

**Artificial Intelligence**

Neural Networks

# **Lesson 16:**

# **Recurrent Networks**

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# Recurrent Networks

- Neural networks with cycles among neurons
  - on individual neurons
  - involving groups of neurons
- Output is generated when stability is reached
- Configuration
  - by construction if structure of the computation to be accomplished is known
  - by extending the error backpropagation algorithm in time to deal with recursion to output stability for each input pattern

# Representing differential equations (1)

- Explicit differential equation

$$x^{(n)} = f(t, x, x', x'', \dots, x^{(n-1)})$$

- Equivalent to a system of differential equations

$$x' = y_1$$

$$y_1' = y_2$$

...

$$y_{n-2}' = y_{n-1}$$

$$y_{n-1}' = f(t, x, y_1, y_2, \dots, y_{n-1})$$

# Representing differential equations (2)

- Recursion representation

$$x(t_i) = x(t_{i-1}) + \Delta t \ y_1(t_{i-1})$$

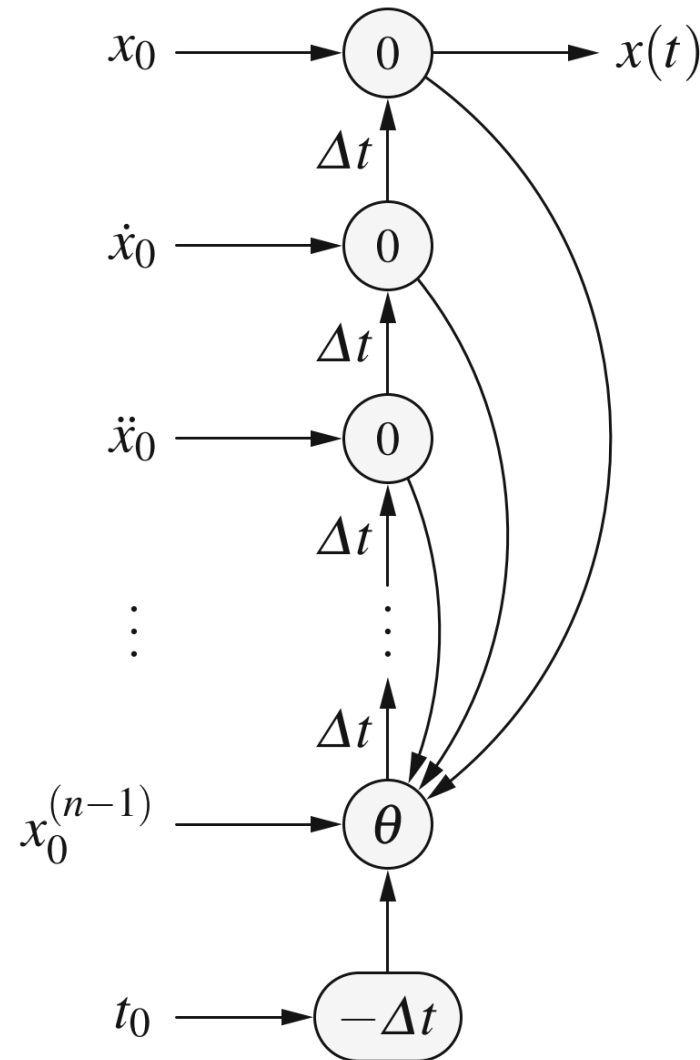
$$y_1(t_i) = y_1(t_{i-1}) + \Delta t \ y_2(t_{i-1})$$

...

$$y_{i-2}(t_i) = y_{i-2}(t_{i-1}) + \Delta t \ y_{i-3}(t_{i-1})$$

$$y_{i-1}(t_i) = y_{i-1}(t_{i-1}) + \\ f(t_{i-1}, x(t_{i-1}), y_1(t_{i-1}), \dots, y_{n-1}(t_{i-1}))$$

# Representing differential equations (3)



# Vectorial Neural Networks

- Recurrent network composed by multiple recurrent sub-networks
- It can be used to compute vectorial differential equations

# Error Backpropagation in Time

- Generalize the configuration of recurrent neural networks when the structure of the computation is not known in advance
- Backpropagation is not directly applicable since loops propagates errors in a cyclic way
- Recurrent network must be **unfolded in time** between two training patterns
- Compute the adjustments of weights by backpropagation on the unfolded network
- Combine the adjustments of the same weight to generate the value for the recurrent network