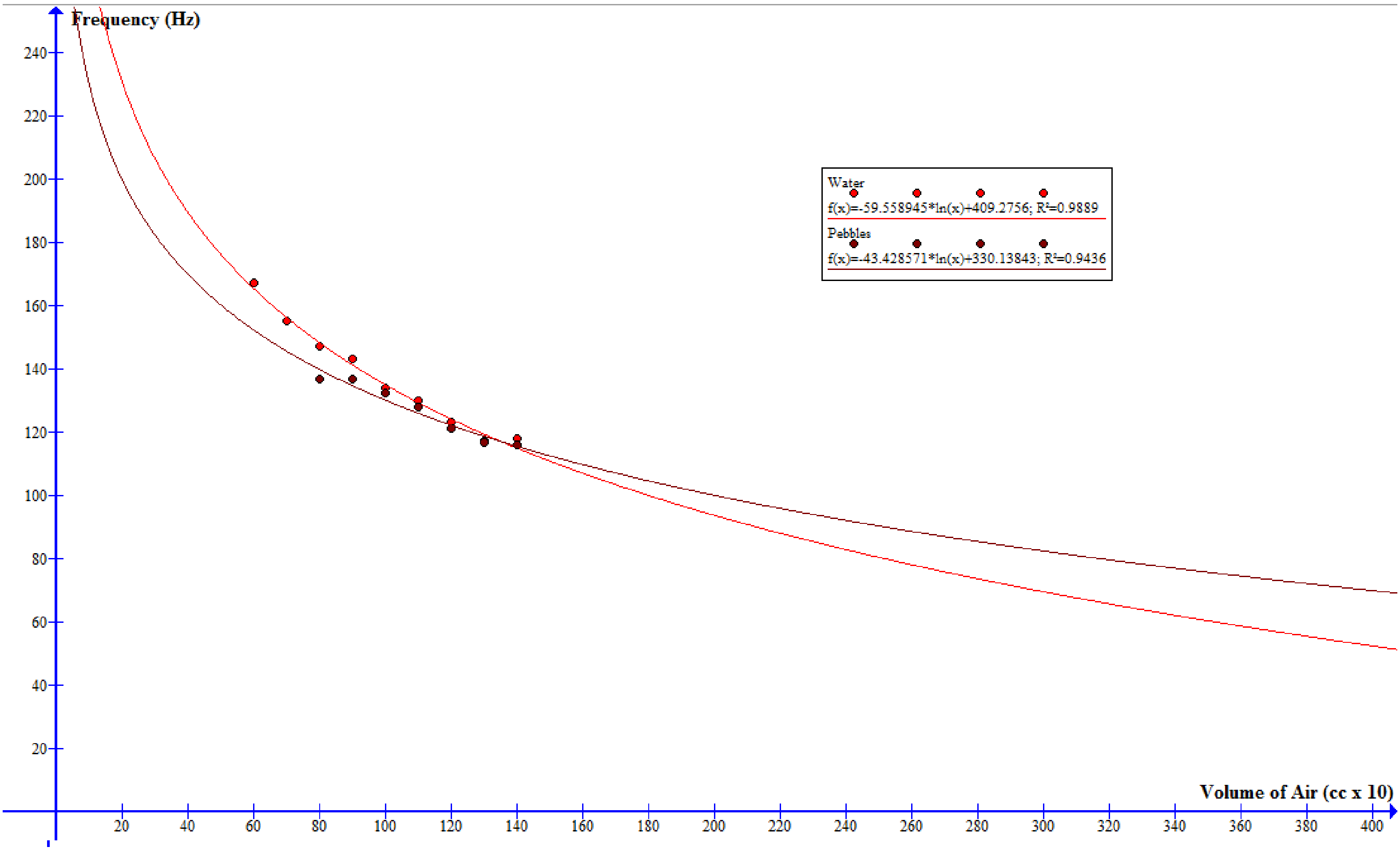
Volume of Pebbles (volume of air) [3]	Frequency	Error (frequency)	Frequency Squared	Error (Frequency squared)
0.75 dl. (0.30 l.) ± 0.03 l.	188 Hz	± 2 Hz	35344 Hz^2	± 752 Hz^2
1.25 dl. (0.25 l.) ± 0.04 l.	208 Hz	± 3 Hz	43264 Hz^2	± 1248 Hz^2
1.75 dl. (0.20 l.) ± 0.05 l.	Full Bottle	Full Bottle	Full Bottle	Full Bottle
2.25 dl. (0.15 l.) ± 0.06 l.	Full Bottle	Full Bottle	Full Bottle	Full Bottle

