The Study of Lock-In Amplifier

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This experiment investigates the operation and applications of a lock-in amplifier through three parts: calibration, low resistance measurement, and mutual inductance determination. The amplifier was calibrated at different gain settings to obtain amplification factors of $\mu_{50} = 148.8 \pm 3.3$ and $\mu_{100} = 286 \pm 6$. Using these values, the resistance of an unknown low resistor was found to be $r = (10.5 \pm 0.5)\,\Omega$. From mutual inductance studies, the coupling between given coils was determined as $M = (157 \pm 5)\,\mu\mathrm{H}$. The results confirm the high sensitivity and linear response of the lock-in amplifier in detecting weak AC signals.

I. OBJECTIVE

- To study the calibration of the Lock-in Amplifier and determine its amplification factor.
- To verify the proportionality of induced emf with current and frequency using mutual inductance measurements.
- To determine the low resistance using Lock-In Amplifier.

II. THEORY

The Lock-in Amplifier (LIA) is a precision instrument used to detect and measure extremely weak AC signals with relatively high noise [1]. It achieves this by using the technique of *phase-sensitive detection* (PSD), which extracts the component of the signal that is phase-coherent with a known reference.

A. Principle of Operation

Suppose the input (signal) voltage and reference voltage are given by:

$$V_{\mathrm{sig}} = V_0 \sin(\omega t + \phi),$$
$$V_{\mathrm{ref}} = V_0 \sin(\omega t),$$

where ϕ is the phase difference between the signal and the reference.

The LIA multiplies these two signals inside a phase-sensitive detector (e.g. AD630 IC). The multiplication gives:

$$V_{\mathrm{out}} = V_{\mathrm{sig}} \times V_{\mathrm{ref}} = V_0^2 \sin(\omega t + \phi) \sin(\omega t).$$

$$V_{\mathrm{out}} = \frac{V_0^2}{2} [\cos\phi - \cos(2\omega t + \phi)]. \tag{1}$$

After passing through a low-pass filter (which removes the high-frequency term $\cos(2\omega t + \phi)$), the DC component of the output is:

$$V_{\mathrm{DC}} = \frac{V_0^2}{2} \cos\phi. \tag{2}$$

Thus, the DC output depends on the phase difference between the signal and the reference. When $\phi = 0$, V_{DC} is maximum, and when $\phi = 90^\circ$, it becomes zero. By adjusting the phase of the reference until the DC output is maximized, the LIA effectively "locks in" to the phase and frequency of the desired signal.

B. Mutual Inductance

When two coils are placed side by side, an alternating current in the primary coil induces an emf in the secondary coil. If the current in the primary coil is:

$$I = I_0 \sin(2\pi f t), \tag{3}$$

then the induced emf in the secondary is:

$$\begin{aligned} V &= -M \frac{dI}{dt} = -2\pi f M I_0 \cos(2\pi f t) \\ &= -2\pi f M I_0 \sin(2\pi f t + \frac{\pi}{2}), \end{aligned}$$

showing that the induced emf is 90° out of phase with the current in the primary coil and is directly proportional to both the current amplitude and frequency.

The Lock-in Amplifier detects this emf, and the DC output (V_{DC}) is proportional to the amplitude of the induced emf:

$$V_{\mathrm{DC}} = \frac{2\mu V_0}{\pi} \cos\phi, \tag{4}$$

where μ is the amplification factor. From the slope of V_{DC} versus applied AC voltage curves, the mutual inductance M can be determined using:

$$\beta = \frac{2\pi M \mu}{R}, \tag{5}$$

where R is the resistance of the primary circuit (Here, $R = 4.8k\Omega$).

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C. Low Resistance

The lock-in amplifier allows precise detection of small voltage signals across a low resistance in the presence of large background noise. When a small AC voltage $V_{\rm AC}$ is applied across a series combination of a known resistor R and an unknown low resistance r, the voltage across r is given by

$$V_r = V_{\rm AC}\frac{r}{R+r}.$$

For $r \ll R$, this simplifies to $V_r \approx V_{\rm AC}(r/R)$. The lock-in amplifier output is a DC voltage proportional to the in-phase component of V_r:

$$V_{\rm DC} = \mu V_r = \mu\, V_{\rm AC}\frac{r}{R}.$$

Thus, the unknown resistance can be determined from the slope of the $V_{\rm DC}$–$V_{\rm AC}$ plot as

$$r = \frac{R}{\mu}\frac{dV_{\rm DC}}{dV_{\rm AC}}, \tag{6}$$

where μ is the proportionality constant obtained from calibration, and $R = 4.8k\Omega$.

