# Study of Rutherford Scattering

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In this experiment, the angular distribution of $\alpha$-particle scattering from thin metal foils was measured in order to test Rutherford's scattering formula and to determine the nuclear charge number of aluminium. The space-corrected counting rates were fitted with the function $f(\theta) = A/\sin^4((\theta - B)/2)$, yielding $A = (7.25 \pm 2.67) \times 10^{-5}$ and $B = (-0.04 \pm 0.46)^\circ$. Noticeable deviations from the fit indicate the presence of systematic effects, mainly due to alignment and finite angular resolution. From the comparison of gold and aluminium scattering at $\theta = +15^\circ$ and $\theta = -15^\circ$, the nuclear charge number of aluminium was obtained as $Z_{\mathrm{Al}} = 13.53 \pm 0.65$ and $14.80 \pm 0.66$, in agreement with the accepted value within experimental uncertainty. This experiment verifies Rutherford's nuclear model of an atom.

## I. OBJECTIVE

1. To record the direct counting rate $N_d$ of $\alpha$ particles scattered by a gold foil as a function of the angle $\theta$.
2. To determine the corrected counting rates $N$ with respect to the scattering distribution in space.
3. To validate the "Rutherford's scattering formula"

## II. THEORY

When energetic $\alpha$-particles are incident on a thin metallic foil, most of them pass through with only small deflections, while a small fraction is scattered by large angles (see Fig. 1). Rutherford explained this phenomenon by assuming that the positive charge and almost the entire mass of the atom are concentrated in a very small, positively charged nucleus. The scattering of $\alpha$-particles is then described as Coulomb scattering in the electric field of the nucleus [1].

Based on this model, Rutherford derived the angular distribution of the scattering rate $N(\theta)$, defined as the number of particles scattered per unit time into a small angular interval around the scattering angle $\theta$. The result is the Rutherford scattering formula:

$$N(\theta) = N_0\, c_F\, d_F\, \frac{Z^2 e^4}{(8\pi\varepsilon_0 E_\alpha)^2}\, \frac{1}{\sin^4(\theta/2)}, \qquad (1)$$

where

- $N_0$ is the incident particle rate,
- $c_F$ is the atomic concentration of the foil,
- $d_F$ is the thickness of the foil,
- $Z$ is the nuclear charge number of the scattering material,
- $E_\alpha$ is the energy of the $\alpha$-particles,
- $e$ is the elementary charge,
- $\varepsilon_0$ is the vacuum permittivity.

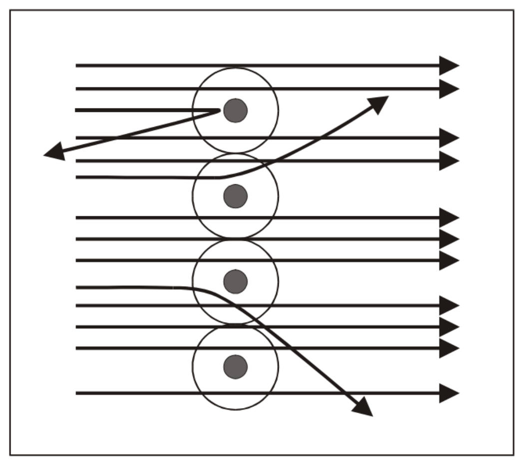

FIG. 1. Schematic illustration of $\alpha$-particle scattering from a thin atomic layer in the Rutherford experiment.

In the experiment, all factors in front of the angular dependence are constant. Therefore, the relevant shape of the angular distribution is given by

$$f(\theta) = \frac{1}{\sin^4(\theta/2)}. \qquad (2)$$

Since the scattering rate decreases rapidly with increasing angle, the data are usually represented on a logarithmic scale. The function has a singularity at $\theta = 0^\circ$, so measurements are only compared with theory for $|\theta| \gtrsim 5^\circ$.

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### A. Measured counting rate and geometrical correction

Experimentally, the number of counts $n(\theta)$ is recorded during a measuring time $t(\theta)$. From this, the direct counting rate is obtained as

$$N_d(\theta) = \frac{n_m(\theta)}{t(\theta)}, \tag{3}$$

where $n_m(\theta)$ is the mean value of repeated measurements.

