STUDY OF LATTICE VIBRATIONS USING ELECTRONIC CIRCUITS

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DATE OF EXPERIMENT - 28/01/26 & 29/01/26

DATE OF SUBMISSION - 04/02/26

Abstract

In this experiment, LC circuits were used as analogues of monoatomic and diatomic lattices to study lattice vibrations & their dispersion relations. The frequency was plotted against phase per unit cell & a cut-off frequency was observed for monoatomic case. For diatomic lattice, two branches corresponding to acoustic & optical modes were observed & frequency band gap was calculated. The results confirm the validity of electrical analogue model for studying lattice vibrations.

Objective :-

i) Build an analogy of mono-atomic lattice using inductors & capacitors & study dispersion relation.

ii) Build an analogy of di-atomic lattice using inductors & two different capacitors & study dispersion relation & band-gap energy.

Theory :-

Consider a spring-mass model of a periodic mono-atomic lattice in one-direction. Atoms having mass 'm' connected by a force constant 'f'. (see fig.)

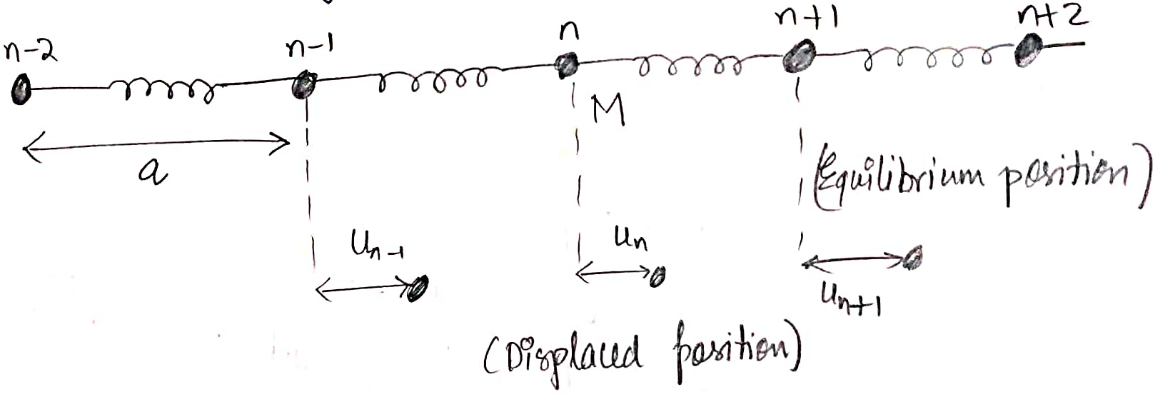

Figure :- 1D Linear Monoatomic lattice with
a → Lattice constant
f → force constant
M → Mass of the atom

The array is assumed to be infinitely long. Assuming only the nearest neighbour interaction, the equation of motion of n^{th} atom is given by :

$$m\ddot{u}_n = f((u_{n+1} + u_{n-1}) - 2u_n) \quad \text{—(I)}$$

Attempting solution to -(I) of the form,

$$u_K = \cancel{Ae^{i(Ka-\omega t)}} \; Ae^{i(nka-\omega t)}$$

$$\ddot{u}_n = -\omega^2 u_n = \frac{f}{m}((u_{n+1} + u_{n-1}) - 2u_n)$$

$$u_K(n,t) = Ae^{i(nka-\omega t)} \qquad u_K(n\pm1,t) = Ae^{i((n\pm1)ka-\omega t)}$$

$$\Rightarrow \quad -\omega^2 e^{inka} = \frac{f}{m}\left(e^{i(n+1)ka} + e^{i(n-1)ka} - 2e^{inka}\right)$$

$$\Rightarrow \quad -\omega^2 = \frac{f}{m}\left(e^{ika} + e^{-ika} - 2\right) = \frac{2f}{m}\left(\cos ka - 1\right)$$

$$\Rightarrow \quad \omega^2 = \frac{2f}{m}\left(1 - \cos ka\right) \Rightarrow \underline{\underline{\omega = \sqrt{\frac{4f}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|}}.$$

$$\boxed{\omega^2 = \frac{2f}{m}\left(1 - \cos\theta\right)}$$

where k - wave vector $= \frac{2\pi}{\lambda} = \frac{\omega}{c}$

c = velocity of propagation

$\theta = ka$ = phase change per unit cell

$$\boxed{\nu_{max} = \frac{\omega_{max}}{2\pi} = \frac{1}{\pi}\sqrt{\frac{f}{m}}}$$

↳ Beyond, this frequency, no transmission occurs. This is analogous to a Low-pass filter circuit which transmits in the range $0 - \nu_{max}$. (see fig.)

