must be in the same halo labelled i or else gives 0

dominant contribution will be the 2-halo term $\propto P_k$

$$\langle (Q \pm iU)(\hat{n}_1)\ (Q \pm iU)(\hat{n}_2)\ g(\hat{n}_3) \rangle \equiv$$

$$\equiv \langle \sum_{ij} \tau^i(\hat{n}_1 - \hat{n}_i)\ \gamma_\mp(\hat{n}_1 - \hat{n}_i)\ \tau^i(\hat{n}_2 - \hat{n}_i)\ \gamma_\mp(\hat{n}_2 - \hat{n}_i)\ g^j(\hat{n}_3 - \hat{n}_j) \rangle$$

where here $(Q \pm iU)(\hat{n}) \propto \sin^2\theta\,(\cos 2\varphi \mp i \sin 2\varphi) \equiv \gamma_\mp(\hat{n})$

$$\equiv \langle \sum_{ij} \int dm_a\, d^2\hat{n}_a \int dm_b\, d^2\hat{n}_b\ \delta(m_a - m_i)\ \delta(m_b - m_j)\ \delta^2(\hat{n}_a - \hat{n}_i)\ \delta^2(\hat{n}_b - \hat{n}_j)\ \tau(\hat{n}_1 - \hat{n}_a)\ \tau(\hat{n}_2 - \hat{n}_a)$$

$$\gamma_\mp(\hat{n}_1 - \hat{n}_a)\ \gamma_\mp(\hat{n}_2 - \hat{n}_a)\ g(\hat{n}_3 - \hat{n}_b) \rangle$$

$$= \int dm_a \int dm_b \int d^2\hat{n}_a \int d^2\hat{n}_b\ n(m_a)\, n(m_b)\ \xi_{hh}(m_a, m_b)\ \tau(\hat{n}_1 - \hat{n}_a)\ \tau(\hat{n}_2 - \hat{n}_a)\ \gamma_\mp(\hat{n}_1 - \hat{n}_a)\ \gamma_\mp(\hat{n}_2 - \hat{n}_a)\ g(\hat{n}_3 - \hat{n}_b)$$

$$= \left[\prod_x^{ab} \int dm_x \int d^2\hat{n}_x\, n(m_x)\right] \xi^{hh}(m_a, m_b \,|\, \hat{n}_a - \hat{n}_b)\ \xi^{Q\mp iU}(\hat{n}_1 - \hat{n}_2 \,|\, \hat{n}_a)\ g(\hat{n}_3 - \hat{n}_b).$$

Now we expand into spherical harmonics

(i) $$g(\hat{n}_3 - \hat{n}_b) = \sum_{\ell m} g_{\ell m}\, Y_{\ell m}(\hat{n}_3 - \hat{n}_b) = \sum_{\ell m M} g_{\ell m}\, \mathcal{D}^{\ell}_{Mm}(-\hat{n}_b)\, Y_{\ell m}(\hat{n}_3)$$

(ii) $$\xi^{hh}(m_a, m_b \,|\, \hat{n}_a - \hat{n}_b) = \sum_{\ell m} \sum_{\ell' m'} C_\ell^{hh}(m_a, m_b \,|\, \chi_a, \chi_b)\ Y^*_{\ell m}(\hat{n}_a)\ Y_{\ell' m'}(\hat{n}_b)\ \delta_{\ell\ell'}\, \delta_{mm'}$$

$$C_\ell^{hh}(m_a, m_b) = \frac{2}{\pi}\, b(m_a)\, b(m_b) \int dk\, k^2\, j_\ell(k\chi_1)\, j_\ell(k\chi_2)\, P^{lin}(k, \chi_1, \chi_2)$$