III. EXPERIMENTAL APPARATUS

A lock-in amplifier, function generator, and oscilloscope were used to perform all measurements. The setup allowed precise detection of in-phase signals even at low signal-to-noise ratios. For calibration, the signal and reference inputs of the lock-in amplifier were driven by the same oscillator through a voltage divider. The DC output $V_{\rm DC}$ was measured for varying $V_{\rm sig}$ to determine the proportionality constant μ.

To determine low resistance, a small known resistor and an unknown low resistance were connected in series with the AC source. The voltage across the low resistance was fed to the lock-in amplifier input, and $V_{\rm DC}$ was recorded versus $V_{\rm AC}$ to determine r.

For finding mutual inductance, two identical coils were used as primary and secondary. The primary was excited by the AC source, and the induced voltage in the secondary was measured by the lock-in amplifier to evaluate the mutual inductance M.

IV. OBSERVATIONS

A. Calibration of the Lock-in Amplifier

The calibration of the lock-in amplifier was performed by measuring the DC output voltage $V_{\rm DC}$ for various input signal voltages $V_{\rm sig}$ at different frequencies and for two gain settings (50 and 100). For an ideal linear response,

$$V_{\rm DC} = \mu\, V_{\rm sig},$$

where μ is the effective amplification factor of the system.

TABLE I. Observed $V_{\rm sig}$ and $V_{\rm DC}$ for different frequencies (Gain = 50).

Frequency (Hz)	$V_{\rm AC}(pp)$ (V)	$V_{\rm sig}(rms$ (V)	$V_{\rm DC}$ (V)
300.5	1.070	0.0011	0.112
	1.505	0.0015	0.186
	2.005	0.002	0.274
	2.500	0.0025	0.325
	3.000	0.003	0.407
610	3.050	0.003	0.413
	2.500	0.0025	0.321
	2.050	0.002	0.250
	1.515	0.0015	0.192
	1.000	0.001	0.104
905	1.000	0.001	0.103
	1.505	0.0015	0.187
	2.000	0.002	0.273
	2.500	0.0025	0.320
	3.050	0.003	0.409
1205	1.030	0.001	0.107
	1.500	0.0015	0.186
	2.015	0.002	0.275
	2.550	0.0025	0.326
	3.050	0.003	0.406
1510	1.030	0.001	0.108
	1.507	0.0015	0.186
	2.015	0.002	0.276
	2.500	0.0025	0.319
	3.050	0.003	0.416

B. Mutual Inductance Measurement

The induced voltage in the secondary coil was recorded for various applied $V_{\rm AC}$ values at different frequencies. The linear relation

$$V_{\rm DC} = s\, V_{\rm AC}$$

was used to determine the proportionality constant $s = dV_{\rm DC}/dV_{\rm AC}$.

C. Measurement of Low Resistance

A low resistance was connected in the signal arm, and $V_{\rm DC}$ was measured for increasing $V_{\rm AC}$ at five frequencies for two gain settings. The slope of the $V_{\rm DC}$–$V_{\rm AC}$ line gives $dV_{\rm DC}/dV_{\rm AC}$, which relates to resistance as:

$$r = \frac{R}{\mu}\frac{dV_{\rm DC}}{dV_{\rm AC}}. \tag{7}$$

TABLE II. Observed V_{sig} and V_{DC} for different frequencies (Gain = 100).

Frequency (Hz)	$V_{\mathrm{AC}}(pp)$ (V)	$V_{\mathrm{sig}}(rms)$ (V)	V_{DC} (V)
305.5	1.015	0.001	0.258
	1.505	0.0015	0.419
	2.000	0.002	0.589
	2.500	0.0025	0.679
	3.000	0.003	0.830
605	1.030	0.001	0.260
	1.505	0.0015	0.418
	2.010	0.002	0.591
	2.550	0.0025	0.692
	3.000	0.003	0.830
900	3.000	0.003	0.829
	2.500	0.0025	0.668
	2.000	0.002	0.531
	1.505	0.0015	0.415
	1.005	0.001	0.252
1200	1.005	0.001	0.252
	1.505	0.0015	0.414
	2.000	0.002	0.583
	2.500	0.0025	0.675
	3.000	0.003	0.830
1500	3.000	0.003	0.831
	2.500	0.0025	0.674
	2.050	0.002	0.536
	1.500	0.0015	0.412
	1.000	0.001	0.250

TABLE III. Observed V_{AC}, V_{DC}, and corresponding slopes for mutual inductance measurement.