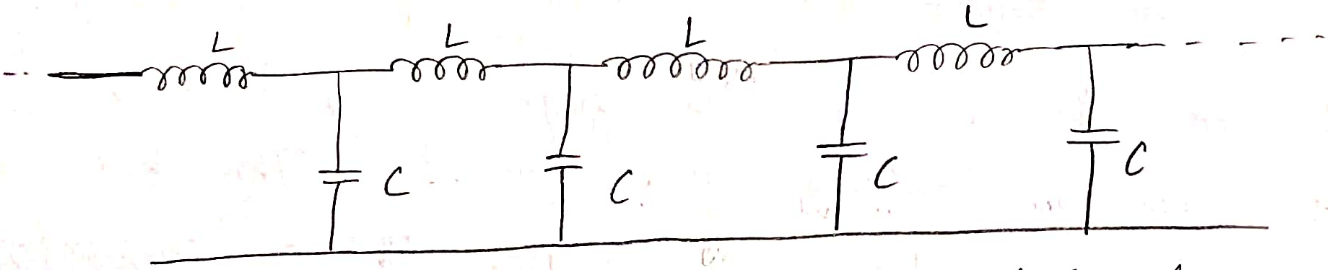

Figure 2 : Electrical Analogue of linear - mono-atomic lattice (Low-pass filter)

The dispersion relation for this circuit is

$$\boxed{\omega^2 = \frac{2}{LC}\left(1 - \cos\theta\right)}$$

where θ = phase change due to 1 section (unit cell) of the filter.

∴ Analog : $(L) \longleftrightarrow f$ & $\left(\frac{1}{C}\right) \longleftrightarrow m$. Studying the phase difference b/w input & output voltages of the circuit as a function of frequency, the dispersion relation may be verified.

* Di-atomic Lattice.

Consider a di-atomic lattice with alternative masses 'm' & 'M' as shown in fig. It can be simulated by a transmission line with alternative capacitors 'C' & 'C_1' as shown in figure.

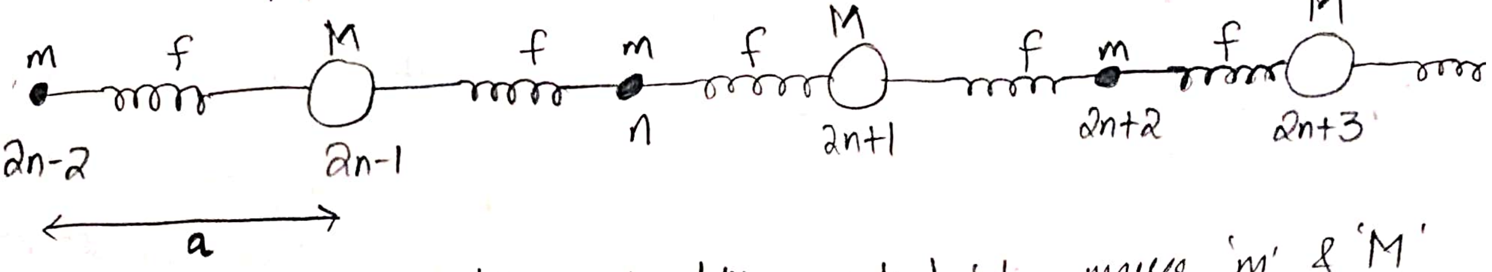

Figure :- Linear - Di-atomic lattice of lattice constant 'a', masses 'm' & 'M' and force constant 'f'

