Study of Balmer series

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The visible Balmer series of hydrogen was analyzed using a diffraction grating spectrometer calibrated with mercury spectral lines. The grating element was determined as $g = (1694.92 \pm 0.02)$ nm, and the Rydberg constant was obtained as $R = (1.084 \pm 0.006) \times 10^7\,\mathrm{m}^{-1}$. These results validate the Bohr model of hydrogen and demonstrate the precision of spectroscopic methods in determining fundamental constants.

I. OBJECTIVE

1. To measure the wavelengths of visible spectral lines in Balmer series of atomic hydrogen.
2. To determine the value of Rydberg's constant.

II. THEORY

A. Atomic Spectra and the Bohr Model

The emission spectrum of an element, consisting of discrete spectral lines, is a result of its quantized atomic energy levels. The hydrogen atom, with its single electron, possesses the simplest such spectrum. The empirical formula describing the wavelengths of its spectral lines was first given by Rydberg:

$$\frac{1}{\lambda} = R_y \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \tag{1}$$

where n_1 and n_2 are positive integers with $n_2 > n_1$, and R_y is the Rydberg constant.

Niels Bohr's model of the hydrogen atom (1913) provided a theoretical foundation for this formula. The model is built on two postulates:

1. Electrons revolve in certain stationary orbits, where their angular momentum L is quantized: $L = n\hbar$, where $n = 1, 2, 3, \ldots$
2. Electrons emit or absorb radiation only when transitioning between these stationary orbits. The frequency ν of the emitted photon is given by $h\nu = E_i - E_f$, where E_i and E_f are the energies of the initial and final orbits, respectively.

B. Derivation of the Rydberg Constant

For an electron (mass m_e, charge $-e$) in a circular orbit of radius r around a proton, the centripetal force is provided by the Coulomb attraction:

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \tag{2}$$

From Bohr's first postulate, the angular momentum is quantized:

$$L = m_e v r = n\hbar \quad \text{where } n = 1, 2, 3, \ldots \tag{3}$$

Solving Eq. (3) for velocity $v = n\hbar/(m_e r)$ and substituting into Eq. (2) yields the quantized orbital radii:

$$\frac{m_e}{r} \left(\frac{n\hbar}{m_e r} \right)^2 = \frac{e^2}{4\pi\epsilon_0 r^2} \tag{4}$$

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2 = a_0 n^2 \tag{5}$$

where $a_0 = 0.529 \times 10^{-10}$ m is the Bohr radius.

The total energy E of the electron is the sum of its kinetic and potential energies:

$$E = \frac{1}{2} m_e v^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \tag{6}$$

Using Eq. (2), $\frac{1}{2} m_e v^2 = \frac{e^2}{8\pi\epsilon_0 r}$. Substituting this and the expression for r_n from Eq. (5) gives the quantized energy levels:

$$E_n = \frac{e^2}{8\pi\epsilon_0 r_n} - \frac{e^2}{4\pi\epsilon_0 r_n} = -\frac{e^2}{8\pi\epsilon_0 r_n} \tag{7}$$

$$= -\frac{e^2}{8\pi\epsilon_0} \left(\frac{m_e e^2}{4\pi\epsilon_0 \hbar^2} \right) \frac{1}{n^2} \tag{8}$$

$$= -\left(\frac{m_e e^4}{8\epsilon_0^2 h^2} \right) \frac{1}{n^2} \quad (\text{since } \hbar = h/2\pi) \tag{9}$$

$$= -\frac{13.6}{n^2}\,\mathrm{eV} \tag{10}$$

When an electron transitions from a higher energy level m to a lower level n, the wavelength of the emitted photon is given by Bohr's second postulate:

$$\frac{hc}{\lambda} = E_m - E_n = \left(-\frac{R_y hc}{m^2} \right) - \left(-\frac{R_y hc}{n^2} \right) \tag{11}$$

$$\frac{1}{\lambda} = R_y \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \tag{12}$$

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where the theoretical expression for the Rydberg constant R_y is identified as:

$$R_y = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} \tag{13}$$

The Balmer series corresponds to transitions where the final state is $n = 2$, yielding visible spectral lines for $m = 3$ (H_α, red), $m = 4$ (H_β green), and $m = 5$ (H_γ, violet).