Frequency (Hz)	$V_{\mathrm{AC}}(pp)$ (V)	V_{DC} (V)	s ($dV_{\mathrm{DC}}/dV_{\mathrm{AC}(rms)}$)
600	7.0	0.040	
	9.05	0.068	
	11.0	0.094	$3.99 \times 10^{-2} \pm 8.3 \times 10^{-4}$
	13.0	0.122	
	15.0	0.154	
905	7.05	0.085	
	9.05	0.124	
	11.0	0.162	$5.73 \times 10^{-2} \pm 1.0 \times 10^{-3}$
	13.05	0.203	
	15.0	0.247	
1205	7.0	0.126	
	9.0	0.177	
	11.05	0.230	$7.53 \times 10^{-2} \pm 1.5 \times 10^{-3}$
	13.0	0.280	
	15.05	0.342	
1500	7.0	0.169	
	9.05	0.235	
	11.0	0.298	
	12.05	0.331	$9.42 \times 10^{-2} \pm 1.7 \times 10^{-3}$
	13.05	0.365	
	15.0	0.438	

TABLE IV. Observed V_{AC}, V_{DC}, and slopes for low resistance measurement (Gain = 50).

Frequency (Hz)	$V_{\mathrm{AC}}(pp)$ (V)	V_{DC} (V)	$dV_{\mathrm{DC}}/dV_{\mathrm{AC}(rms)}$
300	1.025	0.074	
	1.5	0.137	
	2.0	0.207	$3.28 \times 10^{-1} \pm 1.9 \times 10^{-2}$
	2.5	0.242	
	3.0	0.309	
605	1.025	0.074	
	1.505	0.138	
	2.005	0.208	$3.25 \times 10^{-1} \pm 1.9 \times 10^{-2}$
	2.5	0.242	
	3.05	0.312	
905	1.06	0.078	
	1.51	0.138	
	2.005	0.207	$3.23 \times 10^{-1} \pm 1.8 \times 10^{-2}$
	2.55	0.247	
	3.05	0.311	
1205	1.075	0.075	
	1.5	0.137	
	2.01	0.208	$3.27 \times 10^{-1} \pm 2.2 \times 10^{-2}$
	2.5	0.239	
	3.05	0.311	
1505	1.065	0.078	
	1.5	0.137	
	2.035	0.212	$3.27 \times 10^{-1} \pm 2.1 \times 10^{-2}$
	2.5	0.239	
	3.05	0.314	

TABLE V. Observed V_{AC}, V_{DC}, and slopes for low resistance measurement (Gain = 100).

Frequency (Hz)	$V_{\mathrm{AC}}(pp)$ (V)	V_{DC} (V)	$dV_{\mathrm{DC}}/dV_{\mathrm{AC}(rms)}$
305	1.06	0.204	
	1.51	0.323	
	2.005	0.459	$6.34 \times 10^{-1} \pm 3.8 \times 10^{-2}$
	2.55	0.534	
	3.05	0.663	
600	1.09	0.211	
	1.5	0.319	
	2.02	0.461	$6.29 \times 10^{-1} \pm 3.9 \times 10^{-2}$
	2.5	0.523	
	3.0	0.646	
900	1.005	0.190	
	1.505	0.320	
	2.005	0.457	$6.44 \times 10^{-1} \pm 3.8 \times 10^{-2}$
	2.5	0.522	
	3.0	0.656	
1205	1.04	0.198	
	1.51	0.320	
	2.005	0.456	$6.26 \times 10^{-1} \pm 4.0 \times 10^{-2}$
	2.55	0.527	
	3.0	0.645	
1500	1.005	0.189	
	1.5	0.319	
	2.01	0.457	$6.25 \times 10^{-1} \pm 4.1 \times 10^{-2}$
	2.55	0.525	
	3.0	0.644	

