Milikan's Oil Drop Experiment

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The Millikan oil drop experiment was performed to determine the elementary charge and verify the quantization of electric charge. The charge on individual oil droplets was measured using both the dynamic and balancing methods by analyzing their motion under gravitational and electric fields. The values obtained were $e_{\text{dyn}} = (19.4 \pm 1.7) \times 10^{-19}$ C and $e_{\text{bal}} = (20.1 \pm 1.4) \times 10^{-19}$ C. Although these are an order of magnitude higher than the accepted value (1.60×10^{-19} C), the results clearly demonstrated that the total charge on droplets occurs in integral multiples of a smallest unit, confirming the quantization of electric charge.

I. OBJECTIVE

1. To demonstrate that electrical charge is quantised in discrete multiples of the electronic charge e,
2. To measure the value of e

II. THEORY

The Millikan oil drop experiment is designed to determine the elementary electric charge e by observing the motion of charged oil droplets in an electric field. The experiment demonstrates that electric charge is quantized, occurring only in discrete multiples of e [1].

When a small oil droplet is introduced between two horizontal, parallel plate electrodes, it is subjected to the following forces:

- Gravitational force, $F_g = \frac{4}{3}\pi r^3 \rho g$,
- Buoyant force, $F_b = \frac{4}{3}\pi r^3 \rho_a g$,
- Viscous (drag) force, $F_\eta = 6\pi\eta r v$,
- Electric force, $F_E = ne\frac{V}{d}$ (when the field is applied).

Here,

- r = radius of the oil droplet (m),
- ρ = density of the oil ($929\,\text{kg/m}^3$),
- ρ_a = density of air ($1\,\text{kg/m}^3$),
- η = coefficient of viscosity of air ($1.8432 \times 10^{-5}\,\text{kg/m·s}$),
- V = applied potential difference (V),
- d = separation between plates ($5 \times 10^{-3}\,\text{m}$),
- g = acceleration due to gravity ($9.81\,\text{m/s}^2$),
- n = number of electrons (integer),
- e = electronic charge.

A. Dynamic Method

In the absence of an electric field, the droplet falls under gravity. After a short time, it attains a constant terminal velocity v_f, at which the net force on it is zero. Applying Stokes' law for viscous drag:

$$\frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_a g = 6\pi\eta r v_f \tag{1}$$

Simplifying Eq. (1):

$$v_f = \frac{2gr^2(\rho - \rho_a)}{9\eta} \tag{2}$$

When an electric field is applied, a charged droplet experiences an upward electrostatic force. If it moves upward with a terminal velocity v_r, then:

$$ne\frac{V}{d} + \frac{4}{3}\pi r^3 \rho_a g = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_r \tag{3}$$

Combining Eq. (1) and Eq. (3) gives:

$$ne\frac{V}{d} = 6\pi\eta r(v_r + v_f) \tag{4}$$

Hence, the total charge on the droplet is:

$$ne = \frac{6\pi\eta r d}{V}(v_r + v_f) \tag{5}$$

Dividing Eq. (5) by Eq. (2) and simplifying, we obtain:

$$ne = \frac{4\pi g d}{3V}(\rho - \rho_a) r^3 \left(1 + \frac{v_r}{v_f}\right) \tag{6}$$

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1. Cunningham Correction

For very small droplets (radius $\sim 1\ \mu$m), the assumptions of Stokes' law are no longer exact since the droplet size becomes comparable to the mean free path of air molecules. Hence, a correction factor is introduced to account for the slip of air molecules over the droplet surface.

The corrected terminal velocity is:

$$v_f = \frac{2gr^2(\rho - \rho_a)}{9\eta}\left(1 + \frac{C}{Pr}\right) \tag{7}$$

Here,

- $C = 6.17 \times 10^{-8}$ m of Hg·m (Cunningham correction constant),
- P = atmospheric pressure (0.76 m of Hg).