C. Diffraction Grating and Wavelength Measurement

A transmission diffraction grating is used to resolve and measure the wavelengths of these spectral lines. It consists of a large number N of parallel slits per unit length. The grating element is $g = 1/N$, the distance between adjacent slits.

When monochromatic light of wavelength λ is incident normally on the grating, the path difference between waves from adjacent slits is $g \sin\theta$. Constructive interference (principal maxima) occurs when this path difference is an integer multiple of the wavelength:

$$g \sin\theta = p\lambda \quad \text{for } p = 0, \pm1, \pm2, \ldots \tag{14}$$

where p is the order of the diffraction spectrum and θ is the diffraction angle.

III. EXPERIMENTAL APPARATUS

The experimental setup consists of a spectrometer equipped with a transmission diffraction grating and interchangeable spectral discharge tubes (mercury and hydrogen). The discharge tubes are mounted between two high-voltage electrodes and powered by a regulated supply.

For calibration, a mercury vapour lamp is used as the source. After calibration, the mercury lamp is replaced by a hydrogen discharge tube. Collimated light from the spectral tube passes through the diffraction grating, and the diffracted beams are observed through the telescope of the spectrometer.

By aligning the cross-wire of the telescope with the spectral lines on both sides of the central image, diffraction angles θ are measured. These angles are then used to calculate the wavelengths of the hydrogen Balmer lines. The apparatus is shown in Fig. 1.

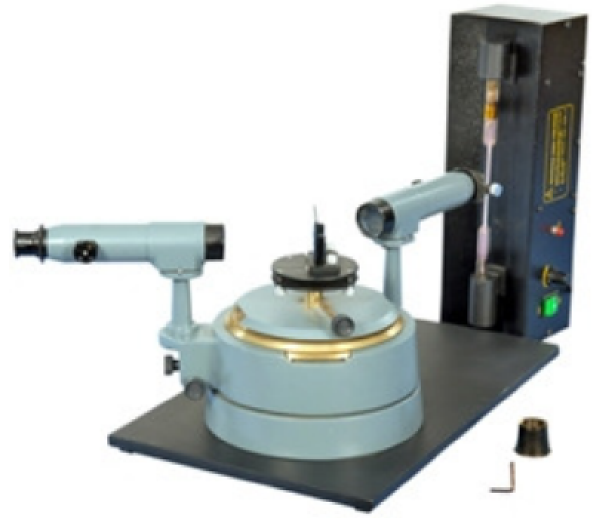

FIG. 1. Diffraction grating spectrometer setup. (Source: Adapted from [1])

IV. OBSERVATIONS

In this experiment, the grating constant g is first determined by using a mercury lamp with known wavelengths. For Hydrogen source, the diffraction angles were carefully measured on both sides using the two verniers. Using the diffraction angle θ for a known λ, g is calculated from Eq. (14), the wavelengths of the hydrogen Balmer lines are determined.

It was noted that the violet spectral line of hydrogen appeared with very low intensity compared to the red and green lines. This is because transitions to higher energy states (such as those responsible for violet emission) have lower transition probabilities,

V. DATA ANALYSIS

The grating element g was obtained from a plot of the mercury wavelengths against $\sin\theta$ (from Table I). Using this calibration, a linear fit of $\frac{1}{\lambda}$ versus $\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ (from Table II) was performed, and the slope yielded the Rydberg constant R.

A. Least square fitting

We analyze the experimental data by fitting a straight line of the form

$$Y = aX + b,$$

where a is the slope and b is the intercept. Applying the least squares method [2] yields the best-fit values of a and b that minimize the squared deviations between the observed and fitted data. The complete computational implementation, including the least-squares routine and error analysis, is provided in the code repository [3].

The uncertainties in slope and intercept obtained from

TABLE I. Observed diffraction angles and calculated $\sin\theta$ for Hg spectrum.