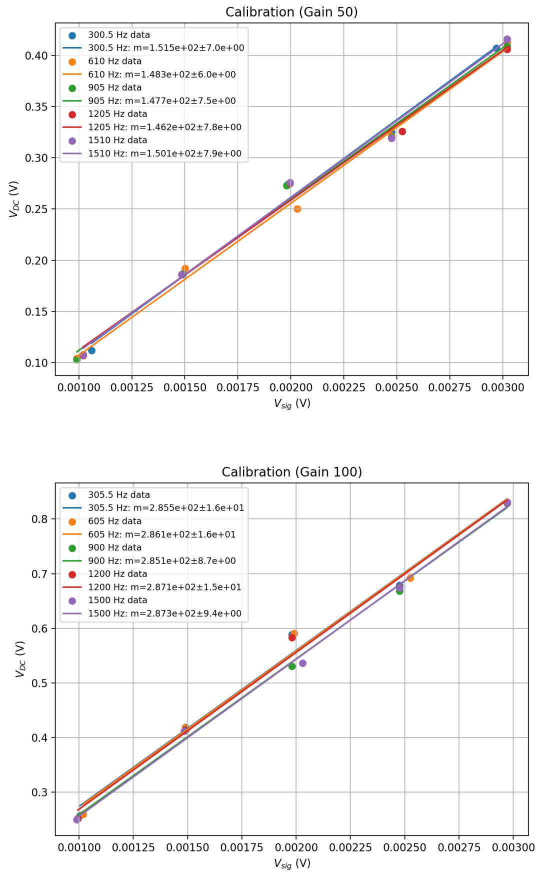


FIG. 1. $V_{\rm DC}$ vs $V_{\rm sig}$ for different frequencies at gains of 50 and 100. The slopes from linear fits give the amplification factors μ.

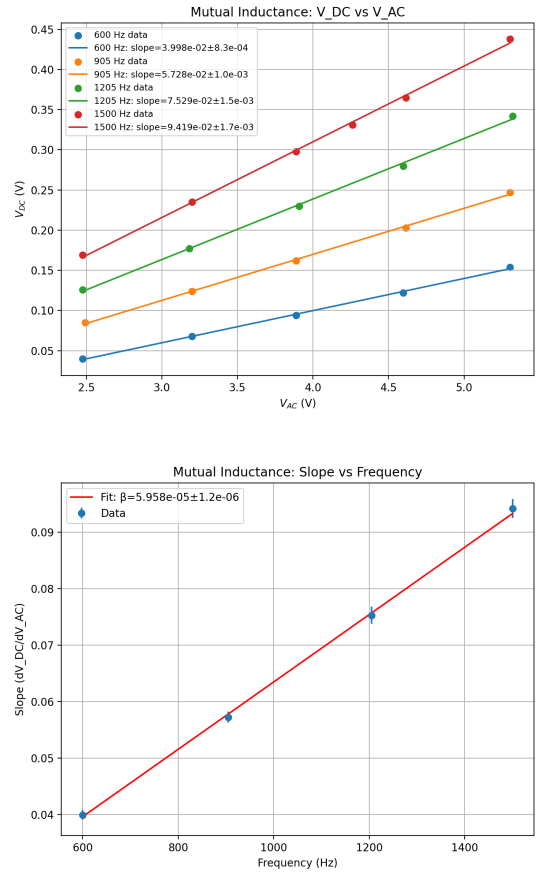


FIG. 2. $V_{\rm DC}$ versus $V_{\rm AC}$ for different frequencies (top), and slope s versus frequency (bottom) used to determine β.

V. DATA ANALYSIS

A. Determination of Low Resistance

From the calibration, the proportionality constant μ was obtained for each gain setting. Using the least squares fitting method, the slope and the uncertainty in slope were calculated. For details and full code, visit the Github repository [2]. From plot V, the amplification factors were determined to be

$$\mu_{50} = 148.8 \pm 3.3$$
$$\mu_{100} = 286 \pm 6$$

Using equation 7, and the plots V, the low resistance is calculated to be,

$$r_{50} = \frac{4.8 \times 10^3 \times 0.326}{148.8} = 10.5\,\Omega$$
$$r_{100} = \frac{4.8 \times 10^3 \times 0.632}{286} = 10.6\,\Omega$$
$$r_{avg} = 10.5\,\Omega$$