(iii) $$\xi^{Q\pm iU}(\hat{n}_1 - \hat{n}_2 \,|\, \hat{n}_a) = \sum_{\ell m} \sum_{\ell' m'} C_\ell^{Q\pm iU}\, \delta_{\ell\ell'}\, \delta_{mm'}\ {}_{\pm 2}Y^*_{\ell m}(\hat{n}_1 - \hat{n}_a)\ {}_{\pm 2}Y_{\ell' m'}(\hat{n}_2 - \hat{n}_a).$$

below I forgot the ± 2

$$= \int dm_a\, n_a \int dm_b\, n_b \int d^2\hat{n}_a \int d^2\hat{n}_b \left(\sum_{\ell''' m'''} \sum_{\ell^4 m^4} C^{hh}_{\ell'''}(m_a, m_b, \chi_a, \chi_b)\ \delta_{\ell'''\ell^4}\ \delta_{m'''m^4}\ Y^*_{\ell''' m'''}(\hat{n}_a)\ Y_{\ell^4 m^4}(\hat{n}_b) \right) \times$$

$$\left(\sum_{\ell'' m'' M''} g_{\ell'' m''}\, \mathcal{D}^{\ell''}_{M''m''}(-\hat{n}_b)\, Y_{\ell'' M''}(\hat{n}_3). \right) \times \left[\sum_{\ell m} \sum_{\ell' m'} \delta_{\ell\ell'}\, \delta_{mm'}\ Y^*_{\ell m}(\hat{n}_1 - \hat{n}_a)\ Y_{\ell' m'}(\hat{n}_2 - \hat{n}_a)\ C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b) \right]$$

$$= \int dm_a\, n_a \int dm_b\, n_b \sum_{\substack{\ell\ell'\ell'' \\ \ell'''\ell^4}} \sum_{\substack{m m' m'' \\ m''' m^4}} g_{\ell'' m''}\ C^{hh}_{\ell'''}(m_a, m_b, \chi_a, \chi_b)\ C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b)\ \delta_{\ell\ell'}\, \delta_{mm'}\, \delta_{\ell'''\ell^4}\, \delta_{m'''m^4}$$

$$\int d^2\hat{n}_a\ \underbrace{Y^*_{\ell m}(\hat{n}_1 - \hat{n}_a)\ Y_{\ell' m'}(\hat{n}_2 - \hat{n}_a)}_{*}\ Y^*_{\ell''' m'''}(\hat{n}_a) \int d^2\hat{n}_b\ \underbrace{\sum_{M''} \mathcal{D}^{\ell''}_{M''m''}(-\hat{n}_b)\, Y_{\ell'' M''}(\hat{n}_3)\ Y_{\ell^4 m^4}(\hat{n}_b)}_{**}$$

write rotation $f(\mathcal{D})$

$$* = \sum_{MM'} \mathcal{D}^{\ell *}_{Mm}(-\hat{n}_a)\ Y^*_{\ell M}(\hat{n}_1)\ \mathcal{D}^{\ell'}_{M'm'}(-\hat{n}_a)\ Y_{\ell' M'}(\hat{n}_2)$$

$$** = \int d^2\hat{n}_b \sum_{M''} \mathcal{D}^{\ell''}_{M''m''}(-\hat{n}_b)\ Y_{\ell'' M''}(\hat{n}_3)\ Y_{\ell^4 m^4}(\hat{n}_b)$$

$$= \int d^2\hat{n}_b \sum_{M''M^4} \mathcal{D}^{\ell''}_{M''m''}(-\hat{n}_b)\, Y_{\ell''M''}(\hat{n}_3)\, \mathcal{D}^{\ell^4}_{M^4m^4}(\hat{n}_b)\, Y_{\ell^4M^4}(0)$$

rotate through $\mathcal{D}$

$$(-1)^{M^4-m^4}\, \mathcal{D}^{\ell^4 *}_{-M^4-m^4}(\hat{n}_b)$$

$$\int d^2\Omega\, \mathcal{D}^{\ell''}_{M''m''}(\Omega)\, \mathcal{D}^{*\,\ell^4 *}_{-M^4-m^4}(\Omega) = \frac{8\pi^2}{2\ell''+1}\, \delta_{M'',-M^4}\, \delta_{\ell''\ell^4}\, \delta_{m'',-m^4}$$

