$$(Q\pm iU)_i(\hat n) \equiv T^i(\hat n)\, \underbrace{\sin^2 x(\hat n)\, e^{\mp 2i\beta(\hat n)}}_{\gamma_\mp^i(\hat n)}$$

$$\langle (Q\pm iU)(\hat n_1)\,(Q\pm iU)(\hat n_2)\, g(\hat n_3)\rangle$$

$$= \Big\langle \sum_{i,j} T^i(\hat n_1 - \hat n_i)\,\gamma_\mp^i(\hat n_1 - \hat n_i)\, T^i(\hat n_2 - \hat n_i)\,\gamma_\mp^i(\hat n_2 - \hat n_i)\, g^j(\hat n_3 - \hat n_j)\Big\rangle$$

$$= \Big\langle \sum_{i,j} \int dM_a\, dM_b \int d^2\hat n_a\, d^2\hat n_b\, \delta(M_a - M_i)\,\delta(M_b - M_j)$$

$$\delta^{(2)}(\hat n_i - \hat n_a)\,\delta^{(2)}(\hat n_j - \hat n_b)\, T(\hat n_1 - \hat n_a | M_a)\,\gamma_\mp^{\xi}(\hat n_1 - \hat n_a)$$

$$T(\hat n_2 - \hat n_a | M_a)\,\gamma_\mp(\hat n_2 - \hat n_a)\, g(\hat n_3 - \hat n_b | M_b)\Big\rangle$$

$$= \int dM_a\, dM_b \int d^2\hat n_a\, d^2\hat n_b\, n(M_a)\, n(M_b)\, \xi_{hh}(M_a, M_b | \hat n_a - \hat n_b)$$

$$T(\hat n_1 - \hat n_a | M_a)\, T(\hat n_2 - \hat n_a | M_a)\, \underbrace{\langle \gamma_\mp(\hat n_1 - \hat n_a)\,\gamma_\mp(\hat n_2 - \hat n_a)\rangle}_{\xi_\mp(\hat n_1 - \hat n_2)}$$

$$g(\hat n_3 - \hat n_b | M_b)$$

$$\xi_{hh}(M_a, M_b | \hat n_a - \hat n_b) \simeq b(M_a)\, b(M_b)\, \xi^{lin}(\hat n_1 - \hat n_3)$$

$$= \Big[\int dM_a\, dM_b \int d^2\hat n_a\, d^2\hat n_b\; b(M_a)\, n(M_a)\, b(M_b)\, n(M_b)$$

$$T(\hat n_1 - \hat n_a | M_a)\, T(\hat n_2 - \hat n_a | M_a)\, g(\hat n_3 - \hat n_b | M_b)\Big]$$

$$\times\, \xi_\mp(\hat n_1 - \hat n_2)\, \xi^{lin}(\hat n_1 - \hat n_3)$$

$$g = N(M_b)\,\delta(\hat{n}_3 - \hat{n}_b)$$

$$= \left[\int dM_a\, dM_b \int d^2\hat{n}_a\, b(M_a)\, n(M_a)\, b(M_b)\, n(M_b)\, N(M_b)\right.$$

$$\left. T(\hat{n}_1 - \hat{n}_a | M_a)\, T(\hat{n}_2 - \hat{n}_a | M_a)\right] \xi_{\mp}(\hat{n}_1 - \hat{n}_2)\, \xi^{lin}(\hat{n}_1 - \hat{n}_3)$$

$$= \left[\int dM_b\, b(M_b)\, n(M_b)\, N(M_b)\right]$$ ← const. ~ galaxy bias × number counts

weighted pol. autospectrum ↙

$$\times \left\{\left[\int dM_a\, d^2\hat{n}_a\, b(M_a)\, n(M_a)\, T(\hat{n}_1 - \hat{n}_a | M_a)\, T(\hat{n}_2 - \hat{n}_a | M_a)\right] \xi_{\mp}(\hat{n}_1 - \hat{n}_2)\right.$$