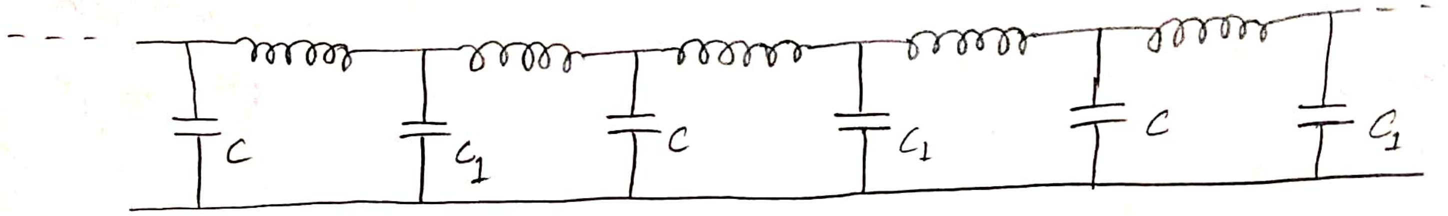

Figure :- Linear diatomic lattice -- electrical analogue.

In this case, there are now two frequencies ω_+ & ω_-, corresponding to a particular wave vector 'k'. This leads to two branches in the ν v/s θ plot. The one corresponding to ν_+ is called optical branch & the one for ν_- is called acoustical branch. The frequency gap b/w these branches depends on $\left(\frac{M}{m}\right) \leftrightarrow \left(\frac{C}{C_1}\right)$.

Apparatus required :-

i) Function generator (signal generator)
ii) Oscilloscope
iii) Breadboard
iv) connecting wires & BNC cables.
v) Inductors
vi) Capacitors (2 types)

~~atomic Lattice~~ :-

Monoatomic (small mass/capacitance) Lattice

$\boxed{N=10}$

No.	Frequency f (kHz)	Phase difference (degree)	Lissajous figure	(phase difference) ~~frequency~~ per unit cell (kHz)
1.	0.025	0°	/	0°
2.	3.25	90°	O	9°
3.	7.00	180°	\	18°
4.	10.65	270°	O	27°
5.	14.35	360°	/	36°
6.	17.70	450°	O	45°
7.	21.30	540°	\	54°
8.	24.65	630°	O	63°
9.	27.95	720	/	72°
10.	31.00	810°	O	81°
11.	34.0	900°	\	90°
12.	36.5	990	O	99°
13.	39.0	1080	/	108°
14.	41.5	1178°	O	117.8°
15.	43.5	1255°	\	125.5°
16.	45.5	1340°	O	134°

Capacitance (C) (nF)

~~C (nF)~~	C_1 (nF)
~~149.68~~	49.01
	47.12
	47.71
	47.77
	47.38
	49.50
	47.93
	48.05
	49.81
	45.88

$C_{avg} = 48.02$ nF

$$\nu_{max} = \frac{1}{\pi}\sqrt{\frac{1}{LC}}$$

$$= \frac{1}{\pi}\sqrt{\frac{10^{-12}}{0.974 \times 48.02}}$$

$$\nu_{max} = 46.54 \text{ kHz}$$

28/1/26

Observations :- ~~C = 150.42 nF~~ (phase diff) Measurements :- $\boxed{N = 10}$ Obse

	Frequency (kHz)	Phase difference ϕ (degree)	Lissajous figure	~~Frequency~~ per unit cell
1.	0.025	0		0
2.	1.695	90°		9°
3.	3.80	180°		18°
4.	5.75	270°		27°
5.	7.90	360°		36°
6.	9.85	450°		45°
7.	11.80	540°		54°
8	13.70	630°		63°
9.	15.655	720°		72
10.	17.30	810°		81°
11.	19.00	900°		90°
12.	20.60	990°		99°
13.	22.10	1080°		108°
14.	23.45	1170°		117°
15	24.75	1260°		126°
16.	25.90	1350°		135°
17.	27.05	1440°		144°

Inductance (L) (mH)	Capacitance (C) (nF)
0.968	144.59
0.982	149.65
0.973	144.24
0.964	150.19
0.980	153.67
0.981	155.49
0.972	147.63
0.974	149.38
0.970	148.81
0.972	149.18

$L_{avg} = \underline{0.974 \text{ mH}}$ $C_{avg} = \underline{149.28 \text{ nF}}$

$$\nu_{max} = \frac{1}{\pi}\sqrt{\frac{1}{LC}}$$

$$= \frac{1}{\pi}\sqrt{\frac{1}{0.974\times10^{-3} \times 149.28\times10^{-9}}}$$

$$\nu_{max} = 0.0264 \times 10^{6} \text{ Hz}$$

$$\Downarrow$$

$$\boxed{\nu_{max} = 26.4 \text{ kHz}}$$

(Monoatomic Lattice)