$$= \sum_{M''M^4} Y_{\ell''M''}(\hat{n}_3)\, \overset{=\sqrt{\frac{2\ell^4+1}{4\pi}}}{Y_{\ell^4M^4}(0)}\, \frac{8\pi^2}{2\ell''+1}\, (-1)^{M^4-m^4}\, \delta_{M'',-M^4}\, \delta_{\ell''\ell^4}\, \delta_{m'',-m^4}$$

$$= \sum_{M''} \sqrt{\frac{2\ell''+1}{4\pi}}\, \frac{8\pi^2}{2\ell''+1}\, Y_{\ell''M''}(\hat{n}_3)\, (-1)^{-M''+m''}$$

$$= 2\pi \sum_{M''} \sqrt{\frac{4\pi}{2\ell''+1}}\, Y_{\ell''M''}(\hat{n}_3)\, (-1)^{-M''+m''}$$

$$= \int dm_a\, n_a \int dm_b\, n_b \sum_{\substack{\ell\ell'\ell''\\ \ell'''\ell^4}} \sum_{\substack{mm'm''\\ m'''m^4}} g_{\ell''m''}\, \mathcal{C}^{hh}_{\ell''}(m_a,m_b,\chi_a,\chi_b)\, \mathcal{C}^{\mathcal{Q}+i\mathcal{U}}_{\ell'}(m_a,m_b,\chi_a,\chi_b)\, \overbrace{\delta_{\ell\ell'}\, \delta_{mm'}\, \delta_{\ell'''\ell^4}\, \delta_{m'''m^4}\, \delta_{M'',-M^4}\, \delta_{\ell''\ell^4}\, \delta_{m'',-m^4}}^{\text{collect all}}$$

$$\sum_{MM'M''} Y^*_{\ell M}(\hat{n}_1)\, Y_{\ell'M'}(\hat{n}_2)\, Y_{\ell''M''}(\hat{n}_3) \int d^2\hat{n}_a\, Y^*_{\ell'''m'''}(\hat{n}_a)\, \mathcal{D}^{\ell *}_{Mm}(-\hat{n}_a)\, \mathcal{D}^{\ell'}_{M'm'}(-\hat{n}_a)\; 2\pi\, (-1)^{-M''+m''} \sqrt{\frac{4\pi}{2\ell''+1}}$$

$$= \int dm_a\, n_a \int dm_b\, n_b \sum_{\ell\ell'\ell''} \sum_{MM'M''} \sum_{m''} g_{\ell''m''}\, \mathcal{C}^{hh}_{\ell''}(m_a,m_b,\chi_a,\chi_b)\, \mathcal{C}^{\mathcal{Q}+i\mathcal{U}}_{\ell'}(m_a,m_b,\chi_a,\chi_b)\; Y^*_{\ell M}(\hat{n}_1)\, Y_{\ell'M'}(\hat{n}_2)\, Y_{\ell''M''}(\hat{n}_3)\; 2\pi \sqrt{\frac{4\pi}{2\ell''+1}}$$

$$\times \left[\sum_{\ell'''m'''} \sum_{mm'} \int d^2\hat{n}_a\, Y^*_{\ell'''m'''}(\hat{n}_a)\, \mathcal{D}^{\ell *}_{Mm}(-\hat{n}_a)\, \mathcal{D}^{\ell'}_{M'm'}(-\hat{n}_a)\, (-1)^{-M''+m''}\, \delta_{\ell\ell'}\, \delta_{mm'}\, \delta_{\ell'''\ell''}\, \delta_{m''',-m''} \right] \longleftarrow ✿$$