Rearranging Eq. (7), we get:

$$\frac{9\eta v_f}{2g(\rho - \rho_a)} = r^2 + \frac{C}{P}r \tag{8}$$

Let

$$\xi = \frac{9\eta v_f}{2g(\rho - \rho_a)}, \qquad \zeta = \frac{C}{2P} \tag{9}$$

Then, Eq. (8) becomes:

$$r^2 + 2\zeta r - \xi = 0 \tag{10}$$

Solving Eq. (10) for r:

$$r = -\zeta + \sqrt{\zeta^2 + \xi} \tag{11}$$

Once r is found, the total charge ne is computed from Eq. (6).

B. Balancing Method

In this method, the droplet is held stationary between the plates by applying an appropriate potential V_b such that the electrostatic force exactly balances the net weight of the droplet:

$$ne\frac{V_b}{d} + \frac{4}{3}\pi r^3 \rho_a g = \frac{4}{3}\pi r^3 \rho g \tag{12}$$

Simplifying Eq. (12), we get:

$$ne = \frac{4\pi g d}{3V_b}(\rho - \rho_a) r^3 \tag{13}$$

The radius r is first obtained from Eq. (11) using the measured free-fall velocity, and then substituted in Eq. (13) to compute ne.

C. Constants Used in Calculations

TABLE I. Constants used in the Millikan Oil Drop Experiment.

Symbol	Quantity	**Value**
g	Acceleration due to gravity	$9.81\,\mathrm{m/s^2}$
ρ	Density of oil	$929\,\mathrm{kg/m^3}$
ρ_a	Density of air	$1.00\,\mathrm{kg/m^3}$
η	Coefficient of viscosity of air	1.8432×10^{-5} kg/m·s
d	Plate separation	5×10^{-3} m
C	Cunningham correction constant	6.17×10^{-8} m of Hg·m
P	Atmospheric pressure	0.76 m of Hg

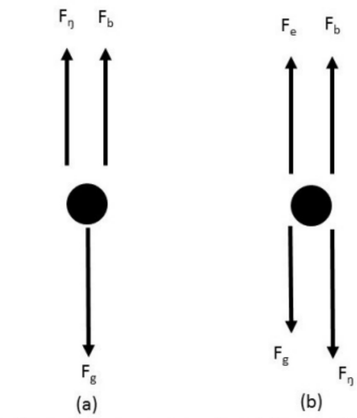

FIG. 1. Forces acting on an oil droplet under (a) free fall and (b) applied electric field conditions.

III. EXPERIMENTAL APPARATUS

The experimental setup consists of an oil drop chamber with two horizontal parallel plate electrodes separated by 5 mm. The upper plate has a fine hole for the entry of oil droplets sprayed using an atomizer. The chamber is illuminated by an LED source, and a microscope with a CCD camera enables observation of droplet motion on a graduated monitor screen. A variable DC power supply applies a uniform electric field between the plates, measured by a digital voltmeter. A timer records rise and fall times of the droplets. The entire assembly is mounted on a leveled base with adjustment screws and a spirit level to ensure horizontal alignment of the plates.

IV. OBSERVATIONS

V. DATA ANALYSIS

The data for free-fall, rise times and balancing voltages were used to determine the total charge on each droplet. The analysis, divided by method, is presented below. For detailed calculation and code, refer to the GitHub Repository [2].

TABLE II. Observations using Dynamic Method
t_f = Free-fall time (s)
t_r = Rise time (s)
v_f = Mean free-fall velocity (m/s)

Droplet	S.no.	t_f (s)	$\overline{t_f}$(s)	t_r (s)	$\overline{t_r}(s)$	Voltage (V)	$\overline{V}$(V)	v_f (10^{-3}m/s)
1	1	13.5	13.2	1.9	2.0	326	326	0.15
	2	13.1		2.0		325		
	3	13.3		2.0		326		
	4	12.9		2.1		326		
	5	13.3		2.0		325		
2	1	10.0	9.9	4.1	4.2	326	326	0.20
	2	10.1		4.3		324		
	3	9.8		4.2		326		
	4	9.9		4.1		326		
	5	9.8		4.1		326		
4	1	8.8	8.8	2.0	1.8	347	348	0.23
	2	8.8		1.7		349		
	3	8.7		1.7		348		
	4	8.5		1.8		348		
	5	9.0		1.9		348		
4	1	7.6	7.6	2.1	2.2	384	383	0.26
	2	7.5		2.2		383		
	3	7.7		2.2		384		
	4	7.4		2.3		383		
	5	7.8		2.3		383		
5	1	7.3	7.4	3.6	3.7	361	361	0.27
	2	7.5		3.8		360		
	3	7.6		3.7		360		
	4	7.4		3.7		361		
	5	7.2		3.9		361		