Colour	λ (nm)	Left Side (deg)		Right Side (deg)		2θ (deg)		Avg. θ (deg)	$\sin\theta$
		Vernier 1	Vernier 2	Vernier 1	Vernier 2	Vernier 1	Vernier 2		
Violet	413	104.15	284.33	132.48	312.27	28.33	27.94	14.07	0.243
Indigo	448	103.25	283.16	133.67	313.57	30.42	30.41	15.21	0.262
Green-1	501	101.22	281.20	135.75	315.52	34.53	34.32	17.21	0.296
Green-2	560	99.12	279.18	137.65	317.43	38.53	38.25	19.19	0.329
Yellow	588	98.05	278.03	138.93	318.68	40.88	40.65	20.38	0.348
Orange	614	97.58	277.52	139.47	319.25	41.89	41.73	20.90	0.357
Red-1	627	96.78	276.73	140.22	320.08	43.44	43.35	21.70	0.370
Red-2	637	96.50	276.52	140.53	320.3	44.03	43.78	21.95	0.374

TABLE II. Observed diffraction angles and $\sin\theta$ for hydrogen Balmer lines

Colour	λ (nm)	Left Side (deg)		Right Side (deg)		2θ (deg)		Avg. θ (deg)	$\sin\theta$
		Vernier 1	Vernier 2	Vernier 1	Vernier 2	Vernier 1	Vernier 2		
Red	670	94.37	274.37	140.98	320.7	46.61	46.33	23.33	0.395
Green	490	100.65	280.6	134.67	314.5	34.02	33.90	16.98	0.292
Violet	440	102.43	282.3	132.65	312.53	30.22	30.23	15.11	0.261

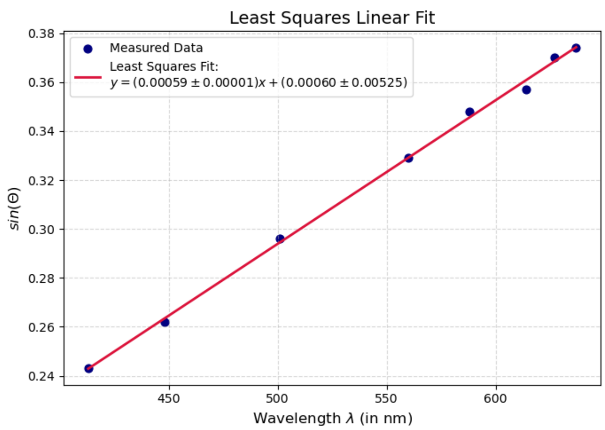

FIG. 2. Least-squares fit of $\sin\theta$ versus λ for mercury spectrum (calibration of grating element).

TABLE III. Least-squares fitting data for calibration plot ($\sin\theta$ vs. λ)

S.no.	x (λ in nm)	y ($\sin\theta$)	x^2	xy
1	413.0	0.243	170569.0	100.359
2	448.0	0.262	200704.0	117.376
3	501.0	0.296	251001.0	148.296
4	560.0	0.329	313600.0	184.24
5	588.0	0.348	345744.0	204.624
6	614.0	0.357	376996.0	219.198
7	627.0	0.37	393129.0	231.99
8	637.0	0.374	405769.0	238.238
Total	4388.0	2.579	2457512.0	1444.321

the graph is calculated as follows:

$$\delta Y = \sqrt{\frac{\sum (Y_n - Y_i)^2}{N-2}}, \tag{15}$$

$$\delta a = \delta Y \sqrt{\frac{\sum X^2}{(N \sum X^2) - \left(\sum X\right)^2}}, \tag{16}$$

$$\delta b = \delta Y \sqrt{\frac{N}{(N \sum X^2) - \left(\sum X\right)^2}}, \tag{17}$$

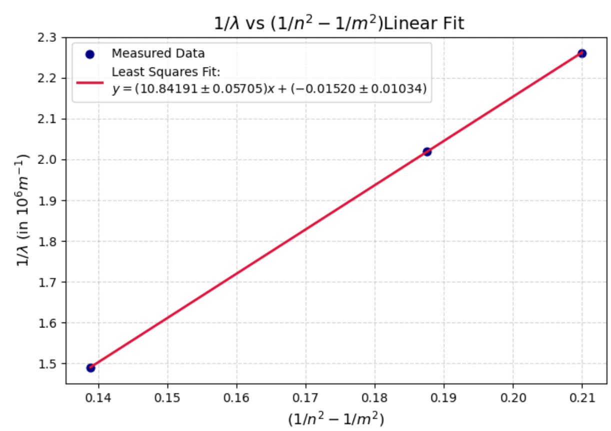

FIG. 3. Least-squares fitting of $1/\lambda$ vs $(1/n^2 - 1/m^2)$ to calculate Rydberg constant.