28/1/26

Observation :- N=5 (Diatomic lattice)

	Frequency f (kHz)	Phase θ (deg.)	Lissajous figure	~~Frequency~~ Phase difference per unit cell (kHz).
1.	0.025	0°	/	0
2.	2.075	90°	O	18°
3.	4.505	180°	\	~~18~~ 36°
4.	6.95	270°	O	~~27~~ 54°
5.	9.25	360°	/	72°
6.	11.55	450°	O	90°
7.	13.75	540°	\	108°
8.	15.70	630°	O	126°
9.	17.65	720°	/	144°
10.	19.55	810°	O Band gap	162°
11.	33.5	990°	O	198°
12.	35.0	1080°	/	216°
13.	36.0	1160°	O	232°
14.	37.0	1240°	\	248°
15.	38.5	1370°	O	274°
16.	39.5	1450°	/	290°

29/01/26

Diatomic lattice.

C	C_1
149.72	47.83
151.23	48.04
150.99	48.62
152.34	47.94
153.69	47.37

$C_{avg} = 151.59$ nF

$C_{1\,avg} = 47.96$ nF

$$\nu = \frac{1}{2\pi}\sqrt{\frac{2}{LC}}$$

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{2\times10^{12}}{0.981\times151.59}}$$

$$\boxed{\nu_1 = 18.46 \text{ kHz}}$$

$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{2}{LC_1}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2\times10^{12}}{0.981\times47.96}}$$

$$\boxed{\nu_2 = 32.81 \text{ kHz}}$$

Inductance

L (mH)

0.992
0.980
0.982
0.984
0.974
0.983
0.986
0.972
0.973
0.986

$L_{avg} = 0.981$ mH

Calcu

(1)

Calculations :

(I) Monoatomic lattice

$C_{avg} = 149.28$ nF $\quad$ $L_{avg} = 0.974$ mH

From $\nu_{max} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 26.4$ kHz.

From the observations / plot, $\nu_{max} = 27.05$ kHz.

% deviation from theoretical value,

$$\% \text{ deviation} = \frac{|(\nu_{max})_{exp.} - (\nu_{max})_{theo.}|}{(\nu_{max})_{theo.}} \times 100\%$$

$$= \frac{(27.05 - 26.4)}{26.4} \times 100\%$$

% deviation = 2.46 %.

(IV) Monoatomic lattice $\quad$ $C_{avg} = 48.02$ nF $\quad$ $L_{avg} = 0.974$ mH.

From $\nu_{max} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 46.54$ kHz.

From observations / plots, we obtain $\nu_{max} = 45.5$ kHz.

% deviation from theoretical value,

$$\% \text{ deviation} = \frac{|(\nu_{max})_{exp.} - (\nu_{max})_{theo.}|}{(\nu_{max})_{theo.}} \times 100\%$$

$$= \frac{|45.5 - 46.54|}{46.54} \times 100\%$$

% deviations = 2.23%

(III) Diatomic lattice

$C_{avg} = 151.59\,nF$ $C_{1avg} = 47.96\,nF$

$L_{avg.} = 0.181\,mH$

Result

From the formulae,

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{2}{LC}}$$

$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{2}{LC_1}}$$

$\nu_1 = 18.46\,kHz$

$\nu_2 = 32.81\,kHz$

From the observations/plot, we obtain,

$\nu_{1_{obs.}} = 19.55\,kHz$

$\nu_{2_{obs.}} = 33.5\,kHz.$

$\therefore$ Frequency band gap,

theoretical, $\Delta\nu = (\nu_2 - \nu_1)_{theo.} = (32.81 - 18.46)\,kHz$
$= 14.35\,kHz$

experimental, $\Delta\nu = (\nu_2 - \nu_1)_{exp.} = (33.5 - 19.55)\,kHz$
$= \cancel{14.95}\ 13.95\,kHz$

% deviation in band gap, $= \dfrac{|(\Delta\nu)_{obs.} - (\Delta\nu)_{theo.}|}{(\Delta\nu)_{theo.}} \times 100\%$