[3] – Bottle 3

APPENDIX E:

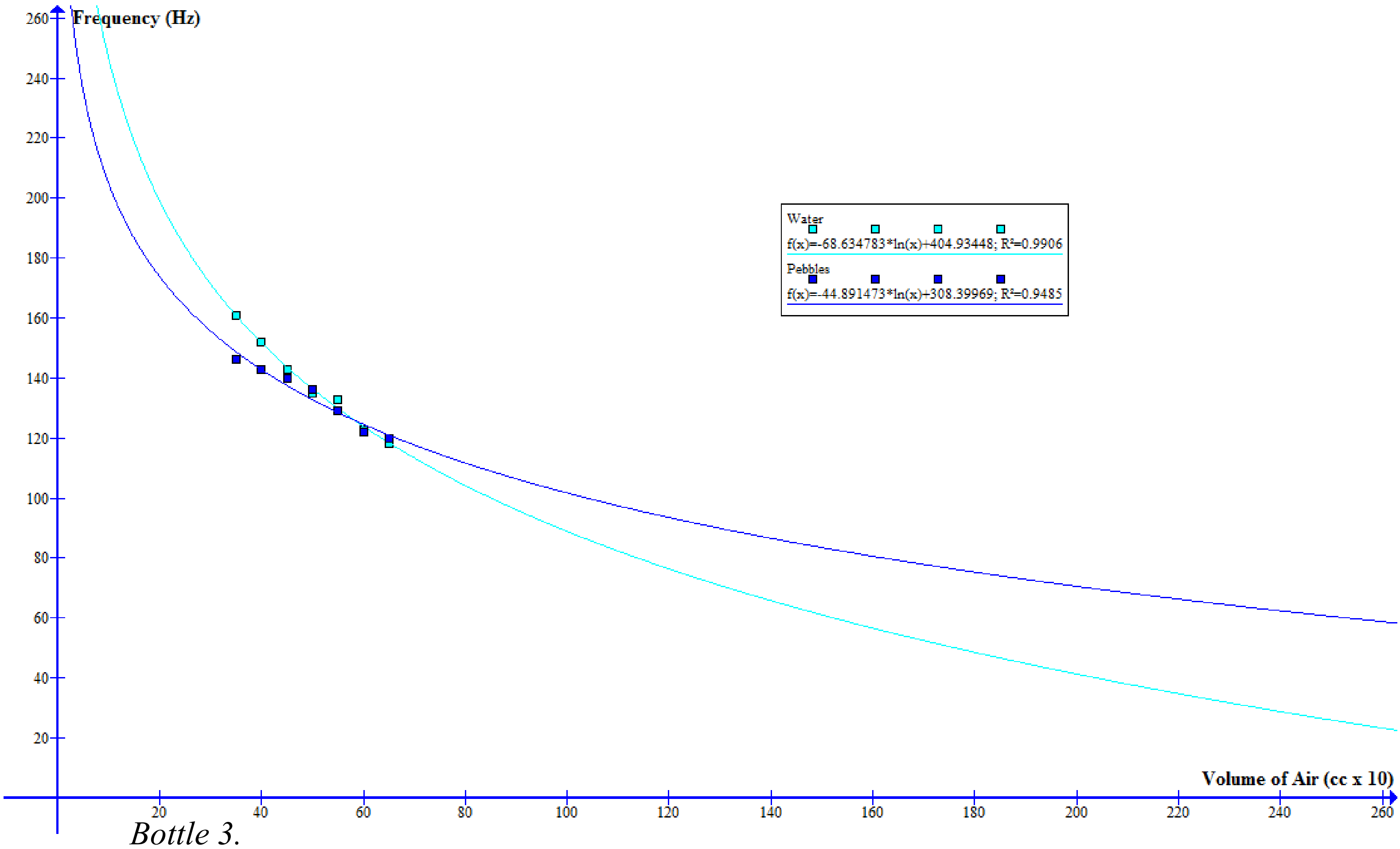
Logarithmic graphs where the lighter lines represents the chamber being filled with water and the darker lines representing the chamber being filled with pebbles.