$$\times\, \xi^{lin}(\hat{n}_1 - \hat{n}_3)$$

Compare w/ $\langle (Q \pm iU)(\hat{n}_1)(Q \pm iU)(\hat{n}_2)\rangle$ which

has only 1-halo term:

$$\langle (Q \pm iU)(\hat{n}_1)(Q \pm iU)(\hat{n}_2)\rangle = \sum_i \langle T^i(\hat{n}_1)\, \gamma^{\hat{n}_i}_{\mp}(\hat{n}_1)\, T^i(\hat{n}_2)\, \gamma^{\hat{n}_i}_{\mp}(\hat{n}_2)\rangle$$

$$= \Big\langle \sum_i \int dM_a \int d^2\hat{n}_a\, \delta(M_i - M_a)\, \delta^{(2)}(\hat{n}_i - \hat{n}_a)$$

$$T(\hat{n}_1 - \hat{n}_a | M_a)\, T(\hat{n}_2 - \hat{n}_a | M_a)$$

$$\gamma_{\mp}(\hat{n}_1 - \hat{n}_a)\, \gamma_{\mp}(\hat{n}_2 - \hat{n}_a) \Big\rangle$$

$$= \int dM_a \int d^2\hat{n}_a\, n(M_a)\, T(\hat{n}_1 - \hat{n}_a | M_a)\, T(\hat{n}_2 - \hat{n}_a | M_a)$$

$$\xi_{\mp}(\hat{n}_1 - \hat{n}_2)$$

So we can identify the second line of the bispectrum calculation as a weighted polarization auto-spectrum.

Schematically:

weighted as above

$$\langle B(\hat{n}_1) B(\hat{n}_2) g(\hat{n}_3) \rangle = C \langle B(\hat{n}_1) B(\hat{n}_2) \rangle \xi^{MM}(\hat{n}_1 - \hat{n}_3)$$

$$= C \, \xi^{BB}(\hat{n}_1 - \hat{n}_2) \, \xi^{MM}(\hat{n}_1 - \hat{n}_3)$$

$$\xi \sum_{\substack{\ell_1 m_1 \\ \ell_2 m_2 \\ \ell_3 m_3}} \langle B_{\ell_1 m_1} B^*_{\ell_2 m_2} g_{\ell_3 m_3} \rangle \, Y_{\ell_1 m_1}(\hat{n}_1) \, Y^*_{\ell_2 m_2}(\hat{n}_2) \, Y_{\ell_3 m_3}(\hat{n}_3)$$

$$= C \sum_{\ell m} C_\ell^{BB} Y_{\ell m}(\hat{n}_1) Y_{\ell m}(\hat{n}_2) \sum_{\ell' m'} C_{\ell'}^{MM} Y_{\ell' m'}(\hat{n}_1) Y_{\ell' m'}(\hat{n}_3)$$

$$= C \sum_{\substack{\ell m \\ \ell' m'}} C_\ell^{BB} C_{\ell'}^{MM} Y_{\ell m}(\hat{n}_2) Y_{\ell' m'}(\hat{n}_3) \, Y_{\ell m}(\hat{n}_1) Y_{\ell' m'}(\hat{n}_1)$$

$$= C \sum_{\substack{\ell m \\ \ell' m'}} C_\ell^{BB} C_{\ell'}^{MM} \sum_{\ell'' m''} \sqrt{\frac{(2\ell+1)(2\ell'+1)(2\ell''+1)}{4\pi}} \begin{pmatrix} \ell & \ell' & \ell'' \\ m & m' & m'' \end{pmatrix} \begin{pmatrix} \ell & \ell' & \ell'' \\ 0 & 0 & 0 \end{pmatrix} Y_{\ell'' m''}(\hat{n}_1) Y_{\ell m}(\hat{n}_2) Y_{\ell' m'}(\hat{n}\ldots$$

Angular bispectrum