$= \dfrac{|13.95 - 14.35|}{14.35} \times 100\%$

% deviation in band gap $= 2.79\ \%$

Results & Discussion :-

i) For a monoatomic lattice, phase difference per unit cell was measured for different frequencies using Lissajous figures & ν v/s θ was plotted. The plot shows that as θ increases, ν reaches a maximum (cut-off) frequency. The experiment was performed for 2 different capacitances circuits representing two different monoatomic lattices with different masses.

ii) Maximum frequency calculated, observed & % deviation is,

for $C_{avg} = 48.02\,nf$, $(\nu_{max})_{calc.} = 46.54\,kHz$ $(\nu_{max})_{obs.} = 45.5\,kHz$ &

for $C_{avg} = 149.29\,nF$, $(\nu_{max.})_{calc.} = 26.4\,kHz$ $(\nu_{max})_{obs.} = 27.05\,kHz$

with a % deviation of 2.23 % & 2.46 % respectively.

iii) For a diatomic lattice, phase difference per unit cell was measured as a function of frequency. The dispersion curve shows two branches: lower frequency branch (acoustic) & a higher frequency branch (optical). The (frequency gap) / (band gap) was calculated & observed to be,

$(\Delta\nu)_{calc} = 14.35\,kHz$ $(\Delta\nu)_{obs.} = 13.95\,kHz$.

with a % deviation of 2.79 %.

iv) The experiment demonstrates the analogy b/w mechanical lattice vibrations & electrical LC networks. Dispersion relation for both monoatomic & diatomic lattices were verified. Existence of cut-off frequency for monoatomic lattice & a band gap for diatomic lattice was observed.

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iii) For a diatomic lattice, phase difference per unit cell was measured as a function of frequency. The dispersion curve shows two branches: lower frequency branch (acoustic) & a higher frequency branch (optical). The calculated & observed (frequency gap) / (band gap) was to be,

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with a % deviation of 2.79 %.

iv) The experiment demonstrates the analogy b/w mechanical lattice vibrations & electrical LC networks. Dispersion relation for both monoatomic & diatomic lattices were verified. Existence of cut-off frequency for monoatomic lattice & a band gap for diatomic lattice was observed.

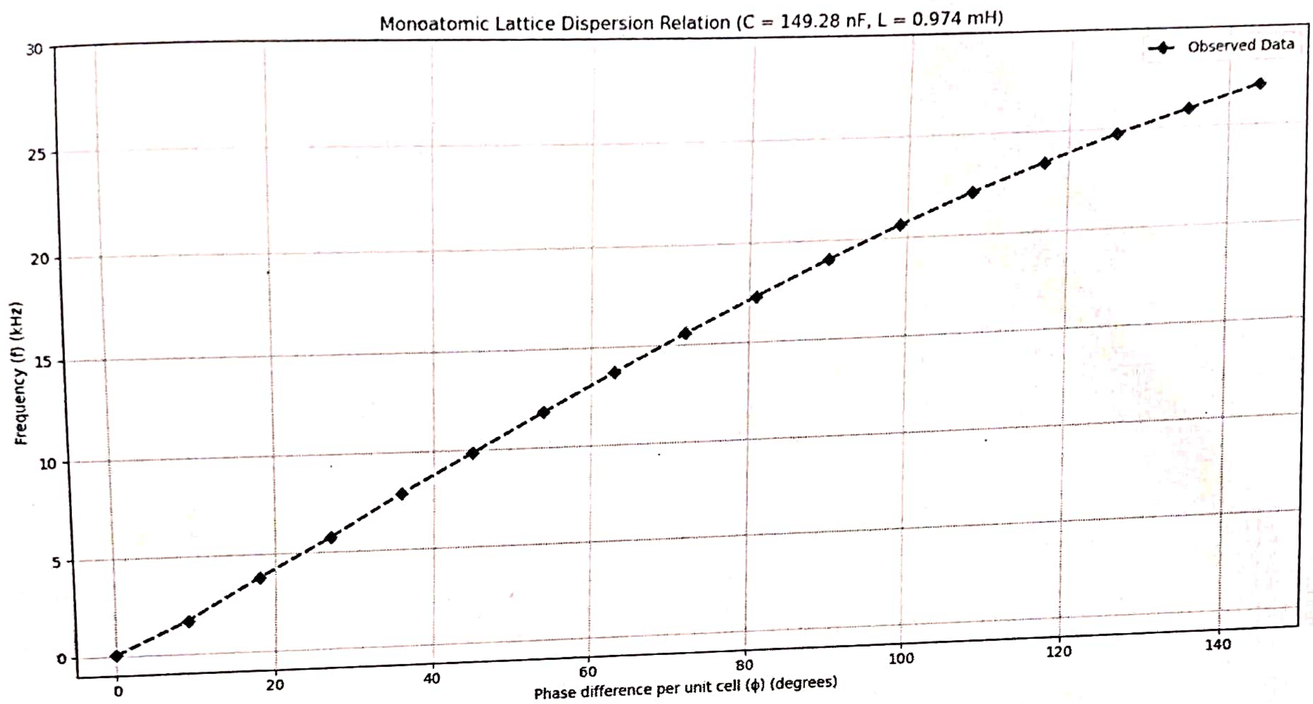
vi) The experimental & theoretical values are within 3% devi[ation] for all cases, hence the experiment matches reasonably with theoretical expectations. Minor deviations can be attributed to the following sources of errors.