Because the detector measures scattering in a plane, while the theoretical formula refers to a three-dimensional angular distribution, a geometrical correction is necessary. A plane angle $\theta$ corresponds in space to a cone with solid angle

$$d\Omega = 2\pi \sin\theta \, d\theta. \tag{4}$$

This leads to the relation between the measured plane counting rate $N_d(\theta)$ and the spatial scattering rate $N(\theta)$:

$$N(\theta) = 2\pi \sin\theta \, N_d(\theta). \tag{5}$$

### B. Fitting function

In practice, small misalignments of the setup or a non-centric position of the source can lead to a slight horizontal shift of the angular distribution. Therefore, the data are fitted with the function

$$f(\theta) = \frac{A}{\sin^4\left(\frac{\theta - B}{2}\right)}, \tag{6}$$

where $A$ is a proportionality constant accounting for the overall scaling of the counting rate, and $B$ represents a small shift of the angular scale.

By fitting this function separately to the positive and negative angle ranges, possible asymmetries and alignment errors of the experimental setup can be observed.

### C. Determination of the nuclear charge number

If scattering rates are measured for two different foil materials (e.g. Gold and Aluminium) at the same angle $\theta$, the ratio of the scattering rates follows from Eq. (1) as

$$\frac{N_{\text{Au}}}{N_{\text{Al}}} = \frac{Z_{\text{Au}}^2 \, d_{\text{Au}} \, c_{\text{Au}}}{Z_{\text{Al}}^2 \, d_{\text{Al}} \, c_{\text{Al}}}. \tag{7}$$

Solving for the nuclear charge number of Aluminium yields

$$Z_{\text{Al}} = Z_{\text{Au}} \sqrt{\frac{N_{\text{Al}} \, d_{\text{Au}} \, c_{\text{Au}}}{N_{\text{Au}} \, d_{\text{Al}} \, c_{\text{Al}}}}. \tag{8}$$

where $d_{Au} = 2\mu m$ and $d_{Al} = 8\mu m$ are the thickness of the metal foils and $c_{Al} \approx c_{Au}$.

## III. APPARATUS

1. Rutherford scattering chamber operated under vacuum (see Fig. 2).
2. $\alpha$-particle source (Am-241) mounted on a swivel arm.
3. Collimator slit and thin metal foil (gold or aluminium) placed in the beam path (see Fig. 3).
4. Silicon semiconductor detector fixed at the chamber wall to detect scattered $\alpha$-particles.
5. Discriminator preamplifier and counter for pulse processing and counting.
6. Vacuum pump and tubing to evacuate the chamber and reduce energy loss in air.

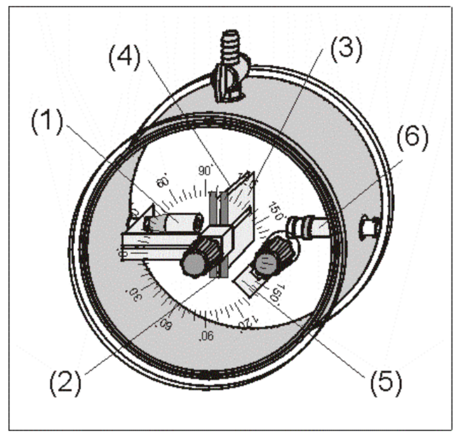


FIG. 2. The scattering chamber: (1) Preparation (2) Holder (3) Gold foil (4) Slit (5) swivel arm (6) detector

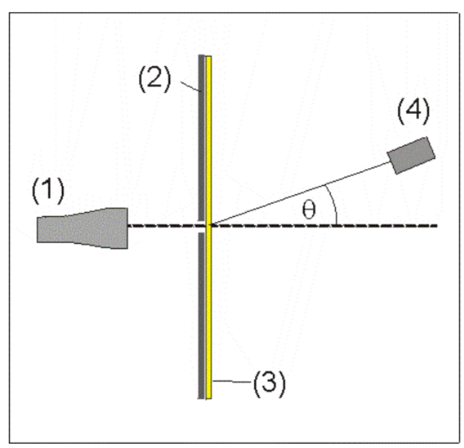


FIG. 3. The scattering geometry: (1) preparation (2) collimator slit (3) gold foil (4) detector

## IV. OBSERVATIONS

TABLE I. Measured data for Rutherford scattering (slit aperture = 5mm).