Bottle 1.

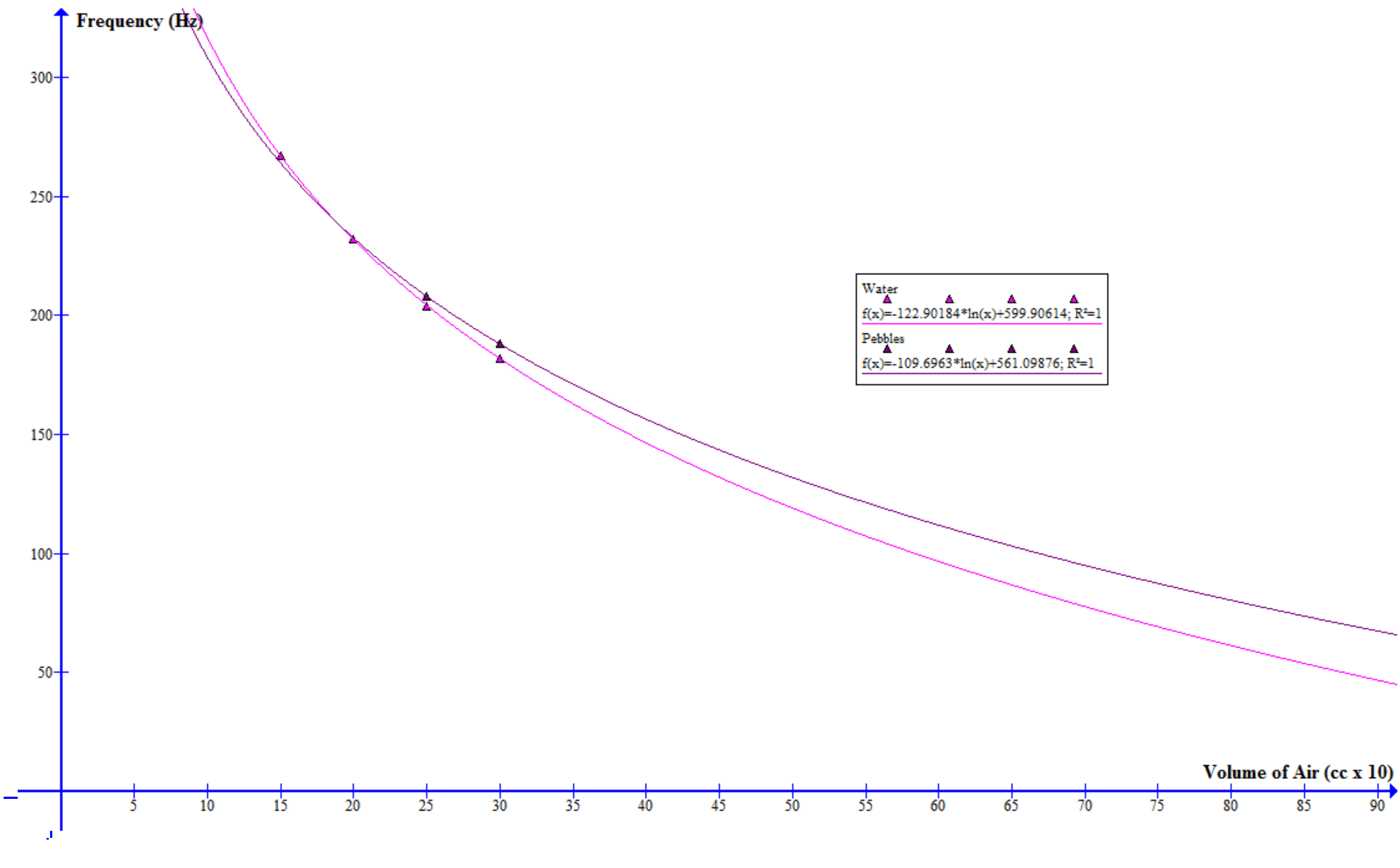


In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?