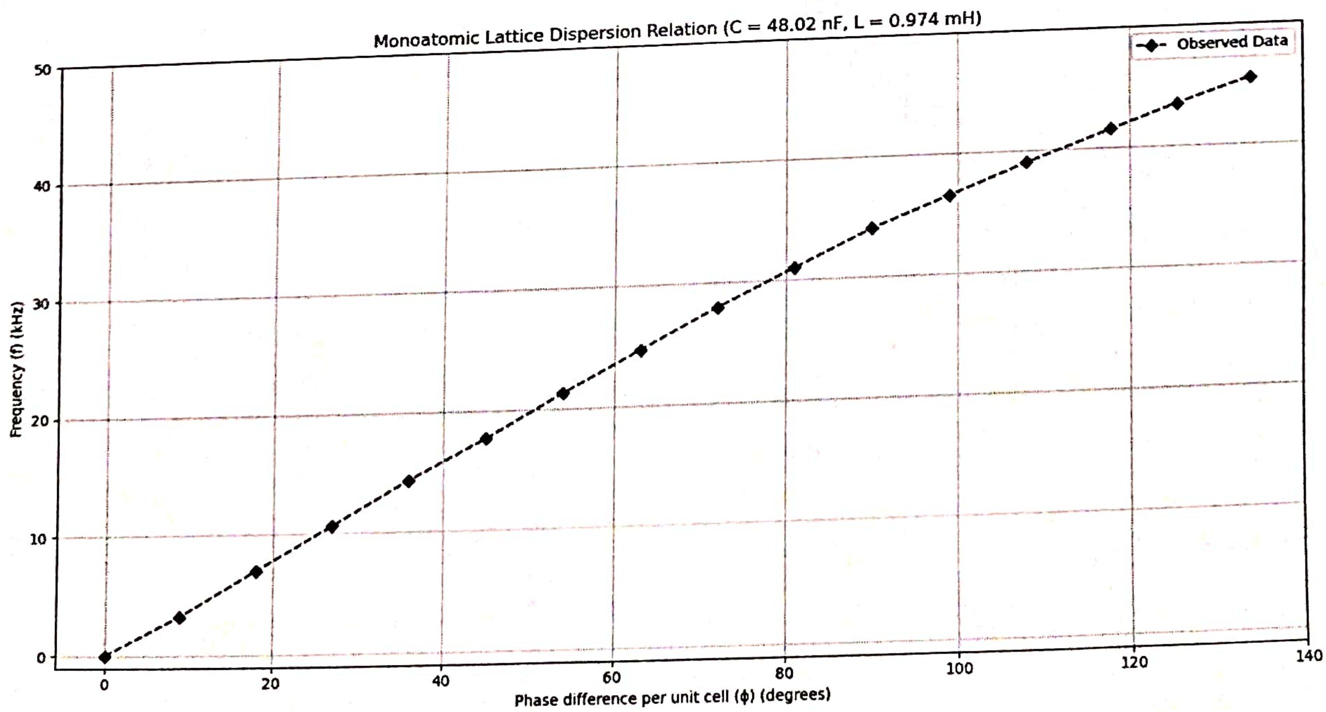
Sources of Error :-

i) Theoretical model assumes an infinite lattice, however we worked with 5 - 10 unit cells in the electrical analog.

ii) Internal resistance of inductors can cause damping in oscillations & alter the measurements.

iii) Heating of components can alter the values of capacitances & inductances, affecting the measurements.