TABLE III. Observations using Balancing Method
t_f = Free-fall time (s)
V_b = Balancing Voltage
v_f = Mean free-fall velocity (m/s)

Droplet	S.no.	t_f (s)	$\overline{t_f}(s)$	V_b (V)	$\overline{V_b}$ (V)	v_f (10^{-3}m/s)
1	1	5.0	4.9	300	300	0.41
	2	4.8		302		
	3	4.9		301		
	4	4.8		299		
	5	4.8		299		
2	1	4.6	4.6	317	318	0.43
	2	4.7		316		
	3	4.7		319		
	4	4.5		318		
	5	4.5		318		
3	1	3.9	4.0	314	314	0.5
	2	4.0		313		
	3	4.1		314		
	4	4.1		312		
	5	4.0		314		
4	1	7.6	7.5	369	369	0.27
	2	7.2		370		
	3	7.6		369		
	4	7.5		368		
	5	7.4		370		
5	1	4.6	4.6	339	340	0.43
	2	4.7		340		
	3	4.5		339		
	4	4.5		341		
	5	4.7		339		

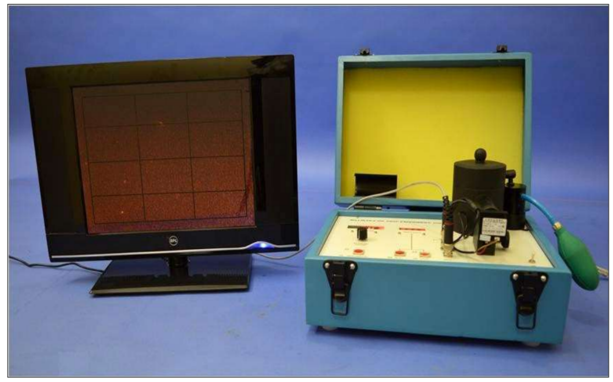

FIG. 2. Image of Millikan oil drop experimental setup.

A. Dynamic Method

The following table shows the intermediate values calculated for each droplet, leading to the total charge, ne.

TABLE IV. Calculated parameters for the Dynamic Method.

Droplet	$\boldsymbol{\xi}$ (10^{-12} m^2)	$\boldsymbol{r}$ (10^{-7} m)	$\boldsymbol{r^3}$ (10^{-18} m^3)	$\boldsymbol{T}$	$\boldsymbol{ne}$ (10^{-18} C)
1	1.36	11.29	1.44	2.39	2.02
2	1.82	13.09	2.24	1.66	2.18
3	2.09	14.07	2.78	2.54	3.88
4	2.37	14.99	3.36	2.26	3.79
5	2.46	15.28	3.57	1.75	3.30

TABLE V. Charge analysis for the Dynamic Method.

Droplet	ne (10^{-18} C)	ne/lowest	n_{eff}	$e = ne/n_{\text{eff}}$ (10^{-18}C)
1	2.01	1	1	2.01
2	2.18	1.08	1	2.18
3	3.88	1.93	2	1.94
4	3.79	1.88	2	1.89
5	3.30	1.64	2	1.65

B. Balancing Method

For the balancing method, the total charge ne was calculated from the droplet's radius and the balancing voltage.

TABLE VI. Calculated parameters for the Balancing Method.