TABLE IV. Least-squares fitting data for Rydberg constant determination ($1/\lambda$ vs. $(1/n^2 - 1/m^2)$).

S.no.	x $(1/n^2 - 1/m^2)$	y ($1/\lambda$ in $10^6 m^{-1}$)	x^2	xy
1	0.139	1.49	0.019	0.207
2	0.188	2.02	0.035	0.3788
3	0.21	2.26	0.044	0.474
Total	0.536	5.77	0.099	1.0603

B. Determination of Grating Element (g)

From the slope of the first plot (wavelength λ vs. $\sin\theta$ for the Hg lamp), the grating element g was determined using the relation

$$g\sin\theta = \lambda \tag{18}$$

$$\sin\theta = \frac{1}{g} \times \lambda$$

Therefore, from the slope (m_1) of the $\sin\theta$ vs λ plot gives the inverse of grating element.

$$g = \frac{1}{m_1} = \frac{1}{0.00059} = 1694.92nm$$

Error propagation:

$$\Delta g = \sqrt{\left(\frac{\partial g}{\partial m_1} \cdot \delta m_1\right)^2} = |\delta m_1| \cdot |\frac{1}{m_1}| = |g| \cdot |\delta m_1|$$

$$\Delta g = 0.02nm$$

C. Calculation of Rydberg Constant (R)

Using the value of g determined above, the wavelengths of hydrogen spectral lines were measured (as shown in table). A linear fit was performed between $\frac{1}{\lambda}$ and $\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$.

TABLE V. Calculated wavelengths of hydrogen Balmer lines using the determined grating element(using eq. 18).

S.no.	Colour	Spectral line	$\sin\theta$	$\lambda_{calc.}$
1	Red	H_α	0.395	669.5
2	Green	H_β	0.292	494.9
3	Violet	H_γ	0.261	442.4

From eq. 12, the slope (m_2) of the graph gives the Rydberg constant. The value of R was thus obtained along with its uncertainty, estimated from the least-squares fitting error.

$$R = m_2 = 10.84 \times 10^6 m^{-1} = 1.084 \times 10^7 m^{-1} \tag{19}$$

Error propagation:

$$\Delta R = \Delta m_2 = 0.006 m^{-1}$$

D. Deviation from literature value

The literature value of Rydberg constant from eq. 13 is $R = 1.097 \times 10^7 m^{-1}$. The percentage deviation of the measured result is:

$$\delta R = \frac{R_{lit.} - R_{calc.}}{R_{lit.}} \times 100\% \tag{20}$$

$$\delta R = 1.2\%$$

VI. RESULTS AND DISCUSSION

Using the mercury spectrum, the grating element was determined as

$$g = (1694.92 \pm 0.02)\,\text{nm}.$$

A least-squares fit of $\frac{1}{\lambda}$ against $\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ yielded the Rydberg constant as

$$R = (1.084 \pm 0.006) \times 10^7\ \text{m}^{-1}.$$

This is in close agreement with the accepted value

$$R_{\text{lit.}} = 1.097 \times 10^7\ \text{m}^{-1},$$

with a relative error of about 1.2%.

The small discrepancy may arise from the following sources of error:

1. Misalignment of the telescope or grating during angle measurement.
2. Low intensity of spectral lines, especially the violet line, which appeared faint and difficult to focus accurately.
3. Temperature-induced drifts in the discharge tube, affecting emission intensities.

VII. CONCLUSION

The wavelengths of the visible Balmer lines of hydrogen were successfully measured using a diffraction grating spectrometer after calibration with a mercury lamp. From the measured wavelengths, the Rydberg constant was determined. The literature value of Rydberg constant was well within the uncertainty of the experimental value.

The experiment thus verifies the quantized energy level structure of hydrogen as predicted by Bohr's theory and demonstrates the effectiveness of diffraction gratings in high-precision spectroscopic measurements.

[1] National Institute of Science Education and Research (NISER), Bhubaneswar, India. *Emission Spectra of Hydrogen (Balmer Series) and Determination of Rydberg's Constant*.

[2] John R. Taylor. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. University Science Books, Sausalito, CA, 2nd edition, 1997.

[3] Aryan Shrivastava. P343—modern-physics-lab. `https://github.com/crimsonpane23/P343---Modern-Physics-lab.git`, 2025.