Bottle 2.



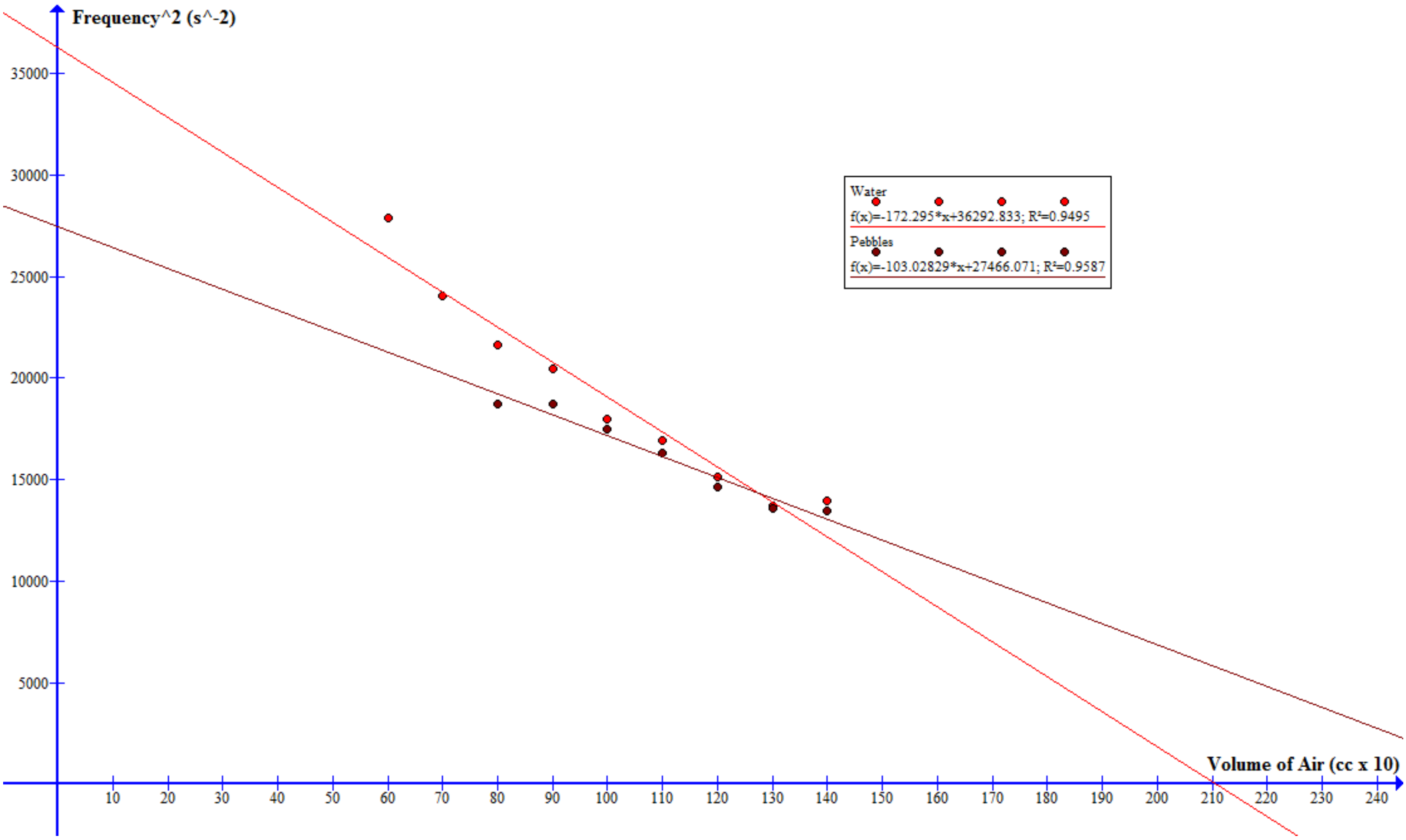
Bottle 3.