Droplet	$\boldsymbol{\xi}$ (10^{-12}m^2)	$\boldsymbol{r}$ (10^{-7}m)	$\boldsymbol{r^3}$ (10^{-18}m^3)	$\boldsymbol{ne}$ (10^{-18}C)
1	3.73	18.92	6.78	2.15
2	3.92	19.39	7.29	2.18
3	4.55	20.94	9.18	1.86
4	2.46	15.28	3.57	1.84
5	3.92	19.39	7.29	2.04

TABLE VII. Charge analysis for the Balancing Method.

Droplet	ne (10^{-18}C)	ne/lowest	n_{eff}	$e = ne/n_{\text{eff}}$ (10^{-19}C)
1	2.15	1.17	1	2.15
2	2.18	1.18	1	2.18
3	1.86	1.01	1	1.86
4	1.84	1.00	1	1.84
5	2.04	1.12	1	2.04

VI. CALCULATION AND ERROR ANALYSIS

From the set of calculated values of e, we obtain the mean and the standard deviation of the measurements. The arithmetic mean is

$$\bar{e} \;=\; \frac{1}{N}\sum_{i=1}^{N} e_i, \tag{14}$$

and the standard deviation is

$$\sigma_e \;=\; \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(e_i - \bar{e}\right)^2}. \tag{15}$$

The uncertainty in the fundamental charge can be estimated by using the standard deviation of calculated values. Instead of using the least count, we use the statistical spread of the observed values. The equation 15 provides the standard deviation. The uncertainty is given by the *standard error of the mean* (SEM) [3].

For the Dynamic Method,

$$\bar{e} = 19.4 \times 10^{-19} C$$
$$\sigma_e = 1.7 \times 10^{-19} C$$

For the Balancing method,

$$\bar{e} = 20.1 \times 10^{-19} C$$
$$\sigma_e = 1.4 \times 10^{-19} C$$

The percentage deviation of the experimental result from the literature value ($e_{\text{lit}} = 1.6 \times 10^{-19}$ C) is calculated as:

For Dynamic method,

$$\%\ \text{Deviation} = \left|\frac{19.4 - 1.6}{1.6}\right| \times 100\% = 1112\%$$

For Balancing method,

$$\%\ \text{Deviation} = \left|\frac{20.2 - 1.6}{1.6}\right| \times 100\% = 1162\%$$

The experimental value deviates from the literature value by $\sim$ 1000%, which is significantly larger than the experimental uncertainty of $\sim$ 10%. Also, the measured value is significantly than the literature value, and the deviation is in 1 order of magnitude. This points at a big problem regarding the experiment. We assume that out of the 5 droplets, one of them will have the charge 'e', which is invalid for such a small sample size. If the experiment involved studying a large number of droplets, then the assumption would be more justified.

VII. RESULTS

From the analysis, the mean values of the electronic charge were found to be

$$e_{\text{dyn}} = (19.4 \pm 1.7) \times 10^{-19}\ \text{C}$$
$$e_{\text{bal}} = (20.1 \pm 1.4) \times 10^{-19}\ \text{C}.$$

Both values are significantly higher than the accepted value $e_{\text{lit}} = 1.60 \times 10^{-19}$ C, showing a deviation of over 1000%. However, the discrete multiples of charge obtained for different droplets confirm the quantization of electric charge.

VIII. CONCLUSION AND DISCUSSION

The experiment verified that electric charge occurs in integral multiples of a fundamental unit, supporting the concept of charge quantization. Although the measured values of e differ greatly from the accepted one, the discrepancy arises mainly from a limited data set, small manual timing errors, and assumptions in droplet selection. Despite these inaccuracies, the experiment successfully demonstrated the quantized nature of charge and the working principle of Millikan's method.

[1] National Institute of Science Education and Research. *Measurement of Elementary charge by Millikan oil drop method*. NISER, 2024. Laboratory Manual for Modern Physics Experiments.

[2] Aryan Shrivastava. P343—modern-physics-lab. `https://github.com/crimsonpane23/P343---Modern-Physics-lab.git`, 2025.

[3] John R. Taylor. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. University Science Books, Sausalito, CA, 2nd edition, 1997.