- $\theta$: scattering angle in degrees
- $t$: gate time in seconds
- $n_m$: mean pulse counts (refer to the attached data sheet for the single readings)
- $N_d$: direct counting rate in $s^{-1}$
- $N$: space-corrected counting rate in $s^{-1}$ (refer to Fig. 4 to see the distrubtion)

| $\theta$ (°) | $t$ (s) | $n_m$ | $N_d$ ($s^{-1}$) | $N$ ($s^{-1}$) |
|---|---|---|---|---|
| -30 | 900 | 141 | 0.157 | 0.493 |
| -25 | 600 | 202 | 0.337 | 0.894 |
| -20 | 200 | 260.3 | 1.3015 | 2.796 |
| -15 | 100 | 717 | 7.17 | 11.659 |
| -10 | 100 | 2234.3 | 22.343 | 24.377 |
| -5 | 100 | 3504.5 | 35.095 | 19.989 |
| +5 | 100 | 3285 | 32.85 | 17.989 |
| +10 | 100 | 1879.3 | 18.793 | 20.504 |
| +15 | 100 | 416 | 4.16 | 6.765 |
| +20 | 200 | 164.5 | 0.8225 | 1.767 |
| +25 | 600 | 125 | 0.208 | 0.552 |
| +30 | 900 | 96 | 0.107 | 0.336 |

TABLE II. Measured data for Rutherford scattering.

- Target: scattering foil material (slit aperture = 1mm)
- $\theta$: scattering angle in degrees
- $t$: gate time in seconds
- $n_m$: mean pulse counts (refer to the attached data sheet for the single readings)
- $N_d$: direct counting rate in $s^{-1}$
- $N$: space-corrected counting rate in $s^{-1}$

| Target | $\theta$ (°) | $t$ (s) | $n_m$ | $N_d$ ($s^{-1}$) | $N$ ($s^{-1}$) |
|---|---|---|---|---|---|
| Au | -15 | 100 | 44.2 | 0.442 | 0.719 |
| Au | +15 | 100 | 26.2 | 0.262 | 0.426 |
| Al | -15 | 1000 | 62 | 0.062 | 0.101 |
| Al | +15 | 1000 | 31 | 0.031 | 0.050 |

## V. DATA ANALYSIS

### A. Angular distribution of counts

The space-corrected counting rates $N(\theta)$ were fitted with the function (see Eq. (6)) which represents the Rutherford angular dependence. From this function fitted using `scipy.optimize.curve_fit` (see Fig. 5), the parameters were obtained as:

$$A = (7.25 \pm 2.67) \times 10^{-5},$$
$$B = (-0.04 \pm 0.46)^{\circ}.$$

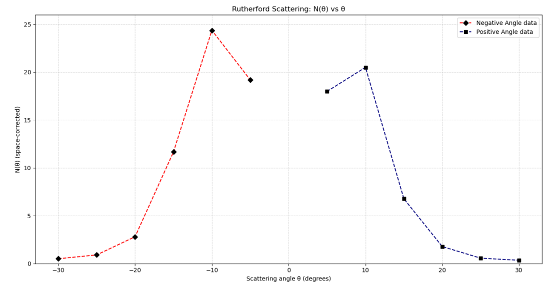


FIG. 4. Variation of space-corrected scattering rate $N(\theta)$ with respect to the scattering angle without fitting.

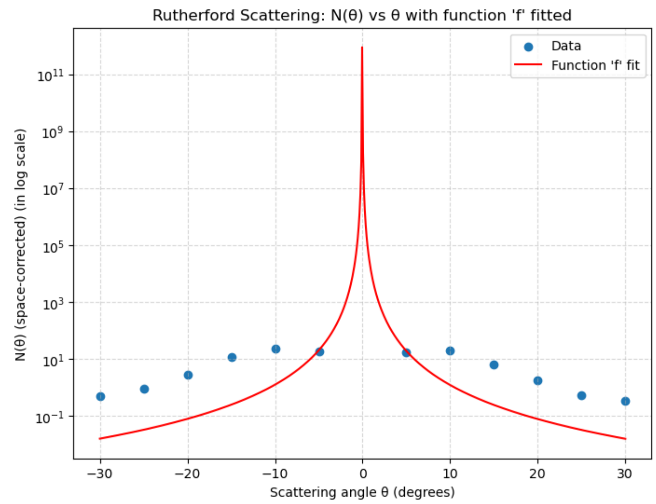


FIG. 5. Space-corrected scattering rate $N(\theta)$ as a function of scattering angle $\theta$ with the fit of a function $f(\theta) = A/\sin^4((\theta - B)/2)$

The small value of $B$ indicates that the symmetry axis of the distribution is very close to $\theta = 0^{\circ}$. There is a small misalignment which causes the counts to be asymmetric about $\theta = 0^{\circ}$. The relatively large uncertainty of $B$ reflects the sensitivity of the function near the singularity at small angles and the limited number of data points close to this region.