APPENDIX F:

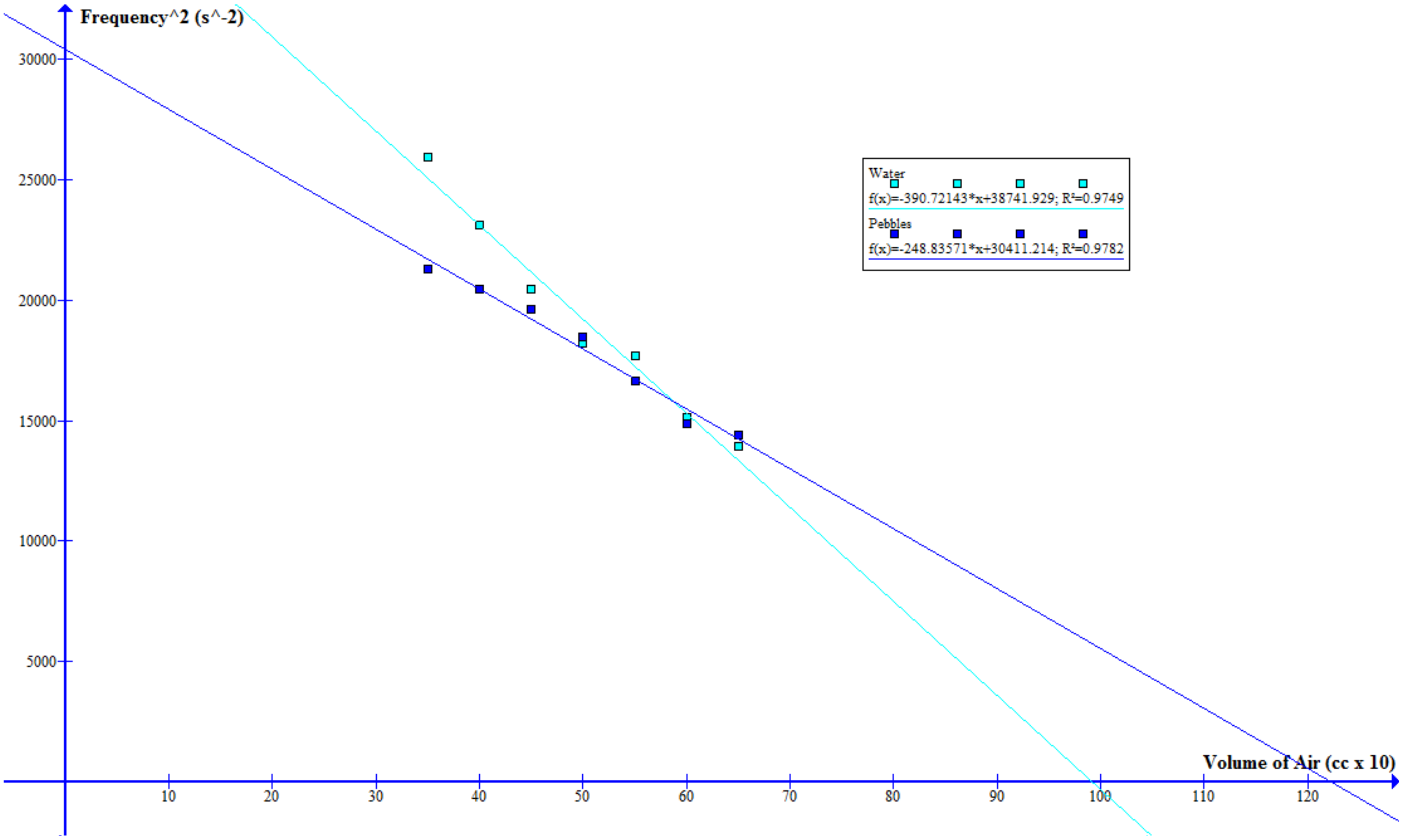
Linear graphs where the lighter lines represents the chamber being filled with water and the darker lines representing the chamber being filled with pebbles.

Bottle 1.



In what way does the introduction of interferences to the volume of a Helmholtz resonator affect the frequency resonated?

Bottle 2.



Bottle 3.