B. Determination of Mutual Inductance

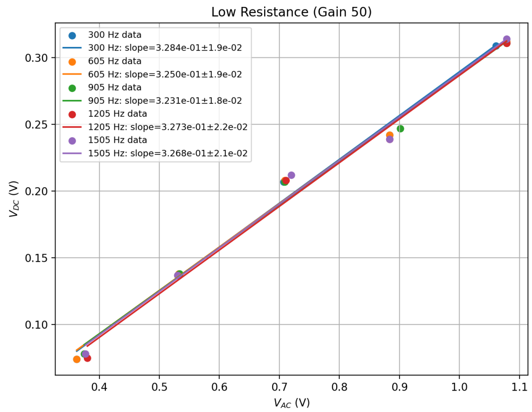
For the coupled coils, to determine Mutual inductance, equation 5 was used, and the slope (β) of the plot slope vs frequency was determined by least squares fit.

$$\beta = (5.9 \pm 0.1) \times 10^{-5} \pm Hz^{-1}$$
$$M = \frac{\beta R}{2\pi\mu}$$
$$= \frac{5.9 \times 10^{-5} \times 4.8 \times 10^3}{2 \times \ 3.14 \times 286} H$$
$$= 157 \mu H$$

VI. ERROR ANALYSIS

A. Uncertainty in Low Resistance

The uncertainty in resistance arises from the errors in the slope of the $V_{\rm DC}$–$V_{\rm AC}$ fit and in the calibration factor μ, which are calculated from the least squares statistical



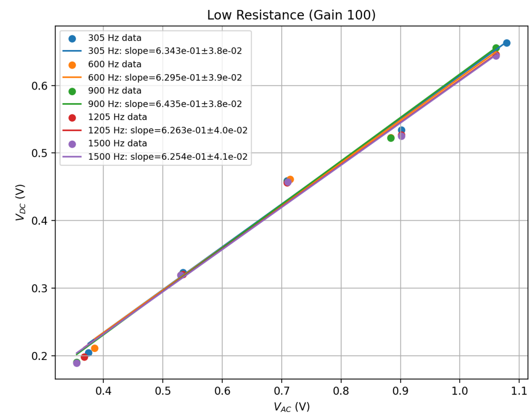


FIG. 3. V_{DC} vs V_{AC} for different frequencies at gains of 50 and 100. Slopes were obtained from least-squares fits to determine the low resistance.

error formulae [3]. The relative error in r is given by,

$$\left(\frac{\Delta r_i}{r_i}\right)^2 = \left(\frac{\Delta\mu}{\mu}\right)^2 + \left(\frac{\Delta(dV_{\mathrm{DC}}/dV_{\mathrm{AC}})}{dV_{\mathrm{DC}}/dV_{\mathrm{AC}}}\right)^2.$$

$$\Delta r_{50} = 0.4\,\Omega$$
$$\Delta r_{100} = 0.4\,\Omega$$
$$\Delta r_{avg} = 0.5\,\Omega$$

B. Uncertainty in Mutual Inductance

The error in mutual inductance M depends on the uncertainties in β, μ, and R:

$$\left(\frac{\Delta M}{M}\right)^2 = \left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2.$$

$$\Delta M = 5\mu H$$

The uncertainty $\Delta\beta$ was obtained from the least-squares fit of the slope–frequency graph, while $\Delta\mu$ was estimated from the linear fit of the calibration data.

VII. RESULTS

The calibration for different gain were calculated to be

$$\mu_{50} = 148.8 \pm 3.3$$
$$\mu_{100} = 286 \pm 6$$

The low resistance was calculated to be

$$r_{avg} = (10.5 \pm 0.5)\Omega$$

The mutual inductance of the coils was calculated to be

$$M = (157 \pm 5)\mu H$$

All derived quantities were found to be consistent within experimental uncertainty.

VIII. CONCLUSION AND DISCUSSION

The lock-in amplifier was successfully calibrated and used to measure both a small resistance and the mutual inductance between two coils. The measured resistance and mutual inductance values were consistent across different frequencies and gains, indicating reliable phase-sensitive detection. Minor deviations can be attributed to contact resistance, phase offsets, and background noise. Overall, the experiment demonstrates the effectiveness of the lock-in technique in extracting weak signals and performing precise electrical measurements.

[1] National Institute of Science Education and Research. *STUDY OF LOCK-IN AMPLIFIER*. NISER, 2024. Laboratory Manual for Modern Physics Experiments.
[2] Aryan Shrivastava. P343—modern-physics-lab. `https://github.com/crimsonpane23/P343---Modern-Physics-lab.git`, 2025.
[3] John R. Taylor. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. University Science Books, Sausalito, CA, 2nd edition, 1997.