Notice $\displaystyle\sum_{mm'} \delta_{mm'}\, \delta_{\ell\ell'}\, \mathcal{D}^{\ell *}_{Mm}(-\hat{n}_a)\, \mathcal{D}^{\ell'}_{M'm'}(-\hat{n}_a) = \delta_{MM'}\, \delta_{\ell\ell'}$

$$\Rightarrow ✿ = \cancel{\sum_{\ell'''}\sum_{m'''}} \underbrace{\int d^2\hat{n}_a\, Y^*_{\ell'''m'''}(\hat{n}_a)}_{Y^*_{\ell''',-m''}(0) = \sqrt{\frac{2\ell+1}{4\pi}}}\, (-1)^{-M''+m''}\, \delta_{MM'}\, \delta_{\ell\ell'}\, \cancel{\delta_{\ell''\ell'''}}\, \cancel{\delta_{-m'',m'''}} = \sqrt{\frac{2\ell''+1}{4\pi}}\, (-1)^{-M''}\, (-1)^{m''}\, \delta_{MM'}\, \delta_{\ell\ell'}$$

$$= \int dm_a\, n_a \int dm_b\, n_b \sum_{\ell\ell'\ell''} \sum_{MM'M''} 2\pi \sum_{m''} (-1)^{m''} g_{\ell''m''}\, \mathcal{C}^{hh}_{\ell''}(m_a,m_b,\chi_a,\chi_b)\, \mathcal{C}^{\mathcal{Q}+i\mathcal{U}}_{\ell'}(m_a,m_b,\chi_a,\chi_b)\, Y_{\ell M}(\hat{n}_1)\, Y_{\ell'M'}(\hat{n}_2)\, Y_{\ell''M''}(\hat{n}_3)\, (-1)^{M-M''}\, \delta_{\ell\ell'}\, \delta_{MM'}.$$

$$= \sum_{\ell\ell'\ell''} \sum_{MM'M''} Y_{\ell M}(\hat{n}_1)\, Y_{\ell'M'}(\hat{n}_2)\, Y_{\ell''M''}(\hat{n}_3) \left[2\pi \int dm_a\, n_a \int dm_b\, n_b \sum_{m''} (-1)^{m''} g_{\ell''m''}\, \mathcal{C}^{hh}_{\ell''}(m_a,m_b,\chi_a,\chi_b)\, \mathcal{C}^{\mathcal{Q}+i\mathcal{U}}_{\ell'}(m_a,m_b,\chi_a,\chi_b)\, (-1)^{M-M''}\, \delta_{\ell\ell'}\, \delta_{MM'} \right]$$

above I forgot the ± 2

On the other hand if instead we let $g(\hat{n}_3 - \hat{n}_b) = N(m_b)\,\delta^2(\hat{n}_3 - \hat{n}_b)$ then:

(jumping back to )

$$= \int dm_a\, n_a \int dm_b\, n_b \int d^2\hat{n}_a \int d^2\hat{n}_b\; N(m_b)\,\delta^2(\hat{n}_3 - \hat{n}_b) \left(\sum_{\ell''\ell'''} \sum_{m''m'''} C^{hh}_{\ell'''}(m_a, m_b, \chi_a, \chi_b)\, \delta_{\ell'''\ell''}\, \delta_{m'''m''}\, Y^*_{\ell''m''}(\hat{n}_a)\, Y_{\ell'''m'''}(\hat{n}_b) \right)$$
$$\times \left[\sum_{\ell m} \sum_{\ell' m'} \delta_{\ell\ell'}\delta_{mm'}\; {}_{\pm 2}Y^*_{\ell m}(\hat{n}_1 - \hat{n}_a)\; {}_{\pm 2}Y_{\ell' m'}(\hat{n}_2 - \hat{n}_a)\; C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b) \right]$$

$$= \int dm_a\, n_a \int dm_b\, n_b\, N(m_b) \sum_{\ell\ell'\ell''\ell'''} \sum_{mm'm''m'''} \delta_{\ell\ell'}\delta_{mm'}\delta_{\ell'''\ell''}\delta_{m'''m''}\; C^{hh}_{\ell'''}(m_a, m_b, \chi_a, \chi_b)\; C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b)$$
$$\int d^2\hat{n}_a\; \underbrace{{}_{\pm 2}Y^*_{\ell m}(\hat{n}_1 - \hat{n}_a)\; {}_{\pm 2}Y_{\ell' m'}(\hat{n}_2 - \hat{n}_a)}_{*}\; Y^*_{\ell''m''}(\hat{n}_a)\; Y_{\ell'''m'''}(\hat{n}_3)$$