### B. Calculation of $Z_{Al}$ and Error Analysis

The nuclear charge number of aluminium was determined by comparing the scattering rates of gold and aluminium at the same scattering angle. From the Rutherford scattering formula, we obtain an expression for Atomic Number of Al (refer Eq. (8)).

Using the measured space-corrected counting rates at $\theta = +15^{\circ}$ and $\theta = -15^{\circ}$, the value of $Z_{\mathrm{Al}}$ was calculated separately for both angles. The uncertainty in the measurement is given by,

$$\delta Z_{Al} = \sqrt{(\frac{\delta Z_{Al}}{\delta N_{Al}} \times \Delta N_{Al})^2 + (\frac{\delta Z_{Al}}{\delta N_{Au}} \times \Delta N_{Au})^2}$$
$$\implies \delta Z_{Al} = \frac{Z_{Al}}{2} \times \sqrt{(\frac{\Delta N_{Al}}{N_{Al}})^2 + (\frac{\Delta N_{Au}}{N_{Au}})^2}$$

where $\Delta N$ are the uncertainty in counting rates [2]. These were estimated through Poisson statistics due to the limited number of observations. The formula used is given by:

$$\Delta N = \frac{\sqrt{\Sigma n_i}}{\tilde{N}.t}$$

where $\tilde{N}$ is the number of observations of counts at that angle, $\{n_i\}$ are the observed pulse counts and $t$ is the gate time.

The Atomic number calculated and the associated uncertainties for two different angles are:

For $\theta = +15^\circ$ :

$$Z = 13.53 \pm 0.65$$

For $\theta = -15^\circ$ :

$$Z = 14.80 \pm 0.66$$

All code related to the plot, fitting and calculations is available on this Github Repository [3].

## VI. RESULTS AND DISCUSSION

The measured space-corrected scattering rates $N(\theta)$ show the expected decrease with respect to the increasing scattering angle, consistent with the Rutherford dependence $\sin^{-4}(\theta/2)$. By fitting the observed data to the Eq. (6), we obtain the parameters,

$$A = (7.25 \pm 2.67) \times 10^{-5}$$
$$B = (-0.04 \pm 0.46)^\circ.$$

The small value of $B$ indicates that the symmetry axis is close to $\theta = 0^\circ$. The larger uncertainty in $B$ arises from the strong sensitivity of the function near small angles and limited data close to the singularity.

From the comparison of gold and aluminium scattering at $\theta = +15^\circ$ and $\theta = -15^\circ$, the nuclear charge number of aluminium was obtained respectively.

$$Z_{\mathrm{Al}}(+15^\circ) = 13.53 \pm 0.65$$
$$Z_{\mathrm{Al}}(-15^\circ) = 14.80 \pm 0.66.$$

The measurement at $\theta = +15^\circ$ consistent with the accepted value $Z_{\mathrm{Al}} = 13$. However, the measurement at $\theta = -15^\circ$ is slightly deviated from the literature value. This deviation can be attributed to the misalignment of the collimator and slit, leading to a displacement of the symmetry-axis from $\theta = 0^\circ$.

## VII. CONCLUSION

The angular dependence of $\alpha$-particle scattering was measured and shows the characteristic trend predicted by Rutherford's scattering formula. The fit of the data has noticeable deviations due to the presence of systematic experimental effects such as alignment errors and limited angular resolution. Despite these limitations, the nuclear charge number of aluminium determined from the scattering data is consistent with the accepted value within experimental uncertainty. This provides qualitative support for Rutherford's nuclear model of the atom.


[1] LD Didactic GmbH. *Rutherford Scattering: Measuring the Scattering Rate as a Function of the Scattering Angle and the Atomic Number.* LD Didactic GmbH, Huerth, Germany. Laboratory manual.

[2] John R. Taylor. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements.* University Science Books, Sausalito, CA, 2nd edition, 1997.

[3] Aryan Shrivastava. P341 - nuclear physics and instrumentation lab. `https://github.com/crimsonpane23/P341-Nuclear-Physics-and-Instrumentation-Lab.git`, 2026.