Math 5740 Homework 3

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Exercise 1. We wrote the Euler-Lagrange equation for the function $P(y) = \int_0^1 F(y, y')\, dx$ as

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}y' - \frac{\partial F}{\partial y}\right) = 0.$$

a) For $F = F(y, y')$, prove the following identity:

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'} - F\right) = y'\left[\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y}\right].$$

b) Use the identity to show that the Euler-Lagrange equation is equivalent to

$$\frac{\partial F}{\partial y'}y' - F = C.$$

Proof

a) To show part a, note the following:

$$\begin{aligned}\frac{d}{dx}\left(\frac{\partial F}{\partial y'}y' - F\right) &= \frac{d}{dx}\frac{\partial F}{\partial y'}y' - \frac{d}{dx}F \\ &= \frac{d}{dx}\frac{\partial F}{\partial y'}y' - \frac{\partial F}{\partial y}\frac{dy}{dx} \\ &= y'\left(\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y}\right).\end{aligned}$$

b) For part b, we have that

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'} - F\right) = y'\left[\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y}\right] = 0 = \frac{d}{dx}\left(\frac{\partial F}{\partial y'}y' - \frac{\partial F}{\partial y}\right).$$

We can integrate the left side with respect to x, and we see that

$$\int \frac{d}{dx}\left(\frac{\partial F}{\partial y'} - F\right) dx = 0 \to \frac{\partial F}{\partial y'} - F = C.$$

It follows then that solving the Euler-Lagrange equation is equivalent to solving for $\frac{\partial F}{\partial y'} - F = C$.

Exercise 2.

Exercise 5. *Proof.* □