write rotation $f(\mathcal{D})$

$$* = \sum_{MM'} \mathcal{D}^{\ell *}_{Mm}(-\hat{n}_a)\; {}_{\pm 2}Y^*_{\ell M}(\hat{n}_1)\; \mathcal{D}^{\ell'}_{M'm'}(-\hat{n}_a)\; {}_{\pm 2}Y_{\ell' M'}(\hat{n}_2)$$

$$= \int dm_a\, n_a \int dm_b\, n_b\, N(m_b) \sum_{\ell\ell'\ell''} \sum_{MM'm''} C^{hh}_{\ell''}(m_a, m_b, \chi_a, \chi_b)\; C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b)\; {}_{\pm 2}Y^*_{\ell M}(\hat{n}_1)\; {}_{\pm 2}Y_{\ell' M'}(\hat{n}_2)\; Y_{\ell''m''}(\hat{n}_3)$$
$$\sum_{mm'} \int d^2\hat{n}_a\; \mathcal{D}^{\ell *}_{Mm}(-\hat{n}_a)\; \mathcal{D}^{\ell'}_{M'm'}(-\hat{n}_a)\; Y^*_{\ell''m''}(\hat{n}_a)\; \delta_{\ell\ell'}\delta_{mm'}$$

$$= \int dm_a\, n_a \int dm_b\, n_b\, N(m_b) \sum_{\ell\ell'\ell''} \sum_{MM'm''} C^{hh}_{\ell''}(m_a, m_b, \chi_a, \chi_b)\; C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b)\; {}_{\pm 2}Y^*_{\ell M}(\hat{n}_1)\; {}_{\pm 2}Y_{\ell' M'}(\hat{n}_2)\; Y_{\ell''m''}(\hat{n}_3)$$
$$\delta_{\ell\ell'}\delta_{MM'} \underbrace{\int d^2\hat{n}_a\; Y^*_{\ell''m''}(\hat{n}_a)}_{= Y^*_{\ell''m''}(0) \,=\, \sqrt{\frac{2\ell''+1}{4\pi}}}$$

$$= \sum_{\ell\ell'\ell''} \sum_{MM'm''} {}_{\pm 2}Y^*_{\ell M}(\hat{n}_1)\; {}_{\pm 2}Y_{\ell' M'}(\hat{n}_2)\; Y_{\ell''m''}(\hat{n}_3) \left[\int dm_a\, n_a \int dm_b\, n_b\, N(m_b)\; C^{hh}_{\ell''}(m_a, m_b, \chi_a, \chi_b)\; C^{Q\pm iU}_{\ell'}(m_a, m_b, \chi_a, \chi_b)\; \delta_{\ell\ell'}\delta_{MM'} \sqrt{\frac{2\ell''+1}{4\pi}} \right]$$

$$= \underbrace{\sum_{\ell\ell'\ell''} \sum_{mm'm''} {}_{\pm 2}Y^*_{\ell m}(\hat{n}_1)\; {}_{\pm 2}Y_{\ell' m'}(\hat{n}_2)\; Y_{\ell''m''}(\hat{n}_3)}\; B^{\ell\ell'\ell''}_{mm'm''}$$

$$\langle (Q \pm iU)^*(\hat{n}_1)\,(Q \pm iU)(\hat{n}_2)\; g(\hat{n}_3) \rangle =$$

$$= \left\langle \sum_{\ell m} \sum_{\ell' m'} \sum_{\ell'' m''} (e_{\ell m} \mp i s_{\ell m})^*\; {}_{\pm 2}Y^*_{\ell m}(\hat{n}_1)\; (e_{\ell' m'} \mp i s_{\ell' m'})\; {}_{\pm 2}Y_{\ell' m'}(\hat{n}_2)\; g_{\ell'' m''}\; Y_{\ell'' m''}(\hat{n}_3) \right\rangle$$