Monoatomic Lattice Dispersion Relation (C = 149.28 nF, L = 0.974 mH)
Observed Data
Frequency (f) (kHz)
Phase difference per unit cell (ϕ) (degrees)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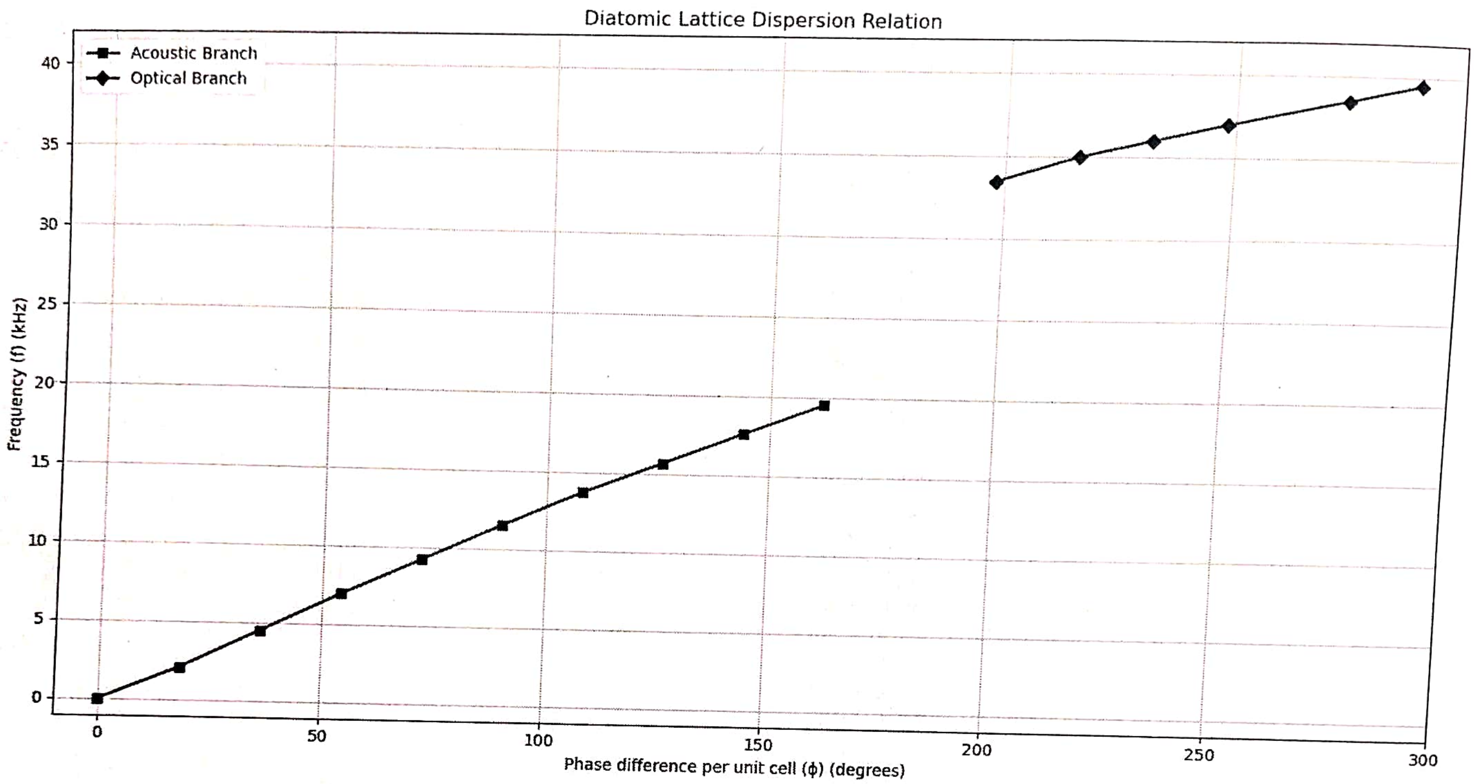
Diatomic Lattice Dispersion Relation
Acoustic Branch
Optical Branch
Frequency (f) (kHz)
Phase difference per unit cell (ϕ) (degrees)
0
5
10
15
20
25
30
35
40
0
50
100
150
200
250
300