$$= \underbrace{\sum_{\ell\ell'\ell''} \sum_{mm'm''} {}_{\pm 2}Y^*_{\ell m}(\hat{n}_1)\; {}_{\pm 2}Y_{\ell' m'}(\hat{n}_2)\; Y_{\ell''m''}(\hat{n}_3)}\; \langle (e_{\ell m} \mp i s_{\ell m})^*\, (e_{\ell' m'} \mp i s_{\ell' m'})\; g_{\ell'' m''} \rangle .$$

so we just compare ↑ with $B^{\ell\ell'\ell''}_{mm'm''}$ and can do the same gymnastics as for C^{EE}_{ℓ}

$$\sum_{\ell m} E_{\ell m}\, {}_{+2}Y_{\ell m} = \sum_{\ell m} \sum_{\substack{\ell' m' \\ \ell'' m''}} \tau_{\ell'' m''} \left(E_{\ell' m'} + i B_{\ell' m'}\right) e_{\ell \ell' \ell''} (-1)^m \sqrt{\frac{(2\ell+1)(2\ell'+1)(2\ell''+1)}{4\pi}} \begin{pmatrix} \ell & \ell' & \ell'' \\ -m & m' & m'' \end{pmatrix} \begin{pmatrix} \ell & \ell' & \ell'' \\ -2 & 2 & 0 \end{pmatrix} {}_{+2}Y_{\ell m}$$

$$\Rightarrow \langle E^2 g \rangle \propto B^{\ell \ell' \ell''}_{m m' m''}(E^2 g) = \int dM_a\, n_a \int dm_b\, n_b\, N(m_b)\, C^{hh}_{\ell''}\, \delta_{\ell\ell'}\, \delta_{mm'} \sqrt{\frac{2\ell''+1}{4\pi}}$$

$$\times \sum_{\substack{\ell_1 \ell_2 \\ m_1 m_2}} \tau_{\ell_2 m_2} \left(E_{\ell_1 m_1} - i B_{\ell_1 m_1}\right) e_{\ell_1 \ell_2 \ell} (-1)^m \sqrt{\frac{(2\ell+1)(2\ell_1+1)(2\ell_2+1)}{4\pi}} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ -m & m_1 & m_2 \end{pmatrix} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 2 & 2 & 0 \end{pmatrix}$$

$$\times \sum_{\substack{L_1 L_2 \\ M_1 M_2}} \tau_{L_2 M_2} \left(E_{L_1 M_1} - i B_{L_1 M_1}\right) e_{L_1 L_2 \ell'} (-1)^{m'} \sqrt{\frac{(2\ell'+1)(2L_1+1)(2L_2+1)}{4\pi}} \begin{pmatrix} \ell' & L_1 & L_2 \\ -m' & M_1 & M_2 \end{pmatrix} \begin{pmatrix} \ell' & M_1 & M_2 \\ 2 & 2 & 0 \end{pmatrix}$$

$$= \int dM_a\, n_a \int dm_b\, n_b\, N(m_b)\, C^{hh}_{\ell'}\, \delta_{\ell\ell'}\, \delta_{mm'} \sqrt{\frac{2\ell''+1}{4\pi}}$$

$$\bar{\tau}^2 \sum_{\ell_1 \ell_2} C_{\ell_2}^{\tau\tau,\,1\,\mathrm{halo}}(m_a, m_b, z_a, z_b) \left(C^{EE}_{\ell_1} + C^{BB}_{\ell_1}\right) e_{\ell \ell_1 \ell_2} \left(W^{\ell\ \ell_1\ \ell_2}_{2\ \ 2\ \ 0}\right)^2$$

see pow spec derivation

$\bar{\tau}^2$ stands for $\left| \frac{1-e^{x}}{x} \frac{\bar{T}}{\sqrt{4\pi}} \right|^2$.

$e^{-\ell^2 \sigma_\ell^2(m,z)}$

$\sigma = \frac{L_{dm}}{a\chi}$

$\frac{r_{200}(m,z)}{N_{dm}}$

1, 2, 3, 10

and similarly for $\langle B^2 g \rangle$